

2007/3/14:
2007/7/5:

:

:

(L.S.O.R.)

$(10 \leq Ra \leq 10^4)$

$(\theta_{\max})$

$(Ra > 50)$

Abstract:

Two – dimensional, steady natural convection in a rectangular cavity filled with a heat generating saturated porous medium has been studied numerically for the case when the vertical walls of the cavity are isothermal and the horizontal walls are either adiabatic or cold. Finite difference method was used to transform the momentum and energy equations from the differential form to the algebraic form; relaxation method was used to solve the momentum equation, while (L.S.O.R.) method was used to solve the energy equation for Rayleigh No. ranges $(10 \leq Ra \leq 10^4)$. Results are presented in terms of the stream lines and isotherms, the maximum temperature in the cavity and intermediate Nusselt number. The thermal convection flow together with the uniform heat generating produces a highly stratified medium at high Rayleigh numbers. The horizontal wall boundary condition changes from adiabatic to cold reduces $(\theta_{\max})$ also it is found that heat transfer increases with increasing

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Rayleigh number while it decrease with aspect ratio. The effect of aspect ratio on heat transfer will appear when ($Ra > 50$).

(Nomenclature)

(L/D)	A
(J/kg.K)	C
(m)	D
(m/s ²)	g
(W/m ² .K)	h
(m ²)	K
(W/m.K)	k
(m)	L
(hD/k)	Nu
(N/m ²)	P
($g\beta KSD^3/2k\nu\alpha$)	Ra
(W/m ³)	S
(K)	T
(m/s) (x)	u
(m/s) (y)	v
(x/D) (x)	X
(y/ D) (y)	Y
(m ² /s) (k/ρC)	α
(1/K)	β
((T-T _c)/(SD ² /2k))	θ
(m ² /s)	ν
(kg/m ³)	ρ
(16) (θ) (ψ)	φ
	ψ

(subscripts)

	ad
	c
	m
	max
(x=0)	o

(Y. Tasak et al) [2]

:

Thermo – Chromic (TLC) (Heat Generation)
(Liquid Crystal) (Geophysical)

(Jue T.C.) [3]

.(Metabolism)

Semi- Implicit Finite Dement) (Abdelhmid Hadim) [1]

(Method
($10^{-10} < Da < 10^{-1}$) ($10^3 < Ra < 10^8$)

(Darcy – Brinkman) –

.(Gralerkin)

($Da \geq 10^{-2}$)

[4] (1992)
(Z.G.Dnand E.Bilgen)

[7,8,9,10,11]
(Isothermal) (10) (0.1) $\cdot (10^8)$ (10^0)

[5] (2002)
(A. khalili, I.S. Shivakumara, M. Huettel)

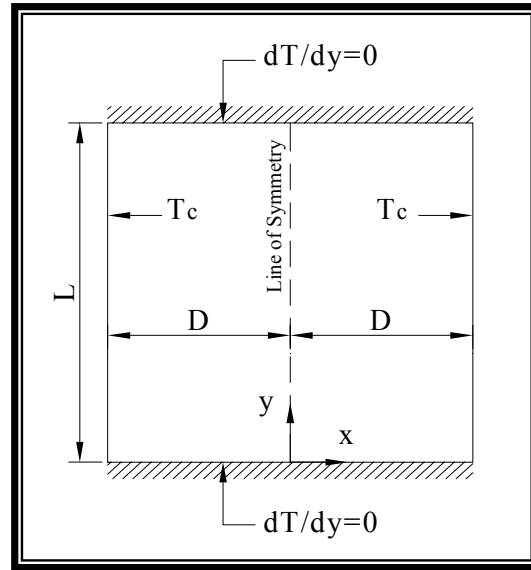
Theoretical Analysis -2

Forchheimer – Extended)
(Darcy
(Galerkin)
(h)
(x)
(h/x)

(Boussinesq) [6]

(M.R. Dhanasekaram et al)

[12,13]



(1)

$$u = -\frac{\alpha}{D} \frac{\partial \psi}{\partial Y} \quad v = \frac{\alpha L}{D^2} \frac{\partial \psi}{\partial X} \quad \dots (6)$$

(1-4)

$$A^2 \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = RaA \frac{\partial \theta}{\partial X} \quad \dots (7)$$

$$\frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} - \frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial X^2} + \frac{1}{A^2} \frac{\partial^2 \theta}{\partial Y^2} + 2 \quad \dots (8)$$

$$Ra = \frac{g\beta KSD^3}{2\nu\alpha k} \quad \dots (9)$$

$$X=0, \quad \psi=0, \quad \frac{\partial \theta}{\partial X}=0 \quad \dots (10)$$

$$X=1, \quad \psi=0, \quad \theta=0 \quad \dots (11)$$

$$Y=0 \text{ and } 1, \quad \psi=0, \quad \frac{\partial \theta}{\partial Y}=0 \text{ or } \theta=D \quad \dots (12)$$

[4] -:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots (1)$$

$$\frac{\partial p}{\partial x} + \frac{\mu}{K}u = 0 \quad \dots (2)$$

$$\frac{\partial p}{\partial y} + \rho g + \frac{\mu}{K}v = 0 \quad \dots (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \quad \dots (4)$$

$$\alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{S}{\rho C}$$

(S)

[4] -:

$$\theta = \frac{(T - T_c)}{SD^2 / 2k} \quad \dots (5)$$

$$a_p \phi_p = a_e \phi_e + a_w \phi_w + a_n \phi_n + a_s \phi_s + Su \quad (16)$$

$$h(T_{\max} - T_c) = -\frac{k}{L} \int_0^L \frac{\partial T}{\partial X} \Big|_{X=D} dy \quad (13)$$

$$(7)$$

$$a_e = \frac{-A^2}{\Delta x^2}$$

$$a_w = \frac{-A^2}{\Delta x^2}$$

$$a_n = -\frac{1}{\Delta y^2}$$

$$a_s = \frac{-1}{\Delta y^2} \quad \dots (17)$$

$$a_p = -\left[\frac{2A^2}{\Delta x^2} + \frac{2}{\Delta y^2}\right]$$

$$Su = RaA(T_e - T_w) / 2 / \Delta x$$

$$(8)$$

$$a_e = \frac{1}{\Delta x^2}$$

$$a_w = \frac{1}{\Delta x^2}$$

$$Nu_{\max} = \frac{2}{\theta_{\max}} \quad \dots (14)$$

$$(T_m)$$

$$Nu_m = \frac{2}{\theta_m} \quad \dots (15)$$

Numerical Solution -3

(Finite Difference)

(Relaxation Method)

Line) (LSOR) ((7)

(Successive Over Relaxation

((8)

: [14]

$$a_n = \frac{1}{A^2 \Delta y^2} \quad \dots (18)$$

(Nu)

[12] .

$$a_s = \frac{1}{A^2 \Delta y^2}$$

:Results and Discussion -4

$$a_p = \left[\frac{2}{\Delta x^2} + \frac{2}{A^2 \Delta y^2} \right]$$

Internal Heat)

$$S_n = -[\psi_e - \psi_w / 2 / \Delta x][T_n - T_s / 2 / \Delta y] \\ + [\psi_n - \psi_s / 2 / \Delta y][T_e - T_w / 2 / \Delta x]$$

: (Generation

($\Delta y, \Delta x$)

.()

(ψ)

($0.5 \leq A \leq 10$)

: (θ)

. ($10 \leq Ra \leq 10^4$)

.(1))

$$\left| \frac{\phi^n - \phi^{n-1}}{\phi^n} \right| < 10^{-4} \quad \dots (19)$$

. : -1

(Ra)

(2)

($Ra < 500$)

($Ra=10$) ($A = 1$)

($Y=0.5$)

(y) (x)

(A)

(Ra)

(1)

()

($Ra = 10$)

$A \leq 1$	31 x 31
$2 \leq A \leq 5$	31 x 121
$A > 5$	31 x 181

(1)

(A)

-4

-5

(Ra=10³)

-3

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(θ_o)

(10)

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(Ra=10³)

.(A = 10) (A = 1)

(A=10) (A=1)

(11)

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(θ_{max})

(Ra)

(θ_{max})

(Adiabatic Horizontal Wall)

(Cold Horizontal Wall)

.(A = 10) (A = 1)

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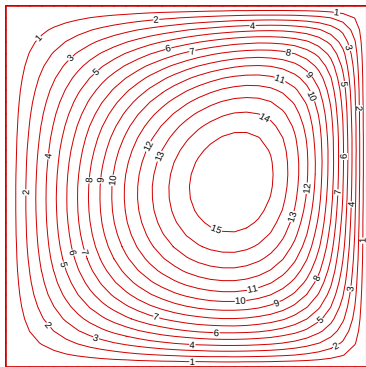
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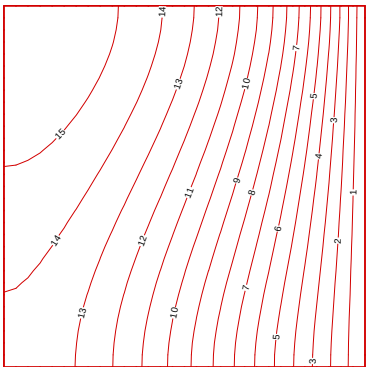
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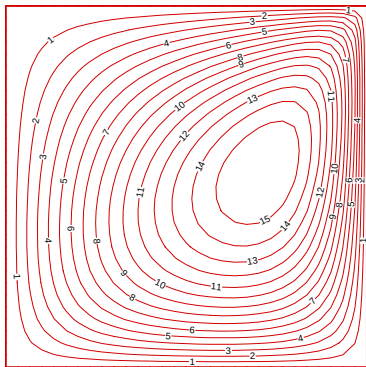
A=1
Ra=10

Level	Stream
15	0.66346
14	0.61923
13	0.574999
12	0.530768
11	0.486538
10	0.442307
9	0.398076
8	0.353846
7	0.309615
6	0.265384
5	0.221153
4	0.176923
3	0.132692
2	0.0884614
1	0.0442307



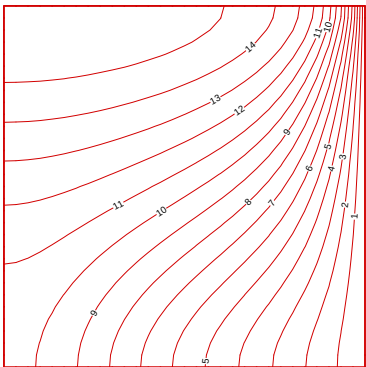
A=1
Ra=10

Level	Temp.
15	0.928082
14	0.86621
13	0.804338
12	0.742465
11	0.680593
10	0.618721
9	0.556849
8	0.494977
7	0.433105
6	0.371233
5	0.309361
4	0.247488
3	0.185616
2	0.123744
1	0.0618721



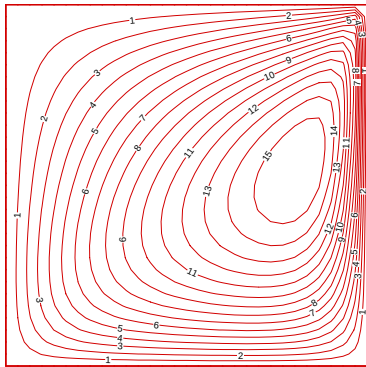
A=1
Ra=100

Level	Stream
15	2.8415
14	2.65207
13	2.46264
12	2.2732
11	2.08377
10	1.89434
9	1.7049
8	1.51547
7	1.32603
6	1.1366
5	0.947168
4	0.757734
3	0.568301
2	0.378867
1	0.189434



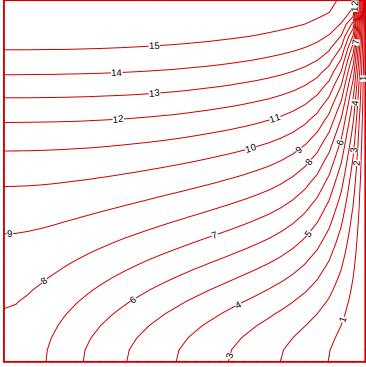
A=1
Ra=100

Level	Temp.
15	0.549821
14	0.513166
13	0.476512
12	0.439857
11	0.403202
10	0.366547
9	0.329893
8	0.293238
7	0.256583
6	0.219928
5	0.183274
4	0.146619
3	0.109964
2	0.0733095
1	0.0366547



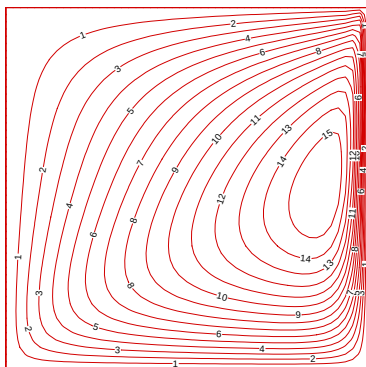
A=1
Ra=1000

Level	Stream
15	7.8361
14	7.3137
13	6.79129
12	6.26888
11	5.74648
10	5.22407
9	4.70166
8	4.17926
7	3.65685
6	3.13444
5	2.61203
4	2.08963
3	1.56722
2	1.04481
1	0.522407



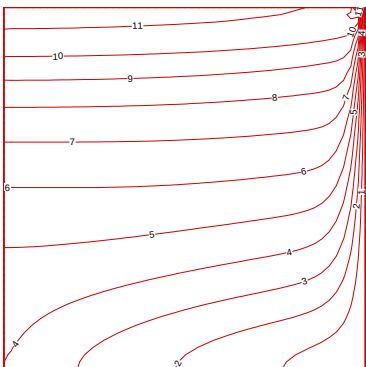
A=1
Ra=1000

Level	Temp.
15	0.252704
14	0.235857
13	0.21901
12	0.202163
11	0.185316
10	0.168469
9	0.151622
8	0.134775
7	0.117928
6	0.101081
5	0.0842345
4	0.0673876
3	0.0505407
2	0.0336938
1	0.0168469



A=1
Ra=10000

Level	Stream
15	19.1252
14	17.8502
13	16.5752
12	15.3002
11	14.0251
10	12.7501
9	11.4751
8	10.2001
7	8.92509
6	7.65008
5	6.37507
4	5.10005
3	3.82504
2	2.55003
1	1.27501



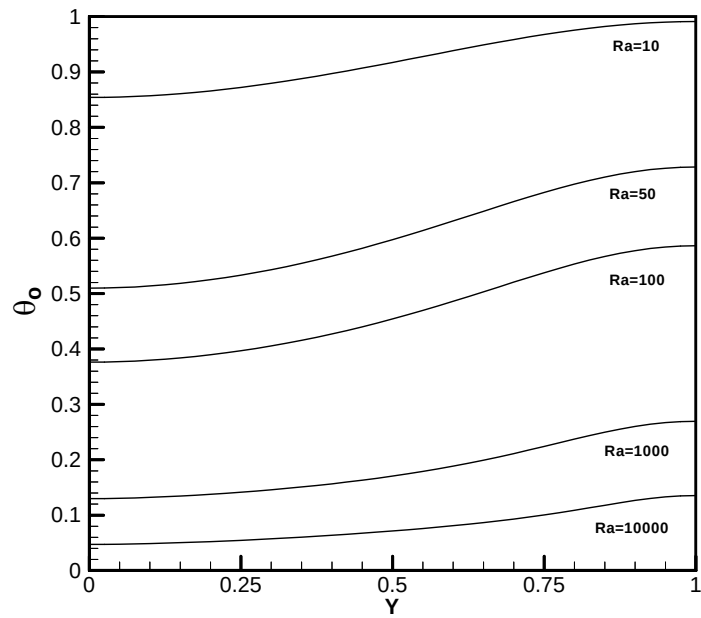
A=1
Ra=10000

Level	Temp.
15	0.170387
14	0.159028
13	0.147669
12	0.13631
11	0.124951
10	0.113591
9	0.102232
8	0.0908732
7	0.079514
6	0.0681549
5	0.0567957
4	0.0454366
3	0.0340774
2	0.0227183
1	0.0113591

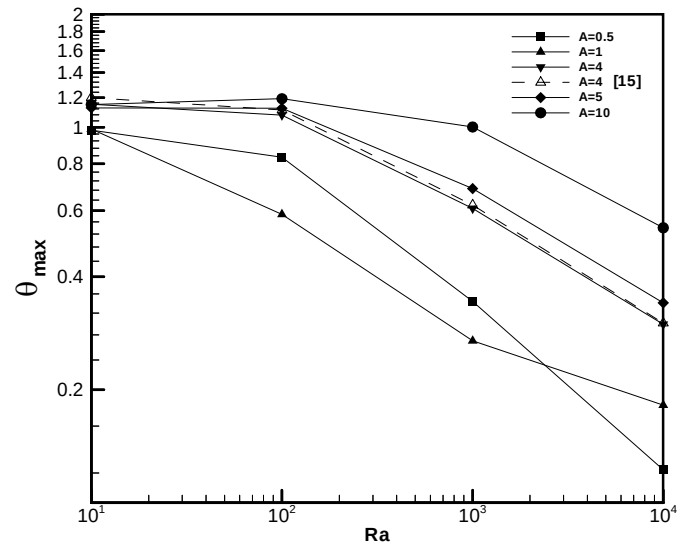
(Ra)

(2)

.()



(Symmetry Line) (θ_0) (3)
 $(A=1)$ (Ra)

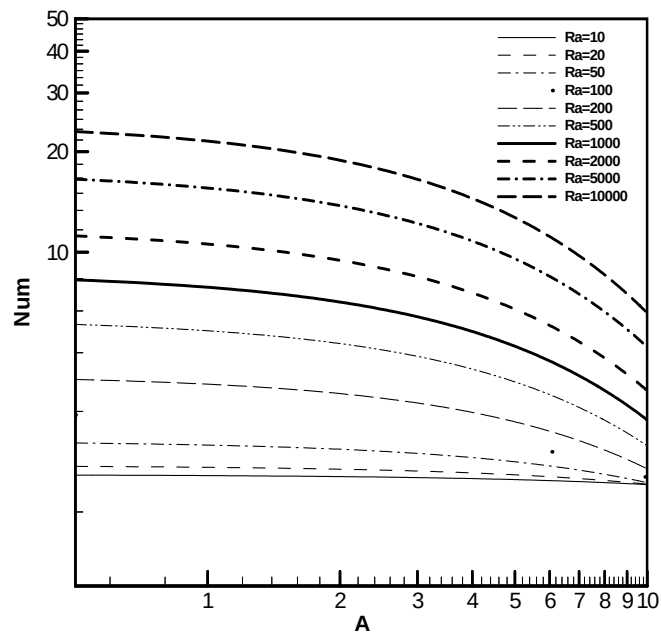


(Ra)

(θ_{max})

(4)

(A)

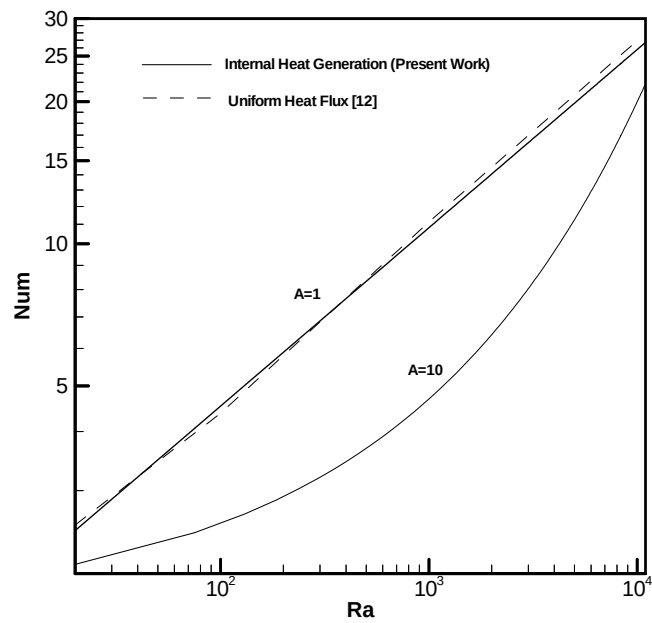


.(Nu_m)

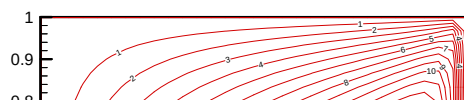
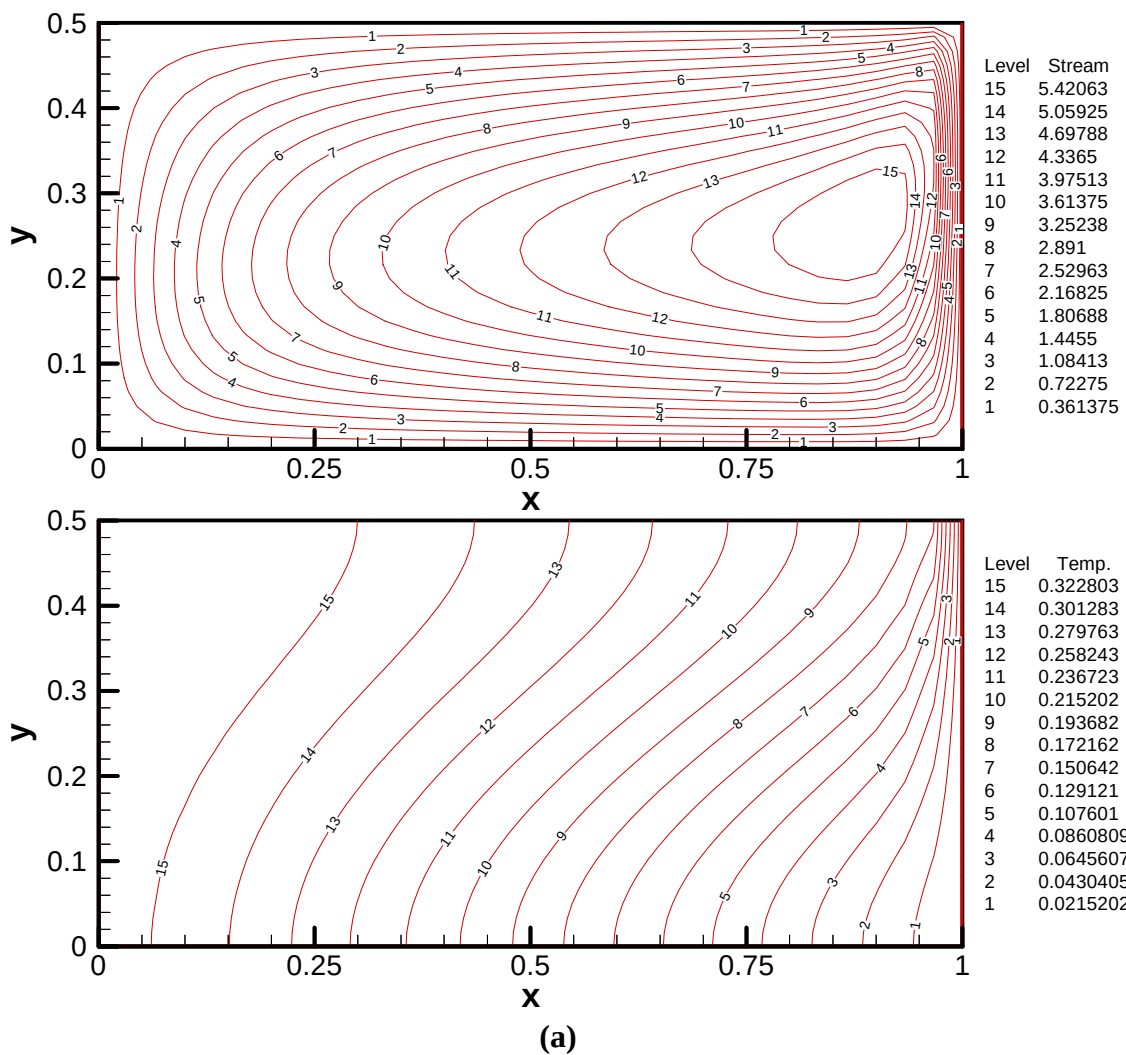
(A)

(Ra)

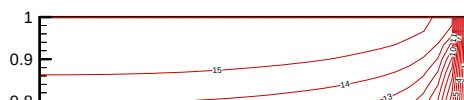
(5)



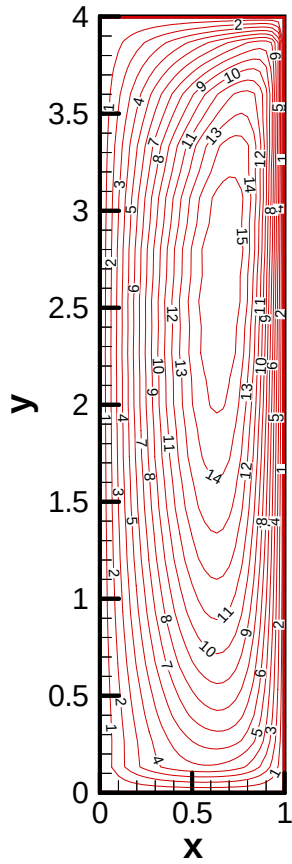
(6) (Nu_m) (Ra) (A=1) (A=10).



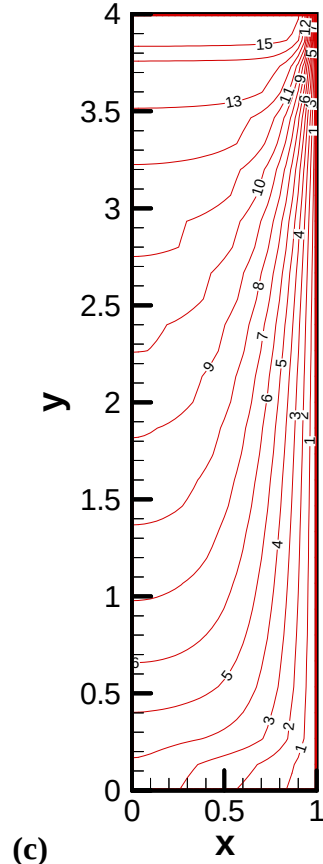
Level Stream



Level Temp

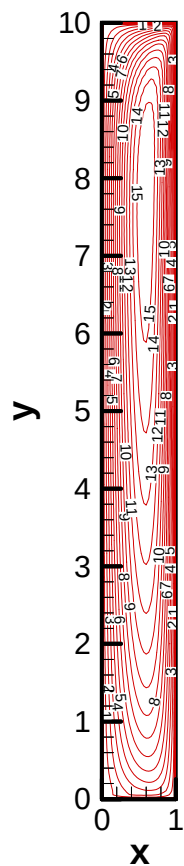


Level	Stream
15	9.98807
14	9.3222
13	8.65633
12	7.99046
11	7.32458
10	6.65871
9	5.99284
8	5.32697
7	4.6611
6	3.99523
5	3.32936
4	2.66349
3	1.99761
2	1.33174
1	0.665871

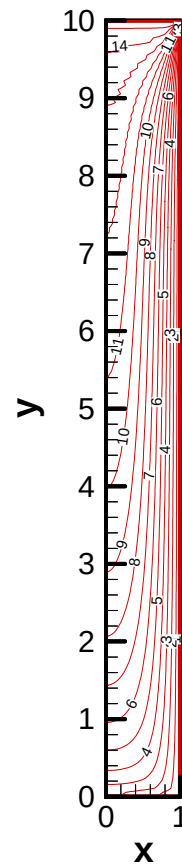


Level	Temp.
15	0.57001
14	0.532009
13	0.494009
12	0.456008
11	0.418007
10	0.380007
9	0.342006
8	0.304005
7	0.266005
6	0.228004
5	0.190003
4	0.152003
3	0.114002
2	0.0760013
1	0.0380007

(c)

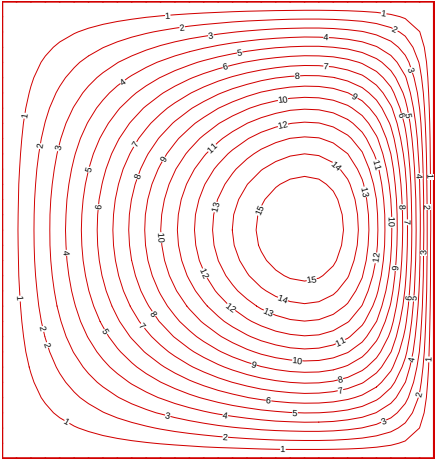


Level	Stream
15	8.86174
14	8.27096
13	7.68018
12	7.08939
11	6.49861
10	5.90783
9	5.31704
8	4.72626
7	4.13548
6	3.5447
5	2.95391
4	2.36313
3	1.77235
2	1.18157
1	0.590783



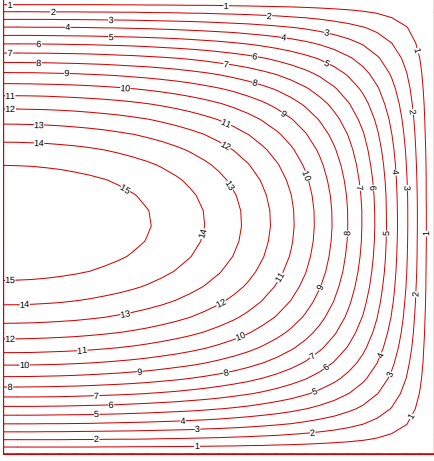
Level	Temp.
15	0.939328
14	0.876706
13	0.814084
12	0.751463
11	0.688841
10	0.626219
9	0.563597
8	0.500975
7	0.438353
6	0.375731
5	0.313109
4	0.250488
3	0.187866
2	0.125244
1	0.0626219

(d)



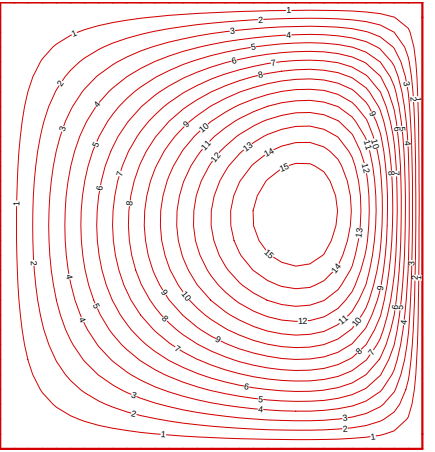
A=1
Ra=10

Level	Stream
15	0.13219
14	0.123378
13	0.114565
12	0.105752
11	0.0969396
10	0.0881269
9	0.0793142
8	0.0705015
7	0.0616888
6	0.0528761
5	0.0440634
4	0.0352507
3	0.0264381
2	0.0176254
1	0.00881269



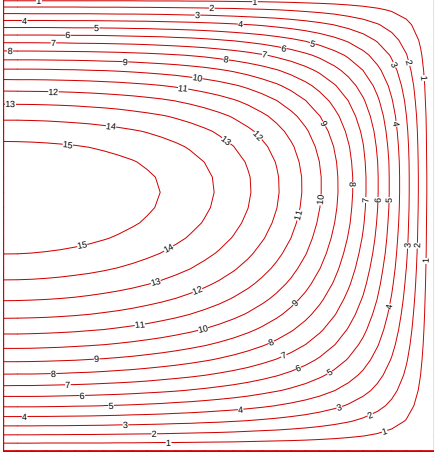
A=1
Ra=10

Level	Temp.
15	0.21326
14	0.199043
13	0.184826
12	0.170608
11	0.156391
10	0.142173
9	0.127956
8	0.113739
7	0.0995214
6	0.0853041
5	0.0710867
4	0.0568694
3	0.042652
2	0.0284347
1	0.0142173



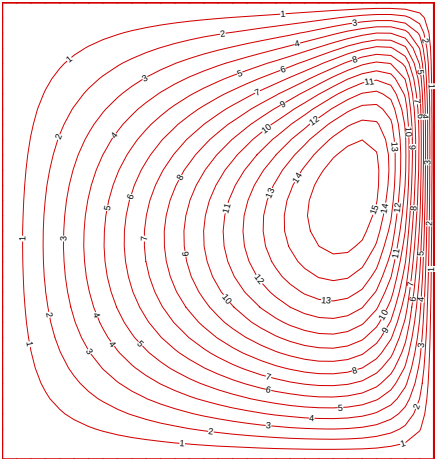
A=1
Ra=100

Level	Stream
15	1.26231
14	1.17815
13	1.094
12	1.00984
11	0.925691
10	0.841537
9	0.757383
8	0.67323
7	0.589076
6	0.504922
5	0.420768
4	0.336615
3	0.252461
2	0.168307
1	0.0841537



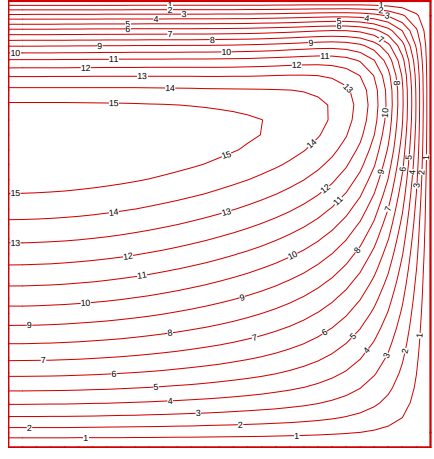
A=1
Ra=100

Level	Temp.
15	0.206776
14	0.192991
13	0.179205
12	0.16542
11	0.151635
10	0.13785
9	0.124065
8	0.11028
7	0.0964953
6	0.0827102
5	0.0689252
4	0.0551401
3	0.0413551
2	0.0275701
1	0.013785



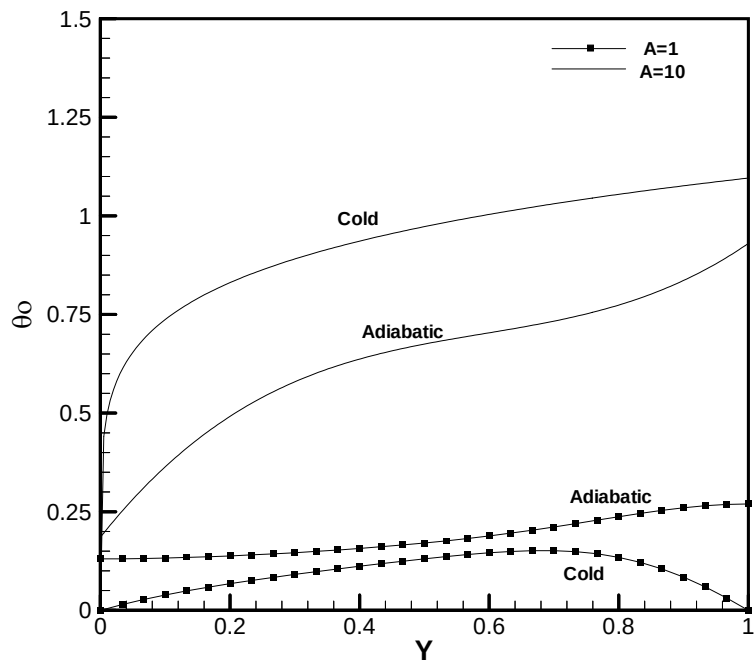
A=1
Ra=1000

Level	Stream
15	6.43742
14	6.00826
13	5.5791
12	5.14994
11	4.72078
10	4.29161
9	3.86245
8	3.43329
7	3.00413
6	2.57497
5	2.14581
4	1.71665
3	1.28748
2	0.858323
1	0.429161



A=1
Ra=1000

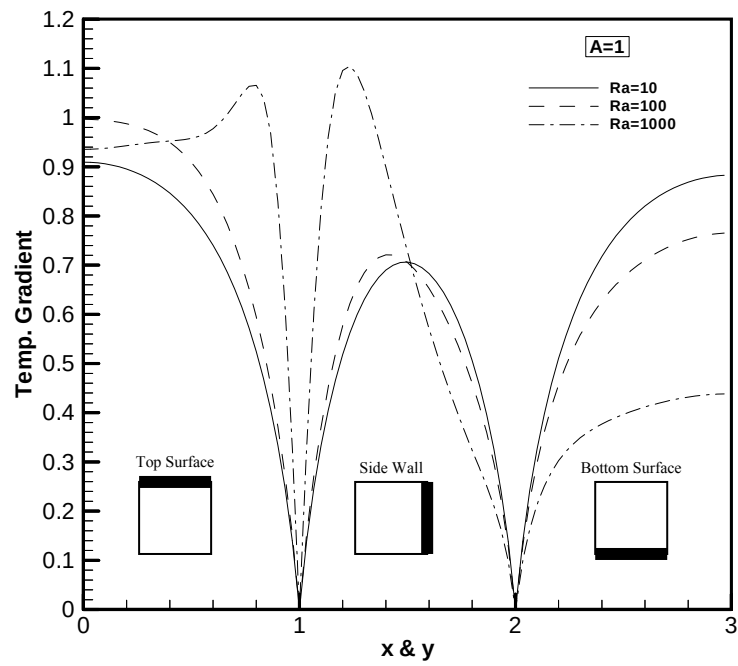
Level	Temp.
15	0.141766
14	0.132315
13	0.122864
12	0.113413
11	0.103961
10	0.0945104
9	0.0850594
8	0.0756084
7	0.0661573
6	0.0567063
5	0.0472552
4	0.0378042
3	0.0283531
2	0.0189021
1	0.00945104



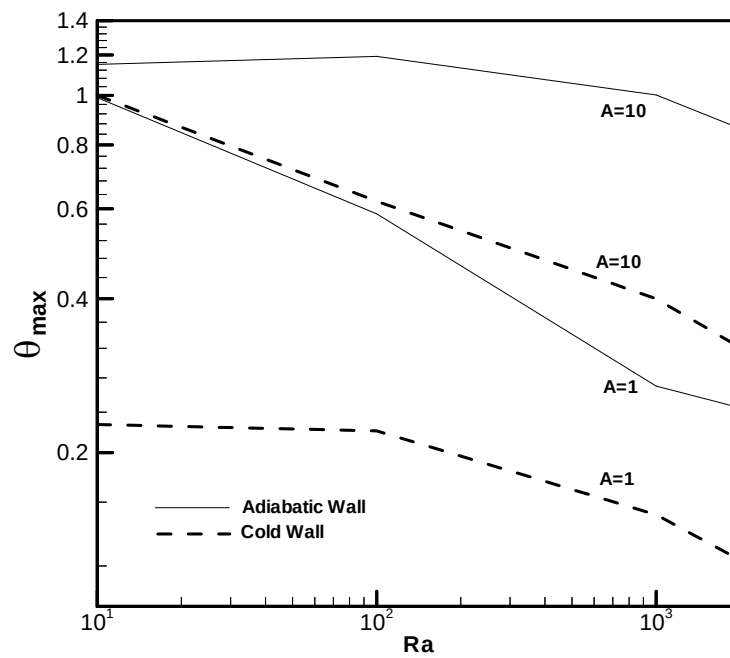
(Symmetry Line)
.(Ra=1000)

(θ_0)
(A)

(10)



(Ra) (A = 1) (9)
 .()



(Ra) (θ_{max}) (11)
 . (A = 10) (A = 1)