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Calculation of MTF For Optical Disk Modulator By Using Fractal Function

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Abstract

Optical modulator is an important component in optical systems. It is a device, which changes the angle between the vision line to the target and coordinate to electrical signal. The optical modulator modulates the optical signal by a frequency depending on the shape and number of sectors. The optical modulator takes various circular shapes due to the need for it. Through this study we have designed an optical modulator consisting of three concentric circles (C_0, C_1, C_2). Each circle is divided to transmittance and oblique sectors, the numbers of sectors chosen equal to (20,40,60) respectively, and increases progressively with increasing the number of circles. And thickness of each circle chosen equal to (Ro=1.5) cm, that means the radius of optical disk is (Rr=4.5)cm.

The central circle was designed using fractal geometry with a modified program to draw and enhance fractal figures including the fractal optical disk. The final shape of the proposed disk was designed using (Auto-CAD) software.

The efficiency of this optical modulator disk was tested by applying the modulation transfer function (MTF), where we found that the results converge, the maximum chopping frequencies in circles (C_0, C_1, C_2) are (2.5,5,10)KHZ at (t=0.004)sec, and minimum chopping frequencies are (0.05,0.1,0.2)KHZ at (t=0.2)sec, and the best modulation at spot light size equal to (2 mm^2), and that the proposed optical modulator disk could be used in optical systems.

حساب دالة الانتقال المعدلة لقرص التضمين البصري باستخدام دالة الهندسة الكسورية

الخلاصة

قرص التضمين البصري عنصر مهم في المنظومات البصرية،وهو عبارة عن آلة تقوم بتحويل الزاوية بين خط النظر إلى الهدف، والمحور البصري إلى إشارة كهربائية، ويأخذ عدة أشكال دائرية وذلك بحسب الحاجة إليه. يقوم قرص التضمين البصري بتضمين الإشارة الضوئية بتردد معين حسب شكل وعدد المقاطع. قمنا خلال بحثنا هذا بتصميم قرص تضمين بصري يتكون من ثلاثة دوائر متحدة المركز (C_0,C_1,C_2)، وكل دائرة من هذه الدوائر تم تقسيمها إلى أزواج من القطاعات الشفافة والمعتمة، تم أختيار عدد القطاعات مساوي الى (20,40,60) على التوالي، وعدد هذه القطاعات يزداد تصاعدياً مع زيادة عدد الدوائر. وتم أختيار سمك كل دائرة من الدوائر الثلاثة مساوي الى (Ro=1.5) cm ، هذا يعني أن نصف قطر قرص التضمين البصري هو (Rr=4.5)cm.

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تم تصميم الدائرة المركزية (C_0) باستخدام دالة الهندسة الكسورية (**Fractal**)، وذلك باستخدام برنامج خاص تم تعديله لرسم وتوضيح الأشكال الكسورية، ومن ضمنها المُقطع الكسوري ، والشكل النهائي لقرص التضمين المقترح تم تصميمه باستخدام برامجيات (**Auto-CAD**).

تمت دراسة كفاءة قرص التضمين البصري من خلال حساب دالة الانتقال المعدلة له ، فقد وجدنا من خلال البيانات والأشكال البيانية لهذه الدالة أن أعلى تردد قطع تم الحصول عليه للدوائر (C_0, C_1, C_2) هو (2.5,5,10)KHZ على التوالي عند الزمن (t=0.004)sec، وأقل تردد قطع تم الحصول عليه هو (0.05,0.1,0.2)KHZ عند الزمن (t=0.2)sec، وأن أفضل تضمين تم الحصول عليه هو عندما يكون حجم البقعة الضوئية مساوي الى ($2\ mm^2$)، وان قرص التضمين المقترح يمكن استخدامه في المنظومات البصرية.

Introduction

Optical modulator means any device used to modify any characteristic of an optical signal (light wave) for the purpose of conveying information [1]. Optical choppers are mechanical devices that physically block a light beam of some type. Rotating optical modulators (choppers) are perhaps the most common form and they are produced by SciTech Instrument Ltd. A metal disc with slots etched into its mounted on a dc motor and rotated. The disc is placed in the path of light beam which will then cause the beam to be periodically interrupted by the blocking part of the disc [2]. Mechanical optical choppers are useful where is not possible to control the light source directly or at the speeds required. For example a standard filament light bulb can be pulsed to a few 100 Hz though the depth of modulation is limited. If it is required to switch the light on an off completely at 20 kHz, then the use of mechanical optical chopper is required.

A crosshair is a shape superimposed on an image that is used for precise alignment of a device. Crosshairs are most commonly a "+" shape, though many variations exist, including dots, spots, circles and chevrons. Most commonly associated with telescope sights for aiming firearms, crosshairs are also common in optical instruments used for astronomy and surveying and also popular in graphical user interfaces as a precision pointer. The crosshair was invented by Robert Hook and date to the 17th century.

The word reticle is from the Latin meaning (net) a network or grid of lines displayed in an optical instrument. The minimum reticle consists of simple "cross-hairs" as described above [3], and also it is called chopper. The use of mechanical optical chopper is required .and the position of the modulator in the optical system as shown in Figure(1).

The modulator has different names which called as:

1-Optical modulator.
2-Chopper.
3-Reticle.
4-Risetar

The optical modulator divides in (q) couple of transparent and opaque spokes as shown in Figure (2). The rotating frequency of optical modulator (fr) is given by[4] :-

$$fr = \frac{wr}{2p} \qquad ....(1)$$

where :-
wr = represent the angular velocity.

The Chopping frequency of the transmitted rays is given by :-

$$fc = q.fr \qquad ....(2)$$

where :-
q = represent the number of pairs of clear opaque segment.

Fractal Function

Euclidean geometry provides a first approximation to the structure of physical object. It describes objects of simple shapes, points, line segments, ellipses, circles, boxes, and cubes that have a few characteristic sizes with dimensions of one, two, and three. This geometry is mainly oriented a round linear, integral system [5].

Non linear shapes and no integral systems are not easily described by traditional Euclidean geometry. These shapes and systems are needed another geometry that is quite different from Euclidean geometry to describe and study these cases. Benoit Mandebort [6] suggested the existence of geometries near to the geometry of nature, known as fractal geometry. Mandelbort (French) coined the term "Fractal" to describe objects that are very "fractured" as clouds, mountains, coaslines, leaves, sun,...........etc. Mandelbort say's: [I coined fractal from the Latin adjective fractus. The Latin verb (frangere) means "to break", to great irregular fragments. It is therefore sensible-and how appropriate for our needs!- that, in addition to "fragmented", fractious should also mean "irregular", both meanings being preserved in fragment].

Mandelbort's fractal geometry is a representational framework that is able to describe the fantastic structures of apparently infinite complexity [7]. Fractal geometry provides both a description and a mathematical model to study the irregular, and many of the seemingly complex shapes found in nature [8].The first steps of the development of a systematic fractal geometry including its graphic aspects were developed by Mandelbrot by using the computer power.

Mandelbrot's famous and pioneering work with fractal geometry and his introduction of two new basic concepts including; first, self-similarity, which is to say that the fractal shapes are to be self-similar and independent of scale or scaling. The general nature of the fractal irregular bumpy structure remains constant through successive magnifications such as is the case for coastlines and mountains. Each small portion when it is magnified can reproduce exactly a large portion. Fractal images exist as the limit of both random and deterministic processes based upon the representation called Iterated Function System (IFS). Second, a fractal has non-integer dimension known as the fractal dimension,

which allows scale independent measurement of such objects, and gives a numerical measure of the degree of boundary irregularity or surface roughness. The fractal dimension is one of the most important concepts in the study of fractals. It is analogous to the concepts of length, area and volume in Euclidean Geometry [9]. And from examples on nonlinear fractals: Mandelbrot set, and Julia set which they shown in Figure (3).

Fractal Modulator Design

Simple non-linear deterministic equations can self-generate irregular outputs. Non-linear deterministic system can be simulated when behavior is linear or nearly non-linear. When it increases, though smooth on short time scales, random and unpredictable behavior can be seen over longer periods.

Let (H(x),h(d)) be a metric space, and let $f : x > x.$ be a function[10].

Let $S \subset X$,then:-

$$f(s)=\{f(x):x\subset s\}.$$

The function f is one-to-one.

If $x, y \in x$, and $f(x)= f(y)$, so $x = y$,then the metric space can be given by the equation:-

$$A^{`} = TA \qquad \text{….(3)}$$

where:-

A is a point in initial area.

A' is a new point in under matrix operation (T)

The matrix (T) is given by:-

$$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \text{…..(4)}$$

The transformation(W) in Euclidean plane can be given by[11]:-

$$W(x, y) = (ax + by + e, cx + dy + f) \qquad \text{…(5)}$$

The points a, b, c, and d define rotation and scaling operations to be applied to the point and are called affine transformation. The e and f points define a translation to be applied to the point. The transformation (W) can be defined in this formula:-

$$W(x) = W\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \qquad \text{…(6)}$$

or

$$W(x) = Ax + T \qquad \text{….(7)}$$

where:-

***Ax* = the matrix** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

***T*= the horizontal vector** $\begin{bmatrix} e \\ f \end{bmatrix}$

By using this concept and (DUG.Nelson) program[12], we have sketched many shapes of fractal optical modulators, one of which Figure (4) is used to design the optical modulator of this paper.

Results and Discussion

At first we have designed optical modulator as shown in Figure (4). This optical modulator consists of three pattern circles (C_0,C_1,C_2). Each circle is divided to transparent and opaque block sectors (q), and the number of sectors increases progressive (nq),where (n) represents the number of circles (n=1,2,3). In this research (q=10) is used. Therefore the numbers of sectors of this three circles are (10,20,30) respectively. It is seen that the number of transmittance and oblique sectors increases with the number of circles (20,40,60) for (C_0,C_1,C_2) respectively.

If the central circle radius is (R_0), therefore the radius of other circles(dics) is (nR_0), (That means the thickness of each circle around the central circle is (R_0)). The central circle (C_0) is designed by using principle of fractal geometry. Special program (Appendix B) is used to sketch the shape of this circle (Fractal Modulator)[12].

Assume that the optical modulator is rotated about its axis, and the rotational velocity is (wr) in (Rev/sec), then the rotational frequency of optical modulator is ($fr = \frac{wr}{2p}$). If the spot light source is incident on the optical modulator, then the spot light makes chopping circle ,the frequency of the chopping light represents the chopping frequency (fc), depending on the number of sectors (q). The value of chopping frequency is equal to ($fc = qfr$) and the chopping frequency of each circle is given by the following relationship :-

$$fcn = n.q.fr \qquad .....(8)$$

where:-

n **is the number of circles.**

q **is the number of sectors.**

fr **is the rotational frequency of the modulator.**

Notice that the chopping frequency of the central circle (C_0) is different from other circles, because of the property of the fractal shapes, as explained in chapter two, It is seen that each part of the fractal modulator is similar to the total shape, that means the spot light is chopped ten times in each part. By using Q-basic program (named RAZAHM) , we calculate the rotational frequency (fr) in (KHZ) for different time(t) (0.004-0.2). From the equation (8), we calculate the chopping frequencies ($Fco, Fc1, Fc2$) of circles (C_0, C_1, C_2), in (KHZ), as shown in table (1).

First we sketched the relation between the change in time(t) and the rotational frequency of the modulator (fr),and the diagram is shown in Figure (6).Then we sketched the relation between the change in time(t) and the chopping frequency of circles (C_0, C_1, C_2), and the diagram is shown in Figures (7,8,9) , It is seen that the frequencies decrease with increased time (t).The relation between the chopping frequency and the number of sectors for each circle at time equal to (0.004,0.2) sec is sketched. The diagrams (10,11) show the chopping frequency (fco) at time (0.004,0.2) respectively. It is seen that the frequency chopped is ten times in each part of fractal shape. The chopping frequencies (fc1,fc2) of circles (C_1, C_2) are shown in diagrams (12,13).

Evaluation of MTF

The MTF is one of the most useful means for characterizing the optical performance of an optical system. For the supposed optical modulator, the modulation transfer function MTF is calculated for each circle (C_0, C_1, C_2) by calculating the transmittance intensity and by using the relationship:-

$$MTF = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \qquad .....(9)$$

where:-

***I_{max}* is transmittance maximum intensity.**

***I_{min}* is oblique minimum intensity.**

First the area of each circle (Sn) is calculated, it is assumed that the optical modulator has diameter equal to (9cm), and radius (Rr) equal to (4.5cm), therefore the central circle (C_0) has radius (R_0) equal to (1.5cm) and the other two circles (C_1, C_2) have thickness equal to (R_0). The area of each circle is given by :-

$$So = (Ro)^2.p = \mathbf{707 mm}^2$$

for C_0

$$S1 = (2Ro)^2.p - 706 = \mathbf{211.9 \ \ mm}^2$$

for C_1

$$S2 = (3Ro)^2.p - 2826 = \mathbf{353.2 \ \ mm}^2$$

for C_2

The circumference of each circle (C_0, C_1, C_2) is (9.42,18.87,28.26)cm respectively. All of these relations are entered to the program **(RAZAHM).**

In this research the transmittance intensity is measured by the movement of spot light size. The measurement operation begin from the center of modulator (Ro=0), and the spot light rotates about the center axis (360^0) with constant radius (1.5 mm). That means the spot light makes ten circles (rings) in each circle (C_0, C_1, C_2). Measurement of transmittance intensity depends on the ratio between the spot size (three different spot sizes (1,1.5,2) mm2 are used) and the area of the sub circle (Scn) of each three circle. By using the program in (RAZAHM), we calculate Imax (the ratio between the spot size and the transmittance area), and Imin (the ratio between the spot size and oblique area). Then we measure the modulation transfer function MTF by using equation (9), (this equation enter to the program). The MTF was calculated for all three circle (C_0,C_1,C_2).

MTF of Central Circle (C_0) (Fractal Modulator)

The fractal modulator (C_0) was designed by using (DUG .Nelson) program . From this program, the iterated function systems (IFS) is found and is given in Table (2) :-

To measure the modulation transfer function (MTF), the (fractal element) must be chosen. Here the fractal element is selected as triangle shape, because the (IFS) codes are transformation of triangle.

Assume the fractal element is equilateral triangle, its dimension is (0.9 mm), and its altitude is (0.779 mm), therefore its area is (3.5 mm^2). The number of triangles in (C_0) is (20),these triangles are divided to (10) transmittance and (10) oblique. The modulation transfer function (MTF) is measured with respect to spot size movement. The spot size rotates about the modulation center axis, and in constant radius (1.5 mm). The spot size movement begins from (0-15)mm. That means there are ten sub circles (rings) in circle (C_0).

By using program (RAZAHM), the area of sub circles (Sn), and area of transmittance and oblique sectors (Scn) are calculated. The maximum intensity (Imax) represents the ratio between the spot size and transmittance area, and minimum intensity (Imin) represents the ratio between the spot size and oblique area. Then by using equation (9), MTF is measured. We chosen three values of spot size (1,1.5,2) mm2, and given in Tables (3,4,5). And the relationship of the mean MTF and spot size of all above cases are given in Figure (17).

Conclusions

The present work has reached the following conclusions

1-The MTF of the supposed optical modulator will be increasing with increasing the spot size and decreasing with increasing Ro.

2-The maximum chopping frequencies in circles (C_0, C_1, C_2) are (2.5,5,10)KHZ at (t=0.004)sec, and minimum chopping frequencies are (0.05,0.1,0.2)KHZ at (t=0.2)sec, and the best modulation at spot light size equal to (2 mm^2).

3-The type of supposed optical modulator can be defined by using the suitable spot size.

4-Circle two can be used as detection modulator by using large size of spot size, and circle one can be used as acquisition modulator by using smaller size than spot size, and the central circle (Fractal) can be used as tracking modulator because it is more accurate.

5-The fractal modulator can be used with another normal optical modulator in optical systems.

6-The fractal function can be used to design the optical modulator, especially for fine optical measurement.

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Table (1) The rotation frequency and chopping frequencies of optical modulator.

t (sec)	**fr(KHZ)**	**Fc0(KHZ)** n=1 q=10	**Fc1(KHZ)** n=2 q=10	**Fc2(KHZ)** n=3 q=10
0.004	0.25	2.5	5	10
0.006	0.1666	1.66	3.333	6.666
0.008	0.125	1.25	2.5	5
0.01	0.1	1	2	4
0.02	0.05	0.5	1	2
0.04	0.025	0.25	0.5	1
0.06	0.0166	0.166	0.333	0.666
0.08	0.0125	0.125	0.25	0.5
0.1	0.01	0.1	0.2	0.4
0.2	0.005	0.05	0.1	0.2

Table (2) The (IFS) codes of fractal modulator

W	A	B	C	d	E	F
-0.901	- 0.828	0.891	-0.781	277.71	127.05	0.125
-0.891	- 0.781	0.901	-0.828	339.32	351.29	0.125
0.891	- 0.781	0.901	0.828	430.15	235.77	0.125
0.901	- 0.828	0.891	0.781	208.17	245.23	0.125
0.901	- 0.828	0.891	0.781	344.98	129.31	0.125
0.891	0.781	-0.901	0.828	231.73	169.63	0.125
0.891	0.781	-0.901	0.828	208.17	242.56	0.125
-0.901	0.828	-0.891	-0.781	430.15	231.84	0.125
-0.901	0.828	-0.891	-0.781	232.86	308.72	0.125
-0.891	0.781	-0.901	-0.828	405.46	324.56	0.125

Table(3) MTF of central circle C_0 at spot size ($1mm^2$)

& The mean MTF is 0.1887.

$Ro(mm)$	**Sn(mm^2)**	**Scn(mm^2)**	**MTF**
1.5	7.060	0.353	0.725
3	21.21	1.059	0.40
4.5	35.34	1.766	0.224
6	49.48	2.472	0.163
7.5	63.61	3.179	0.137
9	77.75	3.885	0.09
10.5	91.89	4.592	0.066
12	106.02	5.298	0.046
13.5	120.16	6.005	0.028
15	134.30	6.711	).0089

The MTF relationship is given in Figure (14).

Table (4) MTF of central circle C_0 at spot size ($1.5mm^2$)

The mean MTF is 0.234

$Ro(mm)$	**Sn(mm^2)**	**Scn(mm^2)**	**MTF**
1.5	7.060	0.353	0.815
3	21.21	1.059	0.461
4.5	35.34	1.766	0.313
6	49.48	2.472	0.241
7.5	63.61	3.179	0.176
9	77.75	3.885	0.137
10.5	91.89	4.592	0.098
12	106.02	5.298	0.06
13.5	120.16	6.005	0.03
15	134.30	6.711	0.009

The MTF relationship is given in Figure (15).

Table (5) MTF of central circle C_0 at spot size (2mm ^{2})

and The mean MTF is 0.3021

Ro(mm)		^{2})	
1.5	7.060	0.353	0.942
3	21.21	1.059	0.652
4.5	35.34	1.766	0.44
6	49.48	2.472	0.32
7.5	63.61	3.179	0.232
9	77.75	3.885	0.169
10.5	91.89	4.592	0.125
12	106.02	5.298	0.081
13.5	120.16	6.005	0.057
15	134.30	6.711	0.032

The MTF relationship is given in Figure (16)

RETICLE SYSTEM SHAPE

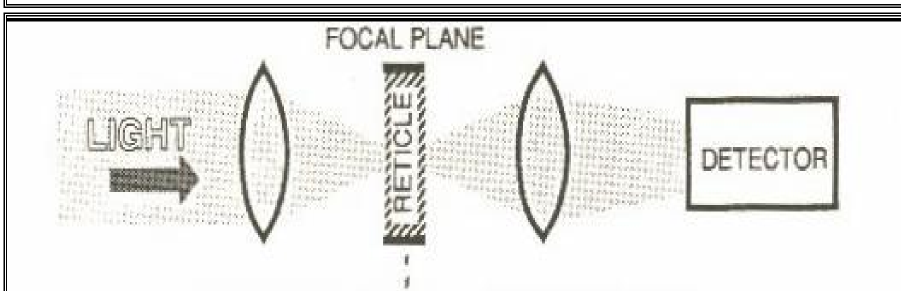


Figure (1) The Position of the modulator in the optical system

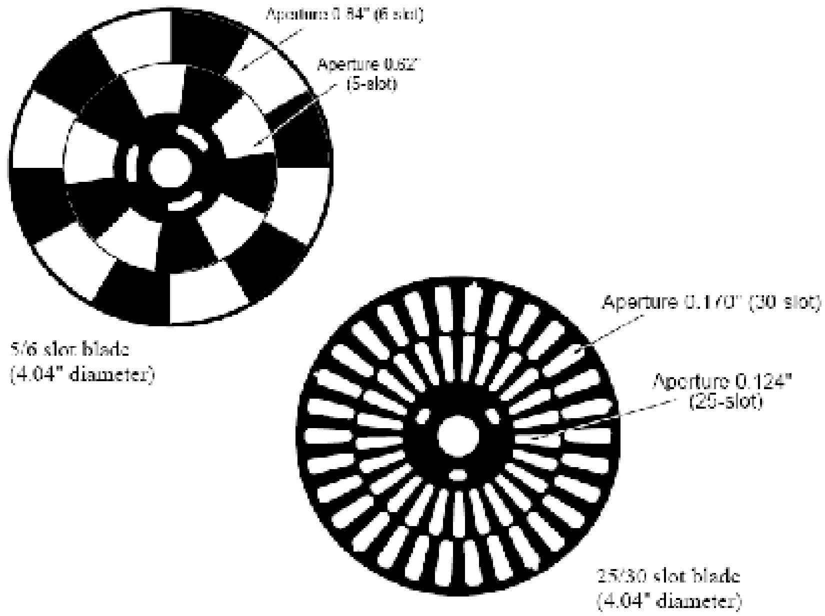
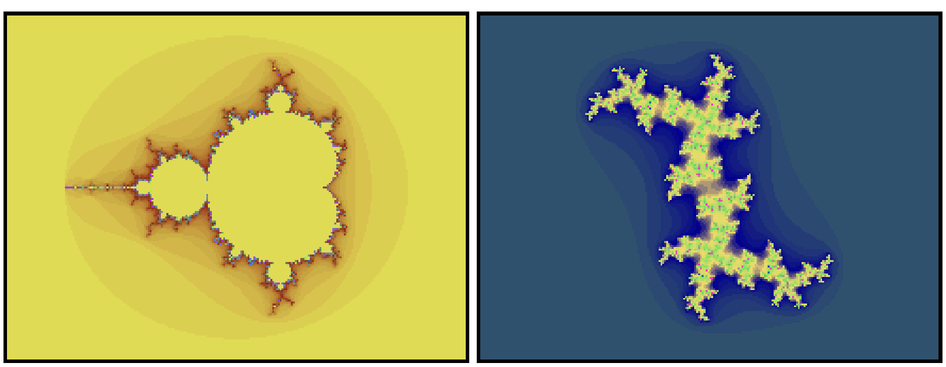


Figure (2) Two types of optical modulators.



(a)- Mandelbrot set **(b)- Julia set**

Figure (3) Samples of non liner Fractal

a- 10-slots Fractal Modulator. **b- 20-slot Fractal Modulator.**

Figure (4) The design of optical modulator

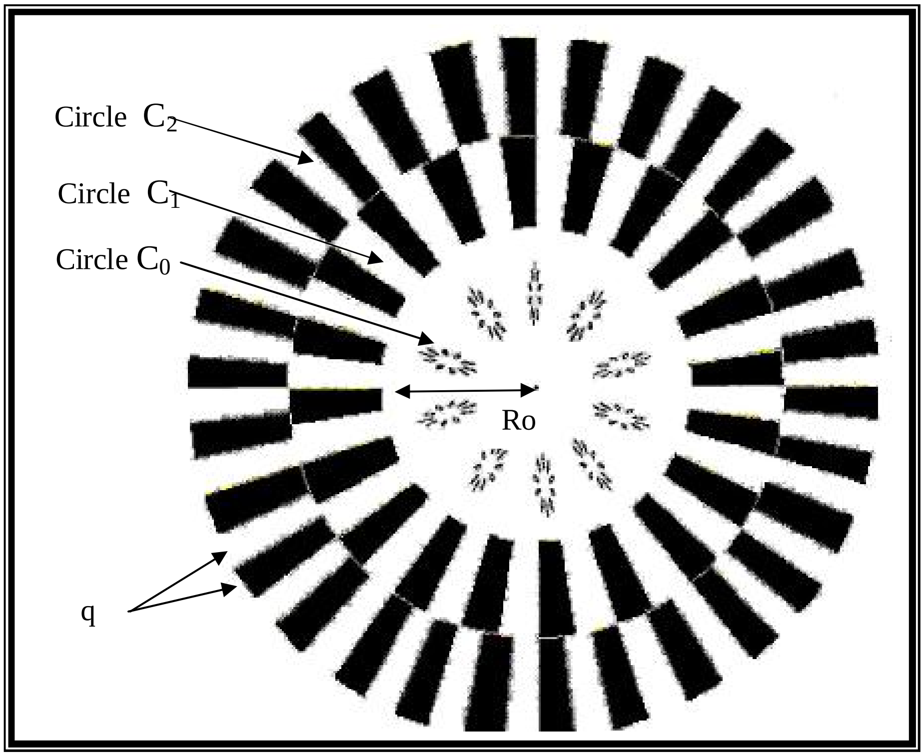


Figure (5) The supposed optical modulator

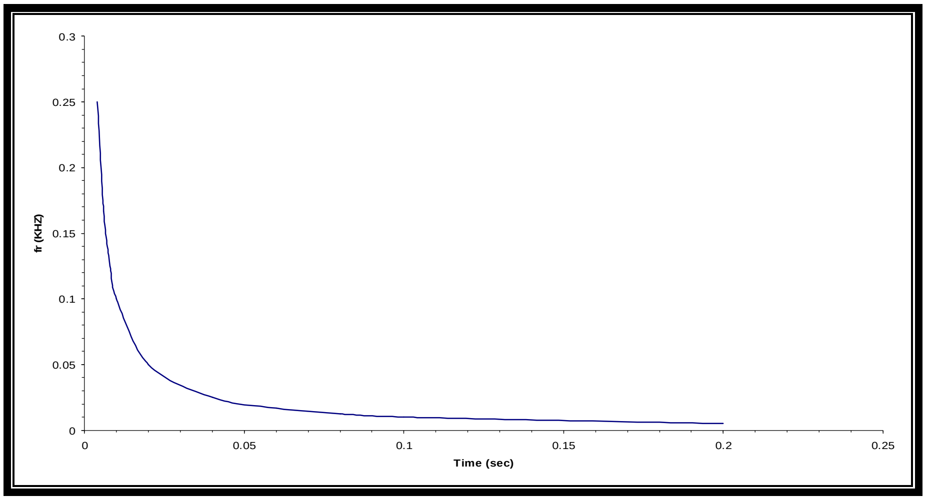


Figure (6) The relation between Rotation frequency (fr) and the change in time

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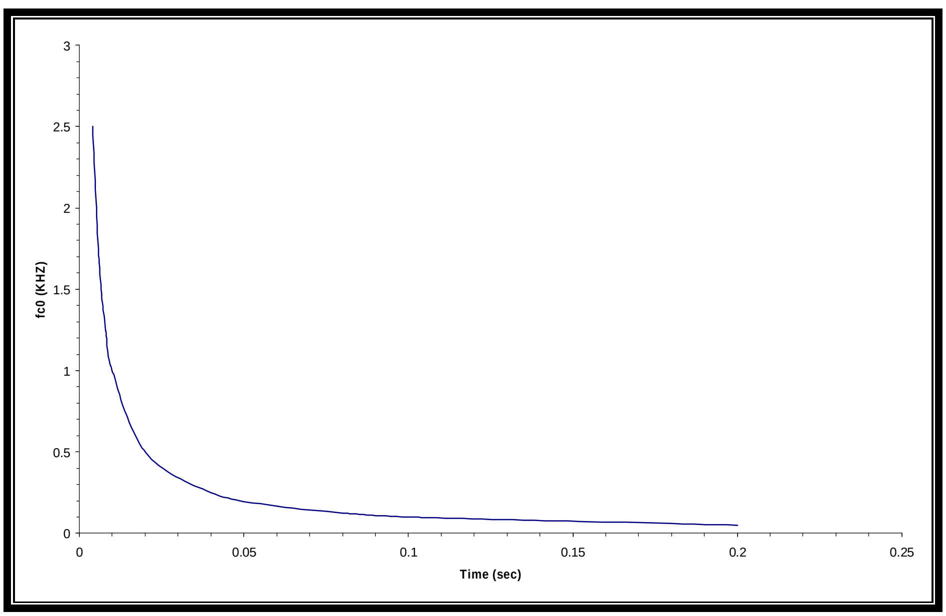

Figure (7) Chopping frequency(fco) of central circle(Co) with different times.

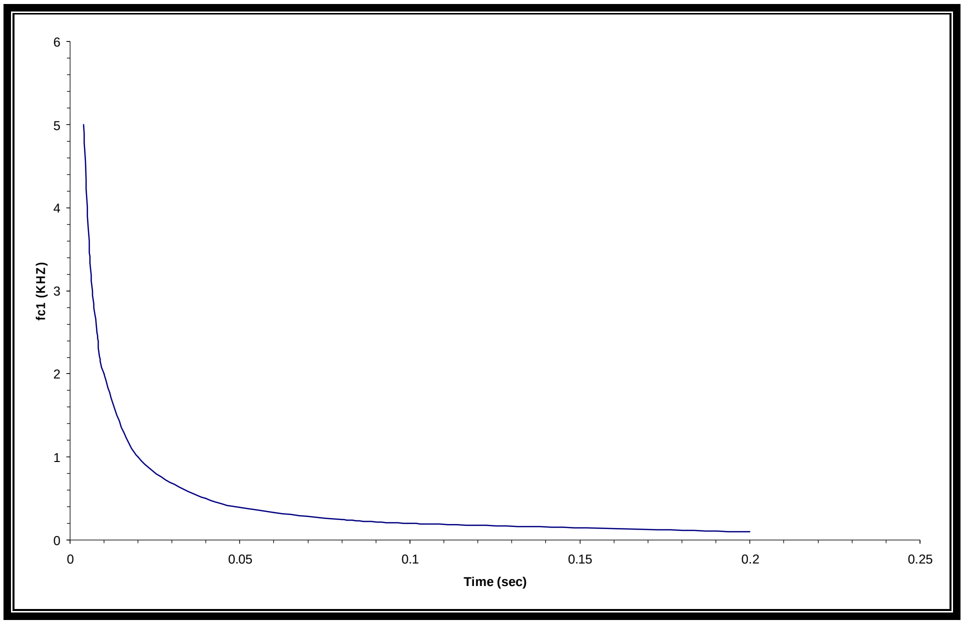

Figure (8) Chopping frequency(fc1) of circle one(C_1) with different times.

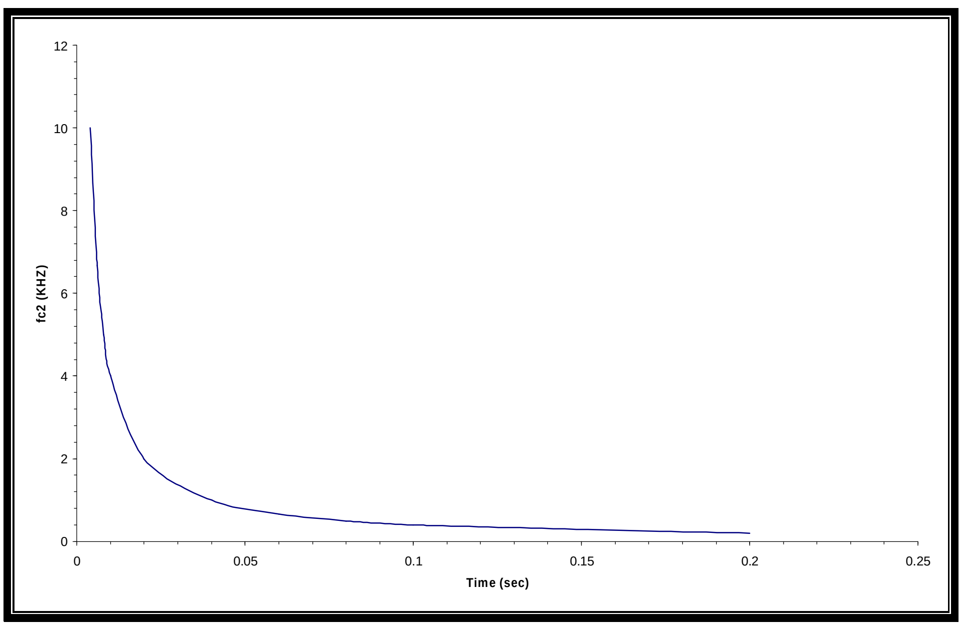

Figure (9) Chopping frequency(fc2) of circle two (C_2) with different times.

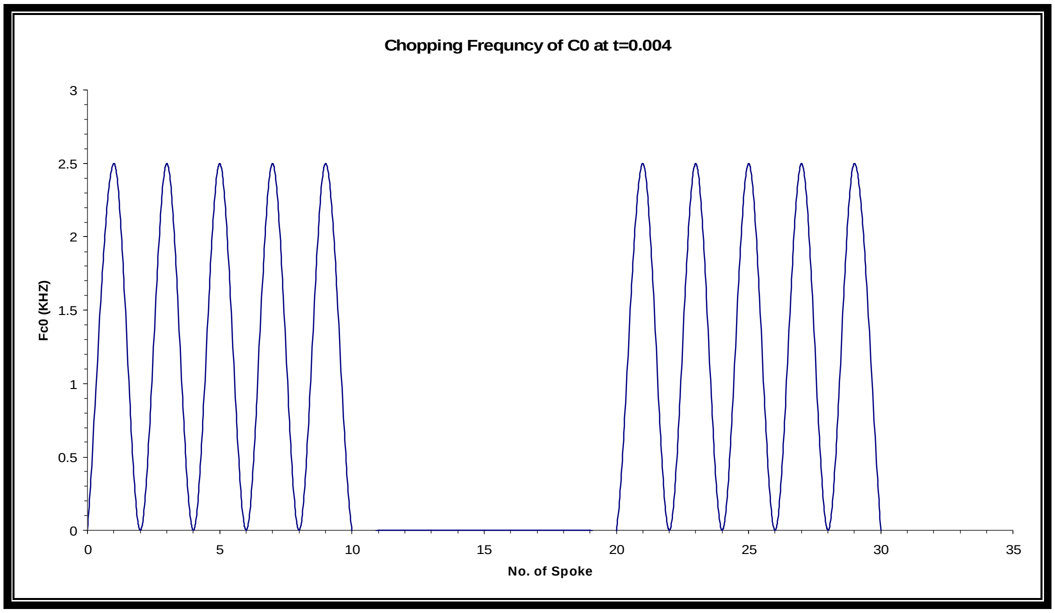

Figure (10) Chopping frequency of C_0 at t=0.004 sec.

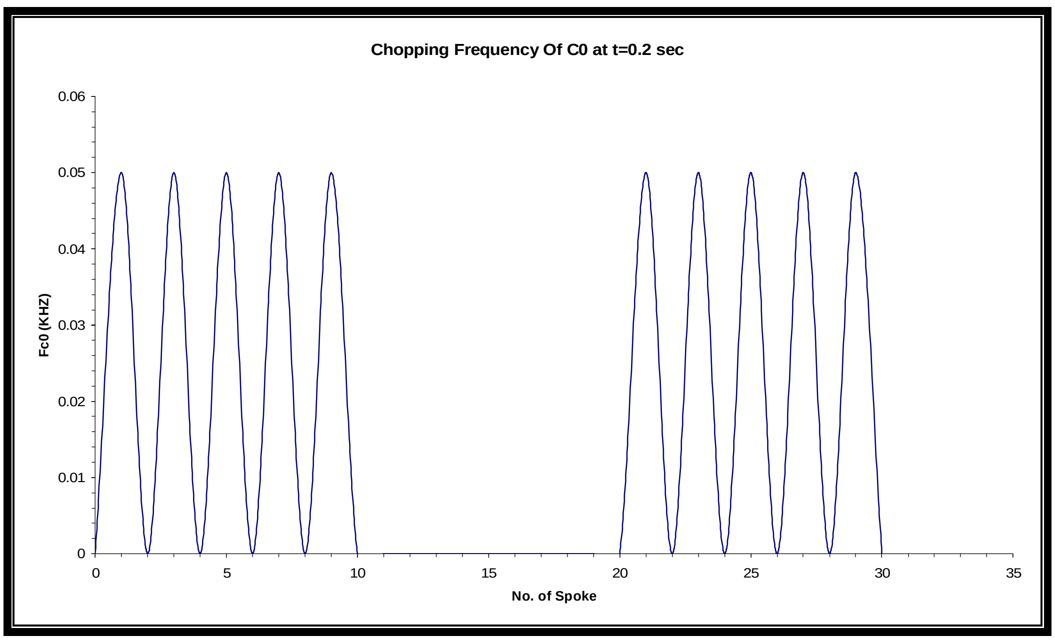

Figure (11) Chopping frequency of C0 at t=0.2 sec.

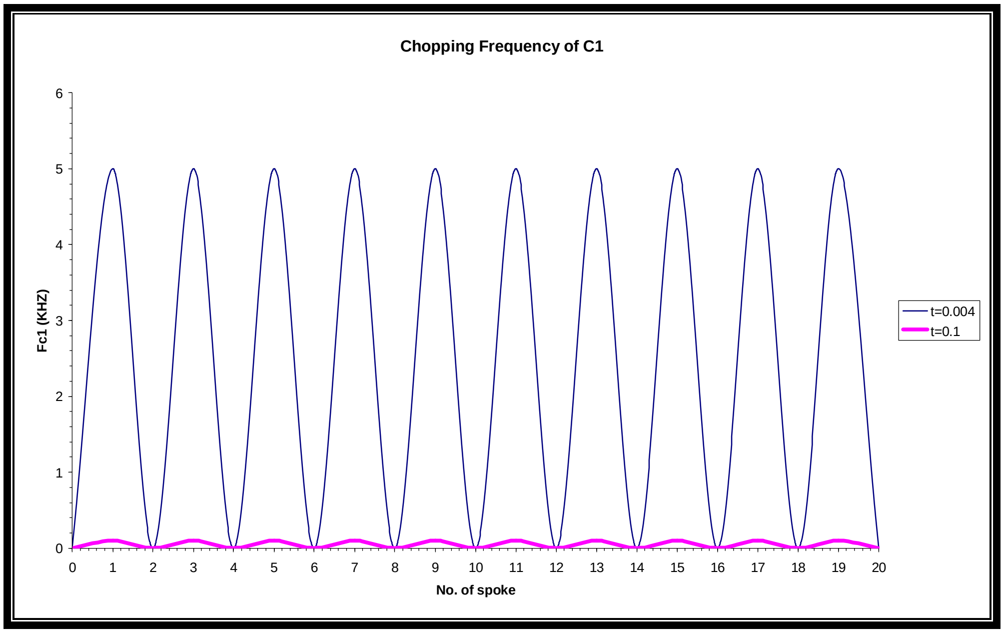

Figure (12) Chopping frequency of C1 at t=(0.2,0.004) sec.

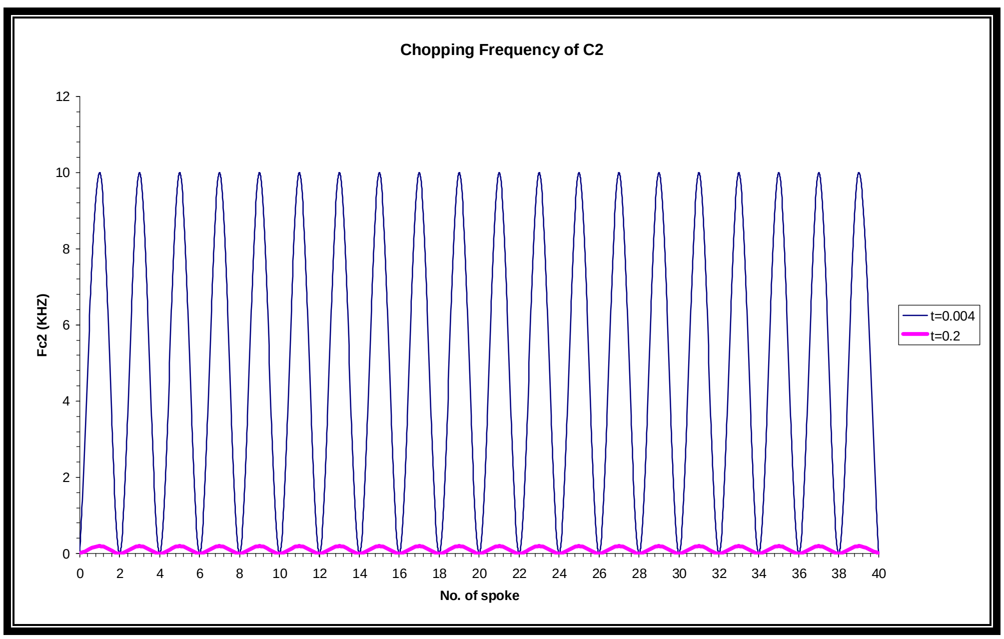

Figure (13) Chopping frequency of C2 at t=(0.2,0.004) sec.

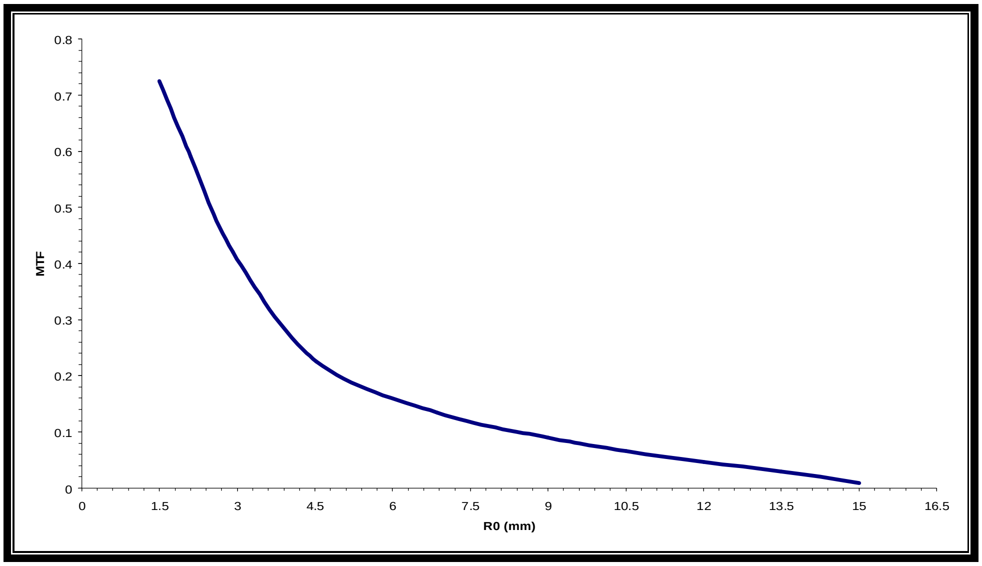

Figure (14) MTF of fractal optical modulator in central circle (C0) at spot size(1mm^2)

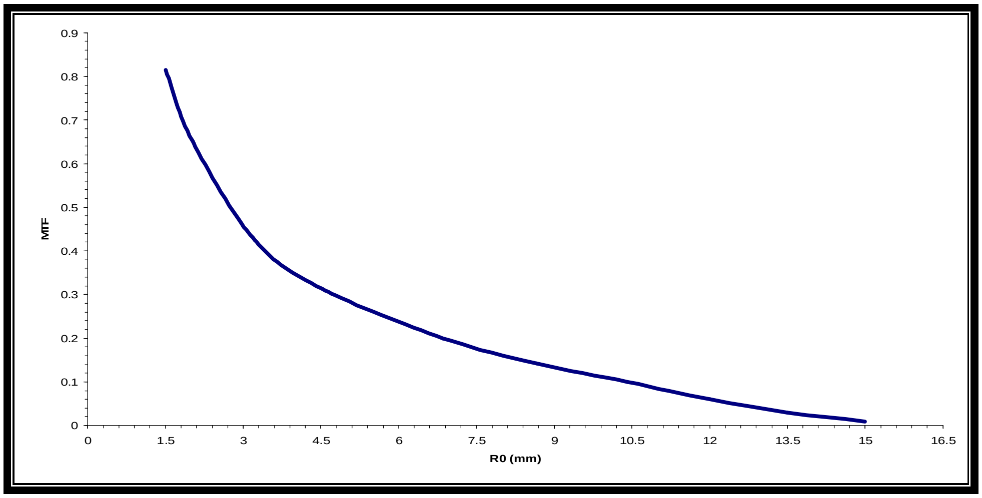

Figure (15) MTF of optical modulator in central circle (C0) at spot size(1.5mm^2)

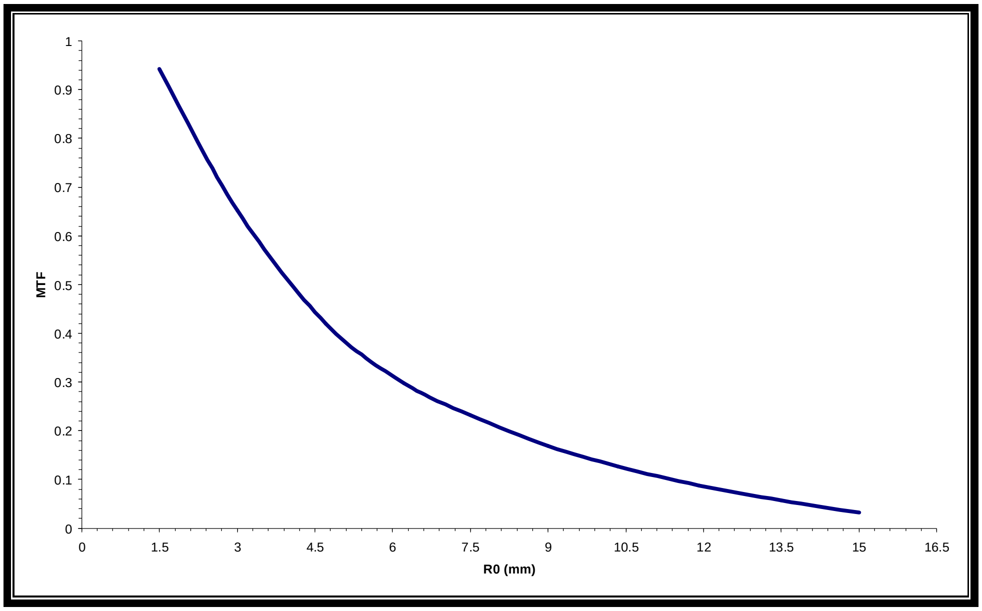

Figure (16) MTF of optical modulator in central circle (C0) at spot size(2mm^2)

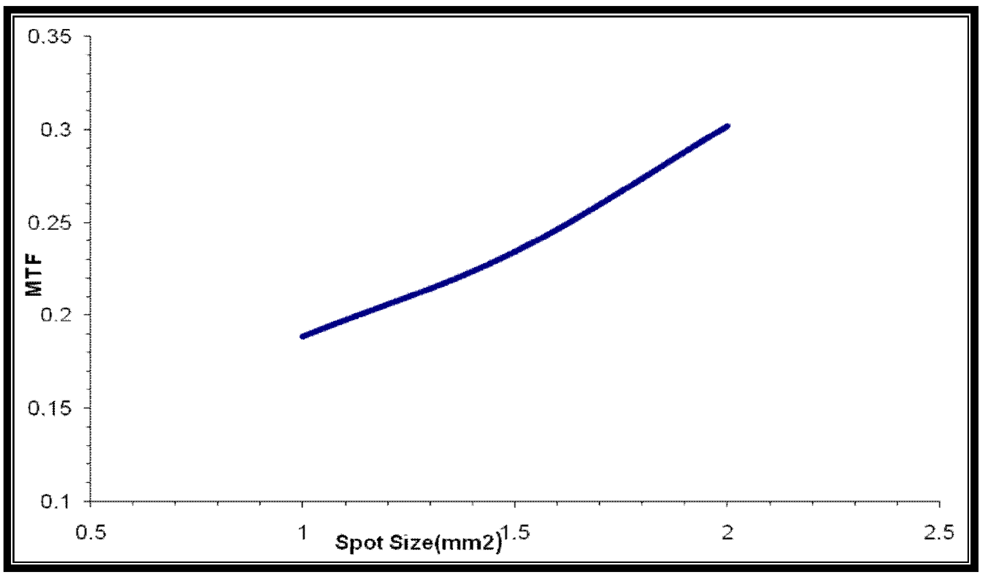

Figure (17) The relationship between the mean MTF and spot size.