An Approach For Forming Spur Gear Tooth Profile

Dr. Safaa H. Abdulrahman* **Dr. Adnan D. Mohammed****

Received on: 14/3/2007
Accepted on: 6/9/2007

Abstract

Due to high working speed requirement in industry of rotating components, gear design development become quite noticeable and rapid in the vicinity of engineering parameters which become to have a large effect on its performance. In this work the spur gear of straight tooth is chosen because of its wide usage in industry, also an equation for joining the involute profile and fillet curve is generated. Variation effect on the shape of tooth profile forming is studied. A suggested method for generating tooth profile is obtained and a computer program is written for drawing the involutes and trochoid curves. The results obtained from the new method are the true continuous involute and trochoid curves in comparison with those obtained from Mechanical Desk software which are approximated curves fitted results.

Keyword:tooth,gear,trochiod,involute

الخلاصة

نتيجة لدخول عنصر السرعة في متطلبات الصناعة لمركبات العناصر الدوارة في المكائن فأن تطـور تصميم التروس قد برز بشكل ملحوظ ومتسارع في مجال المعاملات الهندسية والتي أصبح لها التـأثير الكبير على أداءها. فقد تم اختيار التروس العدلة (ذات السطوح المستقيمة) في هذه الدراسة وذلك بسبب الاستخدامات الواسعة في الصناعة. كذلك فقد تم اشتقاق وتوليد معادلة للربط بـين شـكل الانفوليـوت (Involute profile) ومنحني الفلت (Fillet curve) ، كما تم دراسة تأثير تغير المعاملات الهندسية على شكل تكون السن. لقد تم التوصل إلى طريقة مقترحه لتوليد شكل السن كما كتب برنامج حاسوبي لرسم الانفوليوت ومنحني التروجويد (Trochoid curve) . تمت المقارنة بين الطريقة المقترحه لتوليد السن مع البرنامج التجاري المستخدم في المصدر Mechanical Desk Software والتي تمثل نتائج منحنيات تقريبية.

1.Introduction

Gear design is considered to be one of the most important and complicated fields of mechanical systems because of its wide usage and applications, so this makes their design be reviewed yearly.

Norton,1998[2] developed the first equation for the fillet shape for the spur or helical gears, in which considered it as arcs of circles, while in actual fact it is a trochoid as will be considered in this work. Peter Lynwander, 1983[3] introduced other form of equations for the involute and trochoid curves. Al-Dafai, 1995[4] used FEM to study gear tooth stress then obtained the displacement and stiffness values, Jweeg,M.J,Hussain,I.A[5] investigated the gear tooth shape and its generating rack, based upon both strength and noise consideration using Finite Element Method (F.E.M). Peng et al, 1998[6] their work on the stress concentration at the root fillet depends on the minimum radius of the fillet and general configuration of this part. Jweeg et al 1989 [7] developed the involute tooth initial section and calculated the stress using F.E.M by employing the 8-node Isoparametric elements.AL-Shaibany A.R 2002 [8] studied the effect of geometrical and dynamical parameters on the design of spur gear tooth and have shown the effect of choosing the pressure angle of 25 degree and dynamic load and stresses on the gear performance using

*Electromagnatic Dept.UOT.Baghda-Iraq
**Mechanical Dept.UOT. Baghdad-Iraq

https://doi.org/10.30684/ etj.26.8.5
University of Technology-Iraq, Baghdad, Iraq/2412-0758

FEM. As a conclusion from the previous works mentioned earlier, the fillet was considered as an arc in a circle joined the involute by a suitable curve fitting routine. In the present work an equation for the fillet curve is developed considering the latter as a trochoid. Also a computer program is developed for the matching (meshing) of the involute curve with trochoid.

Construction of Involute Gear Tooth

The involute of a circle is a curve that can be generated by unwrapping a taut string from a cylinder that always tangents to the base circle and the centre of involute curvature is always at the point of tangency as shown in Fig.(1), and to plot the involute, simply pressure angle ϕ values are assumed. A tangent to the involute is always normal to the string, which is the instantaneous radius of curvature of the involute curve [2], [9]. The involute tooth profile can be generated using the following relations [10], [11] as shown in Fig. (1a).

$rp = 0.5 * m * Z$ ---------- (1)

$ra = rp + m + C_f *m$ ---------- (2)

$rf = rp - m * (1.25-C_f)$ ---------- (3)

The involute is generated at the position of the base circle whose radius is given by the equation:

$rb = rp * \cos(\phi) = m * Z * \cos(\phi) * 0.5$(4)

and the root circle radius

$rf = m * (Z-2.5) * 0.5$ ---------- (5)

Gear Tooth Generation

Generating mean that the tool is cutting conjugate form, such a straight sided tooth is sometimes referred to as a rack. As the tool transverse and the work rotates, an involute is generated on the gear tooth flank and a [trochoid] in the root fillet, as shown in Fig. (2).

Fig. (3) is a closer look at a hob tooth. This is a hob of pressure angle ϕ and diametral pitch $\left(\frac{\pi}{TH+TP}\right)$. It is capable of cutting a whole family of gears with the same pressure angle and diametral pitch. Such tools are standard as, for instance, a 20^o pressure angle and (8) pitch (diametral pitch) hob, and are easily obtainable. Gears that are cut using this hob are capable to match (mesh) each other.

The distance (B+Rrt) on the hob tooth is equal to the dedendum of the generated gear tooth, Rrt is the hob tip radius with its center at point O, and B is the vertical distance from point O and the pitch lines as shown in Fig. (3).

TP the hob tooth space, is equal to the tooth thickness of generated gear at gear pitch diameter.

The work piece moves an angle (TP+TH)/rp, where rp is the gear pitch radius. (TP+TH) is the circular pitch of manufacturing gear.

The involute coordinates.

It was shown that the involute can be predicted if the base circle radius and the angle (θ) of involute are known, so the radius of involute curve can be found at any pressure angle ϕ by following equation [10]

$\theta = \tan(\phi) - (\phi)$ ----- (6)

To find the coordinates with respect to the centre of tooth, the tooth thickness at any radius must be known. Let's start at the pitch diameter with pressure angle ϕ_1, a pitch radius rp1, and a circular tooth thickness CTT1. The involute angle can be written as$\theta_1 = \tan(\phi_1) - (\phi_1)$ ------------ (7)

and from Fig.(4), it can be seen that the angle A is:

$$A = \theta_1 + \frac{0.5\,CTT1}{rp_1} \quad \text{--------------} \quad (8)$$

To find the circular tooth thickness at any other radius rp_2, use

$$B = A-(\theta_2) = \theta_1 + \frac{0.5*CTT1}{rp_1} - (\theta_2)$$ ------
(9)

$$\phi_2 = \cos^{-1}\left(\frac{rb}{R_2}\right)$$

$$\theta_2 = \tan(\phi_2) - (\phi_2)$$

$$CTT2 = 2\left(\frac{0.5*CTT1}{rp_1} + (\theta_1) + (\theta_2)\right)$$ ------
(10)

So to find X and Y coordinates of the involute at any radius R_2, use following equations:

$X = R_2 * \sin(B)$ ---------------- (11)

$Y = R_2 * \cos(B)$ ----------------- (12)

The trochoid coordinates.

To find the trochoid coordinates on the desired X-Y system through the center of the tooth, they must be shifted through the angle (w) as in Fig.(2) when the hob traverses a distance (TH+TP) as in Fig.(3), the gear rotates through an angle (TH+TP)/rp; therefore the angle (w+v) between the center of the gear tooth and the center of tooth space is:

$$w + v = \frac{0.5*(TH + TP)}{rp}$$ ------
(1)

Angle v can be calculated as follows:

$$v = \frac{L}{rp}$$ -----------------
(14)

Where L = distance between center hob tooth and point O in Fig.(3)

$$L = \frac{TH}{2} - B*\tan(\phi) - \frac{Rrt}{\cos(\phi)}$$ -----------
(15)

Then

$$w = \frac{0.5*(TH + TP) - L}{rp}$$ -----------
(16)

Fig.(5) shows the trochoid generated by point O at its starting point and after the hob has moved a distance rp. β and the gear has rotated through an angle β the coordinate are:

$Xo = R_O * \sin(T-\beta) = R_O * (\sin(T) * \cos(\beta) + \cos(T) * \sin(\beta))$----------------- (17)

$Yo = R_O * \cos(T-\beta) = R_O * (\cos(T) * \cos(\beta) + \sin(T) * \sin(\beta)$----------------- (18)

$$\cos T = \frac{rp - B}{R_O}$$

Where: and

$$\sin T = \frac{rp \cdot \beta}{R_O}$$

Therefore:

$X_O = (rp.\beta) * \cos(\beta) - (rp-B) * \sin(\beta)$(19)

$Y_O = (rp-B) * \cos(\beta) + (rp.\beta) * \sin(\beta)$ (20)

Fig.(6) shows how to calculate the actual trochoid coordinates adding the hob tip radius rc to the trochoid generated by point O.

$X_T = X_O + rc * \cos(A)$ --------------- (21)

$Y_T = Y_O - rc * \sin(A)$ ----------------- (22)

A is the angle formed by a line normal to the trochoid generated by point z and the Y_T axis which can be found as follows:

To find $dX_O/d\beta$, $dY_O/d\beta$ from equation (19, 20):

$$\frac{dX_O}{d\beta} = -(rp*\beta)*\sin(\beta) + rp*\cos(\beta) - rp*\cos(\beta) + B*\cos(\beta)$$

$$\frac{dY_O}{d\beta} = rp*\sin(\beta) + B*\sin(\beta) + (rp*\beta)*\cos(\beta) + rp*\sin(\beta)$$

and $\frac{dX_O}{dY_O}$ is

$$\frac{dX_O}{dY_O} = \frac{-(rp*\beta)*\sin(\beta) + B*\cos(\beta)}{B*\sin(\beta) + (rp*\beta)*\cos(\beta)}$$ --
(23)

Finally, to obtain the trochoid coordinates with respect to the system through the gear tooth center, refer to Fig.(2)

$$\sin(w) = \frac{X_T + X * \cos(w)}{Y}$$

and

$$\cos(w) = \frac{Y_T - X * \sin(w)}{Y}$$

Then

$X_T * \cos(w) + X * \cos^2(w) = Y_T * \sin(w) - X * \sin^2(w)$ ----------------- (24)

$X = Y_T * \sin(w) - XT * \cos(w)$------ (25)

And

$Y = Y_T * \cos(w) + X_T * \sin(w)$ ------ (26)

The value of (rc), the fillet radius of curvature at fillet tooth (trochoid) is given by:[10]

$$rc = Rrt + \frac{(Addc - e - Rrt)^2}{rp_s + (Addc - e - Rrt)} \quad (27)$$

Where: $Rrt = 0.3 * m$---------------- (28)

$e = Cf * m$ ----------------- (29)

It is possible to add the trochoid and involute curves in complete program to obtain spur gear tooth generation. This program named (program 1) as showing in Fig. (7).

Results.

To explain the effects of varying tooth geometric parameters, nine cases of different pressure angles and correction factors are selected as shown in table (1).

Using the new written program on the above cases, results of spur gear profiles are generated and displayed in Figures (8--16).The profiles shown are the true (non-approximated) curves of involute and trochoid combinations because they are based specifically on the true cutter motion . Also Fig(17) shows tooth profile of case 4 with the Mechanical Desk Top Ver.5[1], and Fig(18) shows the drawn trochiod curve of tooth profile for case 4 with Mechanical Desk Top Ver.5[1] .

Discussion

the generated equation for the involute profile and the fillet curve is used to study the variation of tooth geometric on the shape of tooth profile. Its observed clearly from figure (8-16), how the involute and trochiod curve of tooth profile changes from case 1 to case 9. Case (4) is used for the comparison between the profile resulted from the present developed program with that of the Mechanical Disk Top Software [1]. which shows that this approach gives exact true curve fit, also more accurate curve for trochiod, while from the software curve passes between the plotted point with interpolation method to deduce the profile, also trochiod plotted by an arc at imaginary centre point as in CAD QUST software [13] as shown in fig (17-18).

conclusion.

the constructed computer program using the developed equation for forming the involute and the trochiod of spur gear tooth in comparison with other works shown that present analysis gives a true profile especially at the portion of trochoid, it also showed that the program developed form exact profile because it follows the real path of the cutter that cut's the real profile during the manufacturing process. from that it can be conclude that construction of profile is more accurate than that formed by arc of imaginary centre.

References.

[1]: Mechanical Desk Top Software Ver.5, U.S. and international copyright laws, Autodesk, Inc. ACIS of spatial Technology, inc. 2000

[2]: Norton, Robert L. "Machine Design Handbook" Worcester polytechnic Institute, Worcester, Massachusetts, copyright by Prentice-Hill Inc. New Jersey, 1998

[3]: Lynwander, peter "Gear Drive Systems" American Lohmann

Corporation, Hill side, New Jersey, 1983.

[4]: Al-Dafai, Hatam K.K. "Finite Element Study of Gear tooth stresses with presence of a keyway", Baghdad University , M.Sc thesis, 1995.

[5]: Jweeg,M.J , Hussain,I.A "Finite Element Study of Gear Tooth Stresses with presense of a keyway" ,Baghdad University M.Sc thesis,1995.

[6]: Peng, X.Q., Liu Geng,Wu Liyan, G.R. Lin , K.Y. Lam, " A Stochastic Finite Element Method for Fatigue Reliability analysis of gear teeth subjected to bending" Computational Mechanics 21,253-261, spring-verlegl, 1998.

[7]: Jweeg, M.J, Balsam H. Abdshebib and Nabil, M. Motash, "Investigation of Involute-gear tooth critical section", Journal of Military Engineering College, No.5, 1989.

[8]: AL-Shaidany,A.R,"The effect of Geometrical and Dynamical Parameters on gear design", Military Engineering College,2002.

[9]: Hamilton H. Mabie, Charles F. Reinholtz, "Mechanisms and Dynamics of Machinery", 4th Edition, J. Wiley, New York, 1991.

[10]: Maitra, G.M, "Gear Design Handbook", design Department Rourkela Steel Plant, Rourkela. Copyright by TATA, McGraw-Hill Publishing Company Ltd, India, 1985.

[11]: Gustav, Niemann "Machine Element" McGraw-Hill publishing Company, India, 1960.

[12]: Colbourne, J.R "The Geometry of Involute Gears", Library of Congress cataloging in Publication , copyright by Springer verlage, New York , Inc. 1987.

[13]: CAD Quest Pro. "Involute Gears" Engineer Tips and Tricks page, WWW.cadquest.com /tips/htm 2000.

List of Symbols:

Symbol	Definitions	Units
Ad	**Addendum**	**mm**
A_{ddc}	**Addendum of cutter**	**mm**
Cf	**Correction factor**	
CTT1	**Circular tooth thickness**	**mm**
CTT2	**Arbitrary circular tooth thickness**	**mm**
Ded	**Dedendum**	**mm**
db	**Base circle**	**mm**
df	**Root diameter**	**mm**
m	**module**	**mm**
ra	**Addendum radius**	**mm**
rb	**base circle radius**	**mm**
re	**Curvature radius of tooth fillet**	**mm**
rf	**Root circle radius**	**mm**
R	**Radius of any point**	**mm**
Rrt	**Fillet radius of cutter tooth (hob tip radius)**	**mm**
rp	**Pitch radius**	**mm**
TH	**Hob tooth thickness**	**mm**
tp	**Hob tooth space**	**mm**
v	**Angle between y_T and centre of tooth space**	**degree**
w	**Angle between y_T and centre of tooth space**	**degree**
X	**X-coordinate of involute curve points**	**mm**
X_T	**XT- coordinate of trochoid curve points**	**mm**
Y_T	**YT- coordinate of trochoid curve points**	**mm**
XX	**X- coordinate of trochoid curve points**	**mm**
Y	**Y- coordinate of trochoid curve points**	**mm**
yy	**Y- coordinate of trochoid curve points**	**mm**
Y_T	**YT- coordinate of trochoid curve points**	**mm**
Z	**Number of teeth**	

Greek Symbols		
ϕ	**Pressure angle**	**degree**
θ	**Involute value (Inv)**	

Table (1) The parameters for selected cases of gear tooth design:

Case No.	Module M(mm)	No. of teeth Z	pressure angle ϕ (deg.)	correction factor Cf	coordinate of the weakest point at fillet	
					X (mm)	Y (mm)
1	5	25	14.5	0	4.066	59.382
2	5	25	14.5	0.25	5.236	57.643
3	5	25	14.5	0.5	5.173	59.098
4	5	25	20	0	4.5656	57.15
5	5	25	20	0.25	5.253	58.223
6	5	25	20	0.5	5.489	59.389
7	5	25	25	0	5.437	57.106
8	5	25	25	0.25	5.93	57.864
9	5	25	25	0.5	6.197	58.988

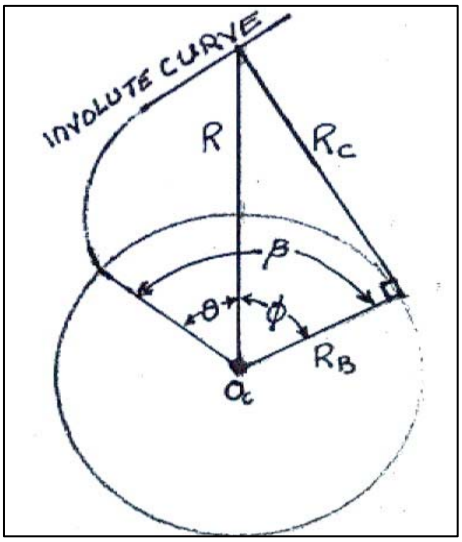

Fig. (1) Involute curve formation

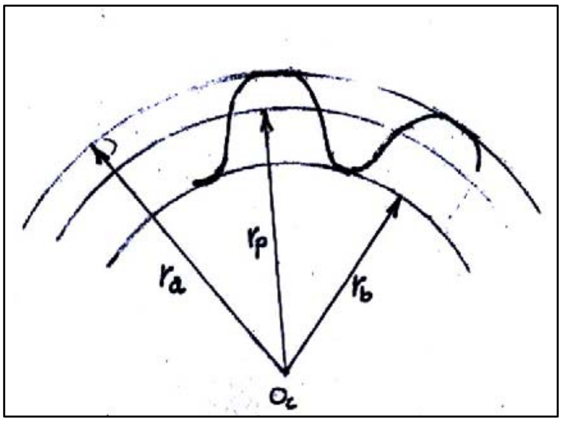

Fig. (1a) radii of circles

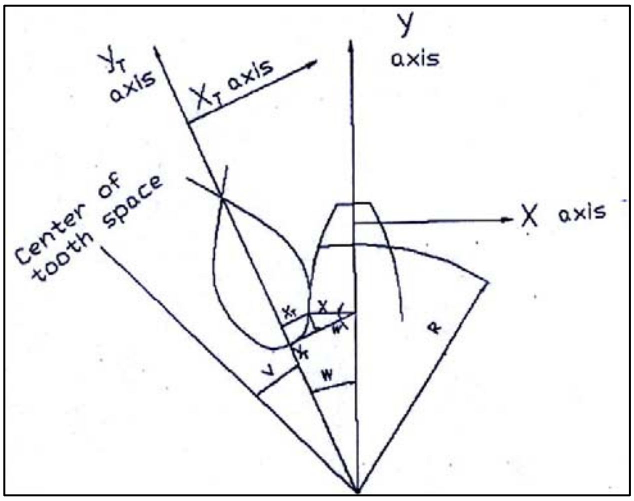

Fig. (2) Root fillet Trochoid and involute curve

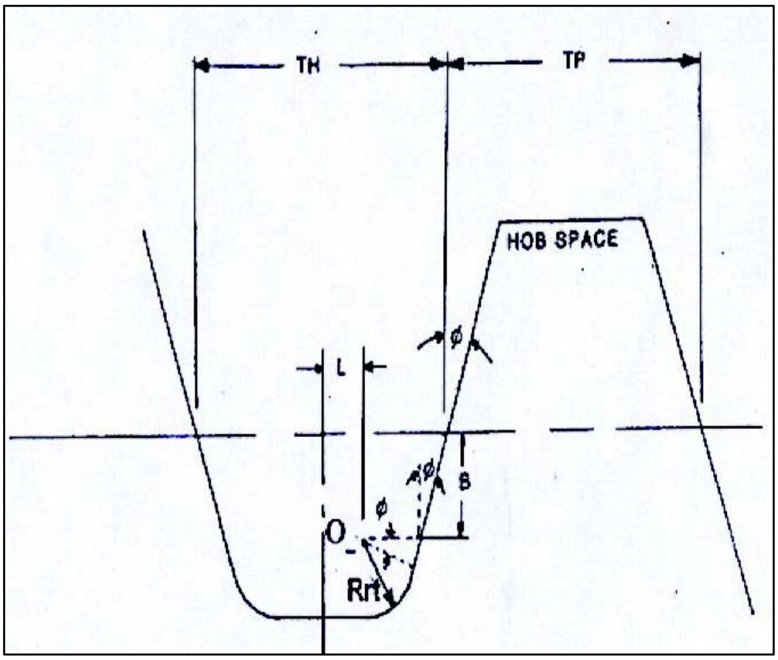

Fig. (3) hob geometry

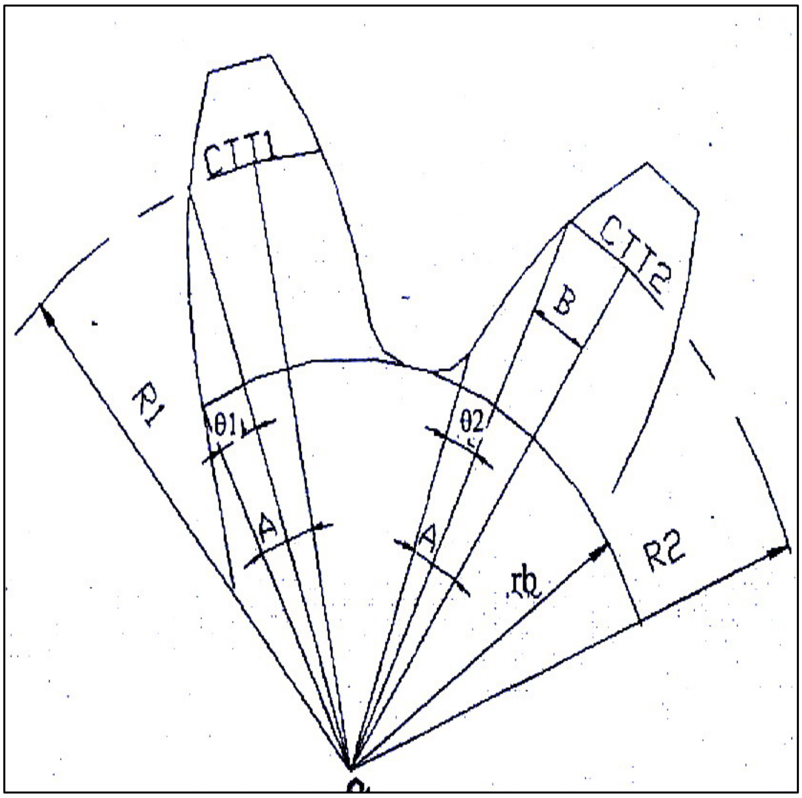

Fig.(4) Tooth thickness calculation

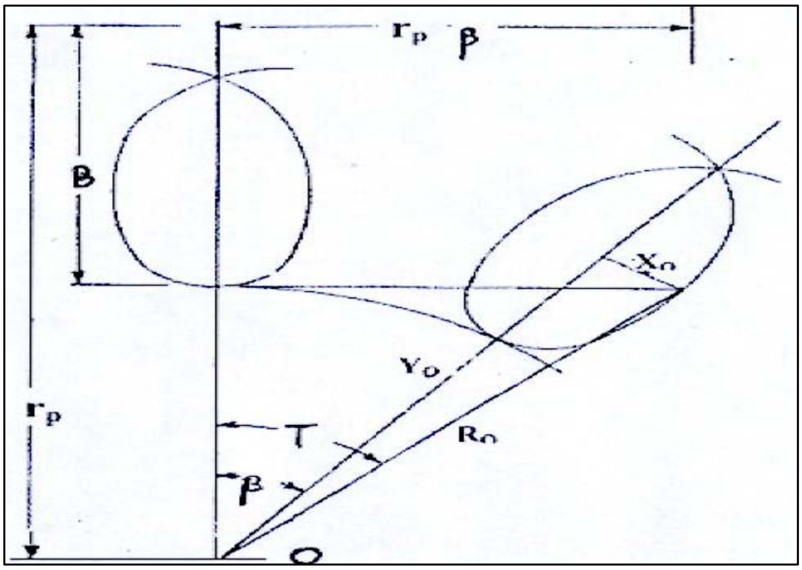

Fig.(5) Trochoid generated by point O

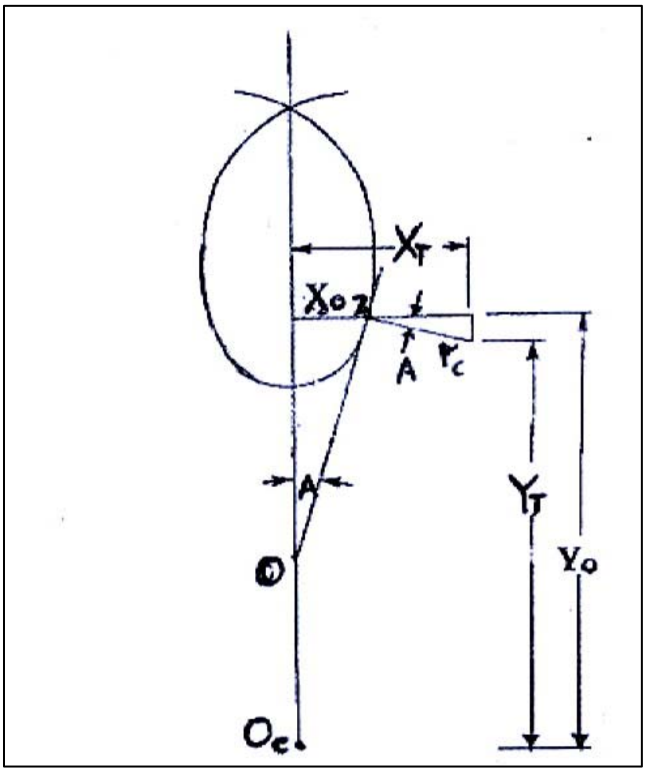

Fig. (6)Trochoid coordinates

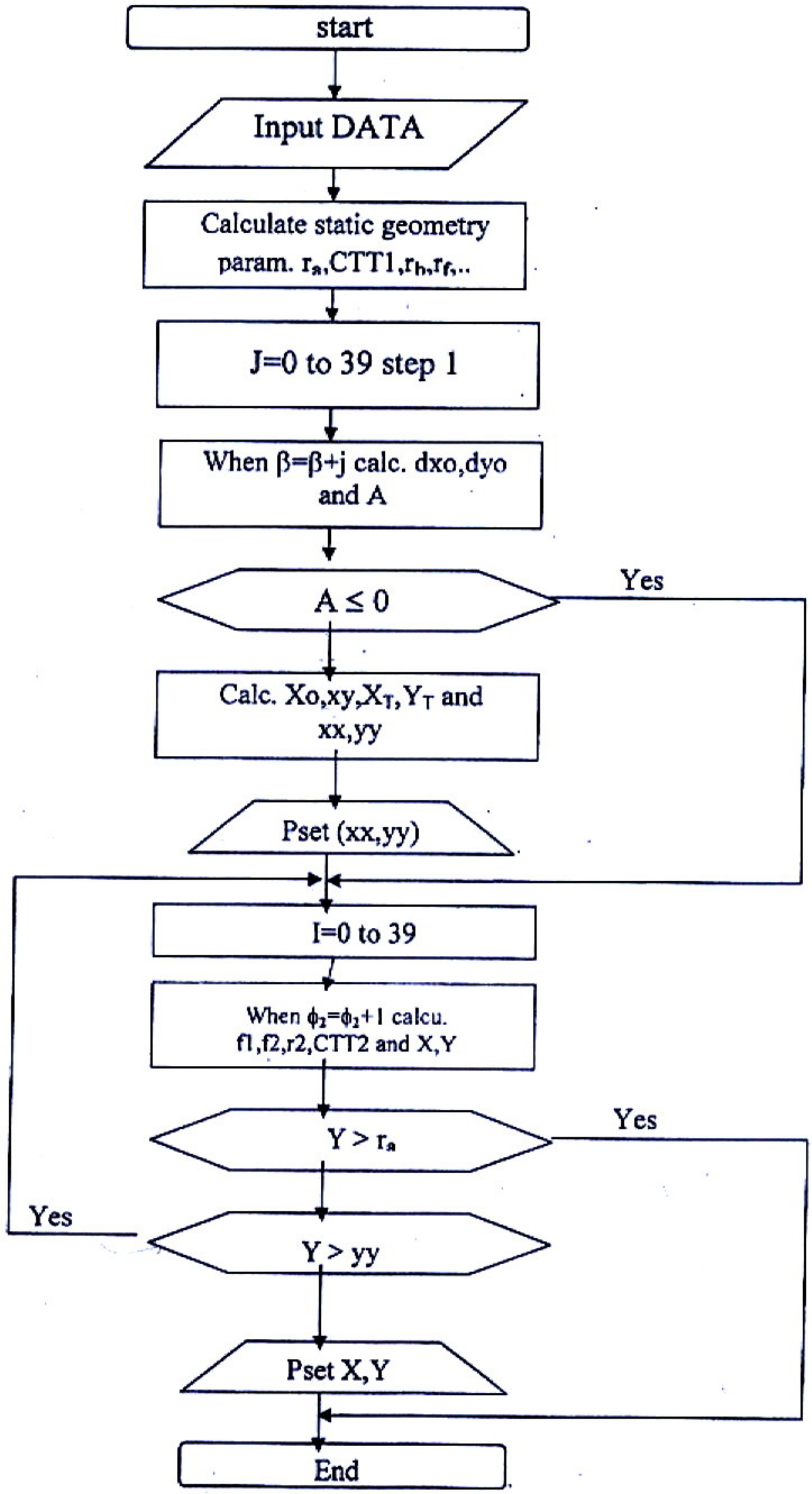
start
Input DATA
Calculate static geometry param. ra,CTT1,rh,rf,..
J=0 to 39 step 1
When β=β+j calc. dxo,dyo and A
A ≤ 0
Yes
Calc. Xo,xy,XT,YT and xx,yy
Pset (xx,yy)
I=0 to 39
When ϕ2=ϕ2+1 calcu. f1,f2,r2,CTT2 and X,Y
Y > ra
Yes
Y > yy
Yes
Pset X,Y
End

Fig.(7) Flowchart of spur gear tooth generation drawing program (program 1)

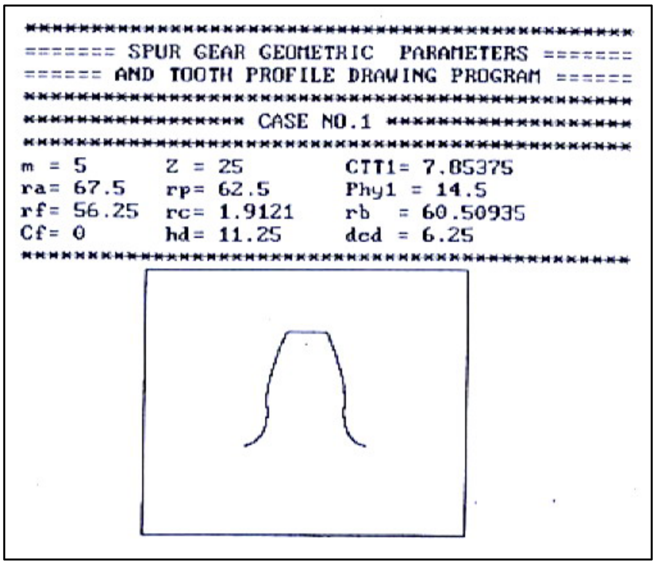

Fig.8

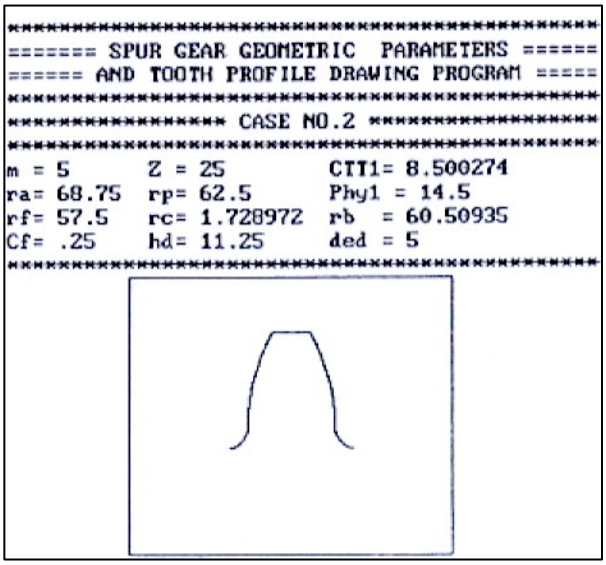

Fig.9

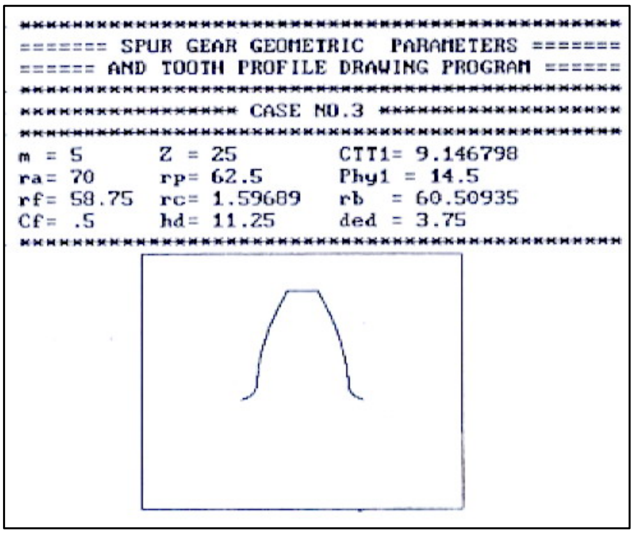

Fig.10

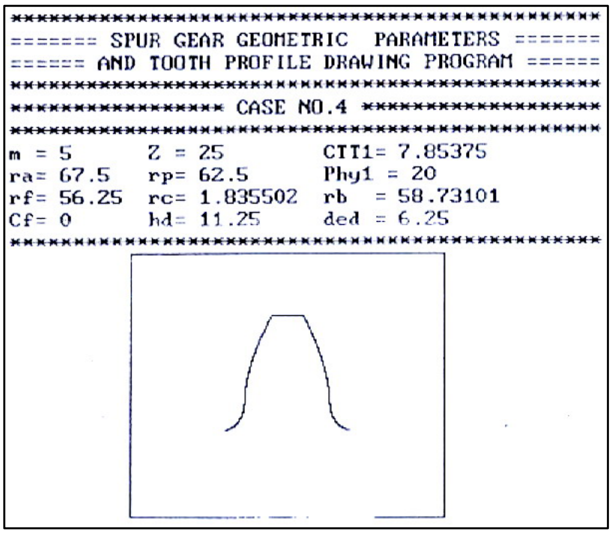

Fig.11

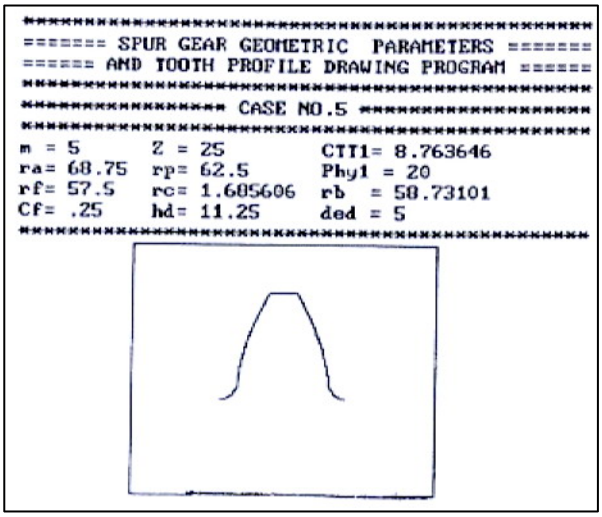

Fig.12

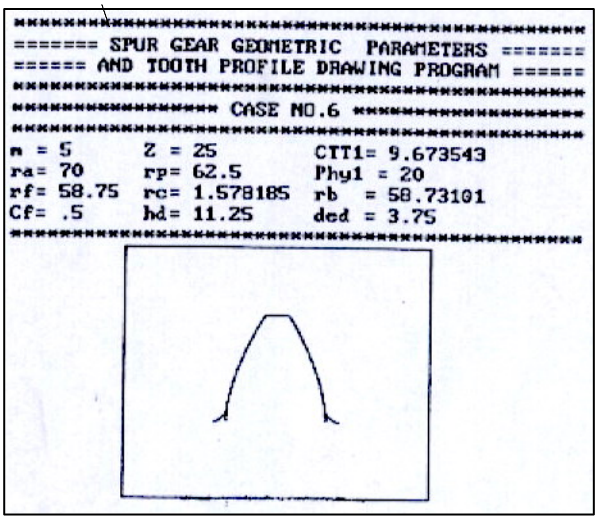

Fig.13

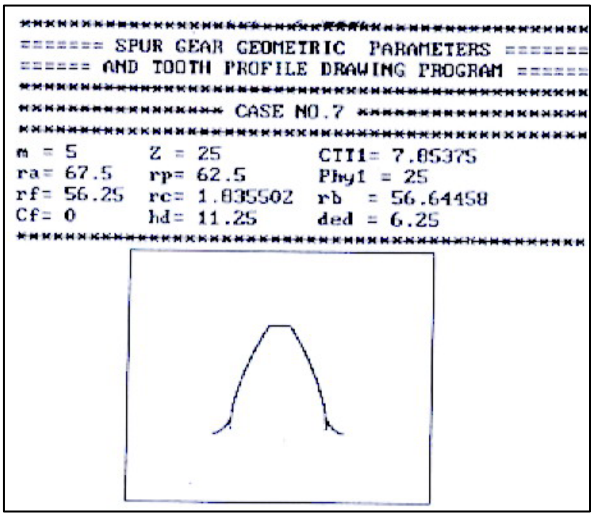

Fig.14

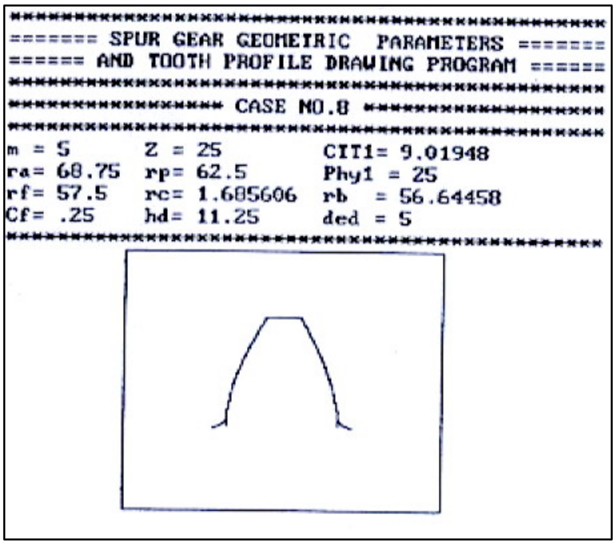

Fig.15

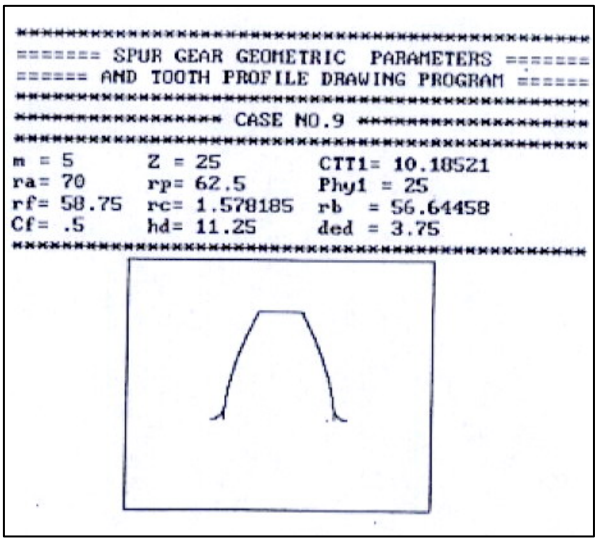

Fig.16

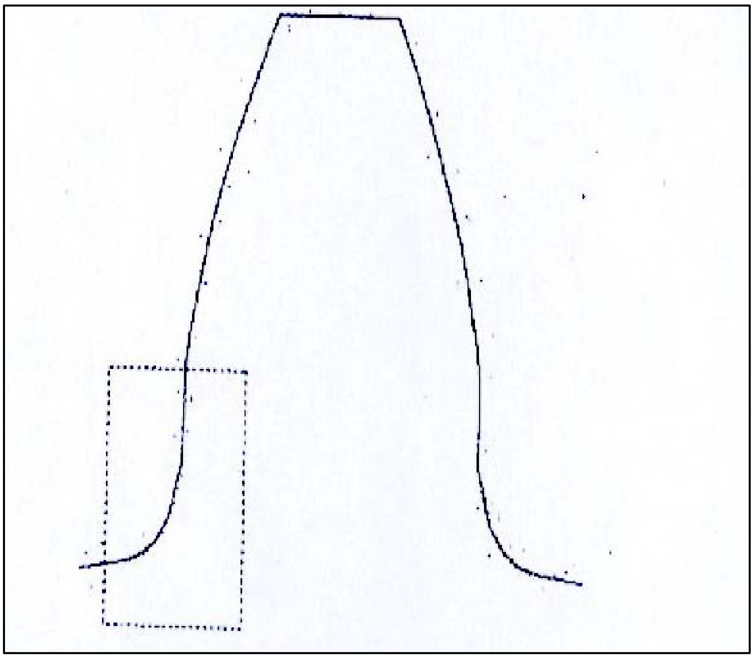

Fig.17 Tooth profile of case 4
From Mechanical Desk Top pakage Ver .5

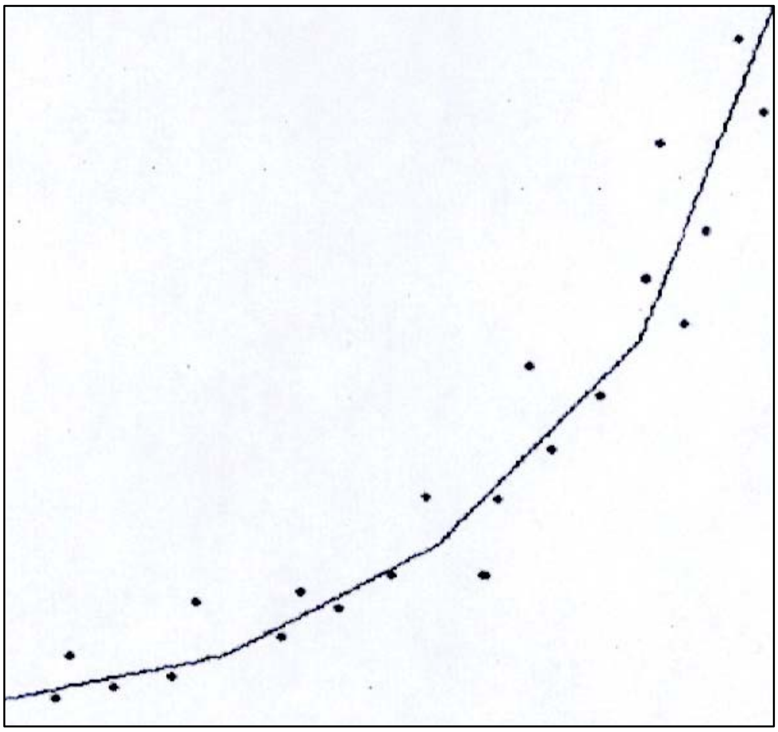

Fig.18Trochiod curve of tooth profile for case 4
From Mechanical Desk Top pakage Ver .5