Modeling of Stress-Strain Relationship for Fibrous Concrete Under Cyclic Loads

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Abstract

A mathematical model to predict the stress–strain behavior of fibrous concrete composites under random cyclic compressive loadings is developed. The envelope unloading strain is used as an index of load history, while the plastic strain and reloading strain are predicted as functions of the envelope unloading strain for both full and partial unloading and reloading. The model is independent of the expression used for the envelope curve. Comparison with cyclic data shows good agreement. The model can be used for completely random loadings, in both the pre–peak and post–peak ranges. It is suitable for both plain and fibrous concrete composites. The model has been built using MATLAB language computer program facilitating the advanced mathematical difficulties of solving and differentiating complex expressions.

In this paper, the monotonic stress–strain curves of Al-Sulayfani model **[1]** for fibrous concrete had been adopted.

Keywords: Cyclic Loading, Fibrous Concrete, Stress-Strain relationship.

نمذجة منحنى الإجهاد–الانفعال للخرسانة الليفية تحت الحمل الدوري

الخلاصة

تم تطوير نموذج رياضي يتكهن بسلوك منحني الإجهاد–الانفعال للخرسانة الليفية المسلحة تحت أحمال الانضغاط الدورية المسلطة عشوائياً. وقد اعتمد الانفعال الذي يرفع عنده الحمل Unloading Strain الواقع على منحني الغلاف Envelope Curve مرجعا يعتمد عليه في حساب كل من انفعال اللدونة Plastic Strain وانفعال إعادة الحمل Reloading Strain سواء في ذلك وضع الحمل أو رفعه كاملاً أو جزئياً. إن النموذج الذي تم اشتقاقه غير مرتبط بنموذج منحني الغلاف للخرسانة، ويمكن الاعتماد عليه مهما كانت طريقة وضع ورفع الأحمال، وهو ملائم للاعتماد في الخرسانة العادية والليفية. تم اشتقاق هذا النموذج بالاستعانة بلغة البرنامج الحاسوبي MATLAB لتذليل كافة الصعوبات المتعلقة بالتعابير المعقدة للرياضيات المتقدمة. اعتمد في البحث على نموذج السليفاني لمنحني الغلاف الأحادي المحور الخاص بالخرسانة الليفية.

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1. Introduction

The main objective of this paper is to develop a simple but complete model to predict and describe the stress–strain behavior of concrete composites under uniaxial compression due to random cyclic loads. Such a model is necessary to determine the response of structures subjected to random loadings such as those induced by earthquakes. This model offers a new way of visualizing the effects of the loading history for any random loading by relating it to a single strain value. Energy dissipation may be predicted by simply integrating the stress–strain response. Fatigue life could also be predicted, provided that the creep effects are properly incorporated.

Several studies had been carried out on the effect of cyclic loads on plain and steel fibrous concrete, and therefore a wide variety of models for the monotonic response of concrete composites are available [2 – 6]. It was found that the envelope curve for cyclic loading coincided approximately with the stress–strain curves for monotonic loading [7]. Some curves obtained experimentally in Ref. [8] are shown in Fig. (1).

Some of the key points in the response to cyclic loading are illustrated in Fig. (2). Point A is termed “unloading point” because it is the point at which unloading begins. Point B' is the corresponding “plastic strain point” when fully unloaded, while point B is the “plastic strain point” when partially unloaded. Point D is a “reloading point” because the reloading curve meets the envelope curve there. Point C is the intersection point of an unloading curve and its corresponding reloading curve and is called a “common point”. Many investigators have used these key points in conjunction with an envelope curve to develop models to predict the stress–strain behavior of concrete under cyclic loadings. The new model proposed here uses the unloading point, the plastic strain, and the reloading point as endpoints of the unloading and reloading curves. The model is developed in three stages. First, the locations of the endpoints of the unloading and reloading curves are determined. Secondly, expressions are presented for the unloading and reloading curves. Thirdly, the case of random loading is considered which is the most important case this paper emphasizes on.

2. Locations of endpoints

The three key endpoints that the proposed model is based on are readily obtained from the data of tests where the unloading and reloading cycles were full or complete. Full unloading begins from the envelope curve and proceeds until the zero stress level is reached. Full reloading begins at the zero stress level and proceeds until the envelope curve is again reached. The unloading and reloading cycles shown in Fig. (1) are complete. The point of unloading on the envelope will always be known, so the other two points will be determined as functions of this unloading point.

2.1 Plastic Strain Point

In previous investigations **[2]** and **[8]**, the plastic strain point was shown to be a function of the unloading strain point. In order to describe this relationship in a general manner for a variety of plain and fiber–reinforced concrete, a normalized expression was sought. It was also necessary that this expression be well behaved at the very large values of strain that can be obtained from fiber–reinforced concrete. The equation used to calculate the plastic strain is **[9]**:

$$\bar{e}_p = \begin{cases} 0.055\bar{e}_m + 0.127(\bar{e}_m)^{3.1} & \text{for } \bar{e}_m \le 2.2 \\ 1.584 + 0.72\,(\bar{e}_m - 2.2) & \text{for } \bar{e}_m > 2.2 \end{cases} \qquad \text{..........................(1)}$$

Where:

$\bar{e}_p = \frac{e_p}{e_0}$: Normalized Plastic Strain.

e_p: Plastic Strain.

e_0: Strain at peak Stress on monotonic curve

$\bar{e}_m = \frac{e_m}{e_0}$: Normalized Unloading Strain

e_m: Unloading Strain on the envelope curve

The aforementioned expression works well regardless of the type or amount of fiber–reinforcement in the concrete.

2.2 Reloading Strain Point

The point at which the reloading curve reaches the envelope can also be expressed as a function of the unloading strain. Otter and Naaman **[6]** suggested to use the following equation for predicting the reloading strain:

$$\frac{e_{m1}}{e_0} = \frac{e_m}{e_0} + k_r \quad \text{...(2)}$$

Where:

e_{m1} = the reloading strain on the envelope curve

k_r = reloading constant

They proposed to use k_r = 0.1 for both plain and fiber–reinforced concrete for large values of strain observed ($\frac{e_{m1}}{e_0} \geq 1$) and k_r = 0 for very small values of strain ($\frac{e_{m1}}{e_0} \langle 1$).

Using the two expressions for plastic strain and reloading strain for a full unloading and reloading, the cycle may be predicted for any given strain at which unloading commences, so the curves to describe the intermediate behavior may be developed with these endpoints.

3. Modeling of Unloading and Reloading Curves

3.1 Unloading Curve

The shape of the unloading curve has been approximated to be characterized by its concavity and single curvature. Many mathematical expressions have been used to describe it including bilinear, trilinear, parabolic and power function (see **Fig.(3)**, **Fig.(4)**, **Fig.(5)**).

In 1998 Bahn and Hsu **[11]** proposed the following power function to simulate the behavior of the unloading curve:

$$\bar{s}_u = C_u \, \bar{s}_m \left| \frac{\bar{e}_u - \bar{e}_p}{\bar{e}_m - \bar{e}_p} \right|^{n_u} \quad \text{...(3)}$$

Where:

$\bar{s}_u = \frac{s_u}{\bar{f}_c}$: Normalized Stress on Unloading Curve.

$\bar{e}_u = \frac{e_u}{e_0}$: Normalized Strain on Unloading Curve.

$C_u \approx 0.95 - 1.0$: Unloading Coefficient.

$\bar{s}_m = \frac{s_m}{\bar{f}_c}$: Normalized Stress at Unloading Point.

$\bar{e}_m = \frac{e_m}{e_0}$: Normalized Strain at Unloading Point.

$n_u = 1 + \sqrt{\bar{e}_p} \quad \geq 1$: parameter expressed as a function of plastic strain because the curvature in unloading curve changes sharply as the strain approaches to the plastic strain.

The paper adopts the aforementioned formula to express the unloading curve for fibrous concrete.

3.2 Reloading Curve

The shape of the reloading curve in compression is more complex. It is characterized by a double curvature with mild concavity in the low stress regions, and a sharp reversal in curvature near the envelope. Many of the existing models simply use a straight line to describe the reloading response (Sinha et al. 1964: **Fig.(3)**). A few models use bilinear (Darwin and Pecknold 1976: **Fig.(4)**) or parabolic expressions (Karsan and Jirsa 1969: **Fig.(5)**), and one model uses a combination of exponential and sinusoidal functions to generate a true reverse curvature (Zhang et al. **[12]**). This paper uses the model proposed by Al-Sulayfani, Bayar **[9]**. To ensure the double curvature and mild concavity in low stress region, the model included a polynomial of 3rd degree as follow:

$$\bar{s}_r = A_r \bar{e}_r^{\,3} + B_r \bar{e}_r^{\,2} + C_r \bar{e}_r + D_r \quad \mathbf{L\ L\ L\ L\ L\ L\ L\ L\ L\ L\ L} \quad (4)$$

Where:

$\bar{s}_r$: Normalized Stress on the Reloading Curve

$\bar{e}_r$: Normalized Strain on the Reloading Curve

A_r, B_r, C_r, D_r : Coefficients of the Reloading Curve

To solve for the coefficients A_r, B_r, C_r, D_r, the following boundary conditions are used:

1. The polynomial passes though the normalized plastic strain point $(\bar{e}_p, 0)$.
2. The polynomial passes though the normalized reloading strain point $(\bar{e}_{m1}, \bar{s}_{m1})$.
3. The slope of the polynomial at the normalized reloading strain point exactly coincides with the slope of the monotonic curve at the same point; i.e. $(E_{r_{(\bar{e}_{m1}, \bar{s}_{m1})}} = E_{c(\bar{e}_{m1}, \bar{s}_{m1})})$.
4. The following expression would be satisfied:

$$\frac{E_{r_{(\bar{\varepsilon}_p,0)}}}{E_{c_{0.45}}} = \frac{1}{(0.95 + 2.78\bar{\varepsilon}_p)}$$

Where:

$E_{r_{(\bar{\varepsilon}_p,0)}}$: the slope of the polynomial at the plastic strain

$E_{c_{0.45}}$: the slope of the monotonic curve at $0.45\bar{f}_{cf}$

$\bar{f}_{cf}$ the ultimate compressive strength of fibrous concrete

Using the expressions mentioned here for the unloading and reloading curves and their endpoints, in conjunction with Al-Sulayfani expression for the monotonic curve, the stress–strain response can be generated for any cyclic loading reaching both the envelope curve and the zero–stress level.

These expressions for cyclic loading can be used for both plain and fibrous concrete (**Fig.(6)**). Only the monotonic response, which is used as the envelope curve, is affected by fiber reinforcement **[13]**.

4. Response to random load history

A model for the stress–strain response under a random load history must keep some record of the load history and make use of it. In order to develop such a model, the cases of partial unloading and partial reloading will be considered. The model presented in this paper may be used to generate the stress–strain response to any random loading history.

The determination of the endpoints (unloading point, reloading point and plastic strain point) are based on the assumption of full reloading and unloading. In order to generalize the model, three other cases should be considered, full unloading with partial reloading, partial unloading with full reloading and partial unloading with partial reloading.

Otter and Naaman **[6]** proposed the two following formulas to interpolate the locations of new reloading point for partial unloading and new unloading point for partial reloading respectively:

$$\bar{\varepsilon}_{m1_{new}} = \bar{\varepsilon}_m + (\bar{\varepsilon}_{m1_{old}} - \bar{\varepsilon}_m)\left(\frac{\bar{\sigma}_m - \bar{\sigma}_{lo}}{\bar{\sigma}_m}\right)^{n_{pm}} \cdots\cdots\cdots (5)$$

$$\bar{\varepsilon}_{m_{new}} = \bar{\varepsilon}_{m_{old}} + (\bar{\varepsilon}_{m1} - \bar{\varepsilon}_{m_{old}})\left(\frac{\bar{\sigma}_{hi} - \bar{\sigma}_{lo}}{\bar{\sigma}_{m1} - \bar{\sigma}_{lo}}\right)^{n_{pm1}} \cdots\cdots\cdots (6)$$

Where:

$\bar{\varepsilon}_{m1_{new}}$: reloading strain for partial unloading and full reloading

$\bar{\sigma}_{lo}$: lowest stress reached during unloading

n_{pm} : partial unloading parameter

$\bar{\varepsilon}_{m_{new}}$: new value of the envelope unloading strain

$\bar{\sigma}_{hi}$: highest stress level attained during reloading

n_{pm1} : partial reloading parameter

These two cases may be combined to produce a model capable of predicting the stress–strain response of concrete under a general cyclic loading. Any combination of full and partial unloading and full and partial reloading can then be handled. The only information required about the previous load history is the current value of the envelope unloading strain. The formulas (3) and (4) would be rewritten to make use of the new interpolated endpoints.

They suggested to use a value of 8.0 for each of n_{pm} and n_{pm1} for best calibration of the model.

Fig.(7) illustrates the stress–strain curve for cyclic partially loading and unloading. It also substantially explains the fatigue failure which is "The phenomenon of concrete rupture when subjected to repeated loadings at a stress substantially less than the static or maximum compressive *strength*" and the fatigue strength which is defined as "the fraction of the static strength that can be supported repeatedly for a given number of cycles". Fatigue strength is influenced by range of loading, rate of loading, load history and material properties [14]. The model shows that the concrete fails after 20 cycles of loading and unloading while subjecting to stress ranges between $0.66\bar{f}_c$ and $0.04\bar{f}_c$.

The most important feature of this model is the use of the envelope unloading strain to keep a record of the loading history. Some properties of the envelope unloading strain as an index of loading should be noted. While unloading is occurring, the envelope unloading strain, which is in some ways a damage index, remains constant. As soon as reloading commences, the value of the envelope unloading strain begins to increase.

A further benefit of the modeling method described herein is that all of the computations are straightforward, and no iteration is required. This facilitates rapid computation of the response for complicated or lengthy loading programs. A computer program based on this model was used to generate the responses shown in this paper. Also the strain energy can be calculated by integrating the area under the stress–strain curve.

5. Summary and Conclusions

The paper presents a model for predicting the stress–stain behavior of concrete under random cyclic loading. The three key points (plastic strain, unloading strain and reloading strain) serve as the endpoints of the unloading and reloading curves., for which expressions are also given. The cases of partial unloading and partial reloading are considered so that completely random loadings may be handled. The model compares favorably to data obtained from various types of cyclic loadings on plain and fiber reinforced concrete.

The envelope unloading strain serves as an index of the loading history, providing a new way of visualizing the behavior of concrete composites under cyclic loadings. The model is simple, general and flexible enough to be adapted for use in describing the behavior of various concrete materials.

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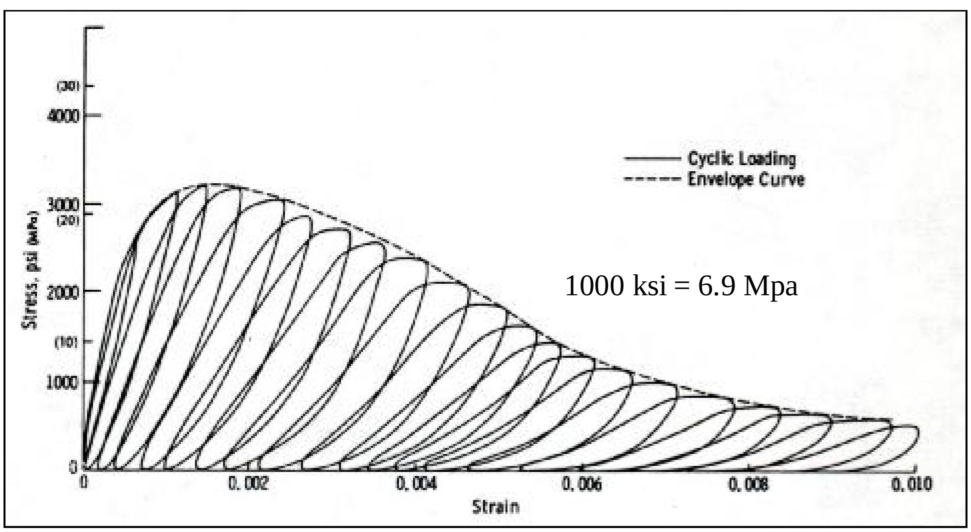

Fig. (1): Stress–Strain Curves for Cyclic Loading on Concrete [8]

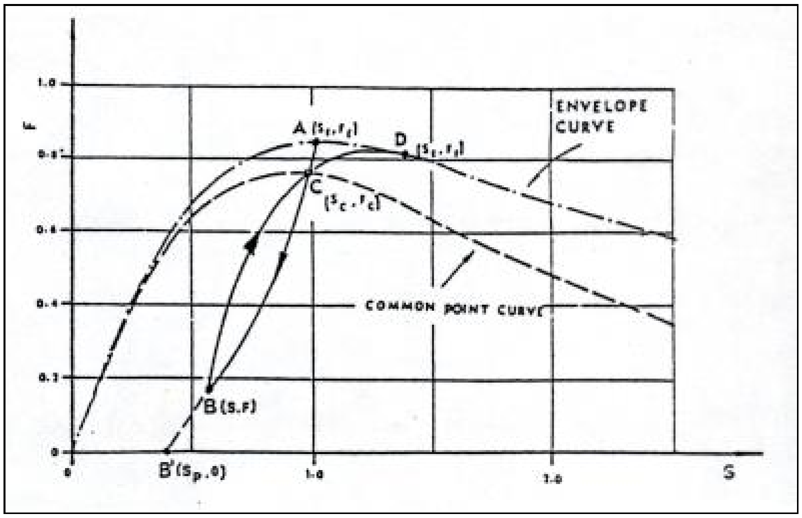

Fig. (2): Key Points of Cyclic Response

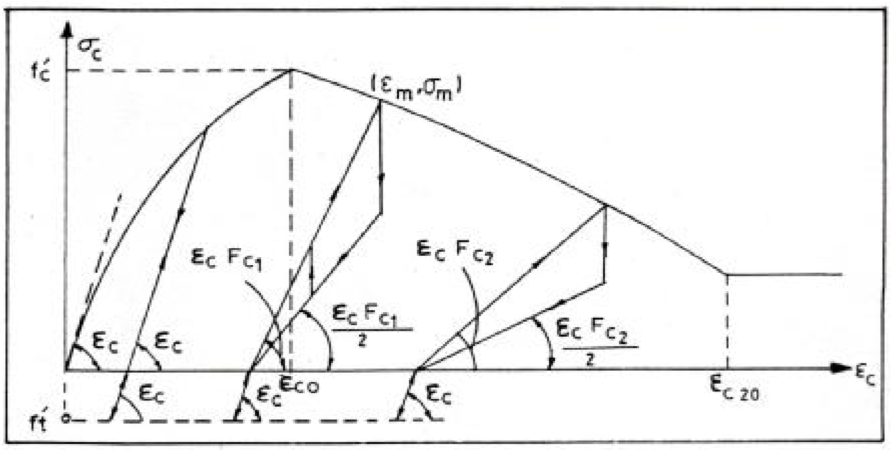

Fig. (3): Bilinear Path of Unloading Curve [5]

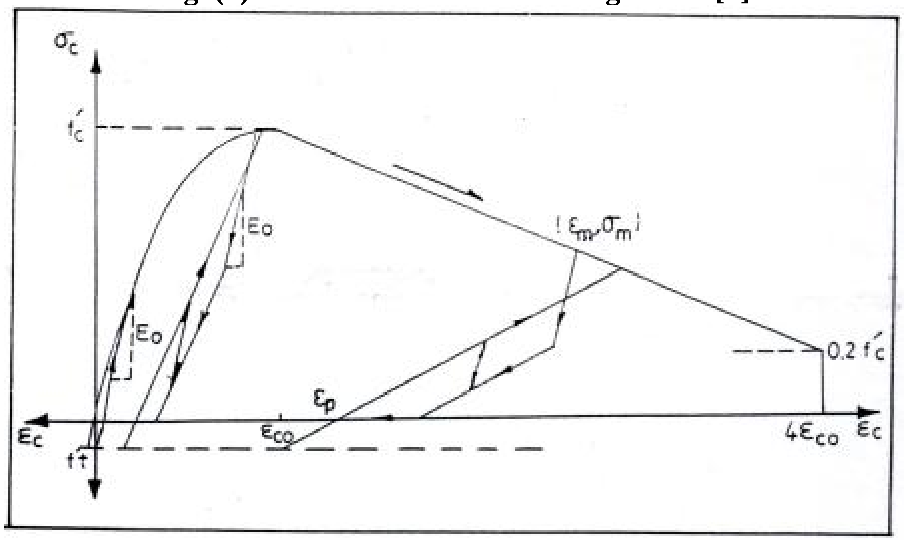

Fig. (4): Trilinear Path for Unloading Curve [10]

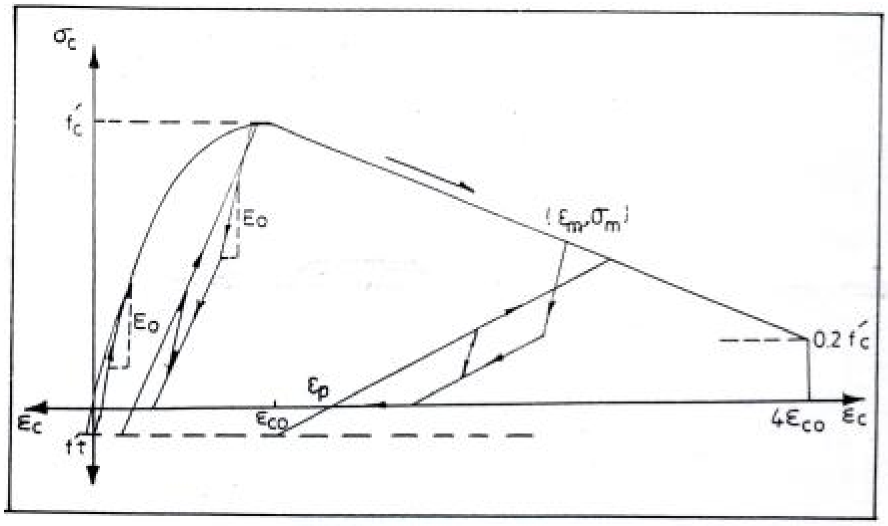

Fig. (5): Parabolic Path for Unloading Curve [2]

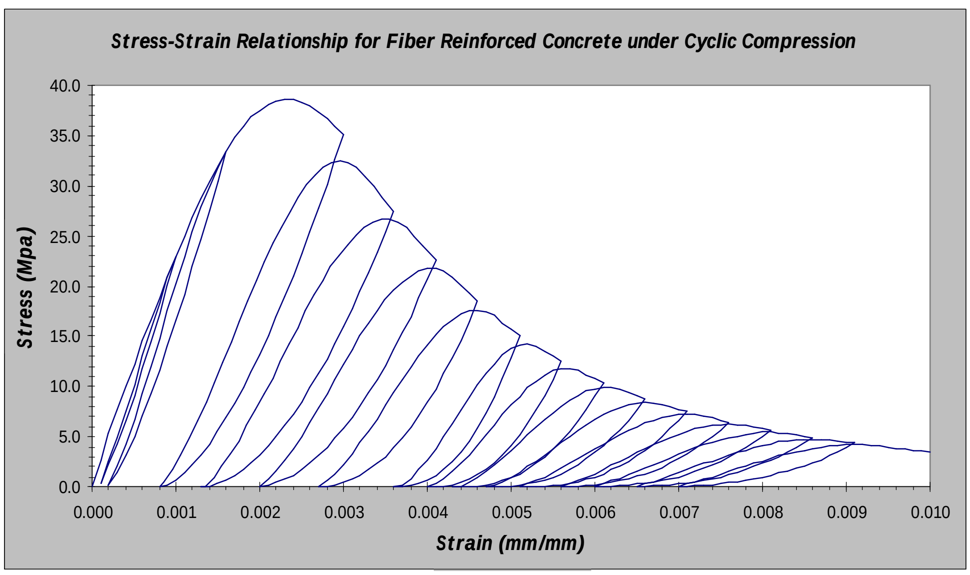

Fig. (6): Present Concrete Model under Cyclic Compression

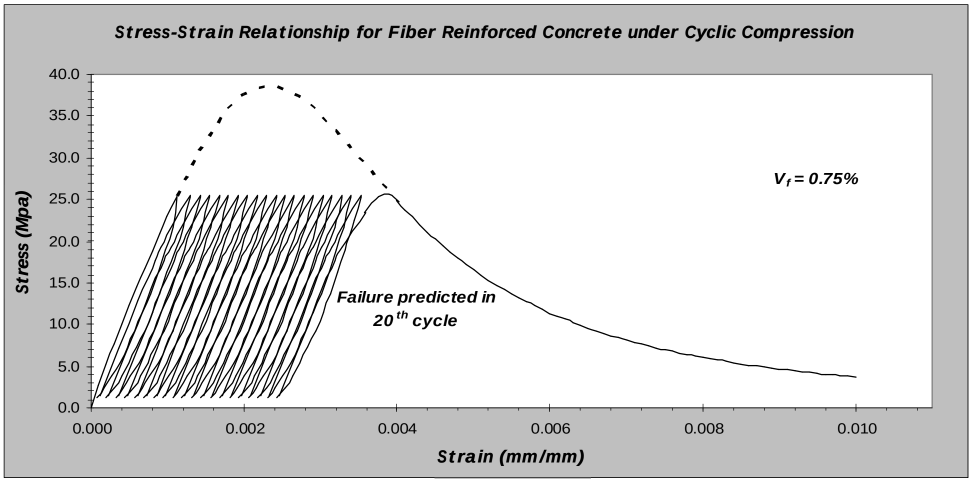

Fig. (7) Partially Cyclic Loading and Fatigue Loading