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A Hydrodynamic Model For Simulation of Unsteady Flow In Storm Sewer Network Systems

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Abstract

In this work, a hydrodynamic numerical model for sewer networks has been developed, Unsteady flow in either tree-type or looped sewer networks systems has been simulated, The Prissmann slot assumption is adopted to extend open channel flow equations to closed conduits under surcharged conditions. The link-node concept with a staggered grid, fully implicit scheme is developed to improve the stability and accuracy of the computation.

The model capable to reduce the order of the sparse matrix equations this has been achieved by treating the sewers of same diameter that connecting at a node with no lateral branching as a superlink, and the ends of the superlinks are defined as superjunctions so that considerable savings in computational effort is achieved. The numerical experiments show that the model is robust and reliable for different configurations of storm sewer networks and gives consistent results under different grid setups. Also, The experiments show an agreement with the results of few published studies.

Keywords: unsteady flow ; storm ; sewer ; closed conduit

نموذج هيدروداينميكي للجريان غير الثابت في شبكات مياه الأمطار

الخلاصة

في هذه الدراسة تم تطوير نموذج بأستخدام الفروقات المحددة لحالة الجريان غيــر الثابــت فــي شبكات المجاري بأعتماد شبكتي الربط من نوع الشجرة (Tree-Type) ومن النوع المتــصل (Loope-Type) . تم أختيار فرضية Prissmann وذلك بأعتماد أسلوب الشق slot في القنــاة المغلقــة للأخــذ بالأعتبار حالات الجريانات الفائضة (Surcharged conditions) . ولأختبار الأســتقرارية والدقــة تــم أختيار طريقة الشبكة المتعاقبة Link-node ، وقد بينت النتائج المستحصلة بأن النموذج المــستنبط فــي هذه الدراسة يمتلك موثوقية عملية ولأشكال مختلفة من الشبكات .كذلك أظهرت النتــائج توافقــاً عنــد مقارنتها مع دراسات سابقة .

Introduction

Flow in storm sewer networks is usually unsteady, non-uniform, turbulent and subject to mutual backwater effects of joining sewers. Traditionally, sewer flow is simulated approximately as steady flow or quasisteady flow from upstream sewer toward downstream in sequence, ignoring the downstream backwater effect.

The basic equations of gradually varied unsteady flow in open channels was firstly proposed by Barrel de Saint Venant in 1871.The one-dimensional form of the equations is used because this form is highly suited to the presentation of the basic concepts of the numerical procedures to be discussed. Extension to irregular surfaces can be accomplished with little difficulty.

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Derivation of these equations is given in standard reference (e.g. Chow, Henderson ,Mahmood and Yevjevich).

Partial differential equations may be solved numerically by replacing derivatives at a point with difference quotients over a small interval, i.e. $\partial f/\partial X$ is replaced by $\Delta f/\Delta x$, where Δx is small. Finite-difference methods are approximate only in the sense that derivatives at a point are approximated by difference quotients over a small interval. The methods can generally give solutions that are either as accurate as the data warrant, or as accurate as is necessary for the technical purposes for which the solutions are required.

Finite-difference schemes may be classified further as either explicit or implicit. In an explicit scheme, the solution at a given time step depends only on the known solution from the previous time step. . In an implicit scheme the solution at a given time step depends on both the known solution at the previous time step and on the adjacent unknown solution at the given time step. The unknowns in the finite-difference equations of an explicit scheme can be evaluated directly, whereas the equations of an implicit scheme are generally solved by iteration Price has shown that the implicit scheme for solving the unsteady flow equations is to be the most efficient and accurate method for flood routing problems.

The analysis of flow in storm sewer networks is mathematically similar to those encountered in a river system or in an interconnected system of channels around a large bay .

Only in recent years have the complete dynamic wave or Saint-Venant equations, which are capable of accounting for the downstream backwater effect and, thus, reversal flow, been adopted in sewer network flow simulation. A French model by SOGREAH [21] used an implicit numerical scheme to solve the dynamic wave equation. The model is proprietary and the details and assumptions of the model are not well documented. The recently added optional routing block EXTRAN of SWMM [26] developed by water Resources Engineers, inc. used an explicit numerical scheme in solving the dynamic wave equation for pipe flow and assumed junction condition considering only the continuity relationship .The method requires very short computational time interval and, thus, it is computationally costly and at times unstable. The Illinoins Storm Sewer system simulation Model (ISS Model) [29] developed in 1973 used the method of characteristics to solve the dynamic wave equations. The ISS Model was developed to simulate flow in existing systems as well as for design (sizing pipe diameters) of new system. Details of the model have been documented elsewhere.

The last study, JI presented numerical model for sewer/channel networks .The Priessmann slot assumption is adopted to extend open channel flow equations to closed conduits under surcharged condition, implicit scheme is developed to improve the stability and speed of computation, the model reduces the order of the sparse matrix equation.

In present study a fully implicit scheme for numerical solution of the partial differential equations was employed, the scheme results in sparse matrix equations for the complete unsteady flow equations. The equations are then solved implicitly for flows and depths

throughout a network system over a time step based on sewer network connectivity and boundary conditions.
The objective of the present study is to introduce a discretization scheme for the complete unsteady flow equations and solving these equations for sewer networks under both free surface and surcharged conditions.

The developed model shall be used to evaluate the operation of the sewer network system for variant inflow hydrographs. Also, the model permits to decide for parallel sewer lines or extension of the main storm sewers , as well as improving their operation by adding loops.

Theory and assumptions

The complete unsteady flow equations for one-dimensional flow under free-surface conditions are used to describe the flow through a sewer/channel system. These equations are a pair of nonlinear partial differential equations: a continuity equation and a momentum equation. The continuity equation describes the mass balance of flow; the momentum equation describes the force balance under dynamic conditions

$$\frac{\partial \mathbf{A}}{\partial \mathbf{t}} + \frac{\partial \mathbf{Q}}{\partial \mathbf{x}} = \mathbf{qo} \qquad \ldots(1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(Qu) + gA\left(\frac{\partial h}{\partial x} - s_0 + s_f + s_L\right) = 0 \qquad \ldots(2)$$

Where
Q = discharge; **A** = flow cross-sectional area; **h** = depth; **u**= velocity; $\mathbf{S_O}$ = bed slope of channel or sewer; $\mathbf{S_f}$ = friction head loss slope; $\mathbf{S_L}$ = local head loss slope; $\mathbf{q_o}$= lateral flow over unit length of channel or conduit; $\mathbf{g}$ = gravitational acceleration; **x** = distance; and **t** = time.

The head loss slope has two components: the frictional head loss slope, S_f, and the local head loss slope, S_L .The friction slope represents the head loss from the shear applied to the wetted perimeter along the longitudinal direction. The local head loss slope represents the energy loss from a sudden change of the flow cross section over a short distance.

Solution of the two partial differential equations requires specification of boundary conditions. The boundary conditions of a system are defined as a stage hydrograph or a relation between the flow and stage where Q = f(H).Lateral flow to a junction is not considered as a boundary condition in the computation, since the flow is already included in the continuity equation. Stage at a boundary junction governs the depth at the end of the boundary conduit when the stage is relatively high (backwater effect). Depth at the end of the conduit is the lesser of the critical depth and the normal depth when the end conduit experiences a free outfall condition.

A few assumptions are made in solving the flow equations for a sewer system. A fictitious Priessmann Slot is introduced to extend the applicability of the one - dimensional unsteady free - surface flow equations to surcharged conditions in closed conduits, The concept is modified so that only the top width of the slot is used in the computation, and that no extra area and wetted perimeter from the slot are introduced. The slot has to be narrow enough so that the geometry of the cross section will not be appreciably changed. The width of the slot in the present model is assumed to be 0.1% of the diameter of sewer under surcharging conditions, as shown in fig (1).

Flooding is considered to occur when a system is overloaded. A flood area assumed on top of a manhole above the ground elevation. From geometry of the flooding area surface area was obtained for different elevation.

Before describing the mathematical formulations for the model, a link-node description was used to represent the sewer network system. The concepts of link, node, superlink, and superjunction are introduced to facilitate interpretation. A link can be either a sewer or a section of an open channel. In the present numerical approach, a link is a basic finite volume, and its length is treated as Δx in the numerical computation. A node can be a manhole, a location where the size or slope of the pipe changes, or purely a computational point to segment a sewer when higher resolution of the computational results is desired. A superlink is defined as a collection of links connected end to end at nodes without branching. The ends of superlinks are defined as superjunctions. A superjunction can be a storage junction, a node with one link connected, a node with more than two links connected, or a node attached to a system boundary. A superjunction can also be a node attached to two links with a noticeable difference in invert elevations.

The discretization of the complete unsteady flow equations is the foundation of getting stable and accurate numerical solutions under a wide variety of physical conditions experienced in sewer network systems. The staggered grid scheme, which is conceptually different from the popularly used four-point scheme, is formulated for solving the completely unsteady flow equations. In establishing the numerical scheme, the physical phenomenon of one-dimensional unsteady flow should be described by the discretized continuity and momentum equations as closely as possible. The momentum equation describes the dynamic force balance in the space dimension. The movement of water volume in. a link is driven by the force governed mainly by the difference between the head at two end nodes of the link. It is reasonable to apply the momentum equation to a link and solve the flow through the link based on the head at the two end nodes. On the other hand, to find the flow at a node can be confusing when lateral flow is applied to a manhole. The continuity equation should be applied around the node rather than a link to avoid this confusion. With the prior considerations, it is physically more appropriate to apply the continuity and momentum equations in a series of staggered control volumes in the computational domain rather than to find flow and head at the same locations, as suggested by the four-point schemes. Applying the staggered grid and implicit scheme to a superlink (e.g., superlink k) with nk links, the depth is solved at Nk nodes, and the flow is solved in nk links subject to the heads at two ends of the superlink. The schematic of the staggered grid is shown in Fig(2).

The link-node setup with the staggered grid implicit scheme is derived in two steps for the solution; The first step is to derive the recurrence relations according to the discretized continuity equation and the momentum equation for each superlink using the staggered grid implicit scheme. The definition of the superlink facilitates the derivation, since there will be no off diagonal nonzero elements in the coefficient matrix formulated for one superlink. A series of superlink recurrence relations relating flows and depths in the links and nodes of each superlink can be derived.

A condensed sparse matrix is created in the second step of the development by applying the continuity equation to all superjunctions in the system using the recurrence relations for each superlink developed in the first step. The order of the matrix set up in this way is equal to the number of superjunctions in the system. This order reduction of the matrix facilitates the solution procedure and speeds up the computation.

The link-node setup with the staggered grid, implicit discretization scheme outlined before is developed in well detailed in Ref.1 . All nonlinear coefficients for the variables during t-t $+\Delta t$ are linearized based on the solutions at time t in the following development. For the momentum equation is linearized by assuming, area **A**, velocity **u**, hydraulic radius **R** and Manning s coefficient **n** are known from the previous time and for continuity equation the top width **B** for link is also assumed to be known from the previous time step.

A new suggestion to be adopted in this work to approximate closely the actual condition, steady uniform flow condition was proposed to compute the flow in each link and depth at each node as initial value for unsteady flow computation.

The initial steady flow in each link equal to the initial lateral flow entering the upstream node. If there is no lateral inflow at node, the initial steady flow in link equal to the initial flow of upstream link.

In practical computation experience, instability of a process for calculating unsteady flow in open – channel is wildly oscillating values, with respect to t and x, of the computed depth and discharge, in implicit scheme, the optimum accuracy is obtained when the finite difference time step, is approximately equal to the space step divided by the kinematics wave speed (Price,1974) .

Then the maximum allowable time step calculation for link i expressed as:

$$\Delta t_i = \frac{A_i \, \Delta x_i}{|Q_i|} \quad \ldots(3)$$

where $Q_i \neq 0$

The maximum allowable time step Δt for network system was taken as the smaller Δt_i that calculated from eq (3) for all links in network system

Numerical tests and verification

Verification of the model has been conducted in two methods. The first method, a computational of total water balance error is used, comparing the error for different grid sizes (Δx).

The second method, the model has been verified by comparing the present model results with other published models for , tree-type sewer network and looped connection sewer network.

A-Tree- type connection sewer network

The network system was used for this numerical test and verification chosen from example suggested by Sevuk and Yen. The network has four inlets, seven sewers linked as a point -type junction with a common crown elevation and free out fall flow at downstream of network system 1 as shown in fig (3).

The network received identical inflow hydrograph at different time at their inlets, in network1, identical flood inflows enter successively into Sewers a1, b1, c1and d1 at t = 0, 4, 8, and 12 min, respectively, representing a hypothetical rainstorm movement towards downstream along the general direction of the network as shown in fig (4).

The inflow hydrographs fig (4) are nearly triangular in shape with a constant bas flow (0.085 m^3/sec). The peak inflow rate (0.425m^3/sec), and the duration 20min.

Sevuk and Yen analyze the two sewer networks by using ISS model, it is dynamic wave routing model for sewer network flow simulation used the method of characteristics to solve dynamic wave equation.

Initial steady flow at (time=0)in each sewer is equal to the summation of flow in sewer that connected to upstream of sewer. e.g. initial flow in sewer **c** equal to 0.085 m^3/sac ,sewer **f** equal to 0.225 m^3/sec and for sewer **g** equal to 0.34 m^3/sec.

In network system fig(3), flood waves from the upstream sewers arrive at the junctions at almost the same times; consequently, the cumulative effect of the flows results in a short-duration and single-peak discharge, and depth hydrographs at the entrance of sewers e1, f1 and g1 as shown in figure (5) .

The flow and depth at entrance of sewers e1, f1 and g1 for network 1 with a different value of space grid (Δx) were also compared as a space grid test and the results compared with the Sevuk and Yen results in the same figures as shown in figure(5) .

The results for three chosen space grid (Δx), 30.48m, 60.96m and 101.6m indicated that the maximum difference for peak flow and depth at entrance of sewers e1, f1 and g1 for three (Δx) chosen not exceed 3% in value, also the water balance error is varying fro -0.15% for Δx=30.48 ,-0.3% for Δx=60.96 and 0.347% for Δx=101.6m.

Comparing the results for the networks 1 with Sevuk and Yen results, as shown in fig(5), shows that the flow and depth hydrographic identical in shape and well agreement in results .

B-Looped connection sewer network

The looped network system 2 was used for this numerical test chosen from a trial network system suggested by Ji in testing his numerical model for sewer network system.

The network in fig (6) has six sewers linked as a point-type junction with a common invert elevation, Manning coefficient for sewers was chosen (n_{full} = 0.0125).

The network receives two inflow hydrograph at two junction A and C, the two inflow hydrograph are triangular in shape with no base flow. The peak inflow rats 0.5 m^3/sec and 0.3 m^3/sec for junction A and C respectively and the duration 2 hour as shown in fig. (7.a). Junctions D and F attached to stage boundary condition defined in fig (7.b).

The reveres flow due to the rising downstream stage partially filled the system at the end of the 2 hours, also that caused a small amount of negative flow through sewers at the first 2 hours that could be neglected. Therefore ,the time of computation started at the beginning of third hour (t_o=7200 sec) where the heads at junctions equal to 10.8 m caused by rising the stage level at downstream the network (junction F and D) and the flows through each sewer were zero. Between the second and fourth hour of the simulation, inflows are applied to junctions A and C while the downstream stage is declining. The system started to drain water through downstream junctions D and F indicated by the positive flow through conduit c and e. The water level in the system was backed up and the system is obviously surcharged during this period. The head at junction C was higher than that at junction E, causing negative flow through sewer f as shown in fig.(8). After 2 hrs of simulation, there is no inflow to the system and the downstream stage was declining to the level below the lowest location in the system. The water volume is drained off completely and the system remained dry during the rest of the simulation. As shown in fig (8) and fig (9).

A series of numerical tests under the same conditions were carried out using different space grid (Δx) and the time step (Δt) according to the equations in sec.3.11, also a serial of numerical tests was done to show the effects of initial time step with respect to total water balance error and to give a reasonable guideline for a suitable initial time step, Δt_{inial}, using in this numerical tests, the results illustrated in table (1).

Fig (8) show the flow at entrance of sewers a, b, c, d, e and f, were compared for three chosen (Δx), 25m, 50m and 100 m and fig (9) shows the head at junctions A, B, C and E for three chosen (Δx), also the results of these cases have been compared with that of Ji[16] results. The results indicate that segmenting a sewer may not have a significant effect on the accuracy of the computation in the range tested.

A minimum water balance error occurred at small space grid (Δx =25) is equal 0 .04% while the water balance error equal 0.9 % at (Δx =100).

The results of the tests suggest that the model can give consistent simulation results under different space grids in the rang tested. It also indicates that the model is robust even under conditions unfavorable to the stability of computation, such as the situation when reverse flow is applied to a dry sewer network system.

Conclusions

The results of the numerical tests suggest that the model can give consistent simulation results under different space grids in the rang tested. It also indicates that the model is robust even under conditions unfavorable to the stability of computation, such as the situation when reverse flow is applied to a dry sewer network system. Segmenting a sewers in networks may not have effect on the accuracy if high inflow is interring the network, that clearly noticed in results of network 2 when it is surcharging, because the value of time step computed from eq.3 is to small. Also , At a given location of a sewer , the maximum discharge need not and usually does not concur with the maximum depth of the unsteady flow. The flow depth corresponding to the maximum discharge is smaller than the maximum depth at a cross section. Thus, sewer design based on maximum discharge dose not guarantee the sewer free from surcharge. Also, the maximum depth of the unsteady flow can occur at the sewer entrance, sewer exit, or another location in the sewer, depending primarily on the downstream backwater effect.

References

[1]Amein ,M. and Fang , C.S. , "Implicit flood routing in natural channels" , Journal of hydraulics division , Vol. 96 , No. HY 12 ,pp. 2481-2500, Dec , 1970 .

[2]-Chaudhry ,M.H. ,and Schulte,A.M., "Computation of steady- state , gradually varied flows in parallel channels" , Canadian Journal of Civil Engineering , Vol. 13 ,No.1, pp.39-45, 1986.

[3]-Fread , D.L., "Numerical properties of implicit four point finite difference equations of unsteady flow" NOAA , Technical Memorandum NWS HYDRO- 18,1974.

[4]-Fread , D.L., " Gradually varied unsteady flow profiles" ,Generalized computer program 723-G2-L7450,Hydrologic Eng. Center ,U.S. Army crops of Engineers, V ICKSBURG, Miss,1977.

[5]- Husain , T., Abderrahman, W.A., Khan ,H.U., Khan , S.M. ,Khan ,A.U.,and Eqnaibi, B.S. , "Flow simulation using channel network model" , Journal of irrigation and drainage , Vol.114, No.3, pp. 424-441, Aug. ,1988.

[6]- Ji , Z., " General Hydrodynamic model for Sewer/ Channel network system" , Journal of hydraulic engineering ,Vol. 124 ,No. 3, pp. 307-315 , March ,1998.

[7]-Joliffe , I.B. , "Computation of dynamic waves in channel networks" ,Journal of hydraulic engineering ,Vol. 110 ,No. 10 pp. 1358-1370 , October,1984.

[8]-Mahmood ,K., and Yevjevich, V. , "Unsteady flow in open channels" , water resources publications , Fort Collins , Colorado , 1975.

[9]- Price , R.K. , "Comparison of four numerical methods for flood routing" , Journal of the hydraulic division, Vol. 100 ,No.HY7 ,pp. 879-899 , July , 1974.

[10]- Roesner, L.A.,Aldrich,J.A., and Dickinson,R.E., "Storm water management model (SWMM) user ' s manual version 4",Cooperative Agreement CR-811607,U.S.Envir. Protection Agency ,Cincinnati , Ohio.

[11]- Sevuk, A.S. , and Yen , B.C. , "Sewer network routing by dynamic wave characteristics" , Journal of hydraulic division , Vol. 108 , No.HY3, pp. 379-398 , March,1982

[12]- Strelkoff, T. , "Numerical solution of Siant Venant equations", Journal of the hydraulic division, vol.96 ,No.HY1,pp.223-252, Jan,1970.

[13]- Yen, B.C. , and Sevuk, A.S. , "Design of storm sewer networks" , Journal of the Environmental Engineering Division , Vol.101, No.EE4,pp.535-553, August , 1975

Table (1) Percentage water balance error for different $\Delta t_{initial}$ and Δx

$\Delta t_{initial}$ (sec) / Δx (m)	10	25	50	100
25	0.57 %	0.04%	0.8%	2.5%
50	1.1%	0.63%	0.2%	1.9%
100	1.4%	0.9%	0.15%	1.5%

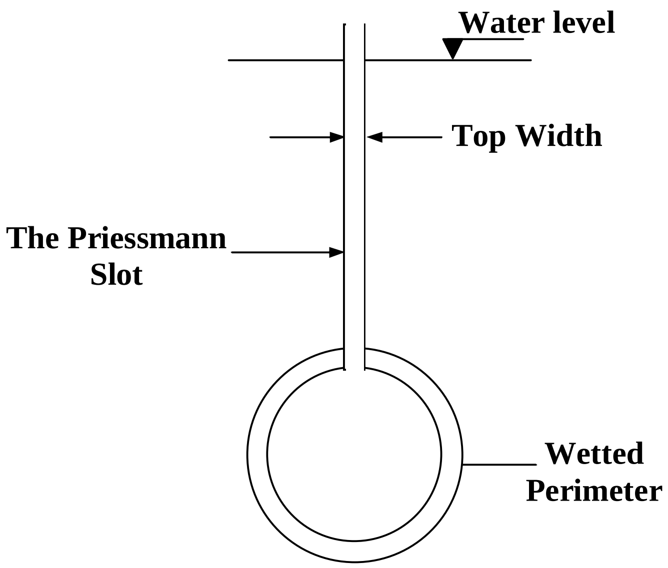


Figure (1) Schematic of priessmann slot

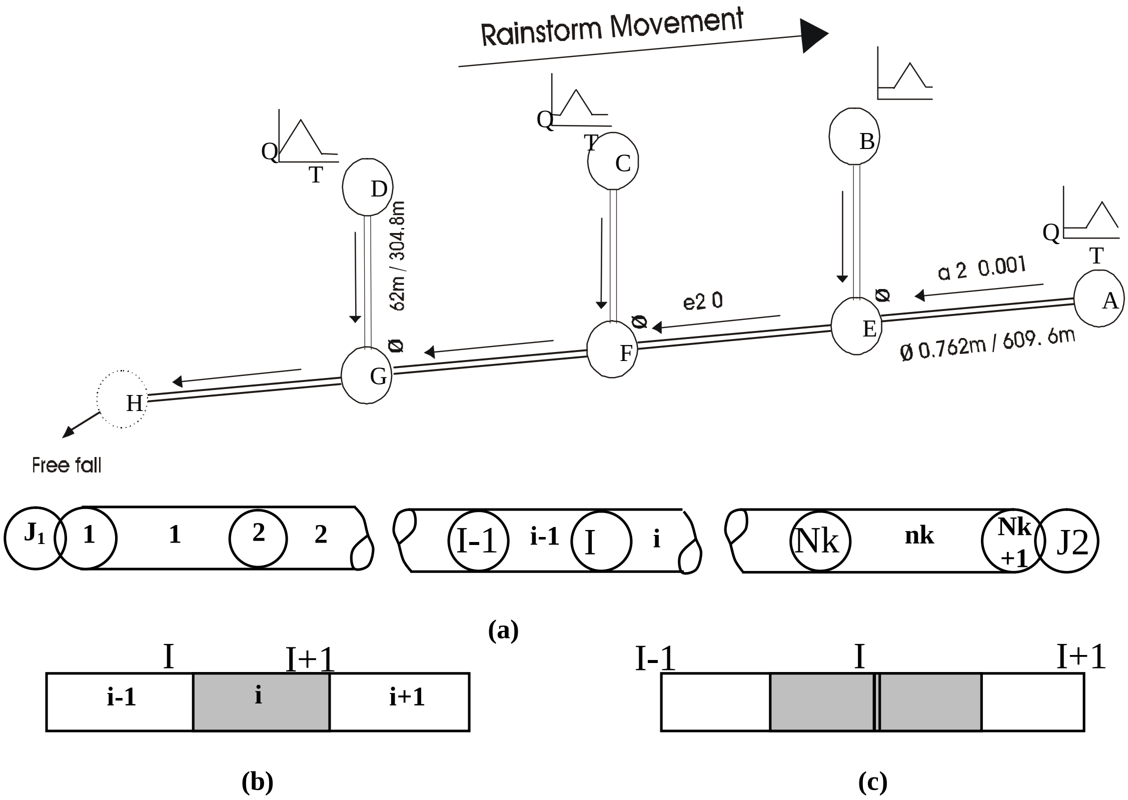
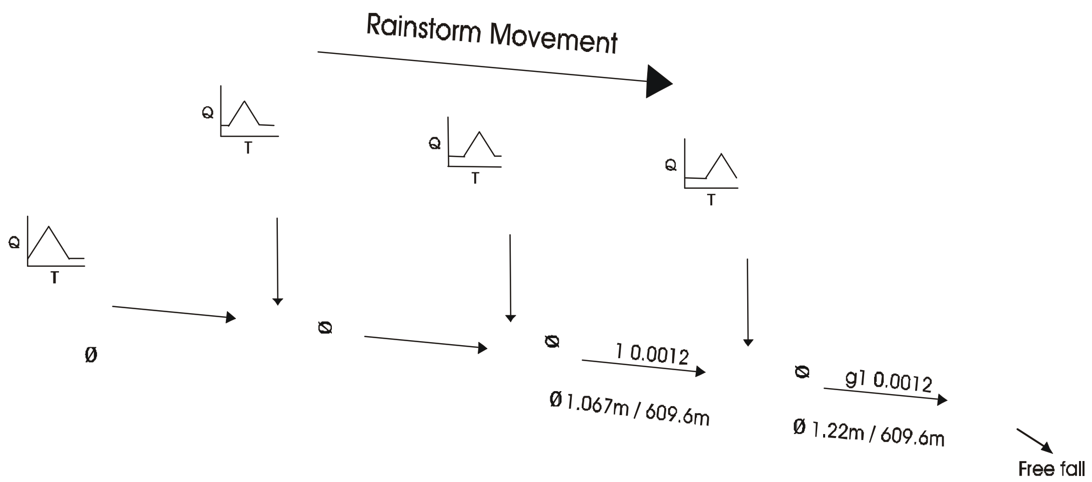


Figure (2) Staggered grid setup of discretized superlink): (a) setup of staggered grid in superlink k; (b) control volume for link i ; (c) control volume for node I.



Figure(3) layout of network 1

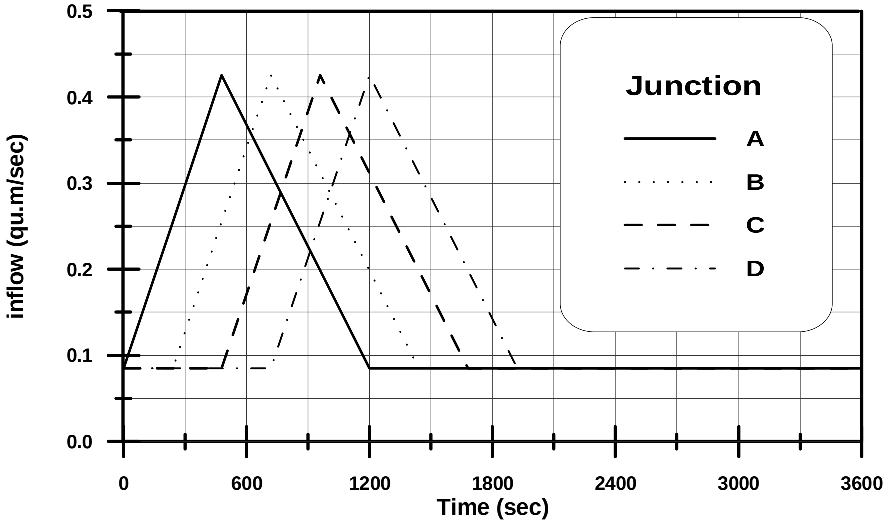
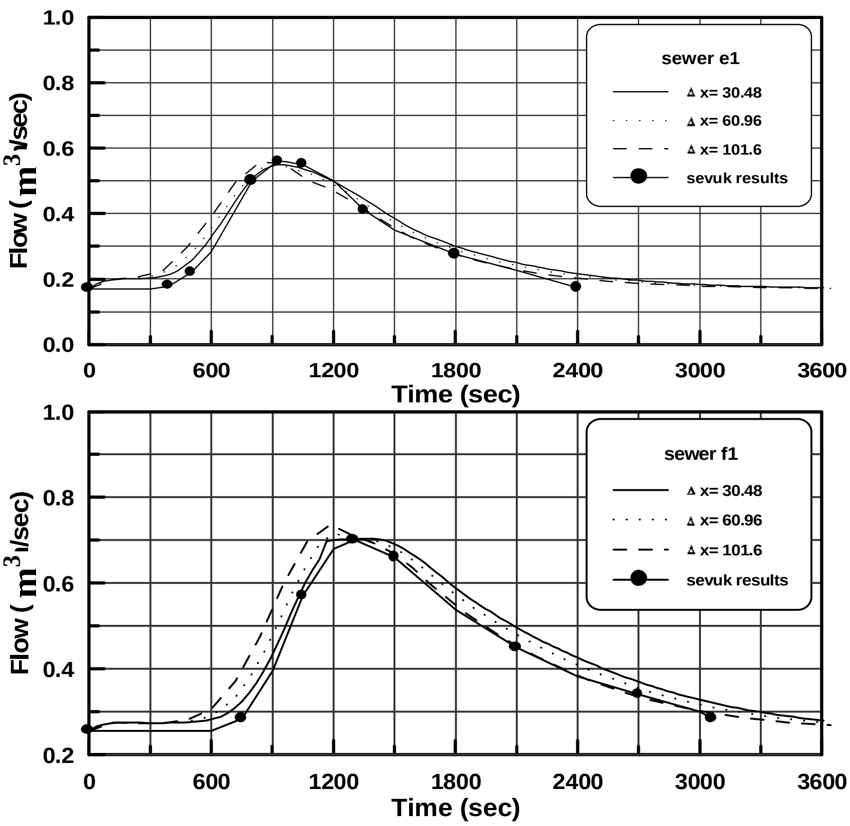


Figure (4) Inflow hydrographs in network 1



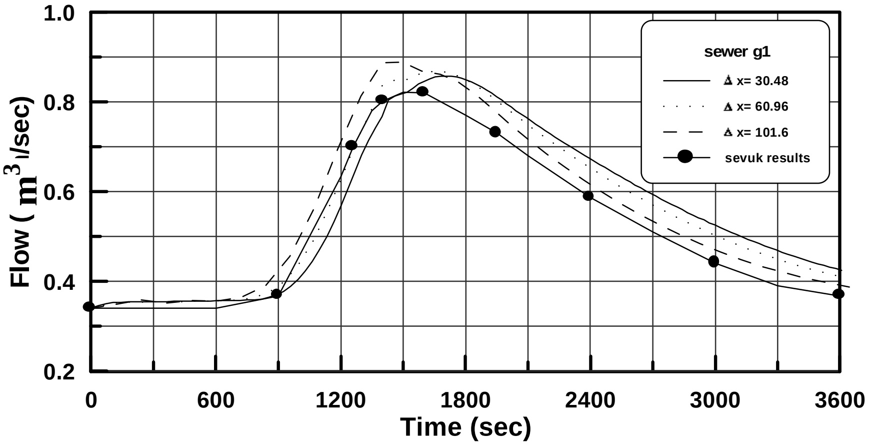


Figure (5) Flow at entrance of sewers e1, f1 and g1 in network 1 for different value of space grid.

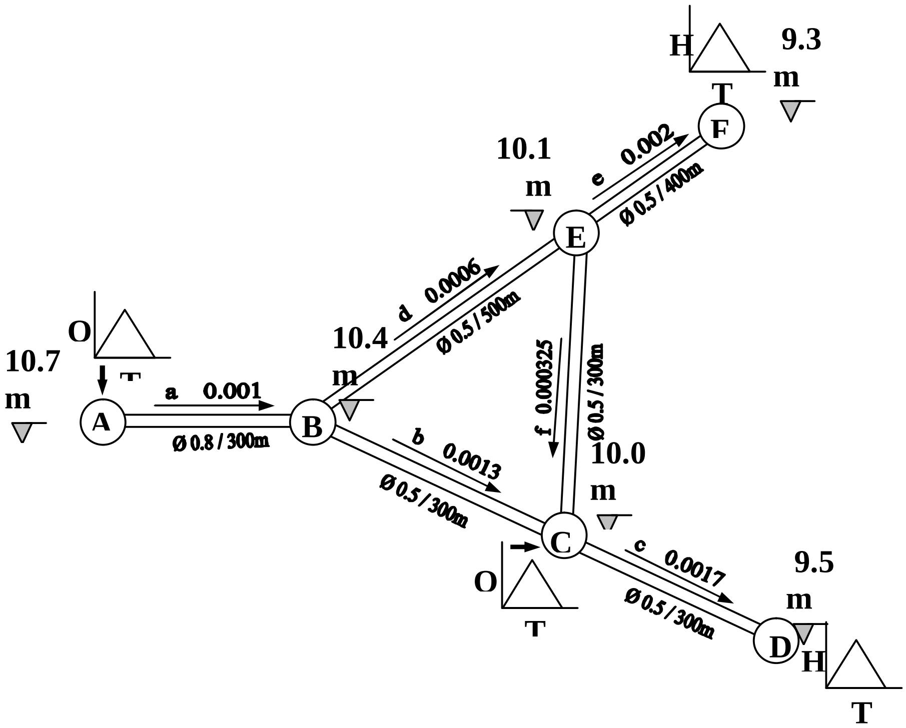


Figure (6) Network 2, looped connection with stage boundary condition at junctions F and D

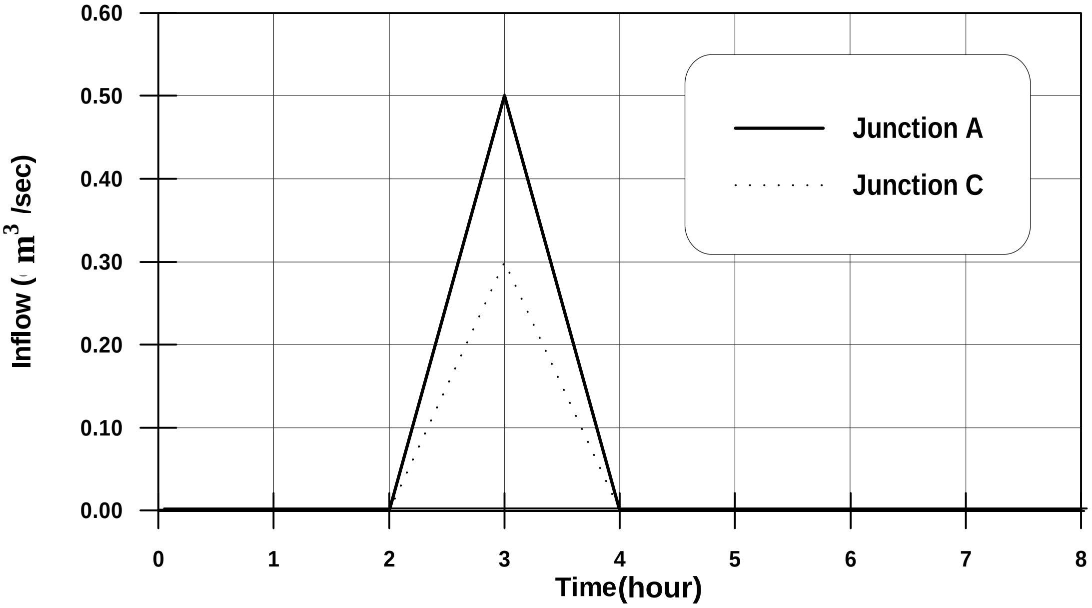


Figure (7.a) Inflow hydrograph in junctions A and C for network 2

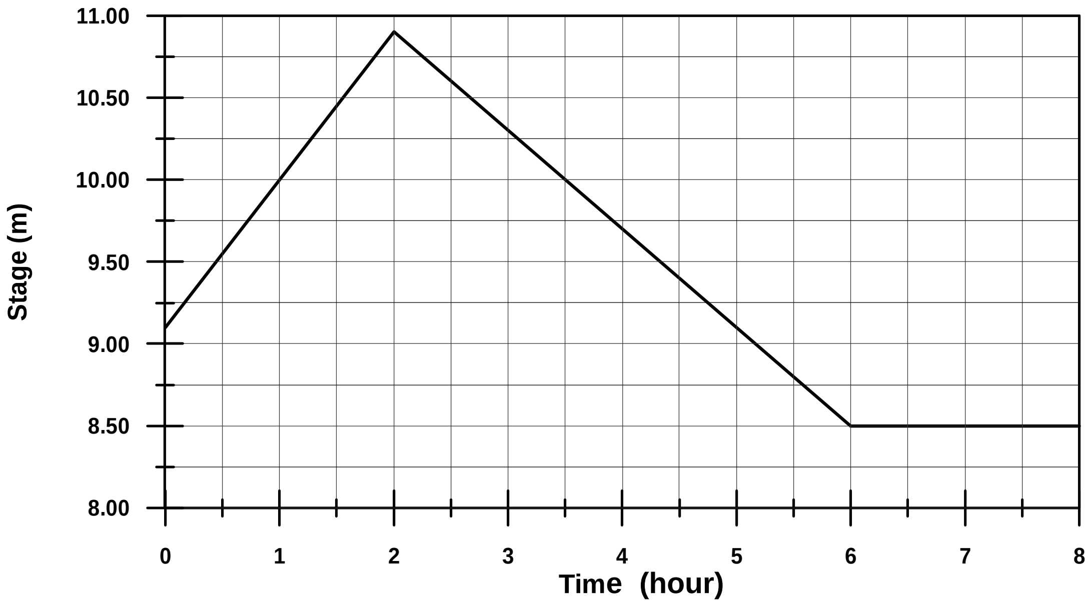


Figure (7.b) Downstream Stage boundary condition at junctions F and D for network 2

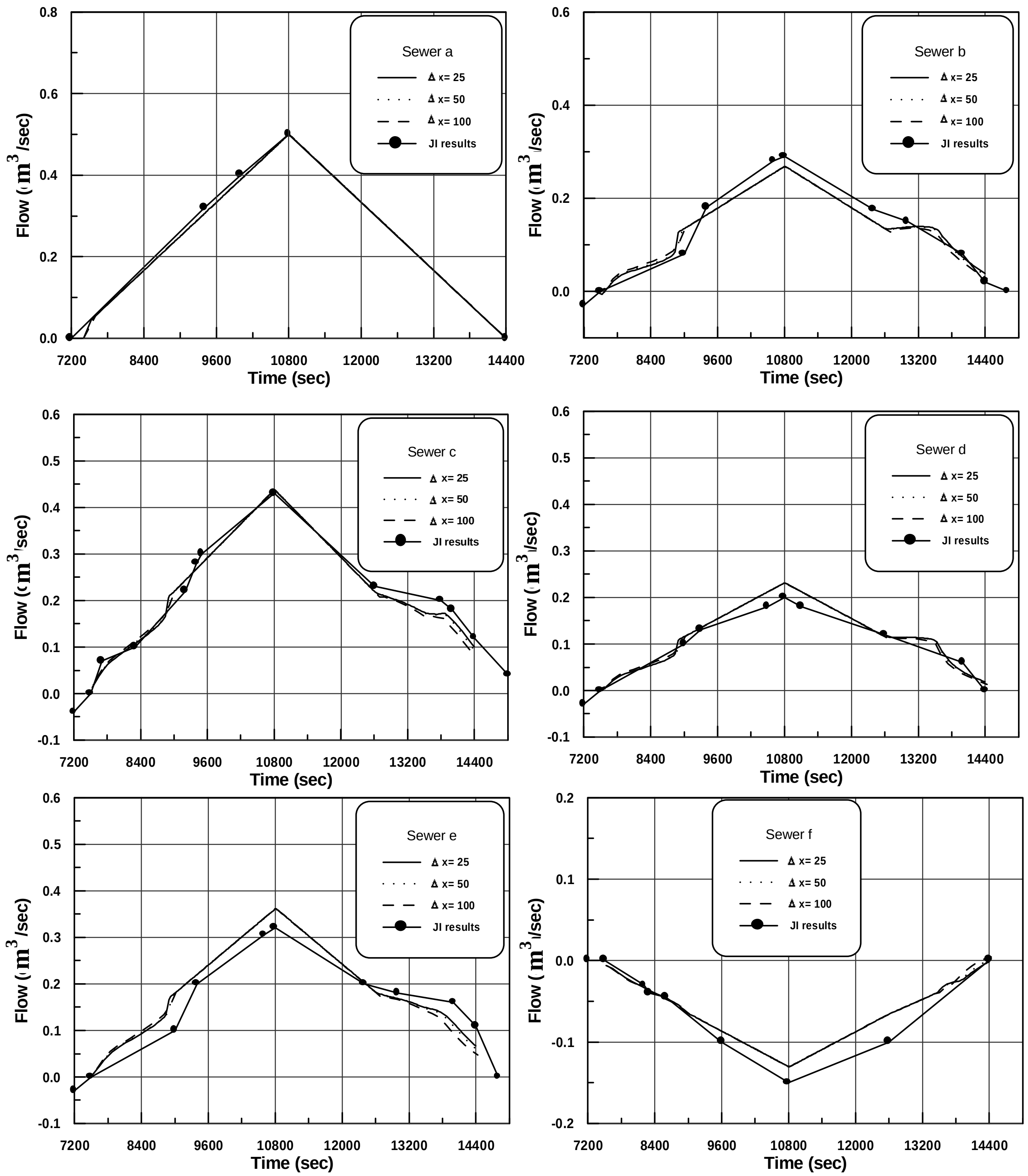


Figure (8) Flow at entrance of sewer a, b, c, d, e and f in network 2

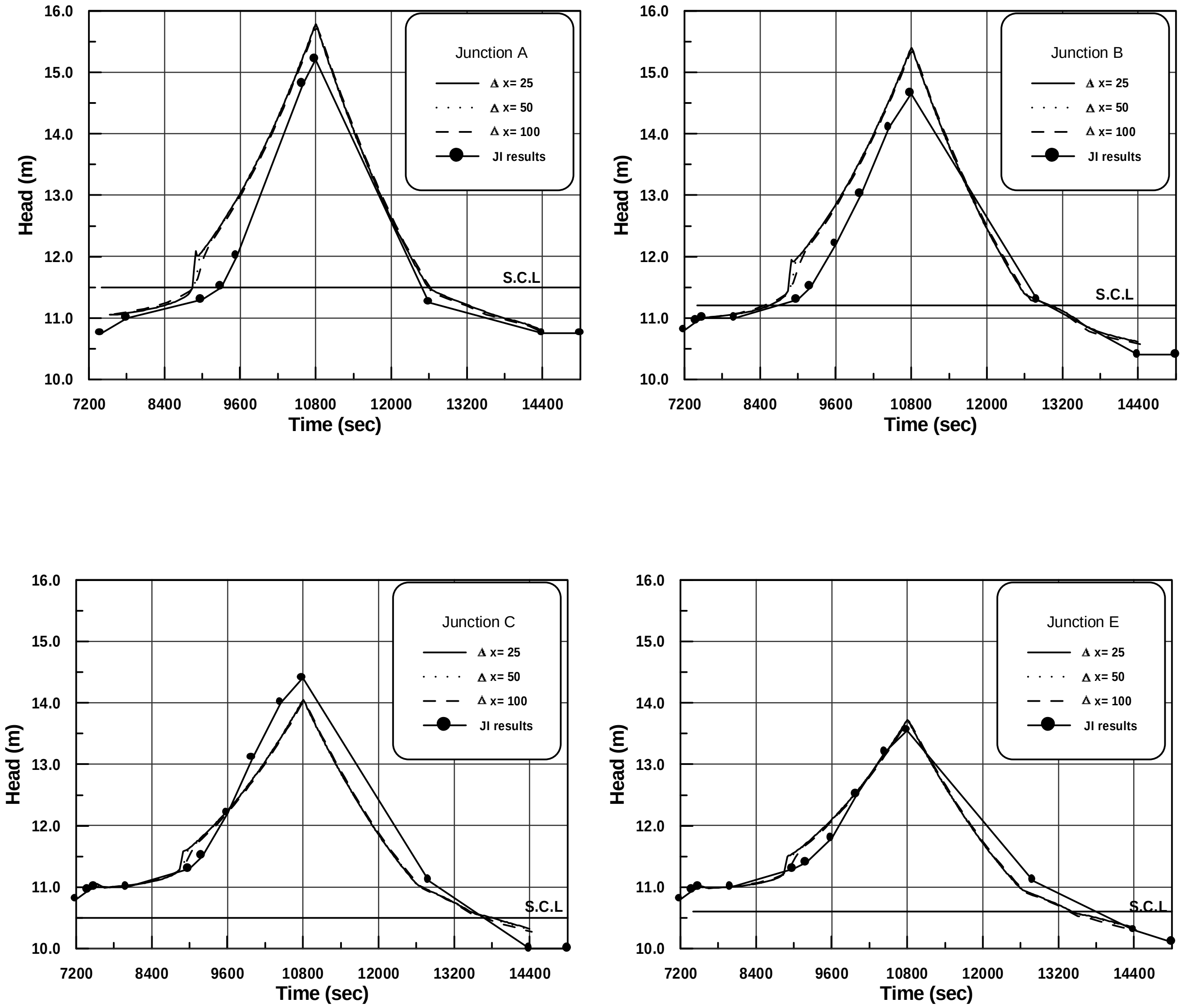


Figure (9) Head at junctions A, B, C and E for network 2

S.C.L – Sewer Crown Level