Eng. & Tech. Journal, Vol. 27, No9,2009

Wavelet and Wavelet Packet Analysis For Image Denoising

Aymen Dawood Salman*

Received on:25/9/2008
Accepted on:5/3/2009

Abstract

The denoising method based on wavelet or wavelet packet is used widely for image denoising. It is one of the most popular methods that depends on thresholding the wavelet coefficients using the Soft threshold. There are many methods used to get the threshold that is used in denoising image. In this paper, the amplitude of threshold is calculated depending on RMS error in order to get the best threshold related with the image information. The denoising results show that the wavelet packet is better than the wavelet method in analyzing the image coefficients of information.

Keywords: Wavelet, wavelet packet, de-noising.

التحليل المويجي والتحليل حزمة المويجي لازاله التشوية من الصورة

الخلاصة

إنّ طريقة إزالة التشويش مستندة على التحويل المويجي أو حزمة التحويل المويجي يَستعملان على نحو واسع لإزالة التشويش للصورةِ. إحدى الطرق الأكثر شعبية تَشْملُ التعتيب لمعاملات التحويل المويجي بَستعمالُ العتبة الناعمة. يوجد العديد مِنْ الطريق التي تُستَعملُ للحُصُول على العتبة التي تستخدم في إزالة التشويه للصورةِ. في هذا البحث تَم احتسابُ قيمة العتبةِ اعتماداً على (RMS Error) للحُصُول على أفضل عتبةِ. لقد أظهرت نتائج إزالة التشويه للصورة بأن حزمة التحويل المويجي أفضل مِنْ طريقةِ التحويل المويجي في تحليلِ معاملات المعلوماتِ الخاصه للصورة.

1. Introduction

An image is often corrupted by noise in its acquition and transmission. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features. In recent years, there has been a fair amount of research on wavelet thresholding and threshold selection for signal denoising [1], because wavelet provides an appropriate basis for separating noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficients are more likely due to noise and large coefficient due to important signal features [2]. These small coefficients can be thresholded without affecting the significant features of the image.

Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing against threshold, if the coefficient is smaller than threshold, set to zero; otherwise, it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the result may lead to reconstruction with the

*Computer Engineering and Information Technology Department ,University of Technology / Baghdad

https://doi.org/10.30684/etj.27.9.8
2412-0758/University of Technology-Iraq, Baghdad, Iraq

essential signal characteristics and with less noise. Since the work of Donoho and Johnstone [1], there has been much research on finding thresholds, however few are specifically designed for images [3].

A generalization of the discrete wavelet transform (DWT) is the discrete wavelet packet transform (DWPT) which keeps splitting both lowpass and highpass subbands at all scales in the filter bank mplementation, thus Wavelet Packet obtains a flexible and a detailed analysis transform. Therefor, the Wavelet Packet transform is used for denoising.

Image de-noising using wavelet or wavelet packet transform consists of three main steps:

1. Wavelet or wavelet packet transform of observed signal.
2. Shrinkage of the empirical wavelet coefficients.
3. Inverse wavelet packet transform of the modified coefficients.

The denoising procedure requires the estimation of the noise level [4].

2. Wavelet and Wavelet Packet Transform

Wavelet Transform (WT) represents an image as a sum of wavelet functions (wavelets) with different locations and scales. The idea behind wavelet analysis is that the signal can be considered as the weighted-sum of overlapping wavelet functions. In fact, any signal of finite bandwidth and finite duration can be completely characterized as a weighted-sum of a finite number of scaled and shifted versions of the underlying wavelet. The concept is similar to Fourier analysis, in which the time series signal can be considered as weighted-sum of sinusoids at various frequencies, with the transform coefficients being the weights. The practical meaning of the wavelet transform of a signal is that each coefficient of the transform is the weight, or relative amount of information (or signal energy) of the wavelet at that particular value of scale, and shift contributes to the overall signal.

The wavelet analysis was performed with the Daubechies 10-coefficient least asymmetric discrete wavelet. Discrete wavelets are not expressible in closed form. Plots of the wavelet and its corresponding scaling function are computed with Daubechies cascade algorithm as shown in Fig. 1.

The discrete wavelet packet transform is a generalization of the discrete wavelet transform. As shown in Fig. 2, each stage of both the wavelet transform and the wavelet packet transform consists of an elemental pair of filters (high-pass and low-pass) that splits the input signal into two decimated orthogonal components. The low-pass output is an approximation of the input signal. The high-pass output contains the details of the input signal that are missing from the approximation. There is no information in the two outputs that overlap, and nothing is lost. The input signal can be exactly reconstructed from the two outputs.

The discrete wavelet transform is implemented by cascading the elemental filter pairs as shown in Fig. 3, while the wavelet packet is implemented by cascading them as shown in Fig. 4. In the wavelet transform configuration, the low-pass output of the preceding stage is fed into an identical copy of the elemental filter pair. Thus, the first approximation is further approximated, and the second set of details consists of the information present in the first approximation but absent from the second. In wavelet parlance, the output of the first high- pass filter is the set of wavelet transform coefficients of the input signal at the finest scale [5].

The output of the next high-pass filter is the set of wavelet transform coefficients of the input signal at the next finer scale. The output of the final low-pass filter is the set of scaling function coefficients. The cascade can be repeated as often as necessary, and all the outputs are orthogonal. There is no fundamental reason why the high-pass output of the elemental filter pair cannot be split as well, and nothing to prevent repeating this process as often as necessary. A filter bank in which at least some of the high-pass outputs are split into other approximations and details implements the wavelet packet transform [6]. If the high-pass output is split whenever the low-pass output is split, then the system is a complete wavelet packet transform. However, it is not necessary for the wavelet packet to be complete. A more efficient representation of the signal may be obtained by leaving out some of the filter pairs in the cascade. Irrespective of how many filter pairs are included, all outputs remain orthogonal.

For any given input signal, there is an optimal configuration of filter pairs that represents most of the input signal information with the fewest output coefficients. This is known as the best basis wavelet packet. The "best basis" is determined by finding the configuration of filter pairs whose output has the highest entropy [5].

DWT for an image as a 2-D signal can be derived from 1-D DWT. The scaling function for 2-D DWT can be obtained by multiplying two 1-D scaling functions: $\Phi(x,y)=\Phi(x)\Phi(y)$. Wavelet functions for 2-D DWT can be obtained by multiplying two wavelet functions or wavelet and scaling function for 1-D analysis. For the 2-D case, there exist three wavelet functions that scan details in horizontal $\Psi^{(I)}(x,y)=\Phi(x)\Psi(y)$, vertical $\Psi^{(II)}(x,y)=\Psi(x)\Phi(y)$, and diagonal directions: $\Psi^{(III)}(x,y)=\Psi(x)\Psi(y)$. This may be represented as a four-channel perfect reconstruction filter bank as shown in Fig.5. Now, each filter is 2-D with the subscript indicating the type of filter (HPF or LPF) for separable horizontal and vertical components. The resulting four transform components consist of all possible combinations of high- and low-pass filtering in the two directions. By using these filters in one stage, an image can be decomposed into four bands. There are three types of detail images for each resolution: horizontal (HL), vertical (LH), and diagonal (HH). The operations can be repeated on the low–low band using the second stage of identical filter bank. Thus, a typical 2-D DWT will generate the hierarchical pyramidal structure shown in Fig.6. The structure of the 2D wavelet-packet transformed matrix M is shown schematically in Fig 7. The right of Fig.7 represents the redundant wavelet packet decomposition x of a vector x at various scales j [7].

Here, we adopt the term "number of decompositions" (J) which is adopted to describe the number of 2-D filter stages used in image decomposition [8].

3. Image Denoising Using Thresholding

One technique for denoising is wavelet thresholding (or "shrinkage"). When data are decomposed using the wavelet transform, filters that act as averaging filters, and others that produce details are used. Some of the resulting wavelet coefficients one correspond to details in the data set (high frequency sub-bands). If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of thresholding is to set all high frequency subband coefficients that are less than a particular threshold to zero. These coefficients are used in an inverse wavelet transformation to reconstruct the data set [9].

There are two thresholding methods frequently used. The *soft-threshold* function (also called the shrinkage function) is shown in Fig.8, that correspond to the following criteria:

$$\hat{X}_{ij} = \begin{cases} Y_{ij} - T & Y_{ij} \geq T \\ Y_{ij} + T & Y_{ij} \leq -T \\ 0 & otherwise \end{cases} \quad (1)$$

It takes the argument and shrinks it toward zero by the threshold T. The other popular alternative is the *hard-threshold* function that is shown in Fig.9. It correspond to the following criteria:

$$\hat{X}_{ij} = \begin{cases} Y_{ij} & |Y_{ij}| \geq T \\ 0 & |Y_{ij}| < T \end{cases}$$

The hard threshold is a method, which keeps the input if it is larger than the threshold T; otherwise, it is set to zero. The wavelet thresholding procedure removes noise by thresholding *only* the wavelet coefficients of the detail subbands, while keeping the low resolution coefficients unaltered.

The soft-thresholding rule is chosen over hard-thresholding for several reasons. **First**, soft-thresholding has been shown to achieve near-optimal minimax rate over a large range of Besov spaces. **Second,** for the generalized Gaussian previously assumed in this work, the optimal soft-thresholding estimator yields a smaller risk than the optimal hard-thresholding estimator. **Lastly,** in practice, the soft-thresholding method yields more visually pleasant images over hard-thresholding because the latter is discontinuous and yields abrupt artifacts in the recovered images, especially when the noise energy is significant. In what follows, soft-thresholding will be the primary focus [10].

4. Experimental Results and Discussions

For different Gaussian white noise levels, the experiment[...]s on which method in *Peak Signa[l to N]oise Ratio* (PSNR) are shown in Table1 for denoising images. The PSNR is defined as

$$PSNR = 20\log_{10}\left(\frac{256}{RMS}\right) \quad \text{.......(3)}$$

where RMS is the mean squared error.

$$RMS \quad err = \sqrt{mean(\hat{f} - f)^2} \quad \text{.......(4)}$$

The noise image with noise level (n=20) is illustrated in Fig 10. The difference between the wavelet and wavelet packet denoised images is subtle. The figures are enlarged in order to focus on a small area of these figures to see the difference.

The results in table (1) and Fig. 10 show that wavelet packet transform method is capable of removing more noise signal than wavelet transform method. Actually, wavelet packet transform method outperforms wavelet method. This can be proved by the "rms error versus threshold points" plot, which is shown in Fig.11 below.

It illustrates the denoising capability of the two methods, and clearly proves that the wavelet packet transform method is better than the wavelet transform method. Also, it tells us where the optimal threshold points are located. For each method, applying the optimal threshold point yields the mininum RMS error. Therefore, a threshold, producing the minimum RMS error, is the optimal one.

5. Conclusions

Image denoising problem can be cast as a 2-D signal estimation problem. We have show that wavelet thresholding is an effective method of denoising noisy signals. The image denoising algorithm uses soft thresholding to provide smoothness and better edge preservation at the same time. Experiments are conducted to assess the performance of wavelet packet transform in comparison with the wavelet transform. The results show that wavelet packet transform removes noise significantly.

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Table (1) PSNR values of denoising images for different test images and noise levels.

image	Noise level	Wavelet transform			Wavelet Packet transform		
		Mim RMS err	Threshold	PSNR	Mim RMS err	Threshold	PSNR
barbara	n=10	7.4431	9	30.7297	6.9896	9	31.2756
	n=20	12.3926	23	26.3016	11.2272	27	27.1595
	n=30	16.2105	41	23.9691	14.4959	41	24.9402
wbarb	n=10	7.3629	9	30.8238	7.1156	9	31.1205
	n=20	11.5406	27	26.9202	11.0067	29	27.3316
	n=30	14.1356	49	25.1587	13.5056	49	25.5547
sinsin	n=10	3.5368	19	37.1954	3.4242	19	37.4772
	n=20	5.3079	47	33.6854	5.2875	47	33.7196
	n=30	7.8434	49	30.2769	7.9191	49	30.3935

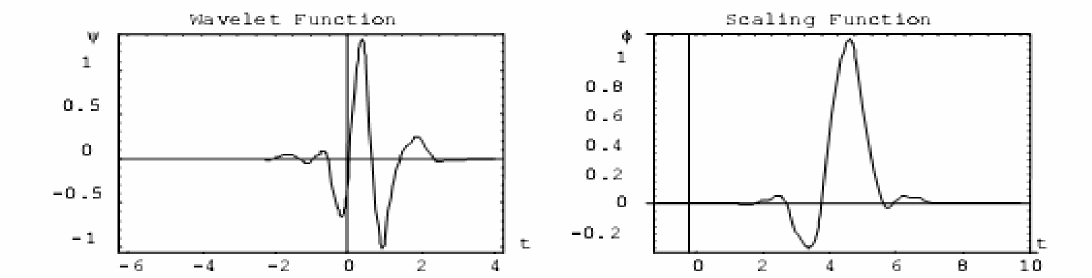

Fig. 1. Wavelet and scaling functions for 10-coefficient least asymmetric wavelet.

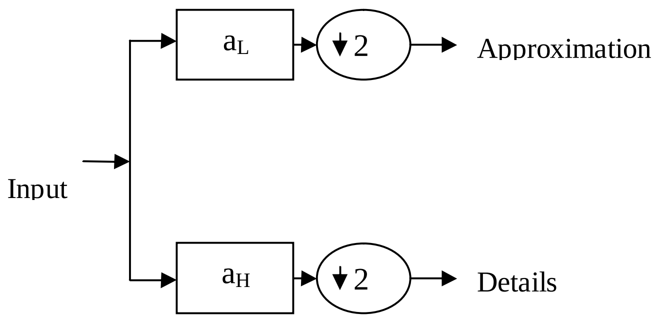

Fig. 2. Elemental filter pair

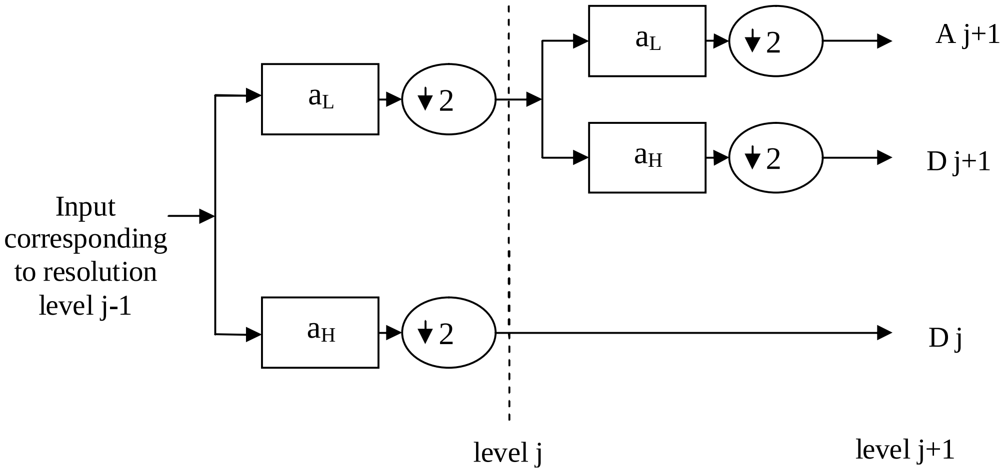

Fig. 3. Implementation of the discrete wavelet transform.

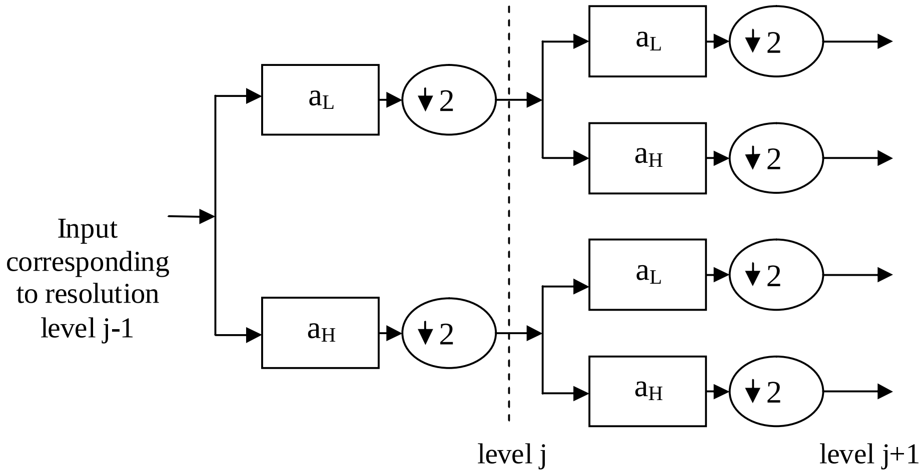

Fig. 4. Implementation of the wavelet packet transform

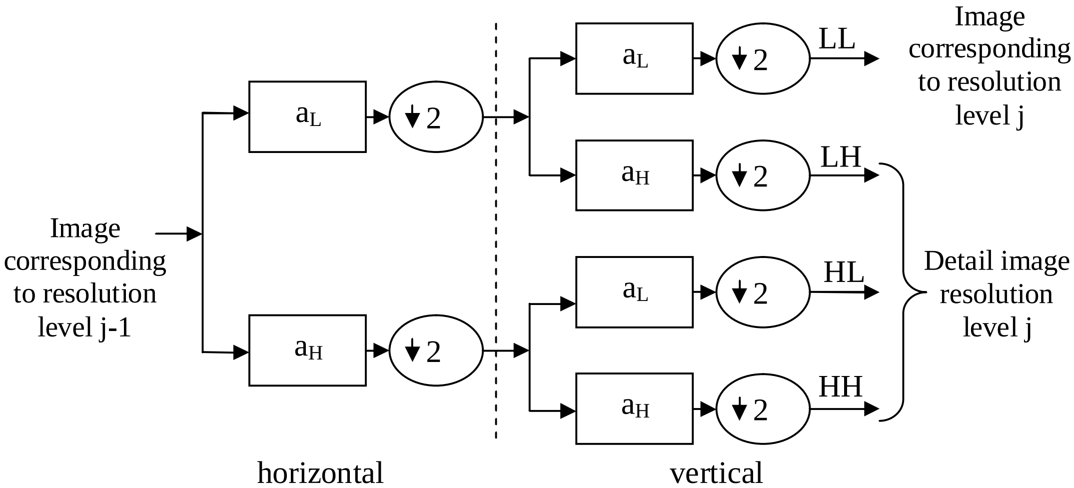

Fig. 5. One filter stage in 2-D DWT.

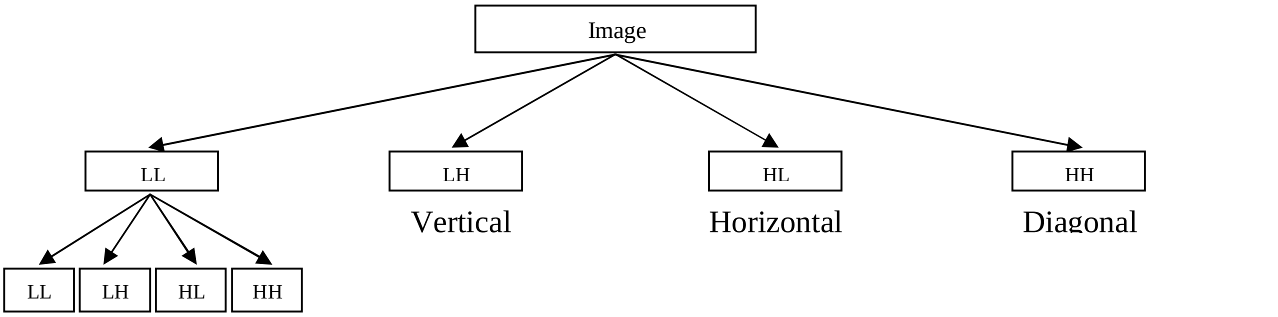

Fig. 6. Analysis / Synthesis of a 2-D wavelet decomposition.

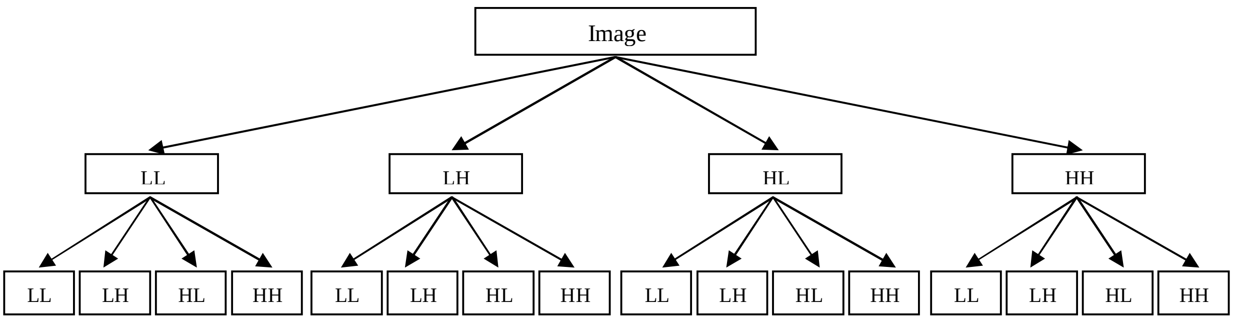

Fig. 7. Analysis / Synthesis of a 2D wavelet packet decomposition.

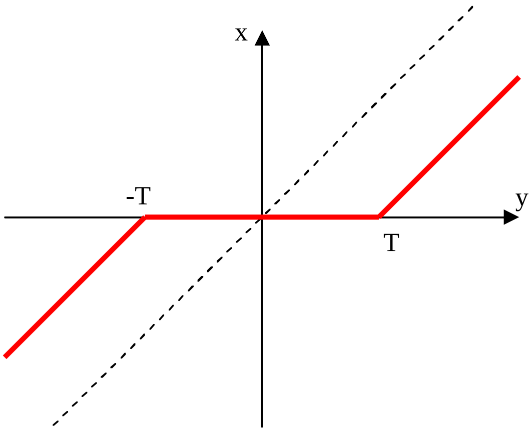

Figure 8: Soft Thresholding.

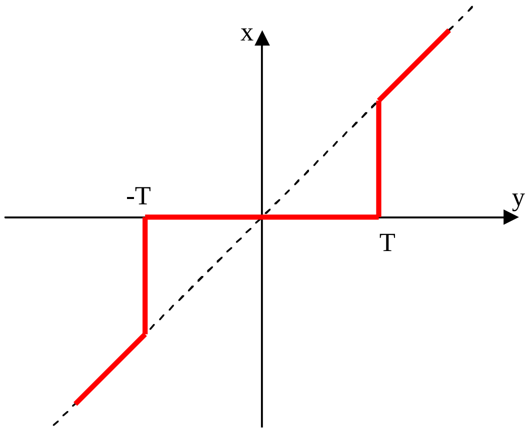

Figure 9: Hard Thresholding.

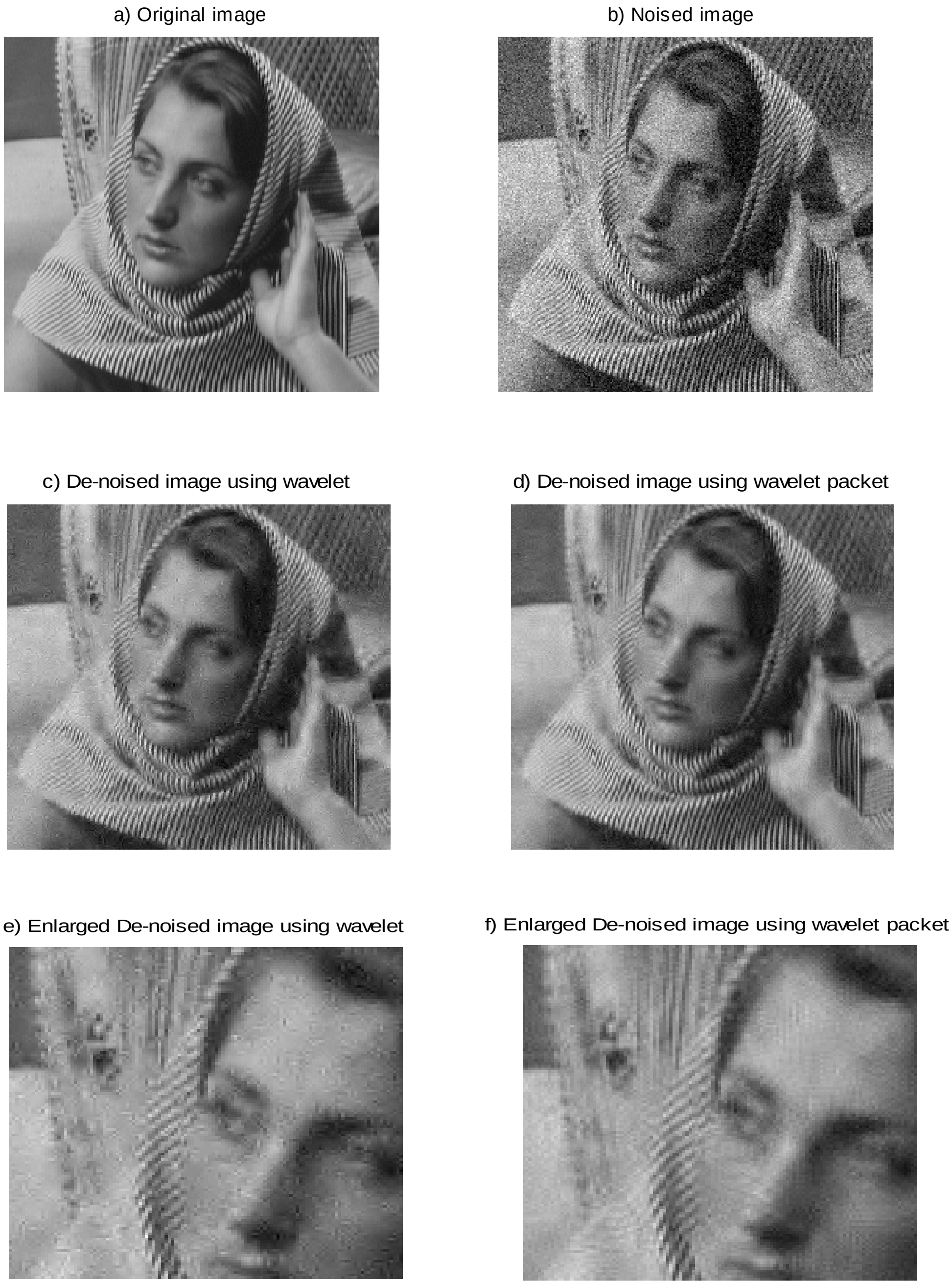

Fig. 10: Comparing the performance of the various methods on *Barbara* with noise level (n=20). (a) Original. (b) Noisy image. (c) De-noised image using wavelet. (d) Denoised image using wavelet packet. (e) Enlarged De-noised image using wavelet. (f) Enlarged Denoised image using wavelet packet.

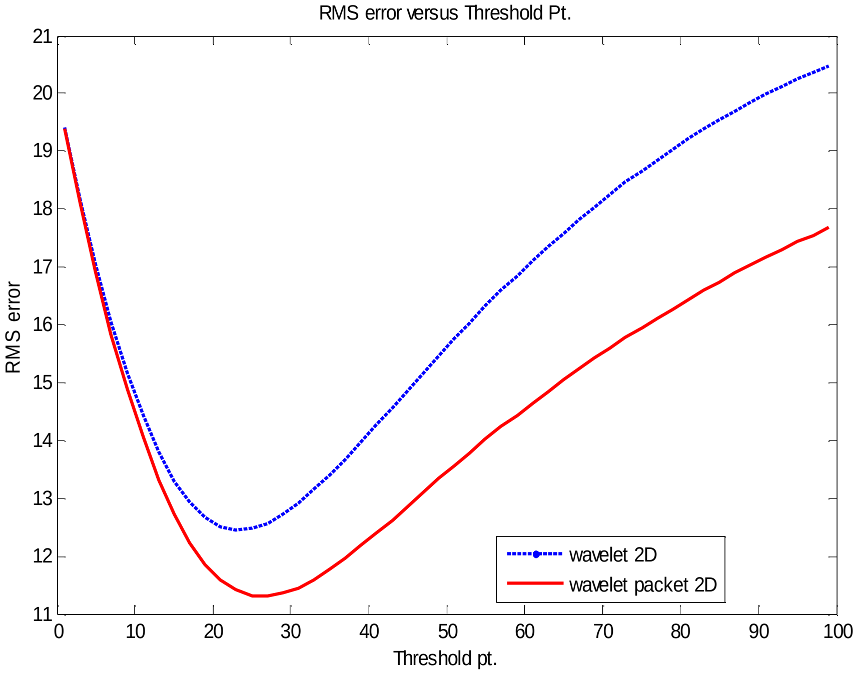

Fig. 11: Illustrates RMS error versus threshold points plot for *Barbara* with noise level (n=20) .