Theoretical Analyses of the Dynamic Behaviour of Composite Cantilever Beam Manufactured From E-glass Polyester

Hayder Moasa Al-Shukri*, Dr. Muhannad Z. Khelifa & Dr. Saad A. Khether***

Received on:17/9/2008
Accepted on:5/3/2009

Abstract

In order to have the right combination of manufactured E-glass polyester composite properties and in service performance, the dynamic behaviour is one of the important properties to evaluate. The dynamic behaviour of E-glass Polyester composite was considered in this study where three laminate types was modelled [0^o, $\pm 45^o$, $0^o/90^o$], also the influences of fibre orientations as well as the stacking sequences of the laminate layers on the natural frequencies and resonance under harmonic conditions were investigated. Commercial Finite Element ANSYS® Release 10.0 package analyses were used to simulate the Modal and Harmonic behaviours of composite cantilever beams in the frequency range of 0 to 1000 *Hz*. The first six modes in this frequency range were extracted and compared in the three laminates. A harmonic simulation was investigated to study its structure response to resonance. The results proved that the [± 45]$_s$ laminate had higher torsional modal frequencies due to its higher shear modulus and is more stable under loading than [0/90] laminate due to the arrangement of the layers.

Keywords: Composite material, Cantilever beam, Model simulations, Natural frequencies, Dynamic behaviour, Finite element method.

التحليلات النظرية لتصرف الديناميكي لعتبة المصنعة من الليف الزجاجي المدعوم بالبولستر

الخلاصة

لغرض الحصول على التركيب الصحيح لخصائص مـادة الليـف الزجـاجي المـدعوم بالبولستر المصنعة خلال أدائها العملي يكون السلوك الحركي من أهمّ الخصائص الواجب تقيمهـا. في هذه الدراسة تم اعتماد السلوك الحركي لمركب البولستر الزجاجي و لثلاثة أنواع من الـصفائح المركبة, [0^o, 45^o, $0^o/90^o$] وبإضافة لتأثيرات اتجاه الألياف وكذلك تتابع ترتيب لصق الطبقات على التردد الطبيعي وظاهرة الرنين تحت حالة الموجة التوافقية. استخدام طريقة العناصر المحددة بالاعتماد على البرنامج التحليلي ANSYS® اصدار 10.0 لمحاكاة التصرف التـوافقي و الـشكلي

* General Directorate for Vocational Education, Ministry of Education /Baghdad
** Electromechanical Engineering Department ,University of Technology/ Baghdad

https://doi.org/10.30684/etj.27.9.14
2412-0758/University of Technology-Iraq, Baghdad, Iraq

لعتبات كابولية ضمن مديات للتردد من 0 إلى 1000 هرتز.اعتمد هذا البحث على الأنماط الأولى الستة لهذه المديات من الترددات وتمت مقارنتها للأنواع الثلاثة من الطبقات. المحاكاة التوافقية كانت لدراسة استجابة الهيكل لظاهرة الرنين. اثبتتْ النَتائِجَ بأنّ الصفائح المركبة $[\pm 45]_s$ لها اعلى َ تردداتُ عند torsional modal بسبب معامل القص الأعلى وأكثـر اسـتقراراً مِـنْ الصفائح المركبة [90/0] عند التحميل وذلك بسبب ترتيبِ الطبقاتِ.

الكلمات المرشدة: مواد مركبة ,عتبة كابولية, محاكاة النموذج, الترددات الطبيعية , السلوك الحركي, طريقة العنصر المحدودةِ.

Nomenclature

E_{xyz}	Elasticity modulus in the global coordinate directions	GPa.
G_{xyz}	Shear modulus in global coordinate directions	GPa.
n_{xyz}	Poisson's ratio in the global coordinate directions	__
e	Element	__
$[M]$	Global mass matrix	__
$[H]$	Global damping matrix	__
$[K]$	Global stiffness matrix	__
$\{F_{(t)}\}$	Global forces vector	__
$[N]$	Shape function for node i matrix	__
$[B]$	Displacement matrix (based on shape functions)	__
$[D]$	Laminate elasticity matrix (orthotropic case)	__
t	Time	sec.
$U_{x,y}$	Structural displacement in the x and y directions	m
$\{u\}, \{\acute{u}\}, \{\ddot{u}\}$	Displacement, Velocity, and Acceleration vectors	__
r_c	Composite density	kg/m^3
w_i	i^{th} Natural circular frequency	rad/sec.
$\{f_i\}$	Eigenvector representing the mode shapes of the i^{th} natural frequency	__
s_x	Stress in the x directions	N/m^2
t_{xy}	Shear stress	N/m^2

Introduction

To avoid structural damage caused by undesirable vibrations and resonance, it is important to determine the natural frequencies of the structure. The mode shapes must also be determined in order to reinforce the most flexible points or to determine the right positions where is necessary to reduce weight or to increase damping [1]. Sahu and Asha (2005) [2], used eight-noded isoparametric quadratic shell element to develop the finite element procedure, to investigate the vibration and stability behaviour of pretwisted panels, the effect of various geometrical parameters like angle of twist, aspect ratio, lamination parameters, shallowness ratio are studied. Tita (2003) [3] worked on the theoretical and experimental dynamic analysis of E-glass reinforced epoxy resin. He used $[0/90]_S$ and $[\pm 45]_S$ laminates in his study. The laminates were fabricated by hand-lay-up process and cut to beam shape specimens. The specimens were used as free-end beams in the vibration measurements. He calculated the mechanical properties of the composite analytically and used them in his simulations of the dynamic properties. He presented his ANSYS® simulation results in contour format showing the mode types and shapes. His experimental results on vibration are in a graphical format demonstrating the frequency response of the two laminates. Colakoglu (2006) [4] performed vibration experiments on 10-layer beam specimens of glass polyethylene composite at a range of temperatures. The vibration was induced by the impact of spherical steel ball hammer. ANSYS® numerical simulations were also used to obtain the frequency response. Teng and Hu (2001) [5] analyzed the design parameters for constrained layer damping structures by employing the Ross-Kerwin-Ungar (RKU) model. They also discussed the effects of temperature, frequency and the dimensions of damped structures on vibration damping characteristics.

The objective of this paper is to contribute to a better understanding of the dynamic behaviour of composite cantilever beams manufactured from E-glass Polyester by using finite element analysis method.

Composite Cantilever Beam

Four layers of E-glass Polyester composite cantilever beam, each layer thickness of 1mm, was used in the simulation of $[0_4]$, $[0/90]_s$ and $[\underline{+}45]_s$ composite laminates were considered in this investigation, E-glass fibres having density of 2.5 g/cm^3 and elasticity modulus of 72 GPa were used as the reinforcing material in polyester matrix [6]. The polyester, Syropole 8340 IS, is an unsaturated resin with catalyst addition, having a density of 1.22 g/cm^3 and rigidity modulus of 2.82 GPa was used as the matrix material. The engineering constants of the orthotropic unidirectional lamina at a volume fraction 22% were extracted from tensile test measurements of unidirectional laminates [7].

Figure (1) shows the element type SHELL99 used in the numerical analysis using the ANSYS® 10.0 commercial analytical tools [8]. Element SHELL99 geometry used to solve analytical problems of thin to moderately thick multilayer composite structures with up to 200 layers. The layers layout of the composite laminates in the simulations is shown in Figure (2). A cantilever beam, 40 cm long and 5 cm wide, was used. The beam is fixed at one end and free to vibrate at the other end as shown in Figure (3).

Vibration Analysis Using the Finite Element Method

Initially the beams were modeled in order to get a first estimation of the undamped natural frequencies and mode shapes. The beams were described using element type SHELL99 shown in Figure (1) which has 8 nodes and it is constituted by layers that are designated by numbers increasing from the bottom to the top of the laminate; the last number quantifies the existent total number of layers in the laminate [3]. To avoid structural damages caused by undesirable vibrations, it is important to determine [9]:

1. the natural frequencies of the structure to avoid resonance;
2. the mode shapes to reinforce the most flexible points or to determine the right positions to reduce weight or to increase damping.

They are also required for harmonic or transient analysis, which allows reviewing the mode shapes of a cyclically symmetric structure by modelling just a sector of it. The harmonic response analysis solves the time-dependent equations of motion for linear structures undergoing steady-state vibration [3]. The general dynamic problem equation of motion for a structural system is,[9]:

$$[M]\{\ddot{u}\}+[H]\{\dot{u}\}+[K]\{u\}=\{F_{(t)}\} \quad \text{.....(1)}$$

For a linear system, free vibrations will be harmonic of the form:

$$[M]\{\ddot{u}\}+[K]\{u\}=\{0\} \quad \text{.....(2)}$$

The elementary matrices above are calculated by using:

$$[M]_e = r_c \int_{vol} [N]^T [N]\, d_{vol} \quad \text{..(3)}$$

$$[K]_e = \int_{vol} [B]^T [D] [B] d_{vol} \quad ..(4)$$

$$[D]^{-1} = \begin{bmatrix} \frac{1}{E_x} & -\frac{n_{xy}}{E_x} & -\frac{n_{xy}}{E_y} & 0 & 0 & 0 \\ -\frac{n_{xy}}{E_x} & \frac{1}{E_y} & -\frac{n_{yz}}{E_x} & 0 & 0 & 0 \\ -\frac{n_{xy}}{E_y} & -\frac{n_{yz}}{E_x} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} \end{bmatrix} \quad ..(5)$$

For the determination of the natural frequencies of undamped system with ***n*** degrees of freedom, the solution is sought by solving:

$$[M]_{N\times N}\{\ddot{u}_c\}_{N\times 1} + [K]_{N\times N}\{u_c\}_{N\times 1} = 0 \quad .....(6)$$

where; $\{u_c\} = [N]^T \{u\}$ .. (7)

$$[N]^T = \begin{bmatrix} N_1 & 0 & 0 & & N_n & 0 & 0 \\ 0 & N_1 & 0 & \dots & 0 & N_n & 0 \\ 0 & 0 & N_1 & \dots & 0 & 0 & N_n \end{bmatrix} \quad .....(8)$$

For a linear system, free vibrations will be harmonic of the form:

$$\{u\} = \{j\}_i \cos w_i t \quad .....(9)$$

Thus, equation (6) becomes:

$$\left(-w_i^2 [M] + [K]\right)\{j\}_i = \{0\} \quad .....(10)$$

This equality is satisfied if either $\{\varphi\}_i = \{0\}$ or if the determinant of ;

$([K] - \omega^2 [M])$ is zero.

This is matrix eigenvalue problem which may be solved for up to ***n*** values of (ω^2) and ***n*** eigenvectors $\{\varphi\}_i$ which satisfy equation (10), rather than outputting the natural circular frequencies $\{\omega\}$.

Table (1) shows the orthotropic mechanical properties of laminates that were used in simulations, they were calculated using the classical lamination theory in MATLAB® program [7].

Modal Simulations

The frequency range in the simulation was 0 to 1000 Hz. The details of the mode types and equivalent frequencies for the first six modes are given in Table (2).

A typical mode shape of the vibration mode at frequency is equal 170.164 Hz as shown in Figure (4), a typical mode shape of the flexural mode is shown in Figure (5) and a typical mode shape of the torsional mode is shown in Figure (6).

A clear illustration of the modes in terms of both displacement and stress contour plots are given. The results of the modal simulations in Table (2) confirm the influence of the stacking sequence of the laminates. The angle ply laminate [$\pm$*45*]$_s$ generally has lower natural frequency, for the first and the consequent modes compared to the unidirectional and cross ply laminates. This is expected since the natural frequencies are related to the stiffness coefficients of the laminate. On in-plane vibration, [$\pm$45]$_s$ laminate has the lowest stiffness coefficients. Similarly, on flexure the [$\pm$*45*]$_s$ laminate has the lowest first flexure frequency since the bending flexure depends on the longitudinal modulus and on the alignment angle of the fibre. All fibres are oriented at angles [$\pm$45^o], in contrast with the [0/90]$_s$ laminate where 50% of the fibres are at [0^o] and with the [0^o]$_s$ laminate where all the fibres are orientated at [0^o].

On torsion the opposite occurs, the [$\pm$45]$_s$ laminate has the largest first torsional frequency. This is also expected since the flexural frequency is related to the shear modulus of the structure and the [$\pm$45]$_s$ laminate has the highest shear on torsion than the other laminates. This is an advantage in using [$\pm$45]$_s$ laminate where torsion is undesirable.

Harmonic Simulations

Any sustained cyclic load will produce a sustained cyclic response (a harmonic response) in a structural system. In order to predict the sustained dynamic behaviour of a structure, Harmonic response analysis can verify whether or not a designed structure will successfully overcome resonance, fatigue and other harmful effects of forced vibration.

The Harmonic response analysis was applied to a composite cantilever beam and its structural response to resonance was investigated. An applied pulse load of 1N at a frequency range (*0-1000 Hz*) was applied in the Z-direction and was used to examine the composite beams reaction to resonance for [0/90]$_s$ and [±45]$_s$ laminates. The frequency range was chosen so as to cover the full frequency range of the modal solutions in modal simulations above.

The pulse was localised at the centre node in the free end edge and the resonance status was monitored at the centre of the beam. The simulations results are extracted and presented in a graphical format for two types layers [$\pm$45^o] and [0^o/90^o] laminates. In figures (7), shaded areas show the effect of frequency responses that gave minimum values of displacement in x and y directions over the beam. The resonance conditions occur frequently at the type [0^o/90^o] laminate as shown figures (A) and (C).When compared between figures (7),we find the resonance occur in the type [$\pm 45^o$] laminate at frequency about 700 Hz. Figures (8) show the relationship between the frequency responses and the rotational displacement in x direction, also show the frequency responses caused rotational displacement in the type [$0^o/90^o$] laminate less than the type [$\pm 45^o$] laminate, due to the arrangement of the layers. Figures (9) and (10) show the relationship between the frequency responses with normal stress and shear stress respectively. From Figures (9) show the type [$\pm 45^o$] laminate is less subjected to high values of normal stress than the type [$0^o/90^o$] laminate, but the behaviour of Figures (10) is opposite Figures (9).

Conclusions

From the results of this work the following conclusions can be obtained:

1-Modal analysis indicated that [0^o]$_s$ and [$0/90$]$_s$ laminate had higher natural frequencies for vibration and flexural modes when compared with the [$\underline{+}45$]$_s$ laminate, this is due to the higher longitudinal elastic modulus of the laminates.

2- The [$\underline{+}45$]$_s$ laminate had higher torsional modal frequencies due to its higher shear modulus.

3-Harmonic analysis confirmed that large increases in normal displacement, rotational displacement, normal stresses and shear stress can occur at the resonance frequencies under dynamic loading conditions and these effects must be considered carefully in the design mechanical structures that use composite materials.

4-The results are in full agreement behaviour with those of Tita [3] who investigated the dynamic behaviours of glass epoxy laminates with similar stacking sequences.

5- The [$\underline{+}45$] laminate is more stable under loading than [0/90] laminate, due to the arrangement of the layers.

References

[1]. Jack R. Vinson," The Behavior of Structures Composed of Composite Materials", Second Edition, Kluwer Academic Publishers,2004.

[2] Sahu S. K. and Asha A.V.," Stability of Laminated Composite Pretwisted Cantilever Panels ", Journal of Reinforced Plastics and Composites, Vol.24 (12), PP. 1327-1334, 2005.

[3]. Tita, V. “Theoretical and Experimental Dynamic Analysis of Fibre Reinforced Composite Beams”, J. of Braz. Soc. of

Mech. Sci. and Eng. Vol. XXV, No.3, pp. 306-310, 2003.

[4]. Colakoglu, M. "Damping and vibration analysis of polyethylene fibre under varied temperatures", Turkish J. Eng. Env. Sci., 30, 351-357, 2006.

[5]. Teng, T.L. and Hu, N.K., "Analysis of damping characteristics for viscoelastic laminated beams", Comput. Methods Appl. Mech. Engrg., Vol.190, pp.3881-3892, 2001.

[6]. Akira K., Stephen W. and Julie W., "Material Characterization of Glass, Carbon, and Hybrid-Fiber SCRIMP Panels", MSc Thesis Stanford University, Department of Aeronautics and Astronautics, 2002.

[7]. Hayder M. Al-Shukri, "Experimental and Theoretical Investigation into Some Mechanical Properties of Glass Polyester Composite under Static and Dynamic Loads", MSc Thesis, Engineering Electromechanics Department, University of Technology, 2007.

[8]. ANSYS® Inc. Theory Reference, Release 10.0, Documentation Chapter 13, Composites, August 2005.

[9]. Shi, Y., Hua, H. and Sol, H., "The finite element analysis and experimental study of beams with active constrained layer damping treatments", *J. Sound Vib.*, Vol.278 (1-2), pp.343-363, 2004.

[10]. Sisemore, C.L. and Darvennes, C.M. (2002), "Transverse vibration of elastic - viscoelastic - elastic sandwich beams: Compression experimental and analytical study", *J. Sound Vib.*, Vol.252(1), pp.155-167, 2002.

Table (1) Mechanical Properties of Composite Laminates

Mechanical Properties	Laminate [0_4]	Laminate [±45]	Laminate [0/90]
E_x (GPa)	18.04	5.03	11.04
E_y (GPa)	3.74	5.03	11.04
E_z (GPa	3.74	6.75	6.75
G_{xy} (GPa)	1.57	4.93	1.57
G_{xz} (GPa)	1.57	1.53	1.53
G_{yz} (GPa)	1.49	1.53	1.53
n_{xy}	0.34	0.37	0.12
n_{xz}	0.34	0.15	0.32
n_{yz}	0.34	0.15	0.32

Table (2) Simulated Modal Frequencies

Natural Modal Frequency (*Hz*)				
Mode	**[*0/90*]$_s$**	**[*0_4*]**	**Mode**	**[*±45*]$_s$**
1st Vibration plane (*x-y*)	14.00	14.36	1st Vibration plane (*x-y*)	7.88
1st Flexural mode	89.52	93.40	1st Flexural mode	49.89
1st Torsional mode	111.61	114.35	2nd Vibration mode	96.81
2nd Vibration plane (*x-y*)	142.6	171.83	2rd Flexural mode	145.39
2nd Flexural mode	267.23	276.58	1st Torsional mode	164.78
2nd Torsional mode	355.03	382.38	3th Flexural mode	307.38

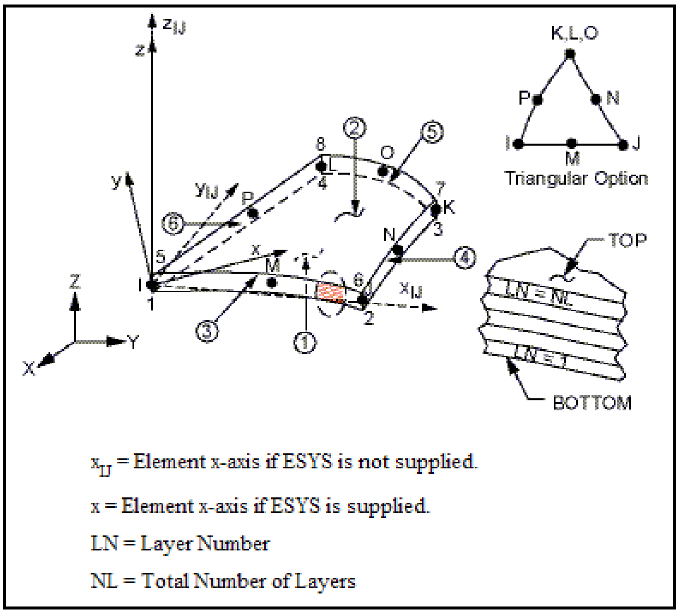


Figure (1) SHELL99 Layered Shell Element Geometry [3]

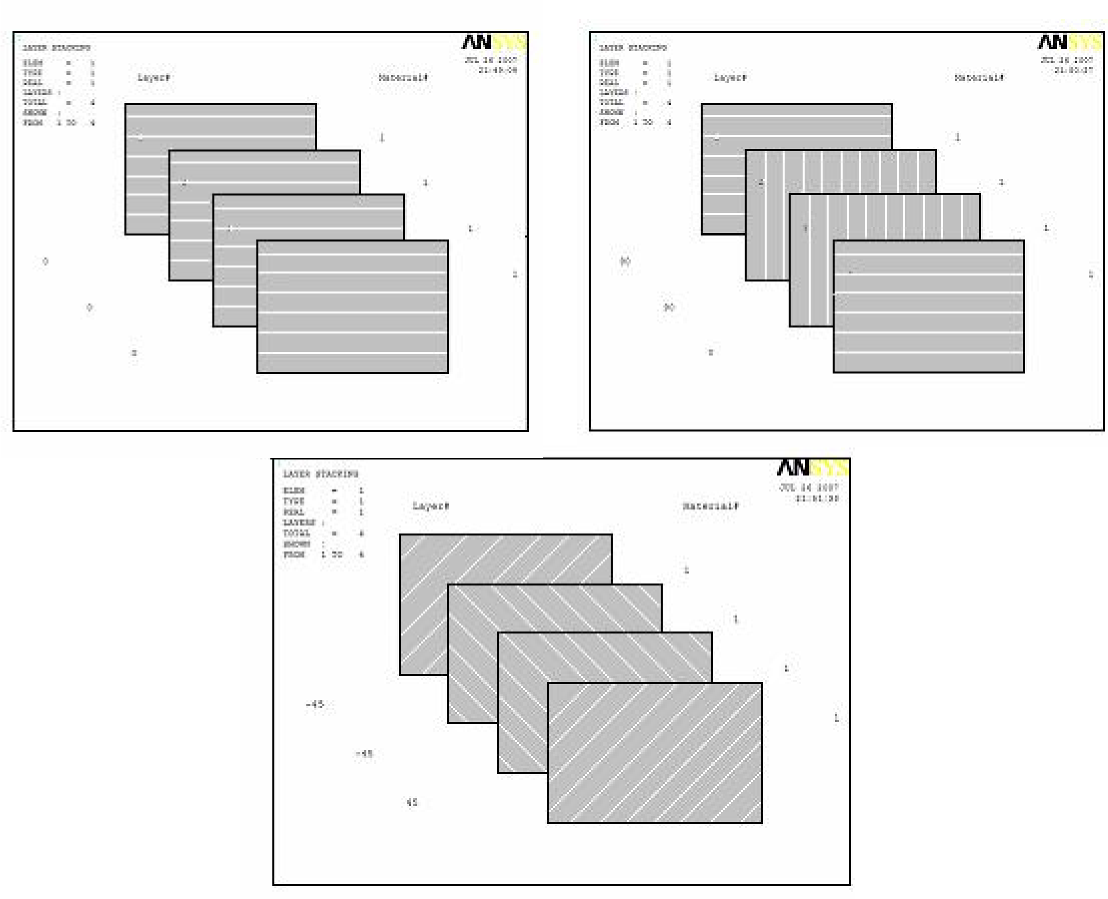


Figure (2) Layers Layout in ANSYS® Simulation

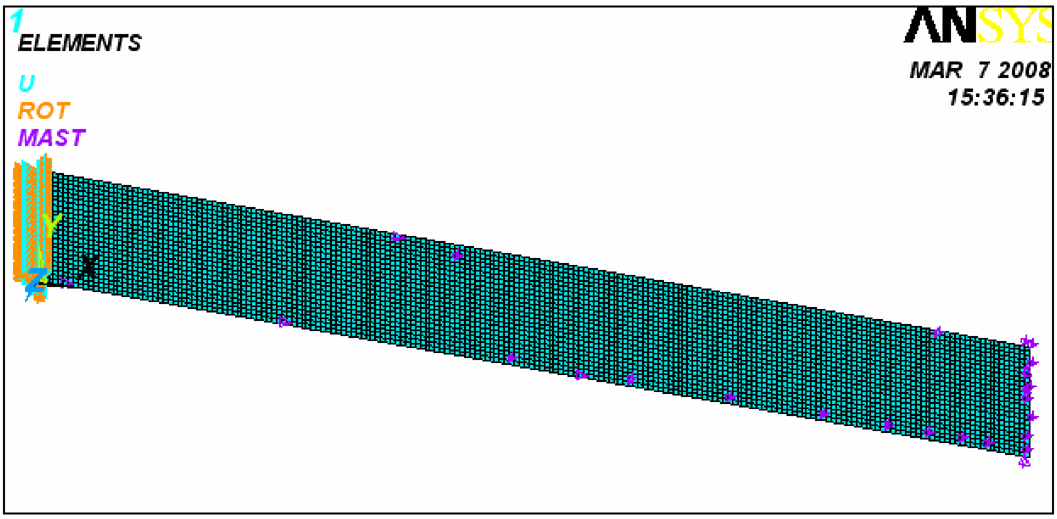


Figure (3) Four Layers of The E-Glass Polyester Cantilever Beam

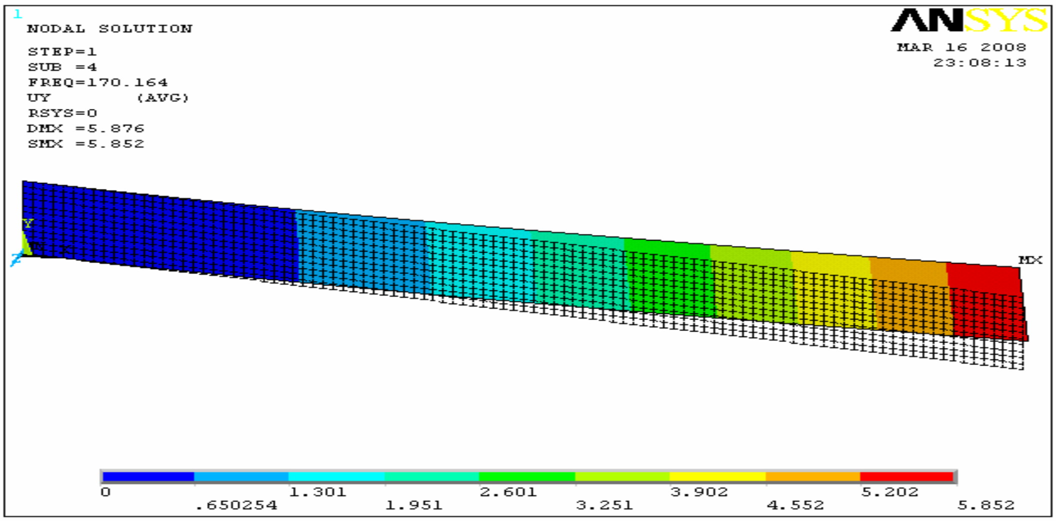


Figure (4) Vibration Mode Shape in 1-2 Plane of the Laminate

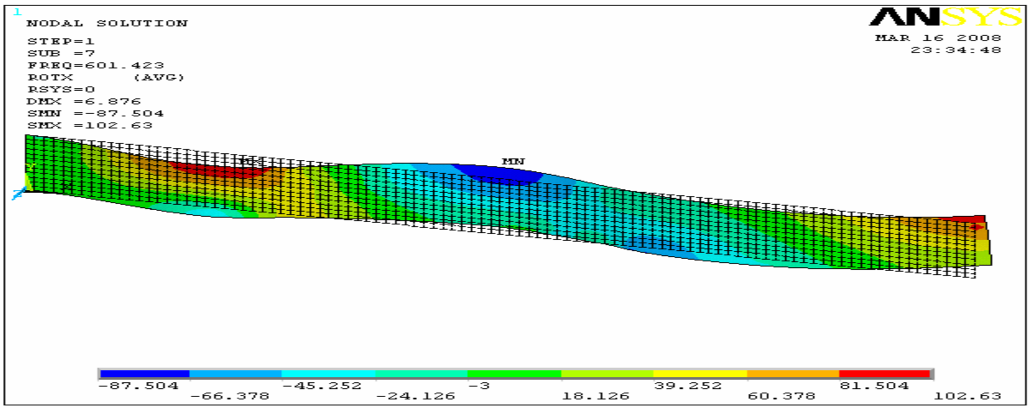


Figure (5) Flexural Mode Shape

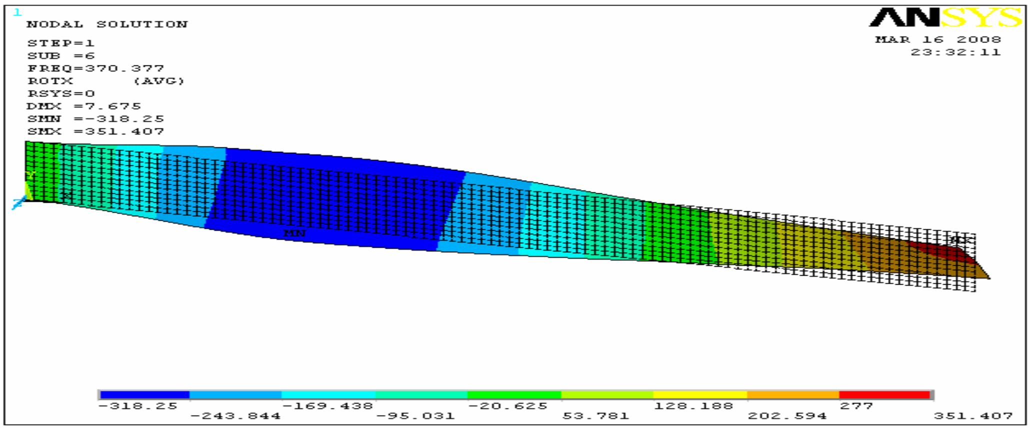
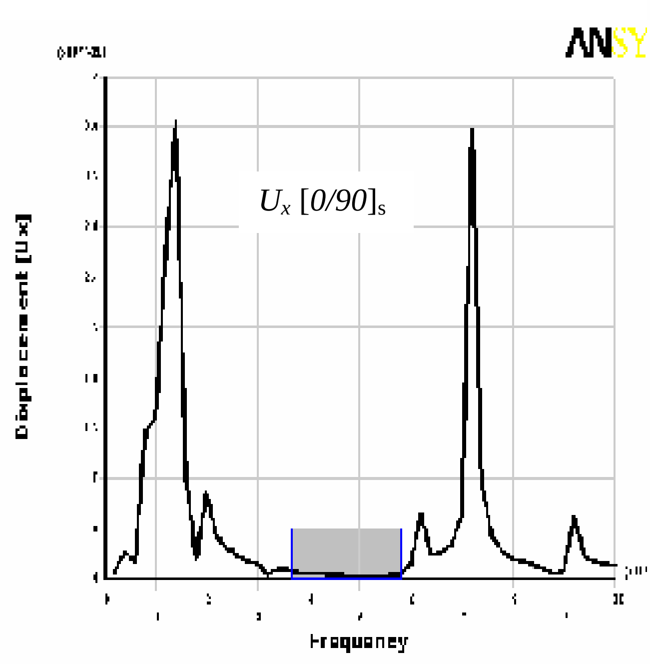
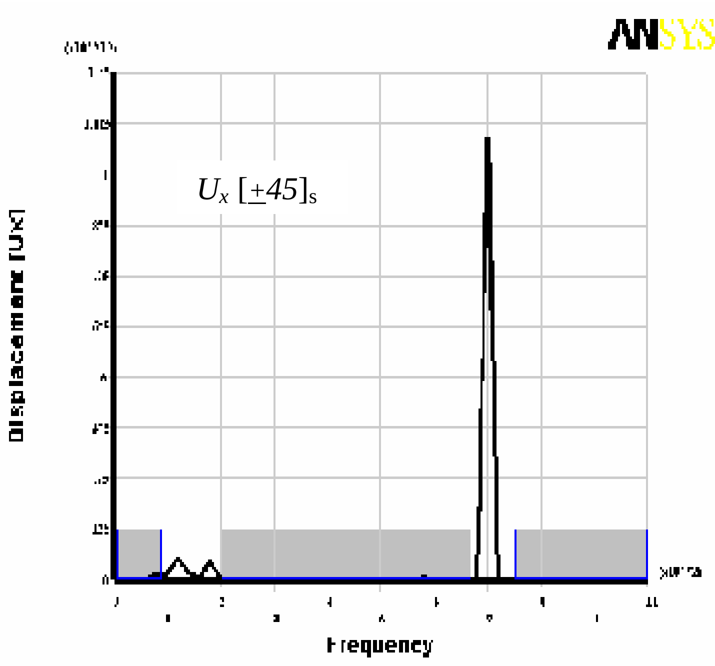


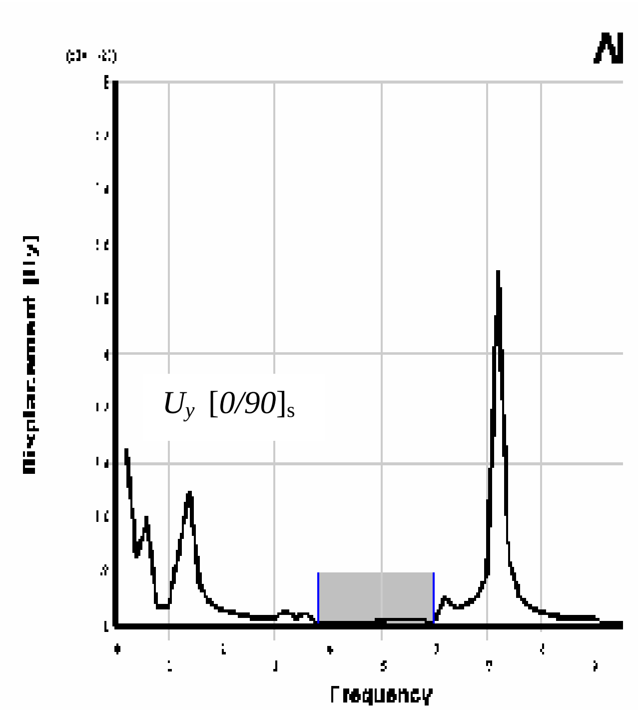
Figure (6) Torsional Mode Shape



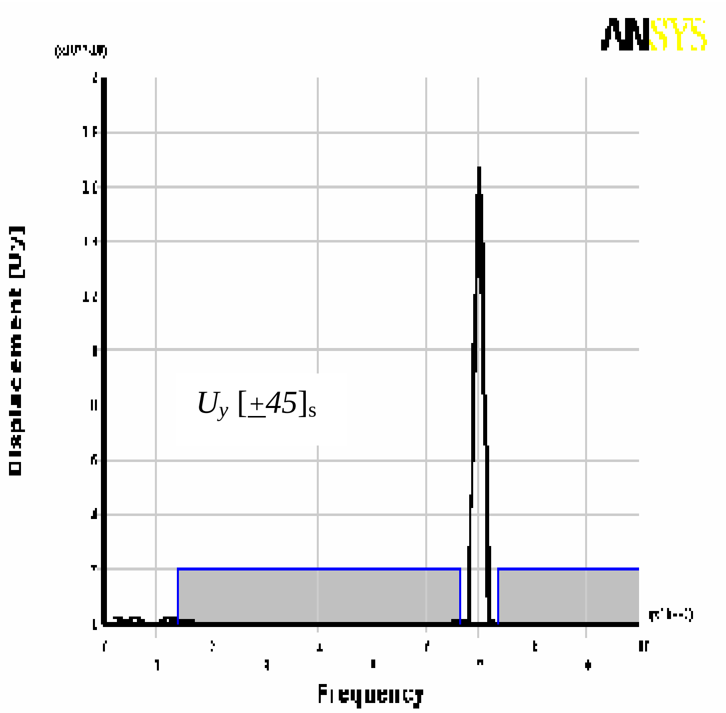
(A)



(B)

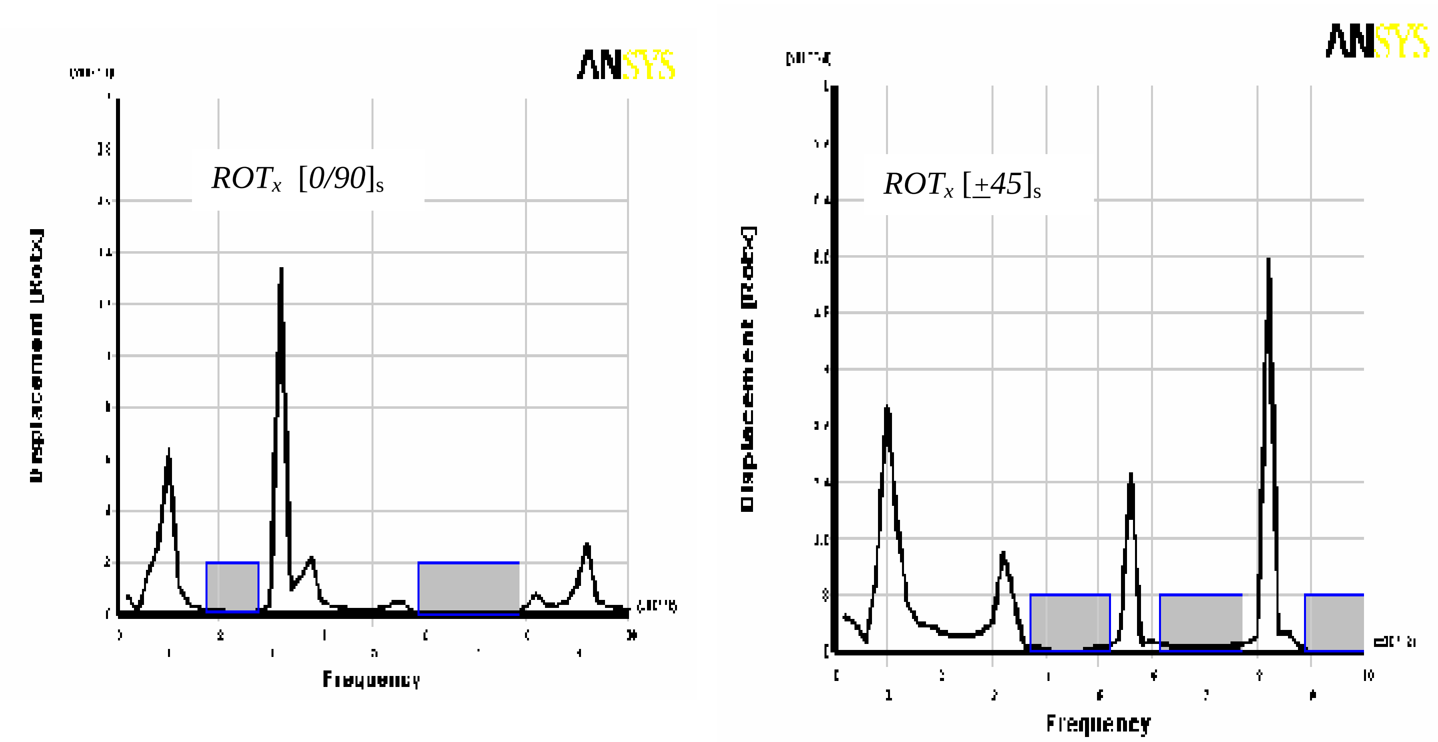


(C)

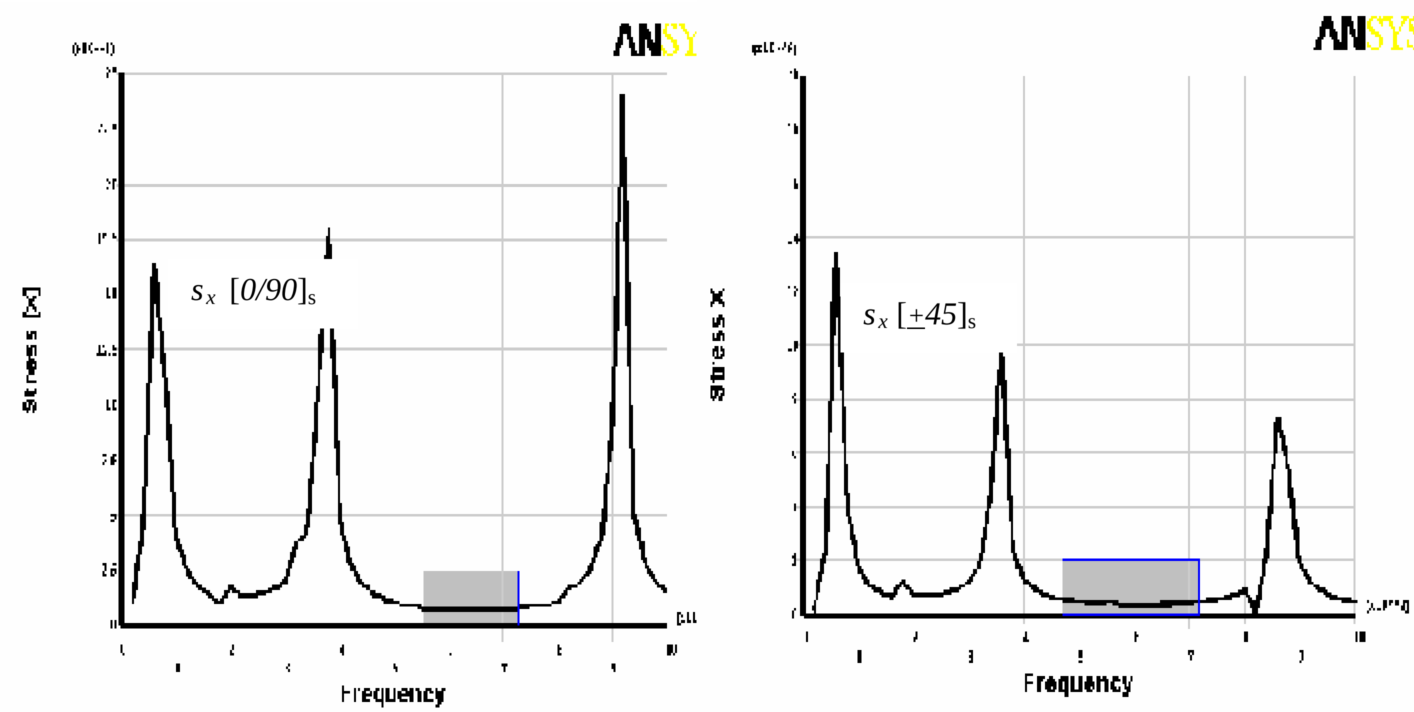


(D)

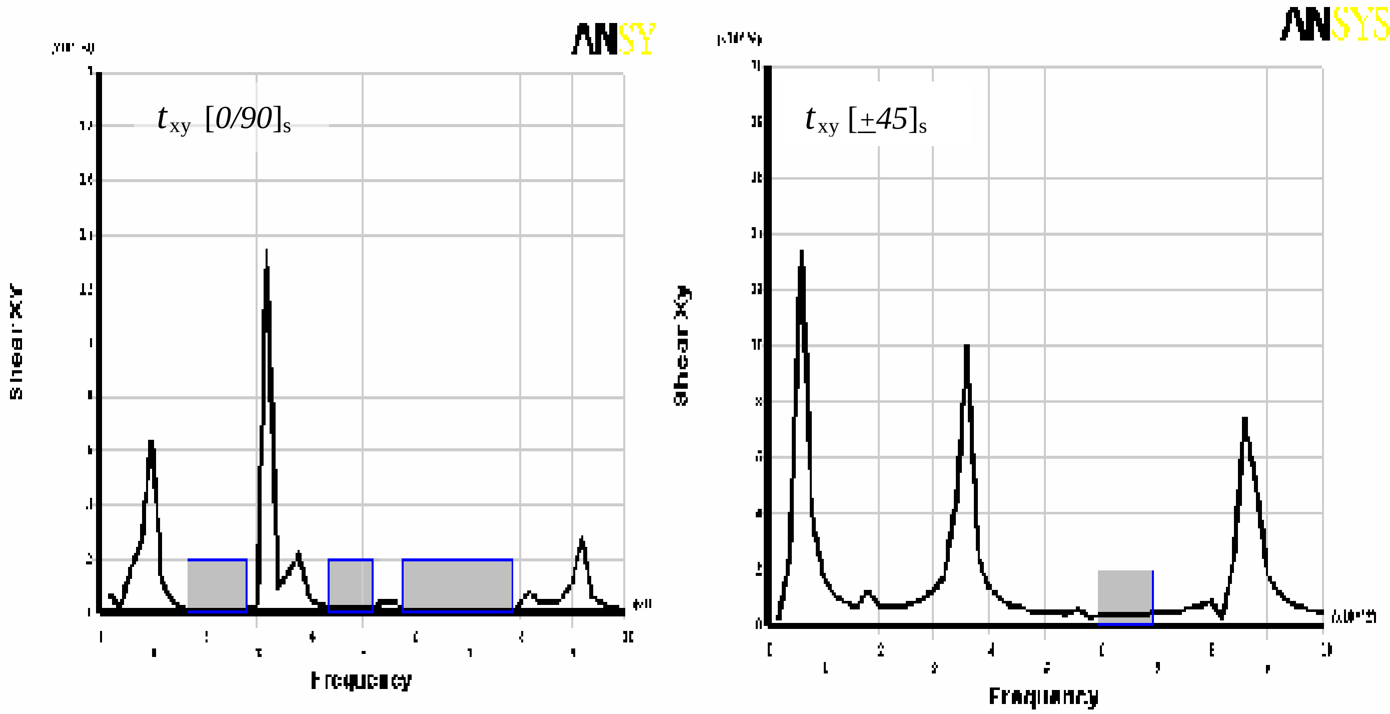
Figures (7) The Frequency Responses (Range 0-1000 Hz) Caused The Displacement (m) in The X and Y Directions



Figures (8) The Frequency Responses (Range 0-1000 Hz) Caused The Rotational Displacement (m) in The X Direction



Figures (9) The Frequency Responses (Range 0-1000 Hz) Caused The Normal Stress (N/m^2) in The X Direction



Figures (10) The Frequency Responses (Range 0-1000 Hz) Caused The Shear Stress (N/m^2) in The XY Direction