# Mathematical Model For Estimation of Desublimated Solid Layer Thickness and Surface Temperature

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#### Abstract:

The thickness of the desublimatd solid layer is an important parameter required for evaluation of such desublimation process, especially in the case of solid layers production needed for special applications. A mathematical model has been derived based on the analogy of heat and mass transfer to calculate the solid layer thickness of the desublimated layer during the operation period. The prediction of thickness from the derived model is based on the data related to the applied experimental conditions as well as the physical properties of the vapour-gas mixture inside desublimator. The calculated thickness from the model has been used to calculate the temperature gradient of the solid layer surface during the run period.

#### Key words:

Desublimation, Solid condensation, Cold trap

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#### **Introduction:**

Desublimation Process was considered one of the important separation processes used in the chemical plants during the decades. Desublimation past encountered in modern chemical technology especially in the catalytic oxidation of aromatic hydrocarbon compounds present in the gaseous phase. This technology was frequently applied in the separation of Phthalic and Maleic Anhydrides from the reaction products. As a purification process, desublimation technique has gained an apparent existence in the purification of Salicylic Acid, Benzoic Acid, Anthracene and Naphthalene. Economic studies shows that desublimation technique becomes competitive with distillation, crystallization and solvent extraction. Certain inherent characteristics give it a distinct advantage over other traditional separation methods (1). Desublimation process has shown a wide attendance in the chemical and petrochemical industries especially in the cases where the distillation technique become practical to separate a mixture containing components that are very close in boiling points and volatilities. In this case the

number of plates is very large and a large reflux ratio has to be used. Separations of ethylene-ethane mixtures, propylenepropane mixtures, styrene-ethyl benzene mixtures and p-xylene - m-xylene mixtures in producing high grade ethylene, propylene, styrene and p-xylene respectively are good examples (8). The desublimation process had been adopted by many research workers . The desublimator were used to remove Tif4 from the reaction zone in the process of titanium extraction from Ilmenite (9). The condensation of gaseous cyanuric chloride had been achieved using desublimator in the process of cyanuric chloride production method (10). The avoidance of de-hydration step has been achieved using desublimation technique in the process of preparing pyromellitic di-anhydride (11). A process for separation of vaporous heavy metal compounds from a carrier gas wherein the heavy metal compounds are cooled and desublimed and are filtered as particles in a filter (12). Vacuum freezing process was considered an important application in the fields of desublimation technique. Several vacuum freezing processes have been introduced by many workers in the desalination field. These processes contains vacuum freezing vapour compression (CFVC), vacuum freezing vapour absorption Vacuum freezing (VFVA). eiector absorption (VFEA), vacuum freezing solid condensation (VFSC), Absorption freezing vapour compression (AFVC), & Vacuum freezing high pressure ice melting (VFPIM) (13) . The vacuum freezing desublimation vapour desublimate vaporization process denoted as (VDDV process) is an improved vacuum freezing process that is useful in separating solvent from solution that contains one or more non volatile solutes (14). There are several cases where the desublimation process is likely to be a

suitable separation method <sup>(6)</sup>, most of them are:

- 1- For unsuitable materials, i.e. temperature or oxidation sensitive at temperatures near to or below the triple point.
- 2- For distillable material, where its desired to produce the solid product by condensation from vapour phase of a preferable crystal type, crystal size or appearance of the product.
- 3- For recoverable non volatile and heat sensitive materials required to be separated from volatile material. Freeze drying for the heat sensitive antibiotic, blood plasma are examples for this case.
- 4- When the material to be separated has a high melting point and at elevated temperature presents many problems such as corrosion of equipment.
- 5- When the most conventional separation methods of distillation, solvent extraction, adsorption, etc. are either impossible or highly expensive (7).

# The Phase Diagram of pure water

The correlation tn = f(p) characterizes the surface borders between the liquid (water) and the solid phase (ice) as shown in (Fig. 1)  $^{(15)}$ .

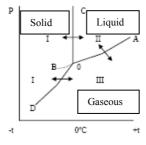


Fig. 1 : Pressure & Temperature relation of pure water

The three phases (liquid, solid, gaseous) exist simultaneously at the point (0). Such points are called triple point of the substances.

Two opposite phenomena are observed when the system temperature changes, melting

(  $I \rightarrow II$  ) and freezing ( $II \rightarrow I$ ).

The line A0D separates the gaseous phase from the solid and the liquid ones. When the values of the parameters change, the following phenomenas are observed:

Evaporation (  $II \rightarrow III$  ),

Sublimation ( $I \rightarrow III$ ),

Liquefying  $(III \rightarrow II)$ 

& Desublimation (III  $\rightarrow$  I).

The water and the water vapour have the property to supercool. The dotted line 0B express the correlation between the temperature and the change of vapour pressure above the supercooled liquid.

# Theoretical Concepts For The Prediction of Solid Layer Thickness and Surface Temperature

The condensation of saturated vapour from a mixture with inert gas in the conventional desublimator occurred on the inside wall and in the main body of the mixture (2). Most of the desublimated particles in the main body of the mixture will be pulled or pushed to the inside wall due to the forced action applied inside the desublimator, which is related to the desublimation technique followed. In the period, the thickness of the desublimated solid layer on the wall is gradually built up due to stable condensation of vapour particles, that will produce a homogeneous solid layer in thickness and dimensions. After some time, the shape of the solid layer will deviate from the homogeneous form, as a result of variables interaction such as that between temperature and pressure in the bulk of desublimator, that will produce unstable system operation.

The form of the homogeneous solid layer and the distribution of temperatures from the bulk of vapour-gas mixture to the bulk of the cooling path is shown in (Figure 2).

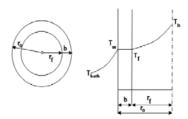


Figure (2): Temperature distribution between the inside bulk of the desublimator and the bulk of water bath

The evaluation of the solid layer thickness and surface temperature is theoretically based on the following assumption in order to simplify the computational procedures.

- 1- Approximately homo-geneous compact circular and cylindrical solid layer without any vacancies in the structure.
- 2- The heat transfer resistance of the tube wall can be neglected because of a thin wall <sup>(3)</sup>.
- 3- The resistance through the gas sublayer near the solid layer to heat and mass transfer is very small; therefore it can be neglected <sup>(3)</sup>
- 4- The temperature of the desublimated solid on the wall is assumed equal to the wall temperature (2)
- 5- The heat and mass transfer process inside the desublimator (cold trap) will be considered as a quasi steady state process.
- 6- The inlet mixture to the cold trap is saturated and homogeneous, composed of inert gas (Nitrogen) and saturated vapour.
- 7- The heat transfer rate approached a steady state even while frost continued to accumulate.
- 8- All condensate settles at the point of formation (2)

- 9- The heat and mass transfer process in the axial direction is very small compared with the transfer in the radial direction; therefore, assumed negligible.
- 10- The vapour pressure of the desublimated solid layer considered very low in comparison with the total pressure of the desublimator (cold trap ) (2)

# 1- Solid layer surface temperature ( $T_s$ )

The general equation for the circular and cylindrical coordinates, is shown as follows (5).

$$\frac{\delta^{2}T}{\delta r^{2}} + \frac{1}{r} \frac{\delta T}{\delta r} + \frac{1}{r^{2}} \frac{\delta^{2}T}{\delta \phi^{2}} + \frac{\delta^{2}T}{\delta z^{2}} + \frac{Q^{2}}{K} \frac{1}{\alpha} \frac{\delta T}{\delta t} \dots (1)$$

For the case of radial transfer, & quasi steady state, equation (1) will be reduced to ordinary differential equation of the form (5):

$$\frac{d^2 T}{d r^2} + \frac{1}{r} \frac{d T}{d r} = 0 \dots (2)$$

Equation (2) can be integrated twice to obtain:

$$T = C_1 \ln (r) + C_2 \dots (3)$$

Where  $C_1$  &  $C_2$  are the Constants of integration

# **Boundry conditions:**

At 
$$r = r_o$$
 ,  $T = T_w$   
Thus;  
 $T_w = C_1 \ln (r_o) + C_2$  ....(4)

 $(T_{\rm f}$ ) will never be equal to  $(T_{\rm b})$ , figure (1) , unless the inside bulk heat transfer coefficient  $(h_{\rm i})$  is infinitely large, thus an energy transfer from the mixture in the bulk to the solid layer surface will be accomplished by ( Newton's convection law with zero temperature difference ) ; therefore, the rate of heat transfer from the bulk of mixture to the solid layer surface at  $(r_{\rm f})$  must equal the rate of conduction to the solid at  $(r_{\rm f})$ . Then ;

$$\begin{split} &h_{i}\left(\,2\,\Pi\,\,r_{f}\,\,L\,\,\right)\left(\,\,T_{b} - T_{f}\,\,\right) = \\ &- K_{s}\left(\,2\,\Pi\,\,r_{f}\,\,L\,\,\right)\,\left(\,\,\frac{d\,T}{d\,r}\,\,\right) \\ &- \left(\,\,d\,r\,\,\right)\,r_{f} \end{split}$$
 
$$& \longrightarrow h_{i}\left(\,\,T_{b} - \,\,T_{f}\,\,\right) = - K_{s}\left(\,\,\frac{d\,T}{d\,r}\,\,\right)\,...(5) \\ &- \left(\,\,d\,r\,\,\right)\,r_{f} \end{split}$$

The differentiation of equation (3) gives the following equation:

$$\left(\frac{dT}{dr}\right) = \frac{C_1}{r}$$

### **Boundry condition:**

At 
$$r = r_f$$
;  
 $(\underline{dT}) = \underline{C}_1$  ....(6)  
 $(\underline{dr})r_f$   $r_f$ 

 $T_f$  could be determined in terms of  $C_1$  and  $C_2$  by substituting  $r_f$  in equation (3)

 $T_f = C_1 \ln (r_f) + C_2$  ....(7) Insertion of equations (6) and (7) in equation (5) gives :

$$h_i (T_b - C_1 \cdot ln (r_f) - C_2)$$
  
= -  $K_s C_1 \dots (8)$ 

Equations (8) and (4) could be solved for  $C_1 \& C_2$  to give the following results :

$$\begin{array}{c} C_1 = \underline{\quad \quad - \quad (T_b - T_w) \quad } \\ \hline ln \left( \begin{array}{c} \underline{r_o} \right) + \left( \underline{\quad K_s \quad } \right) \\ \left( \begin{array}{c} r_f \end{array} \right) & \left( \begin{array}{c} h_i \cdot r_f \end{array} \right) \end{array}$$

$$\begin{array}{c} C_2 = T_w + \underline{ (T_b - T_w) . \ln (r_o) .} \\ \ln (\underline{r_o}) + (\underline{K_s}) \\ (\underline{r_f}) & (\underline{h_i} . \underline{r_f}) \end{array}$$

Insertion of  $C_1$  &  $C_2$  in equation (3) gives an equation for the prediction of temperature as a function of

( r ) for the conduction region between (  $r_f$  ) and (  $r_o$  ) as follows :

$$T = - \underbrace{(T_b - T_w)}_{\ln(r_o) + (K_s)} \ln(\underline{r}_o)$$

Estimation of the solid layer temperature at the position

 $(r_f)$  will be as follows: Suppose that  $T_f = T_s$ Then,

$$T_{s} = -\underline{(T_{b} - T_{\underline{w}})} \underline{ln(\underline{r_{f}})} r_{o}$$

$$ln(\underline{r_{o}}) + (\underline{K_{s}}) r_{o}$$

$$(\underline{r_{f}}) (\underline{h_{i}}, \underline{r_{f}})$$

 $+T_{w}$ 

Since  $r_f = r_o - b$ ; then

$$T_{s} = - \underline{(T_{b} - T_{w})} \\ ln (\underline{r_{o}}) + (\underline{K_{s}}) \\ (r_{o} - b) (h_{i}.(r_{o} - b))$$

$$\begin{array}{c} \ln\left(\underline{r_o} - \underline{b}\right) + \ T_w \\ ( \ r_o \ ) \\ & \dots \end{array} (10)$$

The estimation of wall temperature  $(T_w)$  and the solid layer thickness (b) will enable the evaluation of desublimator solid layer surface temperature  $(T_s)$ .

# 1-1 Estimation of the desublimator wall temperature

At a quasi steady state, the overall heat transfer rate is equal to the transfer rate from the wall to the cooling bath by convection; i.e

$$\begin{array}{ll} Q_{\,\,\text{overall}} &=& Q_{\,\,\text{bath}} & Q_{\,\,\text{overall}} \\ &=& \Pi \,\,.\,\, D_o \,.\,\, L \,\,.\,\, U_a \,\,.\,\, \Delta \,\, T_{lm} \\ &\qquad \qquad \dots \,\, (11) \\ Q_{\,\,\text{bath}} &=& \Pi .D_o \,.\,\, L \,\,.\,\, h_o \,\,.\,\, (\,\,T_w - T_{bath}\,\,) & \dots \\ &\qquad \qquad (12) \end{array}$$

The solution of equations (11) and (12) for  $(T_w)$  gives the following equation :

$$T_{w} = \underline{U}_{a.} \Delta T_{lm} + T_{bath} \dots (13)$$

Where:

$$\Delta T_{lm} = \underbrace{T_b - T_{bath}}_{In \underline{T_b}_{-}}$$

$$T_{bath}$$

Consider  $\Delta T_{lm}$  equal to  $(T_b - T_{bath})$ , therefore:

$$T_{w} = \underline{U_{a}}_{h_{o}} . (T_{b} - T_{bath}) + T_{bath}$$
 .... (14)

# 1-2 Estimation of solid layer thickness (b)

The rate of condensate deposit per unit area per mole could be determined by applying a mass balance in accordance with the approach given by Thompson (2), assuming all condensate settles at the point of formation. The mass balance can be shown as follows:

$$-V \underline{dy}_{dA} = \underline{\rho}_{s} \underline{db}_{t} \dots (15)$$

Equation (15) can be rearranged in the following form by introducing the term (  $h_i / h_i$  )

$$\frac{d b}{d t} = \frac{h_{i.} M_{1.} (-V)}{\rho_{s.}} (h_{i.} d Y) ... (16)$$

The estimation of the term  $(-V \underline{dY})$  will enable the  $(h_i \underline{dA})$ 

integration of equation (16) and then obtaining an equation for thickness distribution estimate through the time.

For the case of saturated vapour – gas mixture, it is necessary to establish whether or not the mixture will remain saturated. This could be achieved by computing the fraction of condensate which comes out as a mist. If this fraction has a positive value, the gas mixture will tend to remain saturated. The fraction of condensate which appears as a mist could be calculated from the following equation

$$\zeta = 1 - \underline{\chi}_1 \qquad \dots (17)$$

Where;

 $\chi_l$  = The rate of diffusion of condensed vapour across the gas sub layer near the desublimated solid layer per unit area

 $\chi$  = Total rate of vapour condensation per unit area

 $\chi_1$  and  $\chi$  could be evaluated in terms of vapour composition and temperature as follows :

$$\chi_1 = \Gamma \cdot (p - p_w)$$

$$= \Gamma \cdot p \cdot (1 - \beta) \quad .... (18)$$

Where ;  $\beta = p_w/p$ 

The relation between heat and mass transfer coefficients had been given by Thompson (2) as follows:

$$j = \frac{\Gamma \cdot p_f}{(G/M_m)} \left[ \frac{\mu}{\rho_m \cdot D_v} \right]^{2/3}$$

$$= \underline{h_{i}} \quad \left[ \underbrace{C_{m} \cdot \mu}_{K} \right]^{2/3}$$

$$\dots (19)$$

Re- arrangement of equation (19) gives the following form:

Substitution of equation (20) in (18) gives the following form:

$$\chi_{1} = \frac{h_{i}}{M_{m} \cdot C_{m}} \left[ \frac{C_{m} \cdot \mu \cdot D_{v}}{K} \right]^{2/3}$$

$$\cdot \frac{p \cdot (1 - \beta)}{p_{f}} \dots (21)$$

In the case of very low vapour pressure of the desublimated solid in comparison with the total pressure of the cold trap, the term (  $p / p_f$  ) Could be expressed by the following form  $^{(2)}$ 

$$\frac{\mathbf{p}}{\mathbf{p}_{\mathrm{f}}} = \frac{1}{\ln(1+y)},$$

Therefore, equation (21) can be converted to the following form:

$$\begin{split} \chi_1 \; &= \frac{h_i}{M_{m_-}} \frac{\left[ \begin{array}{c} C_{m_-} \mu \, . \, D_{v_-} \\ K \end{array} \right]^{2/3}}{\text{$.$ $($1 - $\beta$).} } \ln \left( 1 + y \, \right) \\ &\qquad \dots (22) \end{split}$$

The total rate of condensation could be obtained by achieving a material balance over a unit area as follows (2):

$$\chi = -v \cdot \frac{dy}{dA} , \text{ or}$$

$$\chi = -v \cdot \frac{dy}{dT_b} \cdot \frac{dT_b}{dA} \dots (23)$$

Assuming the gas will remain saturated, and the condensable vapour obeys the perfect gas law ; thus the term (d y / d  $T_b$ ) could be determined using

Modified Clapeyron Equation, which is given by Thompson <sup>(2)</sup> as follows:

$$\frac{dy}{dT_b} = y \cdot (1 + y) \cdot \frac{\lambda}{R \cdot T_b^2} \cdot \dots (24)$$

The term (d T<sub>b</sub> / d A) in equation (23) could be determined by applying a heat balance, i.e., equating the heat transferred

to the inside wall by convection as shown below in (term 1) to the loss in sensible heat of the vapour (term 2) plus the heat of vapour condensing as a mist (term 3), as follows:

$$\begin{aligned} &h_{i} \cdot (\ T_{b} - T_{w}\ ) = \\ & \textbf{(Term 1)} \end{aligned}$$
 
$$V \cdot (\ 1 + y\ ) \cdot M_{m} \cdot C_{m} \cdot \frac{d\ T_{b}}{d\ A}$$
 
$$\textbf{(Term 2)}$$
 
$$+ \lambda \cdot \chi \cdot \zeta$$
 
$$\textbf{(Term 3)} \qquad \dots (25)$$

Rearranging equation (25) gives the following form:

$$\begin{array}{cccc} \underline{d \; T_b} = \underline{h_i \; . \; (\; T_b - \; T_w \;)} & - \; \lambda \; . \; \chi \; . \; \zeta \\ d \; A & V \; . \; (\; 1 + y \;) \; . \; M_m \; . \; C_m \\ & \ldots \; (26) \end{array}$$

Since  $\chi_1 = \chi$ . (1 -  $\zeta$ ); therefore  $\chi$  could be obtained by substituting  $\chi_1$  from equation (22) to give the following:

$$\chi = \frac{h_i}{M_m \cdot C_m \cdot (1 - \zeta)} \cdot \frac{\left[C_{m \cdot \mu} \cdot D_v\right]^{2/3}}{\left[K\right]}$$

$$\cdot (1 - \beta) \cdot \ln(1 + y)$$

$$\cdot \dots (27)$$

Substitute equation (24) and equation (26) in equation (23) to obtain the following form:

$$\begin{split} \chi &= - \ V \ . \ y \ . \ ( \ 1 + y \ ) \ . \\ &\frac{\lambda}{R \ . \ T_b^2} \cdot \frac{h_i \ . \ ( \ T_b - T_w \ ) \ - \lambda \ . \ \chi \ . \ \zeta}{V \ . \ ( \ 1 + y \ ) \ . \ M_m \ . \ C_m} \end{split}$$

Rearrange to obtain the following form:

$$\underline{\mathbf{M}_{m} \cdot \mathbf{C}_{m} \cdot \mathbf{R} \cdot \mathbf{T}_{b}^{2}} = \mathbf{y} \cdot \lambda$$

$$-\underline{\mathbf{h}_{i} \cdot (\mathbf{T}_{b} - \mathbf{T}_{w})} - \lambda \cdot \chi \cdot \zeta$$

Substitute equation (27) in (28) and solving for ( $\zeta$ ) to give the following form

: 
$$\zeta = 1 - \frac{a_1 + a_2}{T_b - T_w + a_1}$$
....(29)

Where;

 $dT_b =$ 

$$a_1 = \frac{\lambda}{M_m \cdot C_m} \cdot \frac{(C_m \cdot \rho \cdot D_v)^{2/3}}{(K)}$$

$$\cdot (1 - \beta) \cdot \ln(1 + y)$$

$$a_2 = \underbrace{R \cdot T^2}_{\lambda \cdot y} \underbrace{\left( C_m \cdot \rho \cdot D_v \right)^{2/3}}_{(K)}$$

$$(1-\beta). \ln(1+y)$$

Substitute for ( $\zeta$ ) and ( $\chi$ ) in equation (26) to obtain the following:

$$\begin{array}{c} \text{d A} \\ \frac{h_{i} \cdot (T_{b} - T_{w})}{\text{V.y.}(1+y) \cdot \frac{\lambda^{2}}{\text{R . T}_{b}^{2}} + \text{V.}(1+y) \cdot M_{m} \cdot C_{m}} \end{array} +$$

$$\dot{V}$$
.y.(1+y). $\frac{\lambda^2}{R \cdot T_b^2}$ + V.(1+y). $M_m$ . $C_m$ 

Or;

$$\frac{d T_b}{d A} = \frac{h_i \cdot (T_b - T_w + a_1)}{V \cdot (1+y) \cdot M_m \cdot C_m \cdot (1+\underline{a}_1)}$$

$$a_2$$

$$\dots (30)$$

The rate of solid deposit could be evaluated as follows:

$$-V \cdot dy = -V \cdot dy \cdot dT_b$$

$$h_i \ d \ A \qquad \qquad h_i \ d \ T_b \ d \ A$$

.... (31)

Substitution of equations (24) and (30) in equation (31) gives the following:

$$- \underbrace{V}_{h_i} \underbrace{d y}_{d A} =$$

$$\frac{\lambda \cdot y \cdot (T_b - T_w + a_1)}{R \cdot {T_b}^2 \cdot M_m \cdot C_m \cdot (1 + \underline{a_1})}.$$

Substitution of equation (32) in equation (16) gives the following form for *Solid Layer Thickness growing Rate*:

$$\frac{d b}{d t} = \frac{h_{i} \cdot M_{1}}{\rho_{s}}.$$

$$\frac{\lambda \cdot y \cdot (T_{b} - T_{w} + a_{1})}{R \cdot T_{b}^{2} \cdot M_{m} \cdot C_{m} \cdot (1 + \underline{a_{1}})}$$

$$a_{2}$$
.... (33)

Integration of equation (33) gives the following form:

$$b = \underline{h_i \cdot M_1}_{p_s}.$$

$$\frac{\lambda \,.\, y \,.\, \left( \,\, T_b - T_w + \,\, a_1 \,\, \right) \,.\, t}{R \,.\, T_b^{\,\, 2} \,.\, M_m \,.\, C_m \,.\, \left( \,\, 1 + \,\, \underline{a_1} \,\, \right)} \,+\, C}{a_2}$$

Where;

C = Constant of integration Initial condition:

at t = 0, b = 0; therefore;

$$b = \underline{h_i \cdot M_1}_{\rho_s}.$$

$$\frac{\lambda \cdot y \cdot (T_b - T_w + a_1) \cdot t}{R \cdot T_b^2 \cdot M_m \cdot C_m \cdot (1 + \underline{a_1})} a_2$$
.... (34)

### 2- Experimental data application:

Solid Layer Thickness and Surface Temperature

application of the derived mathematical model for the experimental conditions related to the work achieved by Ueda (4) has shown a good prediction for solid layer thickness, compared with the thickness measured experimentally. This result has been obtained in the first twenty minutes of run period. The calculated thickness values have deviated from the measured thickness for the period follows the first twenty minutes. This result could be attributed to the tearing of the desublimated solid particles from the surface of solid layer which has occurred due to the forced stream inside the desublimator (Cold trap).

The **comparison** between the solid layer thickness measured experimentally by **Ueda** <sup>(4)</sup> and that calculated from the derived model (based on the experimental conditions related to the work of **Ueda** <sup>(4)</sup>

) for the individual systems of

(Water – Nitrogen) and

(Benzene – Helium) are shown in the figures (3) and (4).



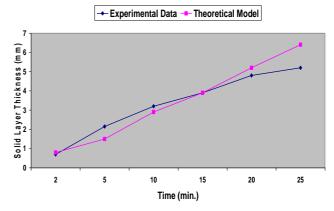


Figure (3) Comparison between mathematical model prediction and experimental data / Water vapour +  $N_2$ 

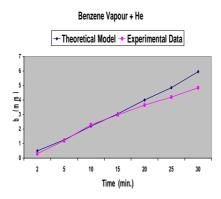


Figure (4) Comparison between mathematical model prediction and experimental data / Benzene vapour + He

The average absolute relative error (AARE) between the measured thickness and predicted thickness from the model for the applied systems related to the work of  $Ueda^{(4)}$  are shown in table (1). Table (1) Average absolute relative error (AARE) between the experimental and surface temperature ( $T_s$ ) with time calculated thickness

	AARE		
System	First 20	Total time of	
	minutes	run	
Water	0.13179	0.14832	
$-N_2$			
Benzene	0.175166	0.17962	
– He			

The application of the estimated solid layer thickness **values** for the system of (Water vapour – Nitrogen) in equation (34) gives **the gradient of solid layer** as shown in figure (5).

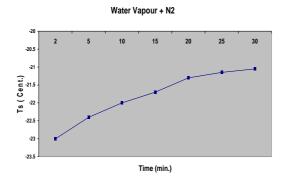


Figure (5) Gradient of solid layer temperature at position (  $r_{\rm f}$  ) with time ( Experimental data – Ueda  $^{(4)}$  )

Fig. 5 shows the desublimated layer surface temperature increase with time up to the limit of quasi constant values, which indicates abstention of vapour droplets to be condensed because of a forced stream inside the desublimator.

### **Conclusion:**

mathematical model has developed to estimate the solid layer thickness and surface temperature of the desublimated solid layer inside the bulk of the desublimator based on the analogy of heat and mass transfer. The calculated values for the solid layer thickness from the derived mathematical model has shown a good agreement experimental data and the thickness measured experimentally related to the work of Ueda (4) for the first 20 minutes of the run period. The deviation in the calculated values occurred for the last period of run. The deviation was attributed to the tearing of desublimated solid particles from the surface of solid layer which is probably due to the forced stream inside the desublimator (Cold trap). The calculated values of solid layer thickness has been used to diagnosis the gradient of solid layer surface temperature through the period of run.

## **Nomenclature**

Syn	abol <u>Description</u>	<u>Units</u>
A	Trap heat transfer area	$m^2$
b	Thickness of deposit	m
$C_{m}$	Specific heat of vapour	
	-gas mixture	J/gm.K°
$D_{o}$	Outside diameter of the	
	desublimator	m
$D_{\rm v}$	Binary diffusivity of the	

vapour-gas mixture	m <sup>2</sup> /sec				
G Mass velocity of the		$T_{c}$	Temperature at the cold trap surface	$C^{o}$	
vapour gas mixture	gm/m.sec	$T_{e}$	Temperature at the	C	
h <sub>i</sub> Inside bulk heat transfer			entrance to the cooling	$C^{o}$	
coefficient J/m <sup>2</sup> .sec.K°		$T_{\rm w}$	section Tube wall temperature	C°	
1 0 4 1 1 11 1 44 6		$T_{\text{bath}}$	Cold water bath	C0	
h <sub>o</sub> Outside bulk heat transfer coefficient		$T_{\mathrm{f}}$	temperature Temperature at position	Co	
J/m <sup>2</sup> .sec.K°			(f)	$C^{o}$	
		V sec	Flow rate of inert gas	gmol /	
K Thermal conductivity of		У	Moles of condensable		
the vapour- gas	mixture	-	gas per mole of inert gas		
J/m.sec.K°		Greek	<u> letters :</u>		
K <sub>s</sub> Thermal conductivity of		ρ	Density of the vapour –	2	
the solid J/m.sec.K°	layer	2	gas mixture Molar density of solid	$gm/m^3$	
L Length of the cold trap	m	$ ho_{ m m}$	Molar density of solid condensate	$mol/m^3$	
M <sub>m</sub> Mean molecular weight	gm/gmol	$\rho_{s} \\$	Density of solid		
M <sub>1</sub> Molecular weight of condensable vapour	gm/gmol	0	deposit Density of vapour	$gm / m^3$ $gm / m^3$	
p Partial pressure of	giii/giiioi	$ ho_{ m v}$ $\mu$	Viscosity of the vapour	giii / iii	
condensable gas	atm.	·	– gas mixture	gm / m .	
p <sub>w</sub> Vapour pressure of desublimated solid		sec φ	Angular direction		
On the wall	atm.	ζ	Fraction of condensate		
		Г	appears as a mist		
Symbol Description	Units	Γ	Mass transfer coefficient	kmol /	
p <sub>f</sub> Partial pressure of va		m <sup>2</sup> .sec.		111101	
at the interface between		λ	Latent heat of	T / 1	
the gas and solid phas Q" Rate of heat generation		П	sublimation per (gmol) = 3.1415927	J / gmol	
r Location from the cen		11	3.1113727		
of the cold trap	m	Refer		a	
r <sub>o</sub> Outside radius of the cold trap	m		adislav Kudela & Michael Jerstanding sublimation tec		
$r_{\rm f}$ $r_{\rm o} - b$		Chemi	cal Engineering, June 23		
R Gas constant	J/mol . K <sup>o</sup>		pages (93-98) [2]. Thompson, W. I., Gaseous diffusion		
t Time	minutes	engine	ering developments, 2 <sup>nd</sup>	edition ,	
T Temperature	K°		, pages (73-82) Bilik, J. , Heat transfer	r in the	
T <sub>b</sub> Cold trap ( Desublimat		desubl	imation of Phthalic Anhyd	lride, The	
bulk temperature	$C^{o}$				

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Mathematical Model For Estimation of Desublimated Solid Layer Thickness and Surface Temperature

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* Keyword: (Fig.1)^{(15)}. )^{(2)} ) ( ; ) -\frac{\text{hj.} (Tb-Tw) - \lambda.\chi.\zeta}{\chi} \chi ...(28)
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Figure(5) Gradient of solid layer temperature at position  $(r_f)$  with time (Experimental data-Ueda $^{(4)}$ )

Desublimation, Solid condensation, Cold trap