Fractal Geometry and its technical mechanisms in the internal space designs.

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Abstract:

Nature governs numerous laws that have been classified by various studies and scientific disciplines. Through the observation of these laws over time, the researcher was motivated to integrate natural engineering principles, specifically fractal geometry, with interior design, considering it a source of inspiration for designing interior spaces. The technical behavior of fractal geometry varies between regularity and irregularity. Since interior design is the creation of an artistic work that relies on the designer's ability to sense and think creatively, it ventures into the realm of imagination with the application of fractal geometry, presenting new content and meaning within the framework of contemporary creative concepts.

From this premise, the research problem was formulated with the following question: What are the mechanisms of fractal geometry, and to what extent can they be applied in the design of interior spaces? The aim of the research is to understand the concept of fractal geometry and uncover its practical mechanisms in designing interior

spaces. The research is contextually defined by studying the concept of fractal geometry and extracting its practical mechanisms in interior space design. Geographically, it focuses on interior spaces in Asian countries and the works of the architect Zaha Hadid from 2013 to 2021.

This was covered in the first chapter. The second chapter included the theoretical framework to identify the standards and components that can be adopted to reach the mechanisms of applying fractal geometry in interior designs. The third chapter covered the research community and methodology, with the researcher following the descriptive method in analyzing the research sample models. Furthermore, the fourth chapter presented the results, conclusions, and recommendations. The most important conclusions were:

1. The mechanism of irregularity can be described as the most effective means to achieve the formal complexity inherent in fractal geometry, as observed in design compositions of architectural formations. This approach aims to introduce elements of ambiguity, surprise, and unpredictability to influence the structural composition of interior spaces.

2. Fractal geometry, with its diverse mechanisms, stands out prominently in shaping forms, leveraging its organic formations found in nature. Multiple mechanisms within fractal geometry can collaborate to construct a formal and design-oriented configuration

that fulfills the aesthetic and structural requirements guided by fractal principles.

Keywords: fractal geometry, Technology, mechanisms, design, interior spaces.

Chapter One: Research Problem and Rationale

1-1.Research Problem: Nature serves as a rich source of inspiration for architectural engineering, influencing natural formations and shapes. Through these elements, applications can be derived to enhance efficiency, sustainability, gravitas, and intrigue within occupied spaces. To expand designers' perspectives through technology, which significantly contributes to integrating natural geometric systems (fractals) with design concepts, enriching the aesthetic and expressive dimensions of interior spaces. This motivated the researcher to articulate the research problem as follows: What are the mechanisms of fractal geometry, and to what extent are they applied in interior space design?

1-2.Research Significance: This study aims to contribute scientifically to the field of interior design by enriching scholarly literature for stakeholders interested in this domain. Furthermore, it aims to affirm the intellectual relationship and its application to support the practical aspects of interior design as a novel approach for exploring theoretical variables (fractals), thereby achieving new dimensions, deepening both intellectual and practical awareness, and employing them aesthetically.

1-3.Research Objective: To comprehend the concept of fractal geometry and reveal its practical mechanisms in the design of interior spaces.

1-4.Research Scope: The study is focused on examining the concept of fractal geometry and extracting its practical applications in interior space design, particularly within interior spaces across Asian countries, including architectural works by Zaha Hadid spanning from 2013 to 2021.

1-5.Definition of Terms:

Fractal Geometry: Derived from the Latin word "Fractus" and corresponding to the verb "Frange," meaning to break or fragment into small pieces. Some relate it to "Fraction," indicating a mathematical fraction[Nazla Khudair,2004,49]. Therefore, fractal geometry refers to mathematical shapes with unique properties such as self-similarity and fractional dimensions, resulting from the iteration of nonlinear equations[Mandelbrot, B. & Frame,2002,3].

Mechanisms: Mechanisms are also known as the means used in the arts to implement a social movement that generates change or a stage that occurs within a certain process. In colloquial language, the concept of a mechanism is used to refer to a methodology or process. In this sense, mechanisms can be physical or abstract and can refer to real or symbolic issues[Ahmed Mukhtar,2012,73].

Technology: A set of materials, tools, machines, methods, means, and systems involved in the design process, aimed at providing a human service. It is obvious that smart technologies vary and differ depending on the functional goals they seek to achieve. Additionally, the final outcome differs based on the components and the extent to which these components influence the overall design process[Ayad Hussein Abdullah,2008,197].

Design: It is the innovation or creation of beautiful, enjoyable, and useful things for humans. It is also the complete process of planning the form of something and creating it in a way that is both functionally and aesthetically satisfying, thereby fulfilling human needs[Ismail Shawqi Ismail,1999,43].

Interior spaces: They are defined as the fundamental units in the process of interior design that reflect a set of perceived and physically embodied relationships, with specific form and meaning. They are known for systems that express functional, aesthetic, and psychological goals[Runak Hashim Ali,2002,4].

Chapter Two: Theoretical Framework

2. Introduction:

Formal organizations are integral to the foundational processes of interior design, where designers seek to materialize design concepts by adopting structural principles and mechanisms that constitute specific design systems. This involves leveraging the dynamics of balance, contrast, and comparison among conflicting values within a dynamic and evolving framework. The designer's creative imagination in shaping the visual aspects of the design system is crucial for success. Therefore, the effectiveness of a designer depends significantly on their judicious selection of expressive discourse mechanisms and their ability to effectively articulate these concepts. This is evident in their technical organization and integration of various architectural design components and elements.

Among the most noteworthy architectural organizational frameworks is fractal geometry, characterized by digital formations governed by mathematical equations that manifest as complex geometric shapes. These shapes exhibit non-similarity in appearance, comprising a geometric structure marked by the overlap of structural elements in diverse configurations and forms. Such formations defy conventional mathematical geometry and typically arise through the use of recursive sequences, presenting a unique design perspective distinguished by its fluidity and dynamic movement. Consequently, fractal forms have evolved into expressive trends and innovative features with a profound visual depth that aesthetically engages and captivates the observer.

2-1.Concept of Fractal Geometry and Design Work:

The essence of design work is embodied through the creative artistic vision of the designer, drawing inspiration from various natural themes in diverse forms. The designer engages with these elements through their imagination, either by mimicking and adhering to their forms or by abstracting them. From these visual data, the designer derives structural laws and building mechanisms that aid in crafting new formal configurations, leveraging modern technologies to fulfill their artistic goals and objectives within spatial environments.

This concept was underscored by Dedkind in 1877, emphasizing the profound understanding of mathematical systems and fundamentals essential for studying natural phenomena. Furthermore, in 1904, Hegle Von Koch introduced a geometrically enriched concept known as the "Koch snowflake" or "Koch curve of snow crystals," its colloquial names before the advent of the term "fractal." This involves an infinite series of shapes with triangular boundaries (see Figure 1), where each iteration adds a triangle with a reduced side length, progressively expanding the perimeter until it asymptotically approaches a finite area while maintaining an infinite perimeter [Ayatollah Mohammad Salah Al-Din,2012,141].

Fractal shapes are numerical constructs defined by mathematical equations, prominently observed in various natural phenomena. They represent complex geometric forms with non-similar appearances, yet governed by unified structural principles. Fractals explore composite geometric structures composed of fractals, describing numerous configurations and arrangements beyond the scope of traditional Euclidean geometry. These formations often emerge through algorithmic iterations, offering a descriptive or visual language for traditional motion techniques. Consequently, fractal shapes have emerged as expressive tools and innovative elements within the realm of visual arts [Kathlean T. Alligoog,2000,149].

2-2.Design Structure and Geometric Forms System: Every design structure has a system[9], and structure is nothing but a system of relationships. Structures define the relationships between elements, governed by laws and systems that constitute transformative actions while preserving structural properties. The system is the essence of design work, through which designers control the coordination of structural relationships, interconnections, and harmonies among design elements, whether shapes, colors, or sizes. The system expresses harmony, coherence, and functional perfection in structural relationships among elements, imparting a sense of unity and structured feeling[Anne , c.,2004,13].

The design system relies on three fundamental characteristics:

1. **Regularity:** Every geometric shape has a specific form with a beginning and an end, where all its parts are interconnected and follow each other regularly[Antoniadis , A.C. ,1990,82]. The designer aims to achieve maximum regularity through ideas or actions in a fluctuating and diverse world[Al-Haidari, Sanaa Saadi,1996,58]. This regularity is the source of beauty in architectural formations, characterized by symmetrical and uniform geometric masses that add aesthetic value[ead , H.,2002,98].
2. **Geometric Characteristic:** Every system has dimensions characterized by geometric shapes and structures represented

by geometric shapes and lines such as triangles, squares, and straight lines. Curved geometric lines are also part of regular lines, like lines of geometric shapes such as circles and their derivatives, with geometric properties subject to mathematical relationships in measurement, proportion, and scale[Outdoor Adrertising,1998,37].

3. **Scale Characteristic**: This refers to how the shapes are perceived in relation to architectural elements. There are two types of scales: mathematical scale and geometric scale used in architectural construction systems. Natural scale measures the size of the shape or space relative to other things in the surrounding structure, while another scale is human scale, measuring the size of the element relative to human dimensions[Lubna Asad Abdul Razzaq,1999,12].

There are several structures used in designing geometric forms, among the prominent ones are:

A. **Formal Structure**: Created in the form of a grid of perpendicular straight lines distributed with equal or horizontal and vertical spaces, where geometric shapes are arranged according to these divisions and their degree of regularity. The formal structure has several patterns:

1. **Repetitive Structure Pattern**: The architectural total area is divided into secondary divisions with the same complete shape

and measurement without adding gaps or spatial divisions left between them. Geometric shapes are uniformly placed around each space.

2. **Gradual Structure Pattern:** Similar to the repetitive structure, except for its structural divisions, which change in shape and size or both in a gradual (sequential) manner[Al-Janabi, Kazim,1991,38].
3. **Radial Structure Pattern:** It is a type of repetitive structure designed in a circular form, with its structural lines revolving around a focal center.

B. **Semi-Formal Structure:** This structure is usually somewhat regular but may include irregular patterns, with or without structural lines to define the arrangement of geometric shapes. It has several patterns:

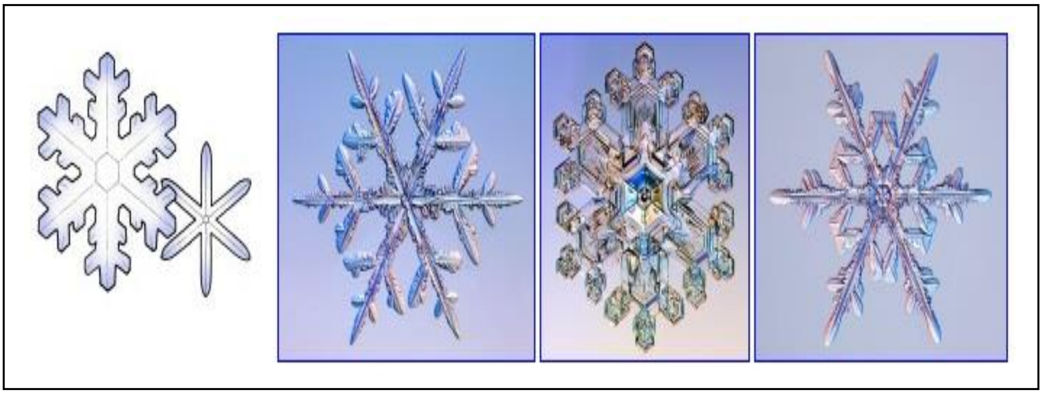
1. **Analogous Structure Pattern:** It lacks the stability of repetitive structure patterns but features similarities between some of its elements[Salima Abdul Rasoul,1999,27].
2. **Central Structure Pattern:** In this pattern, geometric shapes are freely arranged to achieve a concentration effect from a point or an imaginary line predetermined in architectural design. Maximum density crowds around that point or line, either through regular similarity or rotational similarity or

through an illusory progression[Tareq Mustafa Abu Bakr,2002,74].

All these patterns in formal structure have applications in three-dimensional design and composition.

2-3. **Shapes in Nature and Fractal Geometry**: Fractal geometry is characterized by the similarity of the fractions that make up its components. Each part of the shape resembles the shape itself in a complete or approximate manner, with symmetric scales and sometimes approximately symmetrical appearance. This can be observed in fractal geometry's depiction of leafy textures on trees in nature.

Additionally, fractal geometry is found in natural crystals, where nature creates geometric patterns, some regular and some irregular over time, meaning it is autonomous in the measurement gradient, and



sometimes it can stabilize in a state of equilibrium, as shown in figure (1).

The figure (1) illustrates the mathematical geometry in the formation of a snowflake crystal [https://www.arabiaweather.com/ar/content].

Snowflakes, formed in turbulent air, embody a synthesis of fractal geometry's symmetry with the crystallization process of water. This results in the formation of unstable protrusions that branch out, governed by obscure mathematical laws dictating their size, growth rate, and extent of branching [James Clarke,2000,234]. Snowflakes stand as paramount examples of natural geometric phenomena, characterized by rich structural elements that can be innovatively utilized as decorative units in architectural design. Their formation adheres to fundamental geometric principles through physical geometry, manifesting in simple or complex hexagonal layers with balanced and coherent relationships [Islam Mohamed El-Sayed Heba,2007,86].

Moreover, nature exhibits diverse geometric models featuring repetitive fractal patterns. Examples include neuronal cells in the brain, pulmonary structures, arboreal branches, lightning strikes, rivers, and the fractal-like branching patterns of tree leaves. Spiral systems, ubiquitous in nature, are typified by curves that revolve around a fixed point in continuous rotation, representing complete revolutions around an axis from a central point in a conical fashion [Zainab Ali Ibrahim Al-Sayed,1993,13]. See figure (2) depicting this spiral pattern in the conch shell for visual reference. This graphical representation illustrates curves originating from a central hub, spiraling outward and inward, encapsulating the essence of spiral systems [The new encyclopedia,1985,102]. This phenomenon is also observed in cacti plants, where new segments grow in a spiral pattern at a constant angle outward.

Figure (2) Mathematical geometry in the construction of the spiral [http://maaber.50megs.com/issue_february05/epistemology1] and the geometric construction of the cactus [https://www.mosoah.com/health/alternative-and-natural].

2-4.The Practical Mechanisms of Fractal Geometry:

The constructive mechanisms of fractals in artistic forms can be elucidated through a series of operations governing the assembly of elements within the artwork. Design concepts transcend mere aesthetic appeal and spatial impact, extending to the systematic organization and construction of these forms in the design process. These organizational mechanisms are succinctly outlined as follows [Maisa Fikri Ahmed Al-Sayed and Heba,16]:

- **Experimentation:** Experimentation serves as the primary method through which designers shape their ideas and execute their creations. It allows designers to explore and uncover methods of processing and application, generating novel insights and stimulating interest through artistic expression.

- **Assemblage:** The composition of a whole from parts involves the manipulation of representative lines and their movements using fractal-specific software. The designer's role is pivotal in establishing cohesive relationships among these parts, assembling them into coherent wholes that surpass initial conceptions.

Contiguity and Adjacency: While adjacency brings together parts without direct contact, contiguity achieves tactile connection between parts, distinct from mere adjacency. Contiguous shapes manifest in corner or edge contacts, influencing design through geometrical variables such as subject matter, scale, structure, and form.

− **Overlap:** Manifested through the superimposition of multiple parts, overlap results in the partial concealment of one part within another or behind it. This organic intertwining enhances unity and depth perception within the artwork, varying in degrees from partial to complete overlap based on disappearing spaces and the spatial and structural configuration of overlapped parts.

− **Transparency:** Representing a form of overlap where the rear part remains partially visible, transparency allows for layered visualization, enriching spatial complexity and aesthetic diversity within the flat plane of artwork.

− **Reshaping:** Reshaping achieves diversity among parts by varying their spatial dimensions, crucial for defining the true scale of compositional elements in terms of length, width, and thickness. Scale not only dictates shape proportions but also establishes relative relationships within the overall design area.

− **Crashing:** This process involves dismantling and analyzing parts, followed by their reconstitution into new configurations, fostering creative reinterpretations and structural transformations within the artwork.

− **Shortening:** Simplifying and redirecting parts deviates from their original structural paths, streamlining design elements to enhance visual coherence and thematic resonance.

– **Synthesis:** The synthesis of diverse parts integrates them into unified design concepts, balancing fragmentation with cohesive design principles to achieve artistic cohesion and thematic unity.

– **Bending:** Manipulating parts involves fixed and rotational movements, facilitated by specialized fractal engineering software, influencing part orientation and spatial interaction dynamics within the artwork.

– **Rolling:** Rotational movements impart directional and sequential changes to parts, either uniformly or irregularly, enhancing visual dynamism and spatial continuity within the design composition.

– **Repetition:** Iterative incorporation of shapes and motifs reinforces thematic motifs and visual patterns, fostering aesthetic consistency and conceptual depth within the design framework.

In conclusion, the design process hinges on interactive fractal engineering mechanisms that unify and harmonize component elements, culminating in cohesive and organic fractal units. The integrity of these units is pivotal for integrated design outcomes, essential for fostering creative innovation and aesthetic synergy in artistic expression.

2-5.Fractal geometry in nature serves as an inexhaustible wellspring of design inspiration for interior designers. By incorporating natural lines and shapes into their design processes, designers employ organizational techniques to create aesthetically pleasing and sustainable interior spaces. The enduring appeal of these natural elements underscores their role in enhancing both the visual appeal and environmental sustainability of interior designs.

One notable feature of fractal geometry shapes is their organic growth patterns, which manifest in balanced and visually diverse forms. For instance, Figure (3) exemplifies an interior design concept inspired by the shapes of palm trees. Furthermore, Figure (4) illustrates the design of seating units, while Figure (5) explores the conceptualization of spirals and staircase construction techniques within interior environments .

Shape (3) illustrates the application of fractal geometry technique in interior space.

Shape (4) demonstrates the application of fractal geometry in the design of seating units.

Shape (5) illustrates the concept of the spiral and the technique of staircase construction.

Additionally, the design of the Lotus Temple in India in 1986 employed fractal geometry in the depiction of the lotus flower. See Figure (6), designed by the Canadian engineer Fariborz Sahba [https://e3arabi.com/engineering].

Shape (6) illustrates the application of fractal geometry in the design of the Lotus Temple.

The third section: Research Methodology:

3-1. Research Methodology: The researcher employed a descriptive approach to analyze the sample models in order to elucidate the concept of fractal geometry and derive its practical applications in contemporary interior space design, particularly in the works of architect Zaha Hadid. This approach relied on a theoretical framework and its resultant standards, aiming to comprehensively achieve the research objectives.

3-2. Research Community and Sample: The study focused on interior spaces designed with fractal geometry in the Arab world, specifically those created by Iraqi architect Zaha Hadid, totaling seven. A purposive sampling method was utilized from this research community, selecting three models which constitute 43% of the total, aligned closely with the study's objectives. The selection of these research models was based on ongoing research into exemplary interior spaces, showcasing diverse design elements within their interiors and grounded in fractal design principles. These include: Kuwait, Al-Avenues Mosque / Saudi Arabia, King Abdullah Petroleum Studies and Research Center / Qatar, Al-Wakrah Stadium.

3-3. Description and Analysis of the Research Sample:

3-3-1. Description and Analysis of the First Model, Kuwait, Al-Avenues Mosque 2007:

Zaha Hadid's design of the Al-Avenues Mosque represents a contemporary mosque design with a novel symbolic design language that reflects an advanced Islamic discourse suited to the culture and civilization of the twenty-first century. Its form draws inspiration from the natural flow of movement within Earth's space, incorporating dynamism that manifests in repeated scales and diverse stylistic approaches. Additionally, it achieves reduction in fractal structure, thereby introducing visual intrigue, complexity, and aesthetic appeal into the design. See Figure (7).

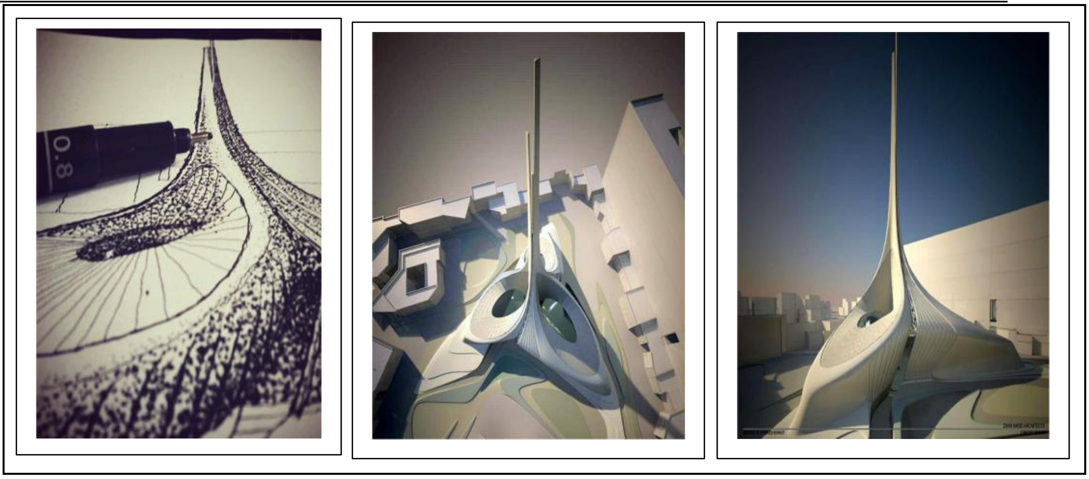


Figure (7) shows the fractal architecture structure of the Avenues Mosque.

3-3-2. Description and Analysis of the Second Model: Saudi Arabia - King Abdullah Petroleum Studies and Research Center (KAPSARC) 2016:

KAPSARC, known by its acronym, is a nonprofit research center specializing in energy economics. It is recognized as one of the fastest-growing development centers globally. Situated north of Riyadh, along King Khalid International Airport Road, adjacent to Princess Nourah Bint Abdulrahman University, KAPSARC dedicates its efforts to the study of all forms of energy and their related economics, policies, and technologies, with the goal of fostering societal well-being both locally and globally. The design concept draws inspiration from the fractal geometry observed in natural sand dunes. This influence is evident in the design through mechanisms such as superimposition, penetration, repetition, and adjacency across multiple axes in the forms of walls and ceilings. The design elements exhibit characteristics of nonlinearity, ambiguity, surprise, and unpredictability, contributing to the dynamic nature of the forms, which are conceived in accordance with fractal geometry. This reflects an idea inspired by nature (sand dunes). See Figure (8).

Figure (8) illustrates the fractal architecture of the King Abdullah Petroleum Studies and Research Center.

3-3-3. Description and Analysis of the Third Model: Qatar – Al Wakrah Stadium 2019:

Designed by architect Zaha Hadid in 2019, Al Wakrah Stadium accommodates 40,000 spectators and stands 48 meters tall. The structural design of the stadium draws inspiration from the sails of traditional dhow boats. The curved roof alludes to maritime navigation, evoking the sensation of being aboard a ship. The roof's design features curved lines and formations inspired by ocean waves, creating a dynamic impression and a sense of continuity through the use of sequences based on fractal geometry. This design approach embodies complexity, ambiguity, and excitement in the spectator experience, as articulated by Zaha Hadid in the architectural concept of the stadium. See Figure (9).

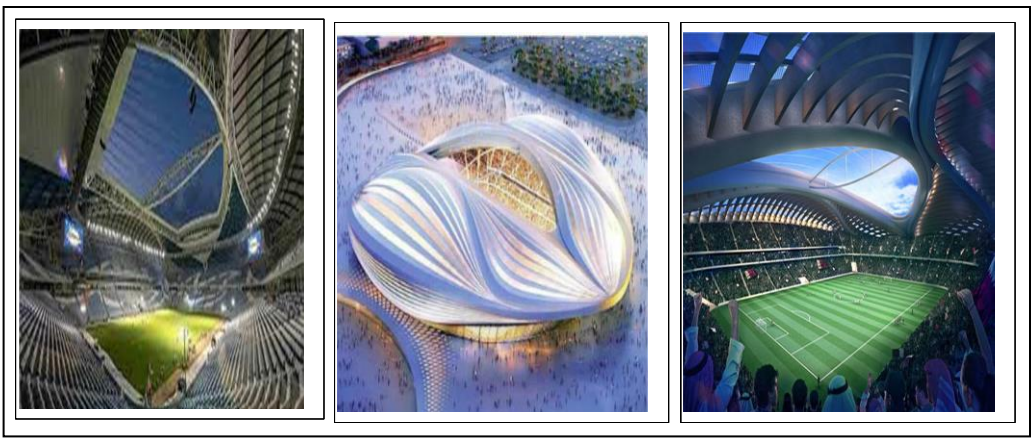


Figure (9) illustrates the fractal architecture of Al Wakrah Stadium.

Chapter Four: Study Results

4-1. Results:

1. The mechanisms of superimposition, penetration, repetition, and adjacency predominantly appear across multiple axes in the design

of forms. Features of nonlinearity, ambiguity, surprise, and unpredictability are evident in the formation of most designed shapes It was achieved at 100% in the second model, 75% in the third model, and 25% in the first model.

2. Flexibility and fluidity are prominently featured in the design samples in various forms. The research samples are characterized by diverse flexibility in design, growth, and adaptability to the type of space and its specific activities At 100% in the first and third models, and 25% in the second model.

3. Dynamism is a clear feature of forms designed according to fractal geometry, achieved through varying relationships within these models. Curved lines and varying levels and measurements of formations enhance the dynamic quality of the shapes, particularly evident in the second model.

4. The properties of design forms, such as size, color, and texture, emphasize the effectiveness of the design concept and the creative structure of the models, resulting in impactful presentations At 100% in the design models.

5. The interior designer emphasized the use of curves and straight lines that mimic the geometric patterns of nature, creating distinctive and contemporary spatial designs It was largely achieved at 100% in the first and third models, while in the second model it was achieved relatively.

4-2. Conclusions:

1. Self-similarity is a significant feature in design structures based on fractal geometry. It involves the repetition of parts or the replication of content at various scales, often accompanied by a gradient in measurement.

2. The mechanism of nonlinearity is highly effective in the complex shapes of fractal geometry, evident in the design formations, contributing to the ambiguity, surprise, and unpredictability in the internal space structures.

3. Fractal geometry, with its various mechanisms, is a prominent feature in the construction of shapes, relying on its organic formations in nature. Multiple mechanisms can collaborate to achieve a design structure that meets the requirements of fractal geometry.

4. The foundations emphasize the display of internal spaces that exhibit the flexibility and dynamism of fractal geometry across multiple axes, despite variations in the shapes and features of internal spaces.

5. Emphasizing the presence of material elements through the imitation of fractal geometry in nature reflects the impact on the perception and understanding of the observer.

4-3. Recommendations:

1. Emphasize the use of specialized software in fractal geometry for interior design studies by designers.

2. Adopt the research findings in the field of contemporary interior design based on fractal geometry as a renewable source of knowledge.

3. Emphasize the importance of natural environmental sources as influential elements in the design process, serving as renewable energy to inspire optimism and joy in observers.

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