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A High Quality Output Voltage for HEPWM of Single Phase AC Motor Drive

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Abstract

A new Harmonic Elimination (HE) PWM method with fast recursive algorithm is used that provide the exact on-line solution to the optimal PWM problem. The proposed algorithm optimization technique is applied to a 3-level unipolar single-phase inverter to determine optimum switching angles for eliminating low order harmonics while maintaining the required fundamental voltage to drive single-phase induction motor with high quality.
The proposed HE method contributes to the existing methods because it not only generates the desired fundamental frequency voltage, but also completely eliminates any number of harmonics. It provides high quality sine-wave output voltage on the induction motor terminals with very low THD.

The high quality sinusoidal output voltage produced by the inverter at different number of switching angles is presented. The complete solutions for 3-level unipolar switching patterns to eliminate the 3rd and 5th harmonics are given. Finally, the unipolar case is again considered where the first 14 harmonics are eliminated.

Keywords: Harmonic Elimination, SHEPWM, THD.

فولتية خارجة عالية الجودة لمسوق محرك التيار المتناوب أحادي الطور يعمل بتضميـــن عرض النبضة بحذف التوافقيات

الخلاصة

تمّ استخدام طريقة جديدة لتضميـــن عرض النبضة بحذف التوافقيات بأستخدام مخطط أنسيابي متكرّر وسريع يجهّز مشكلة التضمين ألمثالي لعرض النبضة بالحل التام والمباشر. تمّ تسليط التقنية المثالية المقترحة الى عاكس أحادي الطور ثلاثي المستويات أحادي القطبية لتحديد زوايا التشغيل المثالية لحذف التوافقيات ذات المرتبة المنخفضة والأبقاء على التوافقية الأساسية للفولتية لسوق محرك حثّي أحادي الطور وبأعلى جودة.
طريقة حذف التوافقيات المقترحة تساهم مع الطرق الموجودة كونها لا تولدّ الفولتية المرغوبة للتردد الأساس فحسب وأنما تحذف بشكل تام أي عدد من التوافقيات. أن التقنية المستخدمة تجهز اطراف المحرك الحثي بموجة فولتية جيبية خارجة وبأقل تشويه ممكن.
تمّ تمثيل موجة الفولتية الجيبية الخارجة من العاكس عند قيم مختلفة لزوايا التشغيل. حيث تمّ إعطاء ألحلول التامّة لنماذج تشغيل أحادية القطبية ثلاثية المستويات لحذف ألتوافقية الثالثة والخامسة. وأخيراً تمّ أعادة أعتبار الحالة مرة أخرى لحذف التوافقيات الأربعة عشر الأولى.

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Introduction

Harmonic elimination (HE) control has been a widely researched alternative to traditional PWM techniques. The Selective HE (SHE) method, which is widely used for efficient inverter control, forms a basis of off-line digital PWM modulation techniques in the power electronics field [1].

A Unipolar PWM waveform consists of a series of positive and negative pulses of constant amplitude but with variable switching instances as depicted in Fig. 1 (as in a power electronic PWM full-bridge inverter). A typical goal is to generate a train of pulses such that the fundamental component of the resulting waveform has a specified frequency and amplitude (e.g., for a constant *V/f* speed control of an induction motor).

Some of the proposed methods for PWM waveform design are: modulating-function techniques, space-vector techniques, and feedback methods. These methods suffer, however, from high residual harmonics that are difficult to control and from limitations in their applicability.

PWM signals are used in power electronics, motor control and solid-state electric energy conversion. The best voltage signal for these purposes is one with a periodic time variation in which amplitudes of selected non-fundamental components of the signal have been controlled to increase efficiency and reduce damaging vibrations.

A sinusoidal PWM inverter, which is a DC-AC power inverter, is used for a wide variety of applications because of its flexibility in driving frequency and voltage in the power electronic field. If the switching frequency is not high, but the control accuracy is good, off-line PWM control is efficient because it optimizes PWM waveforms for harmonic elimination and total distortion.

The Selective Harmonic Elimination (SHE) technique (or Optimal HE technique) forms a basis of the harmonic reduction techniques. This technique consists of synthesizing a PWM waveform by setting its pulse-pattern properly to eliminate selected orders of harmonics.

This is accomplished by solving HE equations which are nonlinear [1]. This technique theoretically offers the highest quality of the output waveform [2].

The optimal HEPWM offers several advantages compared to traditional modulation methods including acceptable performance with low switching frequency to fundamental frequency ratios, direct control over output waveform harmonics, and the ability to leave triplen harmonics uncontrolled in three-phase systems. These key advantages make the optimal HEPWM a viable alternative to other methods of modulation in applications such as ground power units, variable speed drives, or dual-frequency induction heating [3].

A. Optimal HEPWM

At a high power level, to limit switching losses, power switches (e.g., GTO's) can only be switched at low frequencies (typically several hundred hertz). This implies that only a few switching actions may take place within each fundamental period, as far as the fundamental frequency is not very low.

In this case, optimizing the waveform based on specifying an optimal value for each switching instant is necessary for achieving the best modulation result. The high power and cost of the whole system also justify the use of such optimized modulation methods that, in principle, require more advanced (thus more expensive) implementation hardware and software than that of simple carrier-based PWM. The benefits of optimization are also more remarkable, considering the total power of the system. As the total energy of harmonics contained in a PWM waveform is constant that depends only on the fundamental amplitude, regardless of the actual waveform structure, optimizing the waveform implies not eliminating or reducing the total harmonic energy, but altering its distribution among different frequency components.

Considering that low-order harmonics are usually considered to be more harmful than high-order ones, thus they need to be controlled at smaller magnitudes. Hence, roughly speaking, the objective of optimal PWM is to push most harmonic energy into high-frequency regions such that low-frequency harmonics are well attenuated [4].

B. Harmonic Elimination (HE)

One frequently studied optimal PWM method is the HE technique, which aims at the complete elimination of some low-order harmonics [2,5,6]. The underlying principle of HE is that the fundamental and harmonic amplitudes of a symmetrical PWM waveform are nonlinear functions of the *n* switching angles in the first quarter fundamental period. Setting of the fundamental amplitude to a pre-specified value and other *n*-1 low-order harmonics to zero, results in a system of *n* nonlinear equations. The desired optimal pulse patterns can thus be determined by solving these equations. Two major advantages of applying this technique are:

1) If the inverter is used to supply AC power of constant frequency to general AC loads, a filter is usually installed at its output. In this case, when low-order harmonics are eliminated through the modulation of the inverter, only high-order harmonics will appear at the output and need to be attenuated by the filter.

The cut-off frequency of the filter can thus be increased, leading to a significant reduction of the filter size and cost. System efficiency also tends to increase.

2) When used in an AC drive system, eliminating the low-order harmonic voltages leads to great reduction of low-order harmonic torques generated by the motor. Although harmonic torque is the results of interacting between stator and rotor harmonic currents of different orders, higher-order harmonic currents have smaller magnitudes due to the larger impedance that the motor presents to higher-order harmonic voltages. Their contributions to lower-order harmonic torques are thus less significant. Lower-order harmonic torques generated by the motor are thus greatly reduced [4].

The problem of eliminating harmonics in switching inverters has been the focus of research for many years. If the switching losses in an inverter are not a concern (i.e., switching on the order of a few kHz is acceptable), then the sine-triangle PWM method and its variants are very effective for controlling the inverter [7]. On the other hand, for systems where high switching efficiency is of utmost importance, it is desirable to keep the switching frequency much lower. In this case, another approach is to choose the switching times (angles) such that a desired fundamental output is generated and specifically chosen harmonics of the fundamental are suppressed [2,4-6]. This is referred to as *Selective Harmonic Elimination* (SHE) or *Programmed Harmonic Elimination* (PHE) as the switching angles are chosen (programmed) to eliminate specific harmonics.

Specifically, in [2,5,6] the HE problem was formulated as a set of transcendental equations that must be solved to determine the times (angles) in an electrical cycle for turning the switches on and off in a full bridge inverter so as to produce a desired fundamental amplitude while eliminating, for example, the 3^{rd} and 5^{th} harmonics. These transcendental equations are then solved using *iterative numerical* techniques to compute the switching angles [8]. The *Walsh* function method [9] also had been proposed to simplify the process. Recently, on-line computation methods have been proposed to make the technique a more flexible and interactive one, by using

Genetic algorithms [10] and a DSP [4]. The complete solution to the HE problem can be found using the theory of *Resultants* from *Elimination* theory. The solution is complete in the sense that any and all solutions were found [11].

Until now, the transcendental equations characterizing the harmonic content have been converted into polynomial equations, and elimination theory (using resultants) has been employed to determine the switching angles to eliminate specified harmonics, such as 3^{rd},5^{th},...,13^{th} for 3-level unipolar inverters [11]. However, as the number of eliminated harmonics increases, the degree of the polynomials in these equations are large and one reaches the limitations of the capability of contemporary computer algebra software tools (e.g., Mathematica or Maple) to solve the system of polynomial equations by using elimination theory.

To conquer this problem, the switching angles computation with general number *n* of the 3-level unipolar switching scheme is solved by using the proposed fast *recursive* on-line algorithm.

The present work would be to extend the SHEPWM switching scheme in [11] to include more than 7-switching angles per quarter cycle ($n > 7$) or eliminating harmonics more than six.

The problem of the optimal design of PWM waveforms for single-phase inverters [2,5] is examined in this paper.

Optimized PWM Switching Angles

The quarter-wave symmetry assumption in Fig 1, guarantees that the even harmonics will be zero and that all harmonics will be either in phase or anti-phase with the fundamental. Only the odd harmonics exist [3]. Assuming that the PWM waveform is chopped times per half a cycle, the Fourier coefficients of odd harmonics are given by:

$$a_k = \frac{4V_{dc}}{kp} \sum_{n=1}^{\infty} (-1)^{(n-1)} \cos(k\alpha_n) \qquad (1)$$

where k =1,3,5,…, V_{dc} is the amplitude of the square wave and α_n are the optimized switching angles. Amplitudes of any harmonics can be set by solving a system of nonlinear equations obtained from setting (1) equal to pre-specified values.

In the optimal HEPWM method, the fundamental component is set to required amplitude and n-1 low-order harmonics are set to zero. This is the most common approach in electric drives since low-order harmonics are the most detrimental to motor performance. In other applications, like active harmonic filters or control of electromechanical systems, harmonics are set to nonzero values. This task of designing a PWM waveform, the first *n* Fourier series coefficients of which match those of a desired waveform has been the subject of many papers [2,4,5,9]. Often, the Newton iteration method [8] is used to solve the system of nonlinear Eqs. 1. Those methods are computationally intensive for on-line calculations and the storage of off-line calculations leads to high memory requirements. Another approach is to simplify the nonlinear HE equations in order to obtain real-time approximate solutions using modern DSPs [4].

The current paper uses the recursive algorithm [12] with some developments and modifications to reduce the computational complexity for online calculation for solving the PWM harmonic elimination problem without any approximations in the problem statement. Since many PWM applications allow for a computational time frame of a few milliseconds, the developed algorithm will allow for real-time generation of switching patterns with high order.

The optimized unipolar waveform shown in Fig. 1 is assumed to be the quarter-wave symmetric.

The Fourier series of the general quarter-wave symmetric H-bridge inverter output waveform is written as follows:

$$u(wt)=\sum_{k=1}^{\infty}\frac{4V_{dc}}{kp}\left[\sum_{n=1}^{\infty}(-1)^{(n-1)}\cos(ka_n)\right]\sin(kwt) \quad .....(2)$$

where α_n is the optimized switching angles, which must satisfy the following condition: $\alpha_1< \alpha_2 <... \alpha_n ...< \pi/2$.

The optimal PWM problem, as it is considered here, is the design of a PWM waveform $\upsilon(\omega t)$ so that its first Fourier coefficients h_k are equal to prescribed values (1). Therefore, the optimal PWM problem gives rise to the following design equations [2]:

$$\begin{aligned}&\cos a_1-\cos a_2+\cos a_3-...\cos a_n=h_1\\&\cos 3a_1-\cos 3a_2+\cos 3a_3-...\cos 3a_n=h_3\\&\vdots\\&\cos(2n-1)a_1-\cos(2n-1)a_2\\&\quad+\cos(2n-1)a_3-...\cos(2n-1)a_n=h_{2n-1}\end{aligned} \quad ...(3)$$

Given the n values $h_k = k\pi a_k/4V_{dc}$, we have n equations and n unknowns; we would like to find the n unknowns $\{\alpha_1, \alpha_2,..., \alpha_n\}$, with $0 < \alpha_1 < \alpha_2 <....< \alpha_n< \pi/2$.

Transformation of the Optimal PWM Problem

Eqs. 3 can be simplified as was done for example in [13]. Assuming $\beta_i = \alpha_i$ for odd i, and $\beta_i = \pi-\alpha_i$ for even i, we get:

$$\begin{aligned}&\cos b_1+\cos b_2+\cos b_3+...\cos b_n=h_1\\&\cos 3b_1+\cos 3b_2+\cos 3b_3+...\cos 3b_n=h_3\\&\vdots\\&\cos(2n-1)b_1+\cos(2n-1)b_2\\&\quad+\cos(2n-1)b_3+...\cos(2n-1)b_n=h_{2n-1}\end{aligned} \quad ....(4)$$

Using the trigonometric identities; $\cos nt = T_n(\cos t)$ where T_n is the n^{th} Chebyshev polynomial, and changing the variables; $x_i = \cos \beta_i$, we get:

$$\begin{aligned}&T_1(x_1)+T_1(x_2)+T_1(x_3)+...T_1(x_n)=h_1\\&T_3(x_1)+T_3(x_2)+T_3(x_3)+...T_3(x_n)=h_3\\&\vdots\\&T_{2n-1}(x_1)+T_{2n-1}(x_1)+T_{2n-1}(x_3)+\\&\quad...T_{2n-1}(x_n)=h_{2n-1}\end{aligned} \quad (5)$$

As the odd-indexed Chebyshev polynomials are odd polynomials, the PWM equations can be writing:

$$\sum_{i=1}^{n}T_{2k-1}(x)=\sum_{i=1}^{n}\sum_{j=1}^{k}c_{k,j}.x^{2j-1}=h_{2k-1}(1\le k\le n)$$

or

$$\sum_{j=1}^{k}c_{k,j}.s_{2j-1}=h_{2k-1}\quad(1\le k\le n)\quad ...(6)$$

where $s_j=\sum_{i=1}^{n}x_i^{\,j}$ are the sums of powers of $\{x_i\}$. Eq. 6 forms a set of n linear equations for s_{2j-1}, $1 \le j \le n$. Once the values s_{2j-1} are obtained by solving the linear system (6), one has the following problem. Given $\{s_1,s_2, ..., s_{2n-1}\}$, find the solution $\{x_1, x_2, ..., x_n\}$ to the following system of nonlinear equations:

$$\begin{aligned}&x_1+x_2+...+x_n=s_1\\&x_1^{\,3}+x_2^{\,3}+...+x_n^{\,3}=s_3\\&\vdots\\&x_1^{\,2n-1}+x_2^{\,2n-1}+...+x_n^{\,2n-1}=s_{2n-1}\end{aligned} \quad ...(7)$$

Once x_i are obtained, the original variables α_i can be found by letting $\beta_i = \arccos x_i$, $\alpha_i = \beta_i$, for odd i, and $\alpha_i = \pi-\beta_i$ for even i. Due to the symmetry with respect to x_i, any permutation of a solution set $\{x_i\}$ is also a solution set; likewise for β_i. Yet it is necessary to order β_i appropriately such that $0 < \alpha_1 < \alpha_2 <....< \alpha_n < \pi/2$. Note that for odd i, $0 < \alpha_i < \pi/2$, gives $0 < \beta_i < \pi/2$. For even i, $0 < \alpha_i < \pi/2$ gives $0 < \pi-\beta_i < \pi/2$ or $\pi/2 < \beta_i < \pi$. This indicates how to obtain α_i with the desired ordering from β_i: for those values of

$\beta_i \in (0,\pi/2)$ let $\alpha_i=\beta_i$; and for those values of $\beta_i \in (\pi/2, \pi)$ let $\alpha_i = \pi-\beta_i$.

However, the design Eqs. 7 are nonlinear, so obtaining the desired solution $\{x_i\}$ is not so straightforward. In the following sections, this nonlinear system of equations will be closely examined and in Section IV, a systematic procedure is given to obtain the solutions.

For the HEPWM problem, the Fourier coefficients of the PWM waveform $\upsilon(\omega t)$ should match the Fourier coefficients of a pure sine wave. That is, the values h_{2n-1} appearing in (3) are given by $h_1=m$, and $h_{2i-1} = 0$ for $2 \le i \le n$. For this case, the values s_{2i-1} depend on m only and are given by:

$$s_{2i-1} = \frac{m}{4^{i-1}}\binom{2i-1}{i-1}, \qquad (1 \le i \le n) \quad \text{.... (8)}$$

Solving the Optimal PWM Problem

To solve the optimal PWM problem, we will first write the polynomial $P(x)$ having roots $\{x_1, x_2 ..., x_i..., x_n\}$ as:

$$P(x) = \prod_{i=1}^{n}(x - x_i)$$

then the logarithmic derivative is given by:

$$\frac{P'(x)}{P(x)} = \sum_{i=1}^{n}\frac{1}{(x - x_i)}$$

Expanding each term in the sum, one gets:

$$\frac{P'(x)}{P(x)} = \sum_{i=1}^{n}\sum_{j=0}^{\infty}\frac{x_i^{\,j}}{x^{j+1}} = \sum_{m=0}^{\infty}\frac{s_j}{x^{j+1}} = \frac{n}{x} + \sum_{j=1}^{\infty}\frac{s_j}{x^{j+1}} \quad \text{... (9)}$$

where s_j are the sums of the root powers and $s_0 = n$. Integrating (9) gives:

$$\ln P(x) = n \ln x - \sum_{j=1}^{\infty}\frac{s_j}{j\,x^j}$$

Raising e to the power of $\ln P(x)$ and using the last equation gives:

$$P(x) = x^n \exp\left(-\sum_{j=1}^{\infty}\frac{s_j}{j\,x^j}\right) \quad \text{......(10)}$$

In order to generalize procedure, we will obtain an expression similar to (10), but having only odd s_i. To this end, note that:

$$P(-x) = (-1)^n x^n \exp\left(-\sum_{j=1}^{\infty}\frac{s_j(-1)^j}{j\,x^j}\right)$$

therefore:

$$\frac{P(x)}{P(-x)} = (-1)^n \exp\left(-\sum_{j=1}^{\infty}\frac{s_j}{j\,x^j}\left(1-(-1)^j\right)\right)$$

or

$$\frac{P(x)}{P(-x)} = (-1)^n \exp\left(-2\sum_{j=1,odd\,j}^{\infty}\frac{s_j}{j\,x^j}\right)$$

then:

$$P(x) = (-1)^n P(-x) G(1/x)$$

where

$$G(x) := e^{V(x)}$$

$$V(x) := -2\left(s_1 x + \frac{s_3}{3}x_3 + \frac{s_5}{5}x_5 + ...\right)$$

let

$$\tilde{P}(x) = (-1)^n P(-x) \quad \text{..... (11)}$$

then

$$P(x) = \tilde{P}(x) G(1/x) \quad \text{...... (12)}$$

where $\tilde{P}(x)$ is the monic polynomial related to $P(x)$ by negating the roots of $P(x)$.

Eq. (12) is the counter-part to (10). Likewise, by setting like powers of x equal, we can obtain equations that relate p_k and s_i, where p_k is the polynomial coefficients of $P(x)$. However, in order to do this, we need to expand $G(x) = e^{V(x)}$ into a power series of

x. Such a power series for $e^{V(x)}$ can be obtained using the following algorithm. Let:

$$V(x)=\sum_{i=0}^{\infty} v_i x^i$$

and

$$G(x)=e^{V(x)}=\sum_{i=0}^{\infty} g_i x^i$$

If v_i for $0 \le i \le j$ are known, then the values of g_i for $0 \le i \le j$ are given by:

$$g_0 = e^{v_0} \qquad (13)$$

$$g_i = \frac{1}{i}\sum_{k=1}^{i} k v_k g_{i-k} \qquad (1 \le i \le j) \qquad (14)$$

When the first n odd values of s_i are known, then v are known for $0 \le i \le 2n$. [v_{2i} = 0 for $0 \le i \le n$ and $v_{2i-1} = -2s_{2i-1}/(2i-1)$ for $1 \le i \le n$]. Therefore, using the relations (13) and (14), we obtain g_i for $0 \le i \le 2n$, and consequently, we can write out Eq. (12), matching like powers of x, to obtain linear equations from which p_k can be obtained. For example, we write out the expressions for $n = 3$. That is, we are given s_1, s_3, and s_5, and our goal is to find the corresponding monic 3^{rd} degree polynomial $P(x)$, $P(x) = x^3 + p_1x^2 + p_2 x + p_3$.

A Recurrence Algorithm for $P(x)$

The notation $P_n(x)$ will be used to emphasize the dependence of $P(x)$ on n. Specifically, $P_n(x)$ denotes the monic degree-n polynomial associated with the HE problem, with coefficients $p_{n,k}$:

$$P(x) = x^n + p_{n,1}x^{n-1} + \ldots + p_{n,n}$$

With this notation, a recurrence relation:

$$P_{n+1}(x) = xP_n(x) + C_n P_{n-1}(x) \qquad \ldots. \ (15)$$

can be used to compute $P_n(x)$. For the HE problem, the initial conditions can be taken to be $P_0(x) = 1$ and $P_1(x) = x - m$. The coefficients C_n in the recursion can be computed using the following formula:

$$C_n = -\frac{\sum_{k=0}^{n}(-1)^k g_{2n+1-k}\, p_{n,k}}{\sum_{k=0}^{n-1}(-1)^k g_{2n-1-k}\, p_{n-1,k}} \qquad (16)$$

The coefficients $p_{n+1,k}$ are then determined recursively as [this implements (15)]:

$$\begin{aligned} p_{n+1,k} &= p_{n,k}, & k &= 1 \\ p_{n+1,k} &= p_{n,k} + C_n \,.\, p_{n-1,k-2}, & k &= 2, \ldots, n \\ p_{n+1,k} &= C_n \,.\, p_{n-1,k-2} & k &= n+1 \end{aligned} \qquad (17)$$

The Recursive algorithm for computing can be summarized as follows:

Given m and n, the polynomials $P_n(x)$ for are recursively computed as follows:

1) Set $g_0 = 1$ and for $k = 1$ to $2n$, find g_i from Eq. 14.

2) Set $P_0(x) = 1$ and $P_1(x) = x - m$ and for k =1 to n-1 let:

$$C_k = -\frac{\sum_{i=0}^{k}(-1)^i g_{2k+1-i}\, p_{k,i}}{\sum_{i=0}^{k-1}(-1)^i g_{2k-1-i}\, p_{k-1,i}}$$

$$P_{k+1}(x) = xP_k(x) + C_k P_{k-1}(x)$$

Find the roots x_i of $P_n(x)$.

3) Set β_i = arcos x_i, $i = 1,\ldots, n$, with $\beta_i \in (0, \pi)$. For $\beta_i \in (0, \pi/2)$, set $\alpha_i = \beta_i$. For $\beta_i \in (\pi/2, \pi)$, set $\alpha_i = \pi - \beta_i$. Sort the angles α_i.

Fig. 2 illustrates the flowchart of the recursive algorithm for on-line calculation of the optimal switching angles. Using a computer algebra system, such as Maple or Mathematica, this recursive algorithm allows one to obtain $P_n(x)$ as an explicit function of m.

Simulation Results

The computer software package Maple was used to perform all of the above calculations as a first part. The second part of the theoretical calculations involved organizing and analyzing all of the collected optimized switching angles (α). For this purpose, the software package MATLAB was utilized. Using MATLAB, the collected switching angles were organized into look-up tables to be used later in simulations [See Table 1]. Also, MATLAB was used to generate plots of α and *Total Harmonic Distortion* THD versus *m*. The THD mathematically calculated by:

$$THD = \frac{\sqrt{\sum_{k=2}^{\infty} h_k^{\,2}}}{h_1} \quad \text{…. (18)}$$

The computation was done as *m* increased between (0 and 1) resulting in α versus *m*; seem to be almost as straight lines as shown in Fig. 3.

Fig. 3 represents the exact solution of the optimized Unipolar HEPWM switching angles with the variation of the number of switching angles (*n*). We can show that increasing *n* causes decreasing the *m* range. The *m* range for example decreases approximately by 67% when *n* = 15 with respect to *n* = 3. It can seen from the figure that the solution is not continuous for some values of *m*. Note that not all the range of *m* has a solution, for example, in the case of 3-switching scheme (*n* = 3), there are solutions in the interval $m \in [0,0.83]$. On the other hand, for $m \in [0.83,1]$, there are no solutions that solve the Eqs. 1. Also it can be seen that for 15-switching scheme (*n* = 15), there are solutions to the Eqs. 1 in the interval $m \in [0.72,0.78]$ only and there are no solutions for the rest interval. Interestingly, in the schemes with *n* = (8-14), there are (isolated solutions).

Fig. 4 shows the instantaneous unipolar voltage waveforms with optimal HEPWM technique for *n* = 3 (eliminating two harmonics with orders 3^{rd} and 5^{th}) and *n*=15(eliminating fourteen harmonics with orders 3^{rd}, 5^{th}… 29^{th}) and with minimized THD where the number of harmonics to be eliminated = *n* – 1. Therefore, increasing *n* will cause increase in the number of low order eliminated harmonics, which causes to push more harmonic energy into high frequency regions, therefore low frequency harmonics are well attenuated. It can be seen, that the variation of *n* values affect the location of the harmonics in the spectrum, (i.e. the first significant component in the inverter output for *n* = 3 is equal to 7^{th} or 350Hz, while it is equal to 31^{st} or 1550 Hz for *n* = 15) [See also Table 2].

The inverter is loaded by single-phase capacitor-run induction motor with the following ratings: 175Watt, 220V, 1.22 A, and 1275Rpm. The behavior of the motor is explained by the simulation program. By using the equivalent circuit of the motor and the performance equations [14], it can be easy to analyze the performance of the motor operated on an optimal HEPWM 3-level inverter, and using Fourier series to calculate the harmonic currents amplitude to get the frequency spectra for motor current as shown in Fig. 4(d).

Increasing of *n* causes increase of the motor impedance with frequency ($X=2\pi fL$); therefore the harmonics currents decrease for constant harmonic voltage amplitude and as result, the ripple in the instantaneous motor current will decrease with increasing *n* as in Fig. 4(c and d).

Evaluation of the inverter performance can be calculated from the performance factor THD in Eq. 18. THD versus the modulation index *m* for all switching (2-15) is shown in Fig. 5. Fig. 6 (a and b) illustrates the relationship between this factor and *n*. We can see that increasing *n* causes increasing *THDmin* [approximately constant for *n* >7] and decreasing *THDmax*.

Decreasing the harmonic currents with increasing *n* causes decreasing of additional torque pulsations ($Tpulsadd_{min}$) and additional motor losses ($\Sigma Padd_{min}$), as illustrated in Fig. 5(c and d). Notice that, the torque pulsations amplitude decreases with

increasing n until the motor will behave just like motor supplied directly by sinusoidal power supply [See Table 3], so that the motor will be more quite with higher n.
It also can be seen from the figure that, the motor performance (additional torque pulsation and additional motor losses and as a result the overall efficiency η_{max}) will be approximately constant for $n > 14$ [See Fig. 5(c, d and f) and Table 1]. So there is no need to solve the switching angles for $n > 15$.
It should be known that for $n = 3$, the switching frequency equal 7-times the base frequency and equal 31-times the base frequency for $n = 15$. Therefore the switching losses (Psw_{min}) will increases with increasing n as shown in Fig. 5(e).

Conclusions

1) However, increasing the number of α more than 7 per quarter cycle will lead to polynomial equations of higher degree. Therefore, Resultant theory will not be effective of solving these polynomials.
2) This paper presents a contribution to the theory of optimal PWM. It develops and uses a fast algorithm for efficient on-line calculation of PWM switching patterns for general n.
3) The proposed algorithm extends the traditional SHEPWM switching scheme to completely eliminates any number of harmonics to get highest quality motor drive with very low distortion AC waveform.
4) The proposed technique enables the motor to behave just like one supplied directly by sinusoidal power supply and to be more quite with higher switching.
5) The best compromise between high efficiency and high quality of the inverter operation is achieved by the optimal HEPWM technique.

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Table (1) Highest-Quality of the 3-Level Unipolar Inverter and Motor fed from using the Fast Recursive Algorithm with the Optimal HEPWM Technique

Switching No. (n)		2	3	4	5	6	7	8
Deg	α_1	30.2299	21.8958	22.925	18.8804	18.2243	16.3179	15.2280
	α_2	89.7701	36.196	38.2119	28.0493	26.7161	22.7210	20.6901
	α_3		45.6422	47.3323	38.182	36.9936	32.9286	30.7246
	α_4			89.8262	54.7979	53.1178	45.08	41.304
	α_5				58.2133	56.9332	50.0789	46.7849
	α_6					89.9573	66.3199	61.7990
	α_7						67.7067	63.7981
	α_8							89.9137
m		0.86	0.82	0.81	0.8	0.8	0.79	0.79
THD_{min} (%)		31.5599	43.6109	44.6251	47.2747	47.3379	49.1002	48.9801
THD_{max} (%)		754.8863	671.5544	607.9097	555.3122	521.5931	446.6963	433.3106
Psw_{min} (W)		6.8341e-4	0.0011	0.0015	0.002	0.0024	0.0028	0.0032
$\Sigma Padd_{min}$(W)		4.2568	1.3808	1.5275	0.8425	0.8029	0.6061	0.5254
$Tpulsadd_{min}$(N.m)		0.0046	7.925e-4	9.024e-4	3.627e-4	3.361e-4	2.162e-4	1.741e-4
η_{max} (%)		65.5167	66.6819	66.6103	66.9442	66.964	67.0636	67.105
Switching No. (n)		**9**	**10**	**11**	**12**	**13**	**14**	**15**
Deg	α_1	13.7012	12.9885	11.6709	11.3245	10.7385	10.0463	9.5892
	α_2	17.9759	16.7798	14.6469	14.1166	13.1763	12.1834	11.4899
	α_3	27.5374	26.1151	23.4037	22.7352	21.5438	20.1401	19.2215
	α_4	35.7864	33.5178	29.2007	28.2088	26.345	24.3517	22.9765
	α_5	41.6215	39.5223	35.2514	34.2988	32.4852	30.3286	28.9407
	α_6	53.1681	50.1657	43.5472	42.2473	39.5003	36.4866	34.4571
	α_7	55.9845	53.3622	47.2456	46.0973	43.6371	40.6571	38.7927
	α_8	69.5562	66.6928	57.5339	56.1937	52.6482	48.5637	45.932
	α_9	70.371	67.8237	59.3768	58.2066	55.0904	51.167	48.8288
	α_{10}		89.9686	70.9847	70.007	65.8564	60.5466	57.4165
	α_{11}			71.5838	70.7029	67.0006	61.8899	59.1164
	α_{12}				89.9934	79.7012	72.3732	68.9932
	α_{13}					80.0341	72.8293	69.7906
	α_{14}						89.9943	81.2596
	α_{15}							81.5021
m		0.79	0.79	0.79	0.79	0.78	0.79	0.78
THD_{min} (%)		48.803	49.0488	48.7001	48.6333	50.0234	48.3048	49.6866
THD_{max} (%)		352.5778	251.9755	185.042	146.6671	128.137	88.6293	59.0839
Psw_{min} (W)		0.0037	0.0041	0.0045	0.0049	0.0054	0.0058	0.0062
$\Sigma Padd_{min}$(W)		0.3889	0.3521	0.2638	0.2532	0.2361	0.1909	0.1814
$Tpulsadd_{min}$(N.m)		1.117e-4	9.62e-5	6.276e-5	5.899e-5	5.188e-5	3.887e-5	3.519e-5
η_{max} (%)		67.1759	67.1952	67.2416	67.2472	67.2562	67.2802	67.2853

Table (2) the Harmonics Amplitude (h_k) in p.u. with the Number of Switching Angles

n		2	3	4	5	6	7	8	9	10	11	12	13	14	15
p.u	m	0.86	0.83	0.81	0.80	0.80	0.79	0.79	0.79	0.79	0.79	0.79	0.78	0.79	0.78
	h_1	0.86	0.83	0.81	0.80	0.80	0.79	0.79	0.79	0.79	0.79	0.79	0.78	0.79	0.78
	h_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	h_5	0.179	0	0	0	0	0	0	0	0	0	0	0	0	0
	h_7	0.118	0.119	0	0	0	0	0	0	0	0	0	0	0	0
	h_9	0	0.048	0.164	0	0	0	0	0	0	0	0	0	0	0
	h_{11}	0.085	0.152	0.156	0.034	0	0	0	0	0	0	0	0	0	0
	h_{13}	0.061	0.217	0.07	0.179	0.152	0	0	0	0	0	0	0	0	0
	h_{15}	0	0.073	0.189	0.119	0.172	0.106	0	0	0	0	0	0	0	0
	h_{17}	0.057	0.093	0.034	0.141	0.077	0.201	0.161	0	0	0	0	0	0	0
	h_{19}	0.039	0.114	0.089	0.164	0.182	0.024	0.169	0.087	0	0	0	0	0	0
	h_{21}	0	0.039	0.031	0.023	0.054	0.197	0.098	0.202	0.152	0	0	0	0	0
	h_{23}	0.044	0.001	0.044	0.005	0.012	0.074	0.174	0.013	0.177	0.045	0	0	0	0
	h_{25}	0.029	0.011	0.023	0.051	0.012	0.006	0.047	0.198	0.091	0.186	0.145	0	0	0
	h_{27}	0	0.006	0.064	0.09	0.097	0.005	0.013	0.094	0.177	0.101	0.183	0.172	0	0
	h_{29}	0.036	0.027	0	0.015	0.044	0.007	0.005	0.004	0.052	0.169	0.083	0.163	0.138	0
	h_{31}	0.022	0.031	0.072	0.036	0.05	0.054	0.007	0.007	0.01	0.145	0.18	0.122	0.187	0.172

Table (3) Parameters and Specifications of the Proposed Motor

Turn ratio	a_s	1.066		Rated supply voltage	V_1	220	V
Number of pole pair	P	2		Rated current	I	1.215	A
Main winding resistance	R_{1m}	33.5	Ω	Total Power losses	ΣP	85	W
Main winding leakage reactance	X_{1m}	27	Ω	Output power	P_2	175	W
Auxiliary winding resistance	R_{1a}	34.5	Ω	Efficiency	η	67.38	%
Auxiliary winding leakage reactance	X_{1a}	28	Ω	Power factor	P_f	0.9726	
Rotor resistance	R_2	20	Ω	Rated speed	N_r	1275	Rpm
Rotor leakage reactance	X_2	12.5	Ω	Capacitance	C	6	μF
Magnetization reactance	X_m	173	Ω				

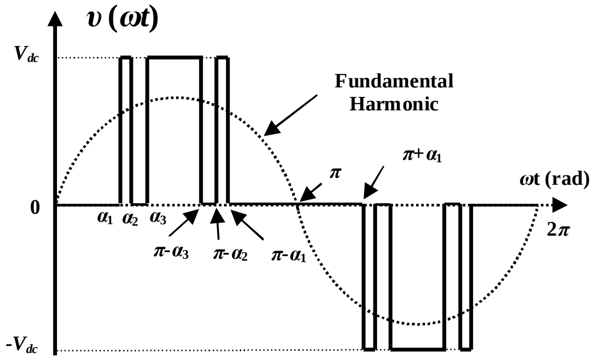


Figure (1) Unipolar PWM Switching Scheme

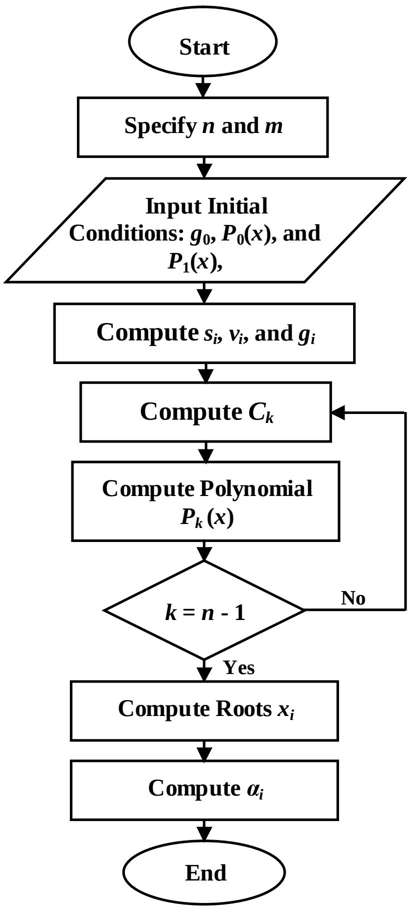


Figure (2) Fast Recursive Algorithm for Optimal HEPWM

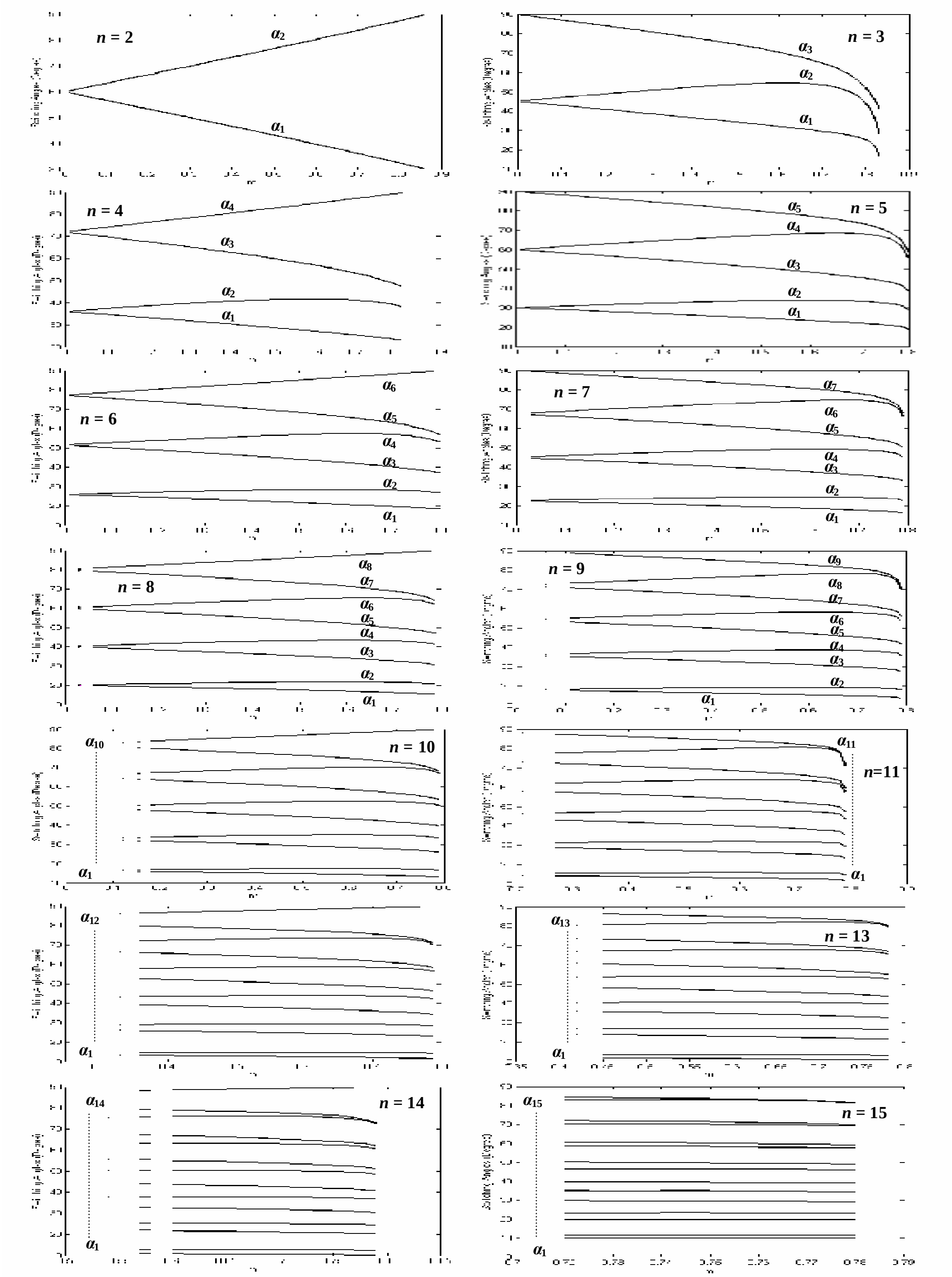
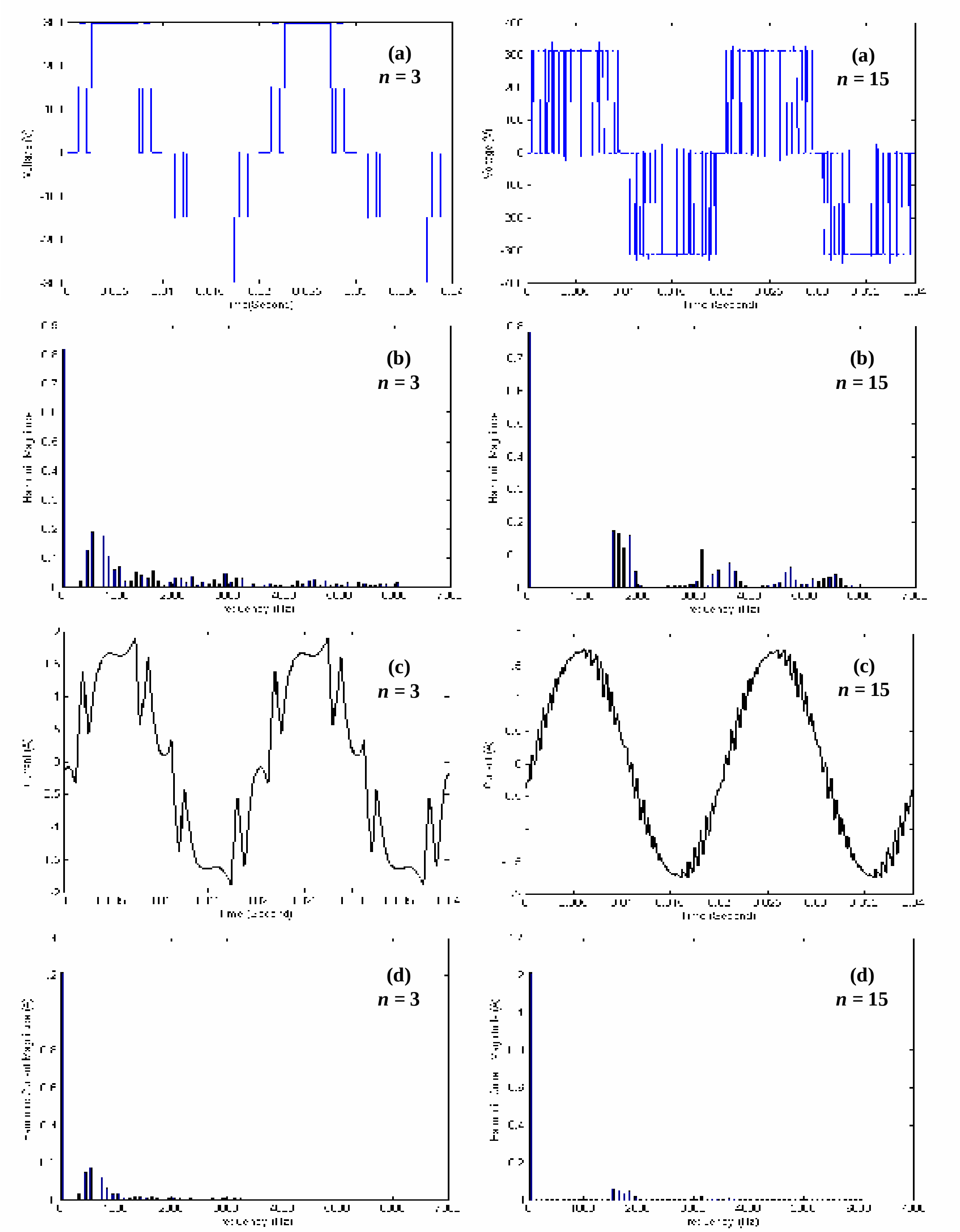


Figure (3) the Solutions of α vs. m with Different Values of n for 3-Level Unipolar Optimal HEPWM Inverter



Figure(4) Shows the Instantaneous Inverter output Voltage Waveform (a), Normalized Voltage Spectra (b), Instantaneous Motor Current (c), and Motor Current Spectra (d) for 3-Level Inverter with i) *n* = 3, *m* = 0.82, and THDmin= 43.6109 to Eliminate the 3rd and 5th harmonics and ii) *n* = 15, *m* = 0.79, and THDmin = 49.3008 to Eliminate the 3rd, 5th,…, 29th harmonics.

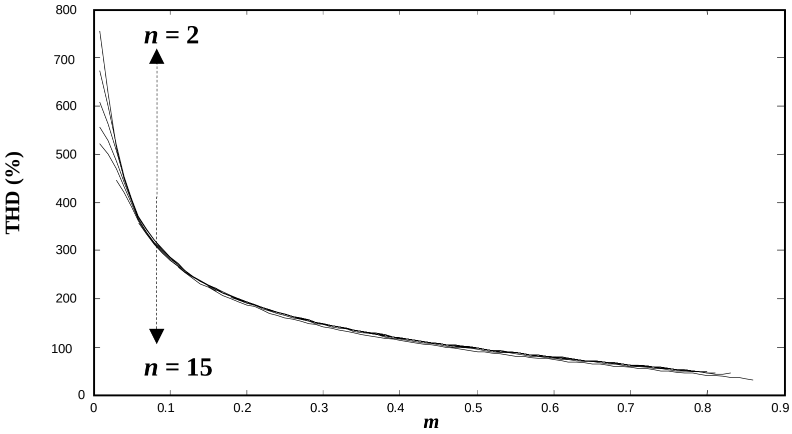


Figure (5) the Voltage THD vs. the modulation index *m* for all Switching (*n*=2-15) of SHPWM

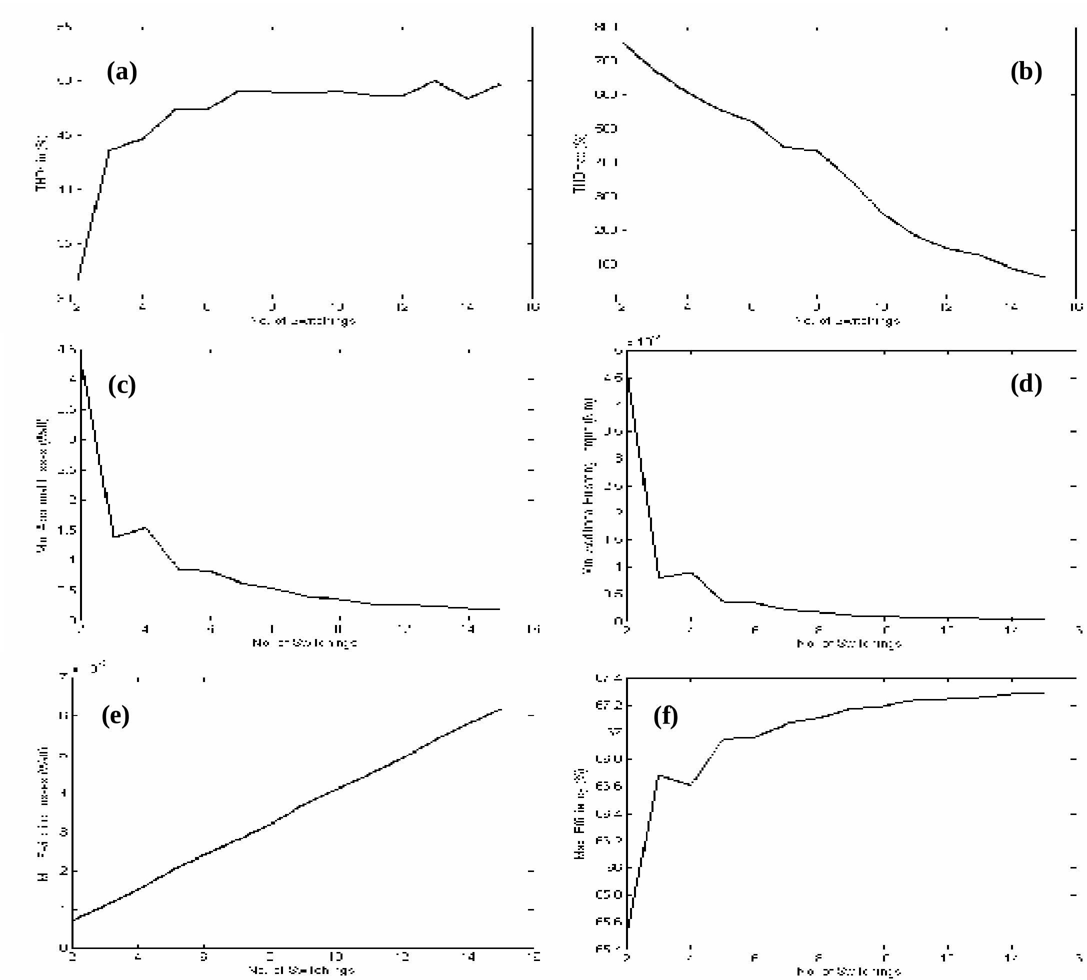


Figure (6) the Quality of the Inverter and Motor Fed from vs. No. of Switching Angles (*n*): (a) THDmin, (b) THDmax, (c) Minimum Motor Additional Losses, (d) Minimum Motor Additional Pulsating Torque, (e) Minimum Switching Losses, and (f) Overall Maximum Efficiency.