Study the Effect of Bearing Type on the Whirl Orbit And Beating Phenomena of Dual Rotor

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Abstract:-

This research presents a study of the effect of bearing type on the whirl orbit. number of loops and the period of beating phenomena of the dual rotor running with different speeds Two type of hearing had been studied one with general case of support and intershaft bearing stiffness represented as springs. While the other one is a bearing with fluid film represented as spring and damper.

Lagrangin equation used to have the matrix of rotor element with (33*33) bound.and the transfer matrix method used to get the point. + felid.bearing matrix and the state vector which connected the ends of rotor elements in the equation of motion.

Mathlap program had been used to solve the matrix and draw the results with takplot program. for verification purposes with another program written in Fortran (77) which used in axis research. a greet mach take place as shown in tables.

Introduction:-

In 1965 [2] Guyan, R.J., presented a research about analysis of rotors by Finite element and at last had stiffness and mass matrices.

And Zienkiewicz [3] reached the complete matrices for stiffness and mass matrices and state victor for baranched systems (Gear pair in the drive system).

In 1964 [4] Nelson and Me Vaugh,gave an example of multi-stepped hallow rotor with Transfer matrix method and calculate the whirl speed for single shaft.

Shiau and Wang 1966 [5] determined the un damped

whirl speed of the Nelson and Me Vaughs rotor as afunction of the spin speed in accordance to the Finite element method.

The Analysis Method:-

The fig. (1) below represents the model of rotor element with two nods, there are (6)degree of freedom (3) for each nod for translational motion and (3) for rotational motion along three perpendicular axis, the displacement at the two end of element can be represented as:-

$$\{d\}^T = \{u_1 \quad v_1 \quad w_1 \quad \alpha_1 \quad \theta_1 \quad \Phi_1 \quad u_2 \quad v_2 \quad w_2 \quad \alpha_2 \quad \theta_2 \quad \Phi_2\} \quad \ldots\ldots (1)$$

where:-

$\left.\begin{matrix} u \\ v \\ w \end{matrix}\right\}$ Translation motion along X.Y.Z

$\left.\begin{matrix} \alpha \\ \theta \\ \Phi \end{matrix}\right\}$ Rotational motion along X.Y.Z

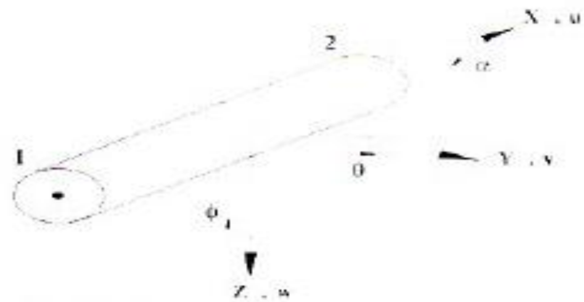


Fig.(1) the model of rotor element

The Axial displacement:-

u (s,t)] { }= Nu d

. . . . (2-a)

Torsional displacement:-

α(s,t) [ ]= { } Nα d

. . . . . (2-b)

Translation displacement:-

{ v(s,t) w (s,t) } = [Nt] { d }

. . . . . (2-c)

The relation between the rotational and translation motion is:

$$\theta = -\frac{\partial V}{\partial S} \qquad \Phi = \frac{\partial W}{\partial S}$$

Using the Potential energy equation for small length of rotor element:-

$$d(P.E) = \frac{1}{2}\begin{Bmatrix} u \\ \alpha \end{Bmatrix}^{T}\begin{bmatrix} AE & 0 \\ 0 & GJ \end{bmatrix}\begin{Bmatrix} u \\ \alpha \end{Bmatrix} ds + \frac{1}{2}\begin{Bmatrix} v \\ w \end{Bmatrix} ds$$

. . . . (3-a)

The Kinetic energy :

$$d(K.E) = \frac{1}{2}\begin{Bmatrix} u \\ \alpha \end{Bmatrix}^{T}\begin{bmatrix} \mu & 0 \\ O & \zeta P \end{bmatrix}\begin{Bmatrix} u \\ \alpha \end{Bmatrix} ds + \frac{1}{2}\begin{Bmatrix} v \\ w \end{Bmatrix}\begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}\begin{Bmatrix} v \\ w \end{Bmatrix} ds + \frac{1}{2}\begin{Bmatrix} \omega a \\ \omega b \\ \omega c \end{Bmatrix}^{T}$$

$$\begin{bmatrix} \zeta P & 0 & 0 \\ 0 & \zeta P & 0 \\ 0 & 0 & \zeta P \end{bmatrix}\begin{Bmatrix} \omega a \\ \omega b \\ \omega c \end{Bmatrix} ds$$

. . . . (3-b)

And by substited the equivalent quintets and integrated along the rotor element , we have:

$$P.E = \frac{1}{2}\{d\}^T([Ku] + [K\alpha] + [Kb])\{d\}$$

$$\text{.......(4-a)} \quad K.E = \frac{1}{2} d^T ([Mu] + [M\alpha] + [Mt] + \{\ \}$$

$$[Mr]) d^T + \frac{1}{2} I_p \omega^2 + \omega d^T [R] d$$

.......(4-b)

Substited in Lagrangin equation:-

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial\{d\}}\right)_i - \left(\frac{\partial K.E}{\partial\{d\}}\right)_i - \left(\frac{\partial P.E}{\partial\{d\}}\right)_i = \{P\}$$

........(5-a)

$$[M]\{\ddot{d}\}_i - \omega[G]\{\dot{d}\}_i + [K]\{d\}_i = \{P\}_i$$

........(5-b)

Where:

$$[K] = [Ku] + [K\alpha] + [Kb]$$

$$[G] = [R] - [R]^T$$

........(6)

$$[M] = [Mu] + [M\alpha] + [Mt] + [Mr]$$

In the same way study the axial force and torsion ,the equation of motion became :-

$$[M]_i\{\ddot{d}\} - [G]_i\{\dot{d}\} + [K]_i = \{P\}_i \qquad \ldots\ldots\ldots(7)$$

Whirl Speed Analysis :-

The whirl speeds can be determined from the solution of eigen value problem resulting from the assembly of all elemental equations, which gives(6):-

[M]([B]-[G]){d} + [K] = {P};

To determine the eigen values of the above problem, we rearrange it as:-

$$[E]\{\dot{y}\} + [F]\{y\} = 0 \qquad \ldots\ldots\ldots(8)$$

where:

$$[E] = \begin{bmatrix} [0] & -[M] \\ [M] & ([C]-[G]) \end{bmatrix}$$

$$[F] = \begin{bmatrix} [M] & [0] \\ [0] & [K] \end{bmatrix}$$

$$\{y\} = \begin{Bmatrix} \{\dot{d}\} \\ \{d\} \end{Bmatrix}$$

The eigen values are complex and in the form:

For undamped systems, the eigen values are purely imaginary. They can be positive or negative positive root indicates that the whirl in the same direction as rotational or spin speed (forward whirl). A negative value of the whirl frequency indicates that the rotor whirl in direction opposite to that of the spin speed (back ward whirl).

Because of the presence of gyroscopic effect the whirl speeds is a function of the rotational or spin speed of the shaft (7).

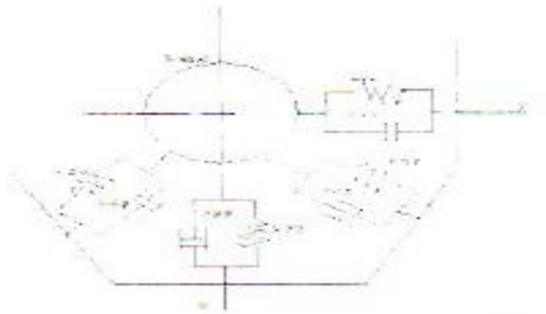
Hydrodynamic Bearings:-

Because of the importunate of the fluid film bearing, we will show the analysis of this type.

Fluid Film Bearing:-

It is commonly use in heavy rotating machines play a significant role in the dynamic behavior of rotors. The thin film that separates the moving surfaces supports the rotor load, The fluid film separates the moving surfaces supports the rotor load, these bearing represents like aspring and damping due to squeeze film effect,fig.(2) shows stiffness and damping(8).

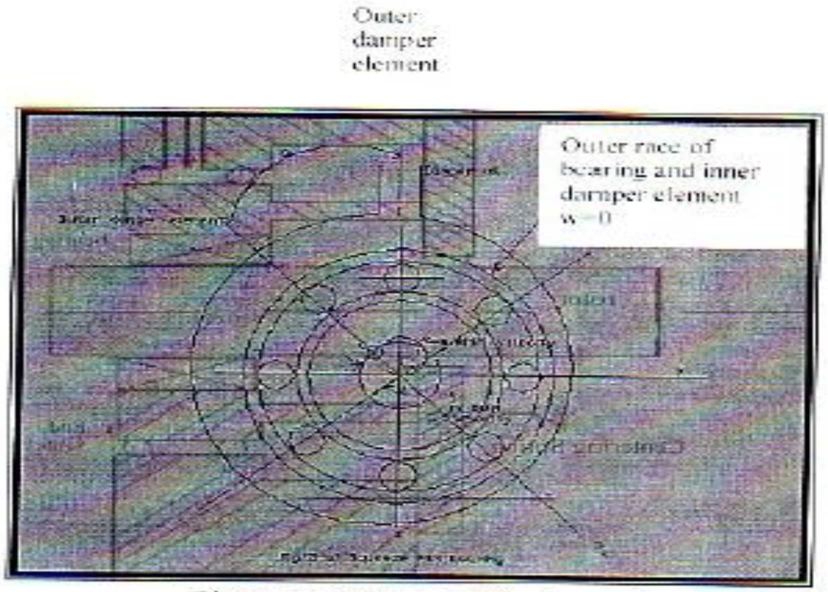
These films significantly alter the critical speeds and out -of balance response of rotors. Similar to the rotor the bearing are fluid film seals which also can effect the dynamic behavior of rotor in pumps further, squeeze film bearing can be employed to limit the while amplitudes of rotors(9).



Fig(2) Stiffness and Damping coefficients
Of fluid film bearing

Squeeze Film Bearings with Orbital motion:-

Amajor application of squeeze film dampers is in the aircraft engine rotors. These rotor are usually supported on roller bearings which offer very little damping. To fight the unbalance film dampers are employed. The are also used to eliminate rotor dynamic instabilities. Fig.(3-a) show the outer race of the bearing (inner damper element) is restring from rotation tabs or keys or by employing a soft centering spring support as in fig.(3-b). fluid film separates the inner and outer elements of the damper. The inner damper element doesn't rotate, therefore, the destabilizing effects that are associated with cross coupled stiffness of the hydrodynamic films are elimenated.Orbital motion is allowed such that ? is equal to the synchronous whirl velocity .



Fig(3-a) Squeeze Flim bearing

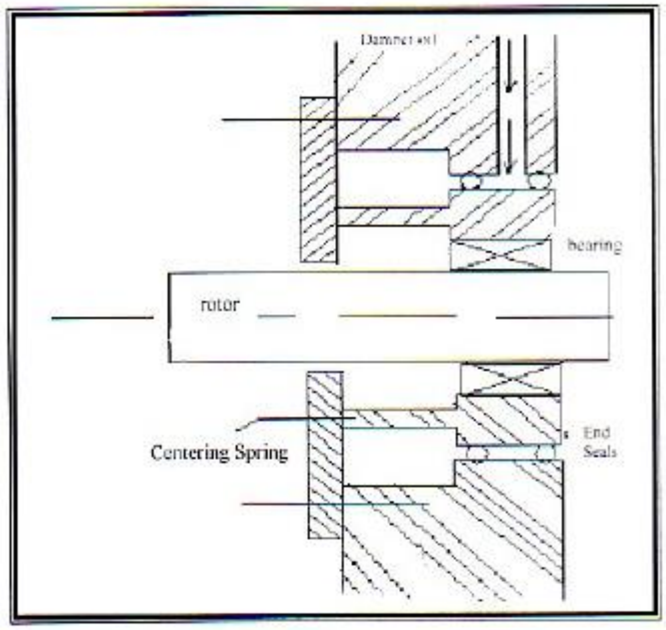


Fig (3-b) Typical squeeze film damper arrangement
with centering spring

Dual Rotor System Analysis:-

With ever increasing demand of larger power and smaller size gas turbine engines for aircraft propulsion.a two spool system with intershaft bearings is becoming a standard layout to accommodate to compressor and turbine rotors.

This sort of layout minimizes shafi deflections caused by rotor unbalance and improves the engine efficiency.

A dual rotor consists of two coaxial rotors, fig.(4) show the inner and outer rotors running at different speeds. Interlined through an intershaft bearing.

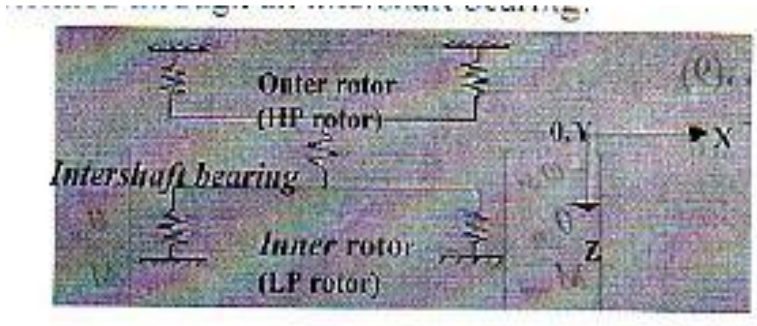


fig.(4) The inner and outer rotors

In this system. The cross-exciting vibration between the inner and outer shafts is effected through the inter shaft by bearing. Thus the dynamic response of each rotor not only depends upon its own unbalance.but also upon the dynamic behavior to the other. And this is what make the dynamic analysis of dual rotor system different from that of a straight rotor.so, the state vector consists of:-

1. The Sine and Cosine components of each state vector quantity, viz, deflection, slope,bending moment and shear force in both X-Z & X-Y planes.

2. Each quantity of the state vector have two components,one to the inner rotor speed and the other to the outer rotor speed (10).

For plane-Z

1- (state vector) contain 33 quantities given in the below:-

$$\{S\} = \begin{Bmatrix} \{S_{2n}\} \\ \{S_{2r}\} \\ \{S_{3n}\} \\ \{S_{3r}\} \\ \vdots \end{Bmatrix}$$

. . . .(9)

where:-

$$\{S_{2n}\} = \begin{Bmatrix} -\omega_{cn} \\ \theta_{cn} \\ M_{ycn} \\ V_{zcn} \\ -\omega_{sn} \\ \theta_{sn} \\ M_{ysn} \\ V_{zsn} \end{Bmatrix} , \quad \{S_{2r}\} = \begin{Bmatrix} -\omega_{cr} \\ \theta_{cr} \\ M_{ycr} \\ V_{zcr} \\ -\omega_{sr} \\ \theta_{sr} \\ M_{ysr} \\ V_{zsr} \end{Bmatrix}$$

$$\{S_{3n}\} = \begin{Bmatrix} v_{cn} \\ \phi_{cn} \\ M_{zcn} \\ -V_{ycn} \\ v_{sn} \\ \phi_{sn} \\ M_{zsn} \\ -V_{ysn} \end{Bmatrix} , \quad \{S_{3r}\} = \begin{Bmatrix} v_{cr} \\ \phi_{cr} \\ M_{zcr} \\ -V_{ycr} \\ v_{sr} \\ \phi_{sr} \\ M_{zsr} \\ -V_{ysr} \end{Bmatrix}$$

2- Field matrix:-

$$[F]_{(33*33)} = \begin{bmatrix} [F] & & & & & & & \\ 0 & [F] & & & O & & & \\ 0 & 0 & [F] & & & & & \\ 0 & 0 & 0 & [F] & & & & \\ 0 & 0 & 0 & 0 & [F] & & & \\ 0 & 0 & 0 & 0 & 0 & [F] & & \\ 0 & 0 & 0 & 0 & 0 & 0 & [F] & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & [F] \end{bmatrix}$$

…………(10)

3- Point matrix:-

$$[P]_{(33*33)} = \begin{bmatrix} [P_w] & & & O & \\ 0 & [P_u] & & & \\ 0 & 0 & [P_u] & & \\ 0 & 0 & 0 & [P_u] & \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

…………(11)

where:-

$$[P_w]_{8*8} = \begin{bmatrix} [p] & 0 \\ 0 & [p] \end{bmatrix}_{p=ww}$$

$$[P_u]_{8*8} = \begin{bmatrix} [p] & 0 \\ 0 & [p] \end{bmatrix}_{p=uw}$$

4- Bearing matrix:-

In similar manner, it can be shown that

$$[B]_{33*33}=\begin{bmatrix} [B_{xz1}] & 0 & [B_{yz1}] & 0 & 0 \\ 0 & [B_{yz1}] & 0 & [B_{xz2}] & 0 \\ [B_{\theta z2}] & 0 & [B_{xz1}] & 0 & 0 \\ 0 & [B_{\theta z2}] & 0 & [B_{\theta z1}] & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

…………….(12)

<u>Results and Discussion</u>

Two kinds of dual rotor system are tested, and study the bearing type effect on the whirl orbit, make acompaer with finite element method solution only, shown in table bellow.

1-First system:-

Fig.(5) shown the model of dual rotor system mounted on isotropic bearings. The charistriez of this system showed in table (1)[1].

Fig (5)
Modeling of dual rotor system (1)

Modeling		Unit
The charis tiestic	The value	
Outer diameter	1.7	Cm
Inner diameter	1.5	Cm
Young's modulus	210 e09	N/m^2
Un balance on each mass	0.001	Kg.m
Outer speed	1000	Rpm
Inner speed	1200	Rpm
Stiffness of support/$K_{support}$	10^6	N/m
Stiffness of shaft/K_{shaft}	10^6	N/m
Intershaft bearing stiffness	0	N/m
Type of four supports	Rigid	
Mass for each disk	5	Kg

Table(1) The charistricz of the system(1)

Results & Discussion of system No.(1)

The response of either rotor will be affected by the unbalance of it's own as well as the other rotor Therefore, the total response at any station is the sum of two harmonic components. This is analogoue to the case of superposition of two different harmonic signals giving rise to periodic beating phenomenon.

For dual rotor the unbalance response component belonging to the inner and outer rotors running at different speeds. The motion will be periodic and if the speed of both rotors are close to each other, beating should be observed.

The periodic of such a beating phenomenon is calculated from (9):-

$$T = \frac{2\pi}{\frac{\omega_o - \omega_i}{p-q}} \qquad \ldots\ldots\ldots$$

..(13)

p & q are smallest integers:

$$\frac{p}{q} = \frac{\omega_o}{\omega_i} \qquad \ldots\ldots$$

..(13)

p & q are smallest integers:

$$\frac{p}{q} = \frac{\omega_o}{\omega_i} \qquad \ldots\ldots$$

.... (14)

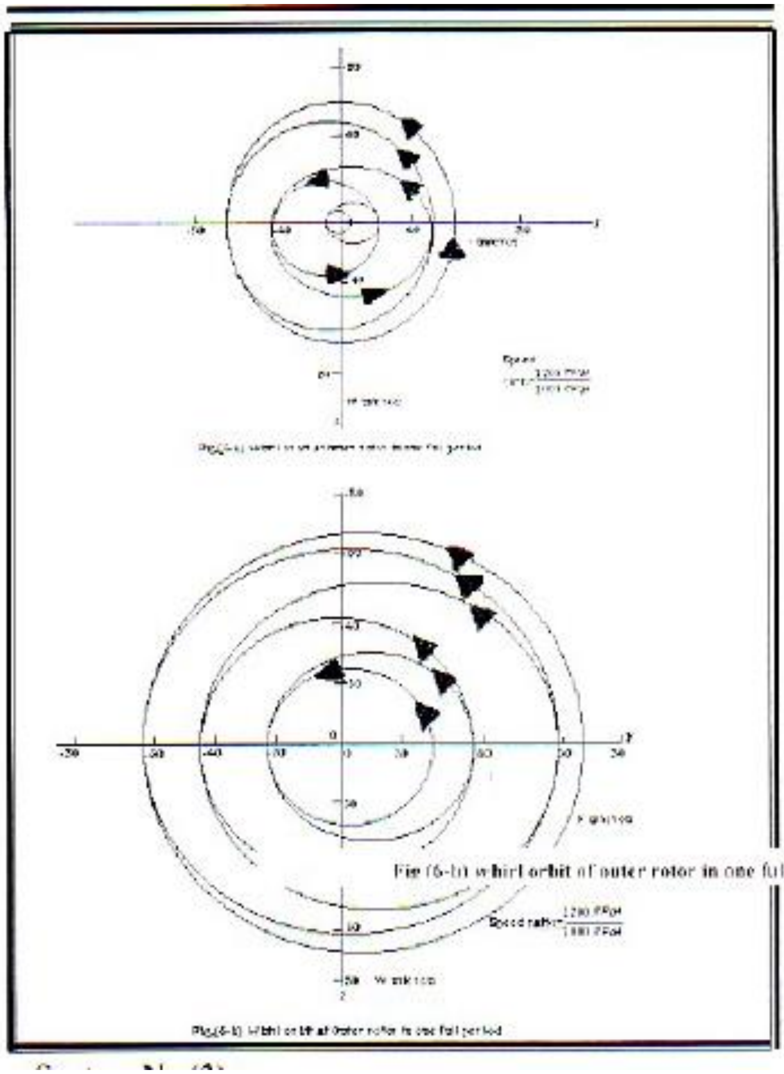
The superposition of response components (w,v) will amount to the combination of two distant orbital motions of different angluar frequencies. The resulting motion will not describe any sigle closed orbit:but instead will exhibit periodicity whose period is given in equ.(13)

Table (2) show the result of system(1).

System(1)/solving method (combined finite element-transfer matrix)		System(1)/solving method (finite element)	
Quantities	The value	Quantities	The value
$\frac{p}{q}$	$=\frac{1000}{1200}=\frac{5}{6}$	$\frac{p}{q}$	
$T=\frac{2\pi*60}{2\pi*200}$	0.3 sec	$T=\frac{2\pi*60}{2\pi*200}$	0.2991 sec
$n=\frac{12oo}{2000}$	6 loops	$n=\frac{12oo}{2000}$	6 loops

Table(2) The compared result of the system(1)

Fig.(6-a) shows the response for the inner rotor while fig(6-6) shows the response for the outer rotor (on full period=0.3 sec).



System No.(2):-

Fig.(7) shown the model of dual rotor system mounted on hydrodynamic bearings. The charistricz of this system shown in table(3)[1].

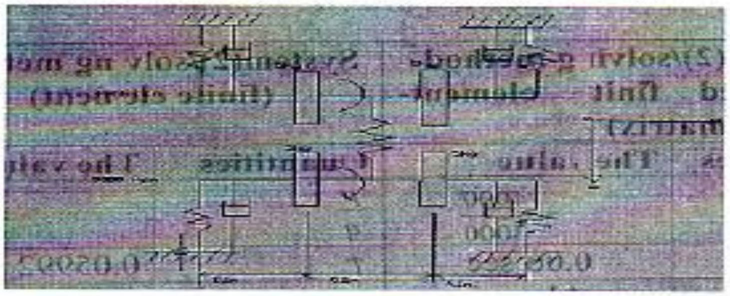


Fig.(7) Model of rotot system (2)

Modeling		Unit
The chari-riestic	The value	
Outer diameter	1.7	Cm
Inner diameter	1.5	Cm
Young's modulus	210 e09	N/m^2
Un balance on each mass	0.001	Kg.m
Outer speed	4000	Rpm
Inner speed	3000	Rpm
$K_{yy} = K_{zz}$	$17.5*10^6$	N/m
$K_{yz} = K_{zy}$	$10.0*10^6$	N/m
$C_{yy} = C_{zz}$	$700*10^3$	KN.s/m
$C_{yz} = C_{zy}$	$400*10^3$	KN.s/m
Mass for each disk	5	Kg

Table(3) The charistriez of the system(2)

Result & Discussion of system no.(2):-

The response obtained is shown in table (4) below,and it has been compared with the result obtained from Fortran program solved by finite element to same system:

System(2)/solving method (combined finit element-transfer matrix)		System(2)/solving method (finite element)	
Quantities	**The value**	**Quantities**	**The value**
$\frac{p}{q}$	$=\frac{4000}{3000}$	$\frac{p}{q}$	
T	0.06 sec	T	0.05992 sec
n	4 loops	n	4.0 loops

Table(4) The compared result of the system(2)

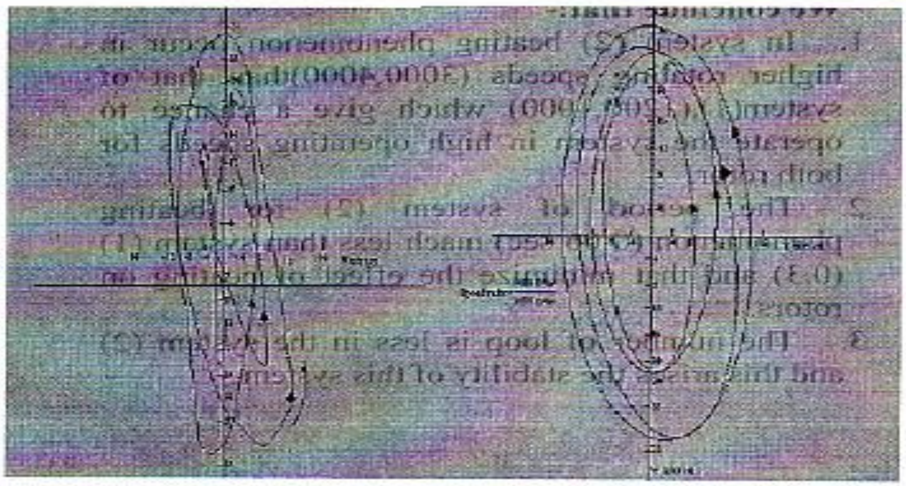


Fig.(8-a) Whirl orbit on outer rotor In one full Fig.(8-a) Whirl orbit on
inner rotor In one full
Period (with hydrodynamic bearing) period (with
hydrodynamic bearing)

Fig.(8-a) shows the response for the inner rotor while

Fig.(8-b) shows the response for the outer rotor (on full period=0.06 sec).

Conclusion:-

We conclude that:-

1. In system (2) beating phenomenon occur in higher rotating speeds (3000,4000)than that of system(1),(1200,1000) which give a chance to operate the system in high operating speeds for both rotor.

2. The period of system (2) for beating phenomenon (0.06 sec) mach less than system (1) (0.3) and that minimize the effect of beating on rotors.

3. The number of loop is less in the system (2) and this arises the stability of this system.

Appendix

Tabl(a) the stumbles

The stumble	The menacing	unit
{d}	Displacement vector	
$[N_a]$	Shape function /axial	
$[N_a]$	Shape function /torsion	
$[N_t]$	Shape function / translation	
A	Sanction area	m^2
E	Young's modules	N/m^2
G	Shear's modules	N/m^2
I	Moment of inertia	m^4
J	Polor Moment of inertia	m^4
{S}	State vector	
[P]	Point matrix	
[F]	Felid matrix	
[B]	Bearing matrix	
JP	Polar Moment of inertia/disk	$Kg.m^2$
[M]	Mass matrix	
Kvv,Kww,Kvw,Kwv	Stiffness coefficients	N/m
Cvv,Cww,Cvw,Cwv	Damping coefficients	N.s/m
[G]	Gyroscopic matrix	
V_y, V_z	Shear force component in y,z direction	N
M_y, M_z	Bending moment component in y,z direction	N.m
W_c, W_c	deflection component in y,z direction	M
Θ_c, Θ_c	slop component in y,z direction	

- **Lateen symbols:-**

unit	The menacing	The stumble
A,θ,Φ	Angler displesment	rad
ζp	Polar moment of inertia for element/length	Kg.m
μ	Mass of element/length	Kg.m
ω	Angler velocity for rotating element	Rad/s
ω a, ωb, ωc	Angler velocity for rotating element in (a,b,c)direction	Rad/s

Higher Symbols:-

i : refer to the imaginary part of complex variable

Lower Symbols :-

Sm: refer to the first rotor(ouret rotor).

Sn: refer to the Second rotor (inner rotor).

الملحق (B)

مصفوفات الكتلة والمرونة

والتأثير الجيروسكوبي للعنصر الدوار

Mass Matrix due to Translation:

$$[M_T]=\frac{\rho\pi(r_o^2-r_i^2)l}{420}\begin{bmatrix}
0 & & & & & & & & & & & \\
0 & 156 & & & & & & & \text{Symmetric} & & & \\
0 & 0 & 156 & & & & & & & & & \\
0 & 0 & 0 & 0 & & & & & & & & \\
0 & 0 & -22l & 0 & 4l^2 & & & & & & & \\
0 & 22l & 0 & 0 & 0 & 4l^2 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & 54 & 0 & 0 & 0 & 13l & 0 & 156 & & & & \\
0 & 0 & 54 & 0 & -13l & 0 & 0 & 0 & 156 & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 13l & 0 & -3l^2 & 0 & 0 & 0 & 22l & 0 & 4l^2 & \\
0 & -13l & 0 & 0 & 0 & -3l^2 & 0 & -22l & 0 & 0 & 0 & 4l^2
\end{bmatrix}$$

Mass Matrix due to Rotation:

$$[M_R]=\frac{\rho\pi(r_o^4-r_i^4)}{120l}\begin{bmatrix}
0 & & & & & & & & & & & \\
0 & 36 & & & & & & & \text{Symmetric} & & & \\
0 & 0 & 36 & & & & & & & & & \\
0 & 0 & 0 & 0 & & & & & & & & \\
0 & 0 & -3l & 0 & 4l^2 & & & & & & & \\
0 & 3l & 0 & 0 & 0 & 4l^2 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & -36 & 0 & 0 & 0 & -3l & 0 & 36 & & & & \\
0 & 0 & -36 & 0 & 3l & 0 & 0 & 0 & 36 & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & -3l & 0 & -l^2 & 0 & 0 & 0 & 3l & 0 & 4l^2 & \\
0 & 3l & 0 & 0 & 0 & -l^2 & 0 & -3l & 0 & 0 & 0 & 4l^2
\end{bmatrix}$$

Torsional Mass Matrix:

$$[M_{\alpha}]=\frac{\rho J l}{6}\begin{bmatrix} 0 & & & & & & & & & & & \\ 0 & 0 & & & & & & & \text{Symmetric} & & & \\ 0 & 0 & 0 & & & & & & & & & \\ 0 & 0 & 0 & 2 & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Axial Mass Matrix:

$$[M_a]=\frac{\rho A l}{6}\begin{bmatrix} 2 & & & & & & & & & & & \\ 0 & 0 & & & & & & & & & & \\ 0 & 0 & 0 & & & & & & \text{Symmetric} & & & \\ 0 & 0 & 0 & 0 & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 2 & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Bending Stiffness Matrix:

$$
[K_b]=\frac{EI}{l^3}\begin{bmatrix}
0 & & & & & & & & & & & \\
0 & 12 & & & & & & & & & & \\
0 & 0 & 12 & & & & & & \text{Symmetric} & & & \\
0 & 0 & 0 & 0 & & & & & & & & \\
0 & 0 & -6l & 0 & 4l^2 & & & & & & & \\
0 & 6l & 0 & 0 & 0 & 4l^2 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & -12 & 0 & 0 & 0 & -6l & 0 & 12 & & & & \\
0 & 0 & -12 & 0 & 6l & 0 & 0 & 0 & 12 & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & -6l & 0 & 2l^2 & 0 & 0 & 0 & 6l & 0 & 4l^2 & \\
0 & 6l & 0 & 0 & 0 & 2l^2 & 0 & -6l & 0 & 0 & 0 & 4l^2
\end{bmatrix}
$$

Axial Stiffness Matrix:

$$
[K_a]=\frac{EA}{l}\begin{bmatrix}
1 & & & & & & & & & & & \\
0 & 0 & & & & & & & & & & \\
0 & 0 & 0 & & & & & & & & & \\
0 & 0 & 0 & 0 & & & \text{Symmetric} & & & & & \\
0 & 0 & 0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
-1 & 0 & 0 & 0 & 0 & 0 & 1 & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Torsional Stiffness Matrix:

$$[K_x]=\frac{GJ}{l}\begin{bmatrix}
0 & & & & & & & & & & & \\
0 & 0 & & & & & & & & & & \\
0 & 0 & 0 & & & & \text{Symmetric} & & & & & \\
0 & 0 & 0 & 1 & & & & & & & & \\
0 & 0 & 0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Gyroscopic Matrix:

$$[C]=\frac{\rho\pi(r_o^4-r_i^4)}{60l}\begin{bmatrix}
0 & & & & & & & & & & & \\
0 & 0 & & & & & & & & & & \\
0 & 36 & 0 & & & & & \text{Skew symmetric} & & & & \\
0 & 0 & 0 & 0 & & & & & & & & \\
0 & -3l & 0 & 0 & 0 & & & & & & & \\
0 & 0 & -3l & 0 & 4l^2 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & 0 & 36 & 0 & -3l & 0 & 0 & 0 & & & & \\
0 & -36 & 0 & 0 & 0 & -3l & 0 & 36 & 0 & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
0 & -3l & 0 & 0 & 0 & l^2 & 0 & 3l & 0 & 0 & 0 & \\
0 & 0 & -3l & 0 & -l^2 & 0 & 0 & 0 & 3l & 0 & 4l^2 & 0
\end{bmatrix}$$

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Not: The references had been arranged according to the year of it (from the new to the old)

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الخلاصة: -

تدول البحث الحالي دراسة تأثير نوع كرسي التحميل على المدارات الدوامة، عدد المدارات والفترة الزمنية اللازمة لحصول ظاهرة التضارب في انظمة متكونة من عدد الاعمدة المتداخلة تدور بسرع دورانية مختلفة نوعين من كراسي التحميل تم دراسة آثارها النوع الأول يضم النوع العام الكراسي التحميل (stiffness) تم تمثيله بنوابض اما النوع الثاني فقد استبدل بكراسي تحميل حاوي على طبقة من المانع (fluid (film) تم تمثيله بتوابض ومحمدات . استخدمت معادلة لاكرائج للحصول على مصفوفة العنصر الدوار متكونة من (٣٣٣٣) واستخدمت المصفوفات الانتقالية للحصول على مصفوفات المجال النقطة ، كراسي التحميل ومتجه الحالة للعمودين الرابطة بين طرفي العنصر الدوار.

كتب برنامج بلغة (Mathlap) لحل النظام واستخدم برنامج (takplot) لرسم النتائج وللتأكد من صحة الحل تم مقارنة النتائج مع نتائج برنامج اخر مكتوب بلغة ((٧٧) Fortran) لاختبار الصلاحية وقد اظهرت النتائج تطابق كبير كما مبين في الجداول (١).