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Effect of modulus of subgrade reaction on reinforced concrete deep beams

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Abstract

Deep beams are one of the widest used members in construction due to their high rigidity and significant bending resistance. It used usually at high rise buildings, bridges, deck slabs and foundations. The beams may rest on soil in many cases like strap footing. Timoshenko beam theory and Fourier series were used for derivatizing the behavior of deep beams which rest on soil grade. Many parameters were investigated in the current study like modulus of subgrade reaction, beams dimensions and loading type. It was concluded that, the rigidity of deep beam is high in amounts that cancelling the effect of modulus of subgrade reaction. The beam width increases the bearing pressure of the soil and working on enhancing the stability of the beam and keeps it rest well on the soil so that increasing the width of beam led to minimizing sinking the beam into the soil so that reducing the deflection. Furthermore, increasing the height of the deep beam leads to minimizing the deflection of beam due to rising the shear resistance capacity of the beam which depends in the first degree on the beam thickness.

Keywords: *Deep beams; modulus of subgrade reaction; Winkler model; concentrated and distributed loads.*

1. Introduction

Soil versus structure interaction is one of the widest problems which faced the designers due to the interaction between them. Too many researchers investigated the behavior of soil in receiving loads. When a loaded structural member deflects, it's deflection will apply a pressure on the soil. The soil will react oppositely to produce a continuous distribution pressure to resist beam deflection. previously many theories were presented for modeling the soil whether it was one or two dimensional [1, 2].
The oldest, simplest model was provided by Winkler (1867), who is initially creates it for railroad trucks examination, it was suggested that, the soil (or any other materials even liquids) behaves as sets of neighboring springs pressing vertically to absorb applied loads, and the vertical soil displacement (y) depends only on the external applied load (p) and subgrade modulus response (K) [1, 3].

Where: $P=K*y$ (1)

This model, in intervening days, has been used to represent many soil-structure interactions. Application of this model involves a solution of a fourth-order degree of a differential equation explained at eq.1 and as explained in figure 1.

$$D \nabla^4 y + Ky = q \quad (2)$$

Winkler model is frequently classified as a one-parameter model. This model firstly faced a narrowness application because the difficulty of determining the factor (K). So that, at 1955, Terzaghi offered a formula to identify this absent factor. The introduced formula suggested that the modulus of subgrade reaction relies upon plate's area resting on the soil. While Vesic at 1961 found that, modulus of subgrade reaction based on soil stiffness, like wise structure stiffness. It is worth to mention that, the unbroken researches show that there is an additional factors effect on the value of (k), such as: soil continuum depth, distribution of load, and any layering effects present in the continuum [4].

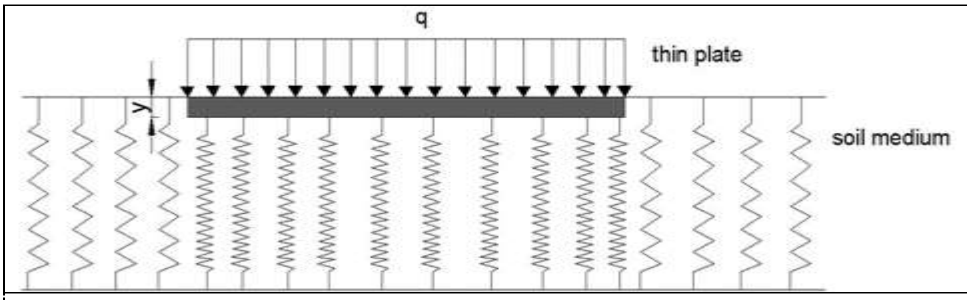

Fig. 1: Depiction of uniformly loaded plate deflection by Winkler model.

This model has some deficiencies. The clearest one is that, Winkler's model shows that the displacement does not extent from the loaded area to the unloaded one. But correctly, the adjoining areas expose to some displacement in the effect of the loaded area [5]. In order to minimize Winkler's model deficiencies and get more accurate results, snag has to be indicated. The snag was that, there is no exactly interaction between the springs (i.e., each spring will behave as a single element). And that which made the displacement in Winkler model does not transform to the neighboring areas. So that, researchers turn to modified the equation in a matter that achieve connection between springs.
Filoneko- Borodich (1940) suggested existing stretched elastic member connect the top face of the springs which expressed by the eq.3.

$$p = kw - T\nabla^2 w \tag{3}$$

Hetėnyi (1950) represented the springs interaction by placing (imbedding) an extra beam has a flexural rigidity D, can deform only in bending. the suggested equation was:

$$p = kw - D * \nabla^2\nabla^2 w \tag{4}$$

Pasternak (1954) approved connecting between the springs by imbedding "shear layer'' which is a vertical incompressible element can only deform laterally. Pasternak equation was as the following [4]:

$$p = kw - G\nabla^2 w \tag{5}$$

Where G: is the foundation shear modulus of elasticity

Valsov-Leontiev [6] in 1966, they supposed a model of two-parameters using the vertical work principle in representing soil continuum. Their model reduced the necessity to determine the factor k, but it increased the poverty for a new parameter γ, which representing the variation in soil deformation. Unfortunately, there is no procedure to compute it.

$$D\nabla^4 w - 2t\nabla^2 w + Kw = q \tag{6}$$

$$\emptyset(y) = \frac{sinh\gamma(H-y)}{sinh\gamma H} \tag{7}$$

Jones and Xenophontos [7] after 17 years, a model introduced by Valsov-Leontiev function in which it used the principle of total strain energy in spite of vertical work in representing soil continuum. It was found a relationship between the ɣ parameter and the structure's vertical displacement resting on the soil. The final equation for their research is: $\phi(y) = e^{-\gamma y}$
Vallaban and Das (1988) introduce model in the name of modified Valsov model. It was detected that the magnitude of ɣ parameter is depending on the ratio between soil stratum depth and beam length. Because of shear parameter existence in this model, the deformation shape will form as circular plate "dish-shape" profile.
Myslecki [8] suggested an approximate method to determine fundamental solution for thin plate on elastic foundation. Several models of foundations (Pasternak, Winkler, elastic half-space) were anatomized. The approximation solutions were obtained through the power analysis series for their Fourier's transform images. In the opposite procedure from images to originals, the renowned solutions of the n-th power of Laplace operator were utilized.
Prakhar Gupta (2015) [9] used Mathlab program to evaluate stresses and displacement of slab on elastic foundation then compared it with Winkler's model. The program gives a good performance reasonable accuracy while using Winkler's model magnitude as 9.25%.
Deep beams are one of the widest members used in high rise buildings, foundations and bridges due to its high rigidity and significant load resistance. The characteristic which helps the deep beam to be stronger than shallow is the high depth of beam in comparing with the wide and length [10–12].
Deep beams at tall buildings (which behaves as a transferring member for the heavy loads and directly bears the force from the upper shear columns or walls), the span-depth ratio has not to be greater than 8 and favourite to range between (3-6) [13]. There are many differences between shallow and deep beams due to the high shear deformation of deep beams and their nonlinearity of straining. So, the conventional elastic beam theory (Bernoulli beam theory) cannot be applicable and have to be replace by Timoshenko beam theory [5]. The simply supported deep beams may satisfy one of the following

conditions so as to behave as a deep beam which are: clear span to total depth less than or equal to four for distributed load (less than or equal to two for point load), and the concentrating load applies at (a) distance within 2h form the support face [1]. The critical section of shear is of (a/h ≤2) where a is the distance from concentrated load to middle line of supporting and d is the total beam depth without cover size [1].
Theoretically, when load applied, the stresses in beams chose the shortest inclined path from load to support. That path can be considered as an interior column impeded in the beam due to large compression stress applied on. So, it will form the bottle shape. The small columns in both sides tries in vainly to push up the support. Thereby, tensile stresses generated parallel to steel bars. Concrete beep beams divide into to general imagination regions (D and B regions) with respect to stresses distribution. D region (which is refers to discontinuity) concentrates near support, load and every sudden change in member cross sectional area. In another word, De st. Venent's theory applied in this region. Which suggested that, the localized effect of concentrated load will disappear about one member depth from load to support. For D region, since the stresses distribution is non-linear then Bernoulli equation theory un-applicable. While B region is the other parts of beam in while the strain develops linearly, that will make Bernoulli theory valid for them [7–11].
Many researchers investigated experimentally and numerically the effect of slab on soil grade like [14–17], beams on elastic foundation [18, 19] and it was concluded from these previous researches that, the magnitude of soil bearing capacity has a significant impact on the overall slab strength. So, this article investigated theoretically the effect of modulus of subgrade reaction on the structural members.

2. Theoretical solution

Fourier series is one of the most theoretical equations which simulates beams and plates deflections because its periodically repeated with time and similar to the deflection curve in case of using sine function. Deep beam member (Figure.2) under point load and distributed load individually has been discussed. Timoshenko beam theory was used for the theoretical analysis). Timoshenko beam theory [20] has been used which satisfied the requirement of deep beams, which are:

- The plane section does not remain plane after bending.
- The normal to the neutral plane after bending will not remain normal to the neutral but have an additional rotation due to high transverse shear deformation.
- Neglecting normal strain along the width.

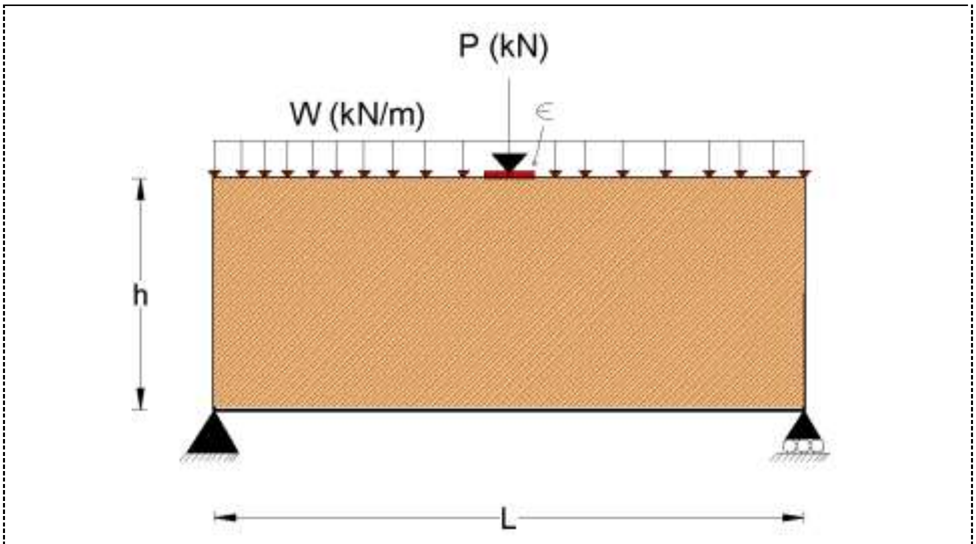

Fig. 2: Deep beam details.

2.1. Deep beam subjected to a point load

To derive the deflection equation of deep beam, several assumptions have to be fixed.
Let $q = \frac{p}{\epsilon}$ and ∈ approaches to be equals zero, and assuming that

$$q(x) = \sum_{m=1}^{\infty} q_m * \sin\frac{m\pi x}{L} \tag{8}$$

$$w(x) = \sum_{m=1}^{\infty} w_m * \sin\frac{m\pi x}{L} \tag{9}$$

$$\frac{d\psi}{dx} = \sum_{m=1}^{\infty} \psi_m * \sin\frac{m\pi x}{L} \tag{10}$$

This assumption is satisfying the boundary conditions of the deep beams which states that the shear at x=0, L equals zero. The governing equation of Timoshenko deep beam theory as following:

$$\frac{d^3\psi}{dx^3} = \frac{q - kw}{EI} \tag{11}$$

$$\frac{d^4w}{dx^4} = \frac{q - kw}{EI} - \frac{1}{c^2GA}\frac{d^2(q - kw)}{dx^2} \tag{12}$$

$$q_m = \frac{2}{L}\int_{L/2}^{\frac{L}{2}+\epsilon} \frac{P}{\epsilon} * \sin\frac{m\pi x}{L} = \frac{2P}{m\pi \in}\left[-\cos\frac{m\pi x}{L}\right]_{\frac{L}{2}}^{\frac{L}{2}+\epsilon} = \frac{2P}{L}\sin\frac{m\pi}{2} \tag{13}$$

Therefore, $q(x) = \sum_{m=1}^{\infty} \frac{2P}{L}\sin\frac{m\pi}{2} * \sin\frac{m\pi x}{L}$ (14)

$$(q - kw) = \sum_{m=1}^{\infty}\left(\frac{2P}{L}\sin\frac{m\pi}{2} - kw_m\right) * \sin\frac{m\pi x}{L} \tag{15}$$

$$\frac{d}{dx}(q - kw) = \sum_{m=1}^{\infty}\frac{m\pi}{L}\left(\frac{2P}{L}\sin\frac{m\pi}{2} - kw_m\right) * \cos\frac{m\pi x}{L} \tag{16}$$

$$\frac{d^2}{dx^2}(q - kw) = \sum_{m=1}^{\infty}\frac{-(m\pi)^2}{L^2}\left(\frac{2P}{L}\sin\frac{m\pi}{2} - kw_m\right) * \sin\frac{m\pi x}{L} \tag{17}$$

By substituting the pervious equations by the governing equation, gets:

$$w(x) = \sum_{m=1}^{\infty}\frac{2P(\frac{1}{EIL} + \frac{(m\pi)^2}{c^2GAL^2}}{(\frac{m\pi}{L})^4 + K(\frac{1}{EI} + \frac{(m\pi)^2}{c^2GAL^2})} * \sin\frac{m\pi x}{L} \tag{18}$$

2.2. Deep beam subjected to a distributed load

The governing equations which the solution will be based on, are listed at equations (), which are satisfy the boundary conditions of the deep beam, the deflection at x=0, L equals zero.

$$q(x) = \sum_{m=1}^{\infty} q_m * \sin\frac{m\pi x}{L} \tag{19}$$

$$w(x) = \sum_{m=1}^{\infty} w_m * \sin\frac{m\pi x}{L} \tag{20}$$

$$\frac{d\psi}{dx} = \sum_{m=1}^{\infty} \psi_m * \sin\frac{m\pi x}{L} \tag{21}$$

$$\frac{d^3\psi}{dx^3} = \frac{q - kw}{EI} \tag{22}$$

$$\frac{d^4w}{dx^4} = \frac{q - kw}{EI} - \frac{1}{c^2GA}\frac{d^2(q - kw)}{dx^2} \tag{23}$$

$$q(x) = \sum_{m=1}^{\infty}\frac{4q}{m\pi} * \sin\frac{m\pi x}{L} \tag{24}$$

$$w(x) = \sum_{m=1}^{\infty}\frac{4q\ (\frac{1}{EI} + \frac{(m\pi)^2}{c^2GAL^2}}{m\pi(\frac{m\pi}{L})^4 + K(\frac{1}{EI} + \frac{(m\pi)^2}{c^2GAL^2})} * \sin\frac{m\pi x}{L} \tag{25}$$

3. Results and discussions

3.1. Deep beam exposed to a point load

Deep beam model was solved using Fourier series model with a totally length equals 1 m, compressive strength 25MPa, subjected to a point load equals 10 kN and passion’s ratio equals 0.2. several parameters were discussed like the modulus of subgrade reaction, beam thickness and beam width.

It was concluded from results at Table.1 that, the modulus of subgrade reaction has no significant effect on the beam deflection which means that, the beam is strong enough to be not affected by how the soil was strong. Also, increasing the height of the deep beam leads to minimizing the deflection of beam due to rising the shear resistance capacity of the beam which depends in the first degree on the beam thickness (as mentioned at Table 2.). When comparing between the previous literature, it could be noting that, the results matches with the reference [21].

Table 3. investigated the effect of increasing beam width. It could be concluded that, the wider beam gives little deflection due to distributing the load over a large area of soil which prevent the beam to sink into the soil significantly.

Table 1: Impact of modulus of subgrade reaction

Deep beam 100*400 section size						
K (KN/m^3)	320000	128000	80000	24000	12000	4800
deflection	0.017021	0.017021	0.017021	0.017021	0.017021	0.017021

Table 2: Influence of increasing beam depth

Deep beam (beam width=100 and K=128000)				
h	500	400	350	600
deflection	0.051064	0.06383	0.072948	0.042553

Table 3: Effect of increasing beam wide on the deflection values

Deep beam (h=400 mm)				
b	50	100	150	200
deflection	0.12766	0.06383	0.042553	0.031915

3.2. Distributed load subjected on a deep beam

After solving the derived equations related with the deep beam under distributed load, it can be concluded from the solutions (Table.4 and Table .5) that, the modulus of subgrade reaction has no effect on the beam deflection due to the high rigidity of the beam, also, increasing the beam depth leads to minimizing the displacement due to that, after each incrementing in beam depth the bending capacity decreases while the shear capacity rises. It could be also concluded from the results (shown in Table.6) that; beam width has a significant effect on the deflection.

Table 4: Impact of modulus of subgrade reaction

Deep beam 100*400 section size						
K (KN/m3)	320000	128000	80000	24000	12000	4800
deflection	1.485	1.485	1.485	1.485	1.485	1.485

Table 5: Influence of increasing beam depth

Deep beam (beam width=100 and K=128000)				
h	500	400	350	600
deflection	0.887168	1.485319	2.063242	0.603

Table 6: Effect of increasing beam wide on the deflection values

Deep beam (h=400 mm)				
b	50	100	150	200
deflection	2.970496	1.485319	0.990228	0.742677

3.3. Impact of concrete compressive strength

As expected, for both concentrated and distributed loads, enhancing concrete compressive strength led to minimizing the deep beam deflection due to increasing the strut strength which is considered as the bath of stresses that crosses through.
The minimizing in deflection (due to rising *f'c*) seems to be noticeable for concentrated load more than the distributed load model which means that the shear stresses in the point load model were too concentrated and heavy on the strut and such rising in strength worked on avoiding sinking the model into the soil. In another word, if the beam was rigid enough, then it will not deflect a lot under the subjected load and will resist the load by itself without depending on soil strength. The increasing in concrete compressive strength and its effect on minimizing the deflection was also proved by literature investigations [21]. Noting that, the results which listed in Figure 3 are for modulus of subgrade reaction equals 128000 kN/m which means that the soil type is crash stone with soil [3].
The reduction in deflection behaves linearly for the both models in a confidence coefficient equals 96%. The equations of reduction were listed within Figure 3.

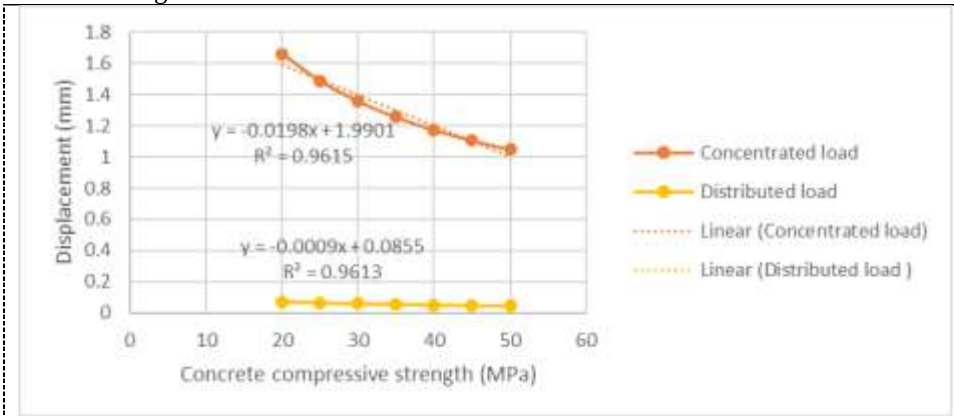

Fig. 3: Compressive strength influence of the deep beam behavior.

4. Conclusion

The study discussed the deflection of deep beams which rests on soil grade under point and distributed loads. Timoshenko beam theory was used for calculating the model displacement. After deriving and solving the equations it was concluded that, the rigidity of deep beam is high in amounts that cancelling the effect of modulus of subgrade reaction. The beam width increases the bearing pressure of the soil, working on enhancing the stability of the beam and keeps it rest well on the soil. And because of that, when studying varying width sizes, it was noted that, increasing the width of beam led to minimizing sinking the beam into the soil which reduces the deflection. Furthermore, increasing the height of the deep beam leads to minimizing the deflection of beam due to rising the shear resistance capacity of the beam which depends in the first degree on the beam thickness. Also, it was concluded that, if the beam was rigid enough, then it will not deflect a lot under the subjected load and will resist the load by itself without depending on soil strength.

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