

## Simulation of the Acousto-optic Interaction in an Optical Bistable Device

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### Abstract

In this paper, a computer-aided simulation of the relation between peak phase delay and laser intensity, at different values of the Klein-Cook parameter of the fourth order diffraction, was presented. The numerical results describing the sound-light interaction in LiNbO<sub>3</sub> crystal with and without feedback were presented and discussed. Also, the relations between laser intensities and the acousto-optic frequency and the values of Klein-Cook parameter were studied. The results were applied to an optical bistable device using MATLAB software.

**Keywords:** Sound-light interaction; Acousto-optic; Peak Phase delay; Bistable Optical Device;.

Q

Q

### 1-Introduction

An acousto-optic modulator (AOM) consists of a small piece of crystal or glass to which a piezoelectric transducer is bonded. When a voltage waveform is applied to the transducer, an AOM can be made to intensity modulate, frequency shift or deflect a laser beam due to the acousto-optic effect[1].

Basically an AOM has the ability to modulate light waves by electrical signals. In an AOM, an interaction between sound and light is formed, known as acousto-optics interaction. Through the action of a piezoelectric transducer, electrical signals are converted into sound waves propagating in the AOM. The pressure in the sound wave causes perturbations in the index refractive of the medium, which in turn creates a phase grating in the material and splits the incident laser light into various diffracted orders. A bistable device has a capability to generate two different

outputs for a given input[2]. The nonlinearity associated with the AOM and the feedback causes bistability in the overall system.

### 2- Computer-Aided Simulation Without Feedback

A more complete mathematical model of acousto-optic interactions than either the diffraction grating or wave-vector diagram representations is the multiple plane wave interaction models. The theory represents the light and sound fields as plane wave decompositions and describes their interaction as a multiple scattering of plane waves. According to the theory, the complex amplitude of the  $n$ -th order diffracted plane wave propagating along  $\theta_n = \theta_{inc} + 2n\theta_B$ ,  $\tilde{E}_n$ , is given by the following set of infinitely-linked complex differential equations [3,4]:

$$\begin{aligned} \frac{d\tilde{E}_n}{d\zeta} = & -j\frac{\hat{\alpha}}{2} \exp\left[-\frac{jQ\zeta}{2}\left[\frac{\theta_{inc}}{\theta_B} + (2n-1)\right]\right] \tilde{E}_{n-1} \\ & -j\frac{\hat{\alpha}}{2} \exp\left[\frac{jQ\zeta}{2}\left[\frac{\theta_{inc}}{\theta_B} + (2n+1)\right]\right] \tilde{E}_{n+1} \end{aligned} \tag{1}$$

where:  $\alpha$  represents the peak phase delay through the medium and is given by  $\alpha = Ck_0SL/2$  where  $C$  is the strain-optic coefficient of the medium and  $S$  is the amplitude of the sound wave propagating along the  $x$ -direction,  $\zeta$  is the normalized position inside the acousto-optic cell and is equal to  $z/L$  in case of light propagating in the  $z$ -direction through a cell of length  $L$ ,  $\theta_{inc}$  is the angle of incident plane wave, and  $Q$  is Klein-Cook parameter given by  $Q=2\pi\lambda Lf_a/nV_a$  where  $L$  is the crystal length,  $f_a$  is acoustic wave frequency and  $V_a$  is acoustic velocity.

A set of equations that only accounts for four diffracted orders can also be extracted from equation(1):

$$\begin{aligned} \frac{dE_{-1}}{d\zeta} = & -j\frac{\hat{\alpha}}{2} \exp\left[\frac{jQ\zeta}{2}\left(\frac{\theta_{inc}}{\theta_B} - 1\right)\right] E_o \\ \frac{dE_o}{d\zeta} = & -j\frac{\hat{\alpha}}{2} \exp\left[\frac{jQ\zeta}{2}\left(\frac{\theta_{inc}}{\theta_B} - 1\right)\right] E_{-1} \\ & -j\frac{\hat{\alpha}}{2} \exp\left[\frac{jQ\zeta}{2}\left(\frac{\theta_{inc}}{\theta_B} + 1\right)\right] E_o \\ \frac{dE_1}{d\zeta} = & -j\frac{\hat{\alpha}}{2} \exp\left[\frac{jQ\zeta}{2}\left(\frac{\theta_{inc}}{\theta_B} + 1\right)\right] E_o \\ & -j\frac{\hat{\alpha}}{2} \exp\left[\frac{jQ\zeta}{2}\left(\frac{\theta_{inc}}{\theta_B} + 3\right)\right] E_2 \\ \frac{dE_2}{d\zeta} = & -j\frac{\hat{\alpha}}{2} \exp\left[\frac{jQ\zeta}{2}\left(\frac{\theta_{inc}}{\theta_B} + 3\right)\right] E_1 \end{aligned}$$

With initial condition  $E_0(\zeta = 0) = E_{inc}, E_1(0) = 0, E_{-1}(0) = 0$  and  $E_2(0) = 0$ .

### Simulation of the Bistable Optical Device (With Feedback)

A numerical simulation of the AO device is used to predict the general behavior of the output intensity and to our knowledge treatment of the device with the feedback. Bistability occurs when non-linearity is introduced to the system. For the four-diffracted order system developed in this paper, non-linearity can be caused by a feedback. For this purpose, the first order intensity is amplified, a bias voltage ( $\alpha_0$ ) is added and feedback the AOM input feedback [2]. Thus, the laser beam would be intensity modulated by its feedback. This feedback signal has a recursive influence on the diffracted light intensities until a steady state occurs. The situation is shown in figure (1).

In the simulation of the bistable device, the effective  $\alpha$  scattering the light in the acousto-optic cell is given by the feedback equation [4-5]:

$$\hat{\alpha} = \hat{\alpha}_0 + \beta |E_1|^2 \tag{2}$$

where  $\beta$  denotes the product of the amplifier gain constant and the quantum efficiency of the photodetector(PD).

Note that the non-linearity involving only two diffracted orders is a sine squared function :

$$I_1 = |E_1|^2 = I_{inc} \text{Sin}^2(\hat{\alpha}/2) \tag{3}$$

where  $I_{inc} = |E_{inc}|^2$  is the incident intensity and we have a system with a nonlinear

input ( $\hat{\alpha}$ ) –output ( $|E_1|^2$ ) relationship.

The steady-state behavior of the system is then given by simultaneous solution of equations (2) and (3). The

hysteric behavior can be numerically found by MATLAB software.

### Results and Discussions

The solutions of the four-order system are difficult to be obtained analytically. The Runge-Kutta numerical method for solving boundary value problems was chosen for solving the four-order diffraction problem because it is easy to implement from scratch in MATLAB because of its high accuracy.

The simulation results for four order diffraction is shown in figure (2a). It shows that the predicted intensity of the zero diffraction has first maximum when the value of Cook-Klein parameter is about one. The intensity of the first order diffraction is slightly less than other diffraction orders. This is because of some light being diffracted into the minus first and second orders as well. Subsequent intensity peaks of the first order become lower as more and more light gets diffracted into higher orders.

This result can be easily verified by summing the intensities of non-zero orders as shown in figure(2b). The result, of course, should be exactly equal to the incident intensity (in our simulations  $I_{inc} = 1$ ). The amount of variation from this value can be used to calculate the error introduced by the numerical method used to solve the equations system.

The value of Klein-Cook parameter equal to one is not an interface between the two modes of diffraction (single and multiple). Decreasing Klein-Cook parameter lower than one assumes that the interaction is occurred in the multiple diffraction region. Scaling more than one, the interaction is supposed to occur in the single diffraction (Bragg diffraction) region. In fact, several publications have supposed that the range(1-10) of Q parameter is a combined range, i.e., it can be considered as single or multiple diffraction according to the required application and accuracy. In such combined range, the intensities of higher

diffraction orders are weak or negligible as the Q parameter being higher.

As Q parameter is lower than one, the relative intensity of the first and zeroth diffraction orders is high as well as that of the minus first order. Whereas the relative intensity of the second order is low and this is an ideal case as shown in figure(3). This means the occurrence of multiple (Raman-Nath) diffraction when Q parameter being lower than (0.1).

On the other hand, figures related to the Q parameter more than 1 explained extreme decreasing in the relative intensity of the first and second diffraction orders as well as for the zeroth and minus first orders especially when Q parameter being more than 10, as shown in figure(4). This supposes good agreement between the instrumentation and mathematical solution.

The four-order diffraction model can be used to predict a variety of other features of acousto-optic interaction. One interesting result is to predict what happens when the voltage waveform applied to the transducer increases. A plot of the first order intensity versus frequency is shown in figure(5). It can be noticed that the first order intensity increases approximately linear with acoustic wave frequency because the device operates in the Raman-Nath region. Again, we have seen that the first order intensity approaches a maximum and remains nearly constant as the frequency increases.

Acousto-optic modulators can be operated in two regions. The Klein-Cook parameter can be used to determine the region a device is operating in. Generally, a device is said to be operating in the Raman-Nath region for  $Q < 1$ . Operating is said to be in the Bragg region for  $Q > 1$ .

Because the Klein-Cook parameter is dependent of frequency, we can also plot the intensity of the first order diffraction versus Q using the same MATLAB software and generating a Q parameter. The results in figure(6) show the two modes of operation of the AOM. For  $Q < 1$ , the first order diffraction intensity

increases approximately linear with Q parameter. For  $Q > 1$ , however, the first order intensity approaches a maximum and then remains nearly constant as the frequency increases.

The result obtained by MATLAB software with feedback gain ( $\beta$ ) of 2.6 about is shown in figure (7). Note from figure that a gradual increase in input bias  $\alpha_0$  produces a steady increase in the output intensity  $|E_1|^2$ , which represents the lower stable state (AB) until reaching a critical value at point (B) where the output switches up to the higher stable state (BC). Decreasing the input, the output does not immediately fall but remains on the upper branch of the curve (the higher stable state, (CD)) until the input is reduced to a lower critical value (DA), at which the output switches down.

For a feedback gain of 3.5, it takes the curve as shown in figure (8). Figure (9) shows a simulation run for first order at different value of feedback. It is noticed from the figures that hysteresis width differs for different values of the gain. This width increases as the feedback gain ( $\beta$ ) increases. Also, the effects of the Klein-Cook parameter value on the width of the hysteresis curve and high values of feedback ( $\beta$ ) constant require more step points.

**Conclusions:**

As concluding remarks, the intensity decreases as Klein-Cook parameter increases at the first-order diffraction, while at the other orders, the intensity increases with the increasing value of Klein-Cook parameter. The total intensity of all orders is exactly equal to the incident intensity. In the simulation of a bistable device, the hysteresis width increases as the feedback gain increases due to the nonlinearity properties of the medium tested (LiNbO<sub>3</sub> crystal).

**Reference:**

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 [2] Banerjee, P. P., Poon, T.-C., "Principles of Applied Optics", New York:Richard D. Irwin,Inc. 1991, 203, 213.  
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 [4] M. R. Chatterjee and J. -J. Huang, "Demonstration of acousto-optic bistability and chaos by direct nonlinear circuit modeling," Applied Optics, vol. 31, pp. 2506- 2517, 1992.  
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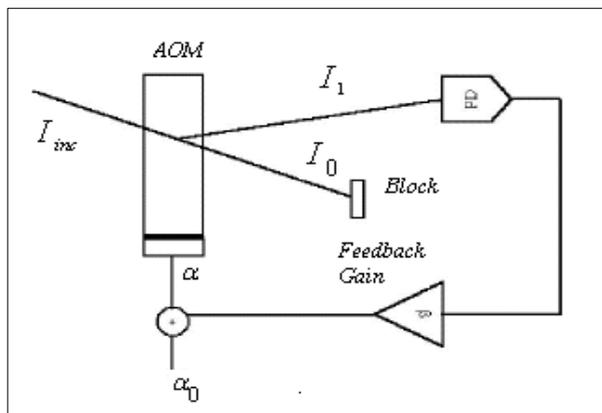
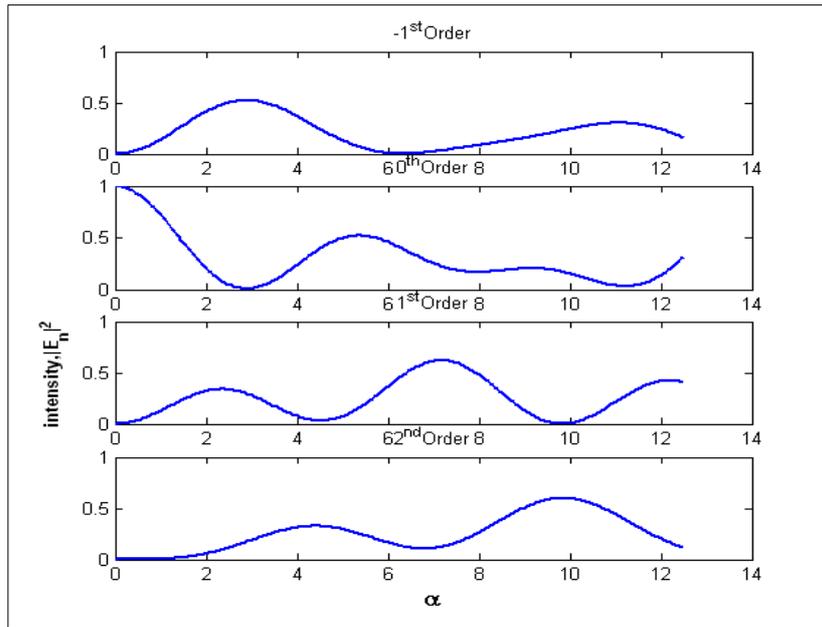
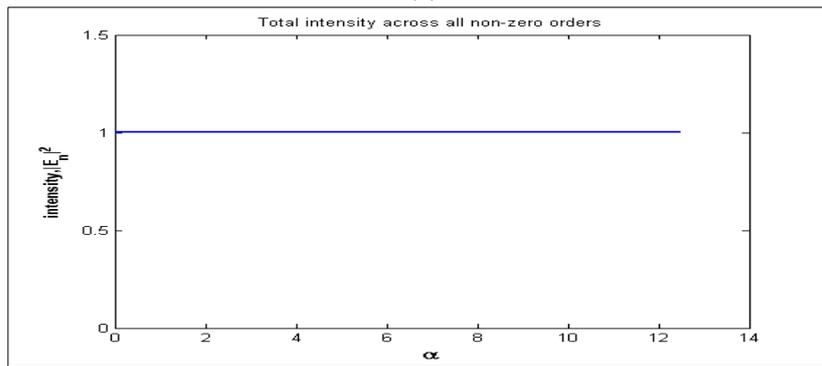


Figure (1): Principle of Bistable Optical Device with first order Feedback

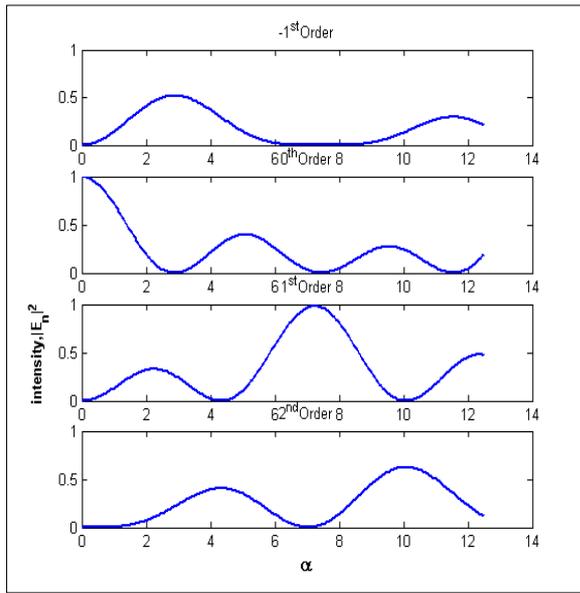


(a)

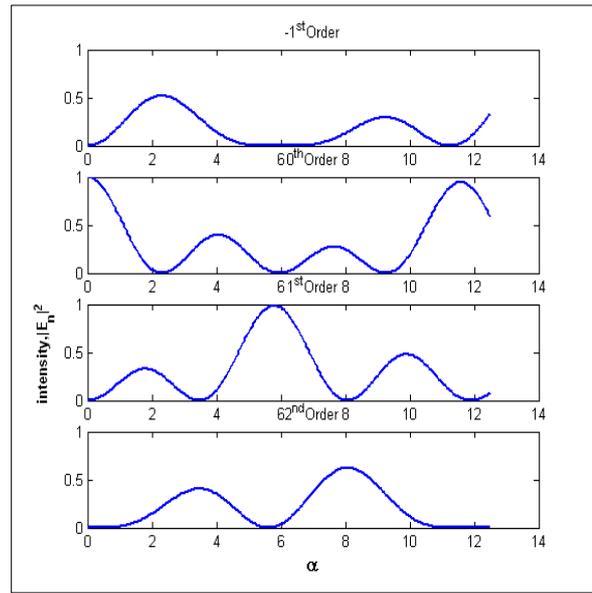


(b)

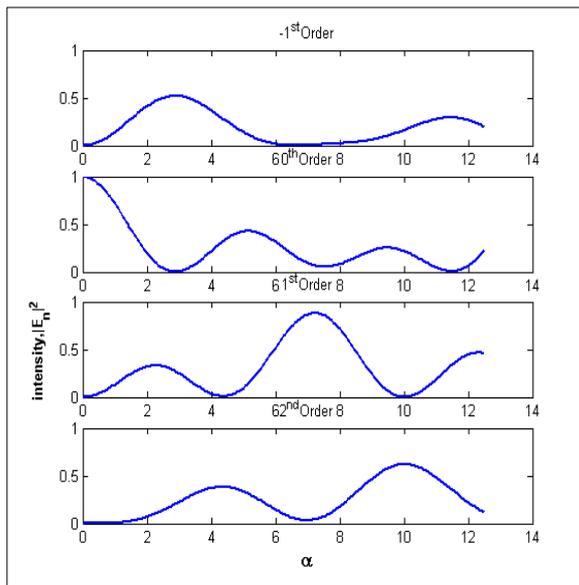
Figure (2): Solutions for (a) four order diffraction at  $Q=1$  and (b) Total calculated intensity across all non-zero



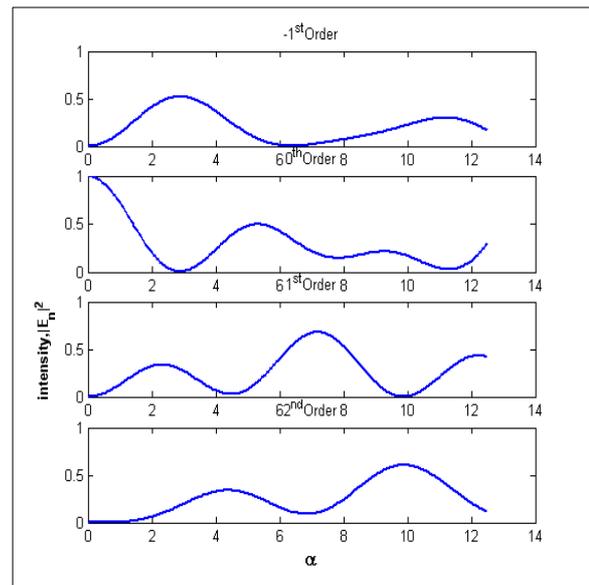
(a)



(b)

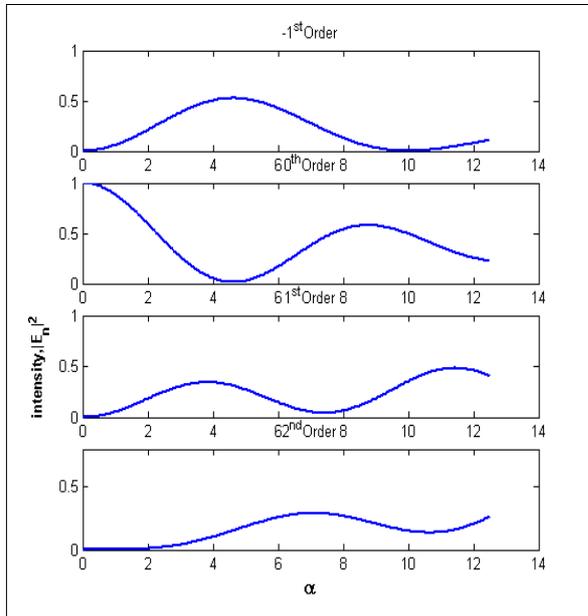


(c)

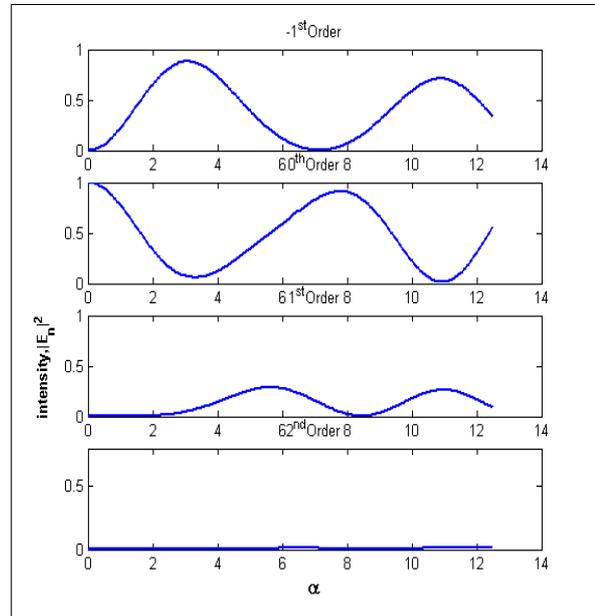


(d)

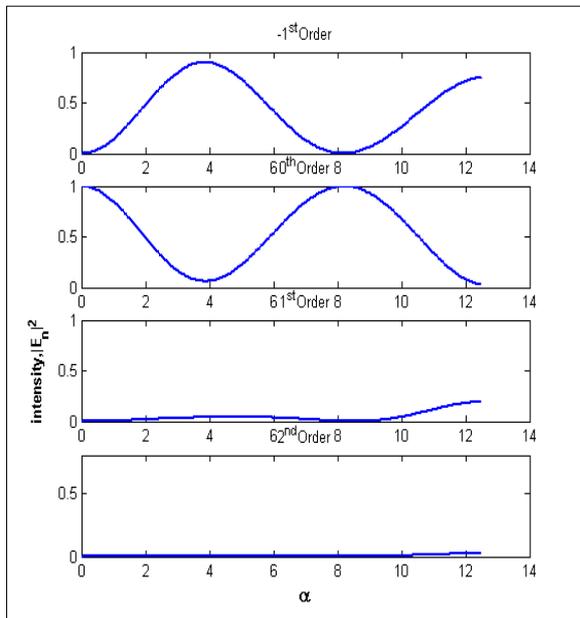
Figure (3): Solution for four order diffraction when Q is equal to (a)0.01, (b)0.1 ,(c)0.3 ,(d)0.8



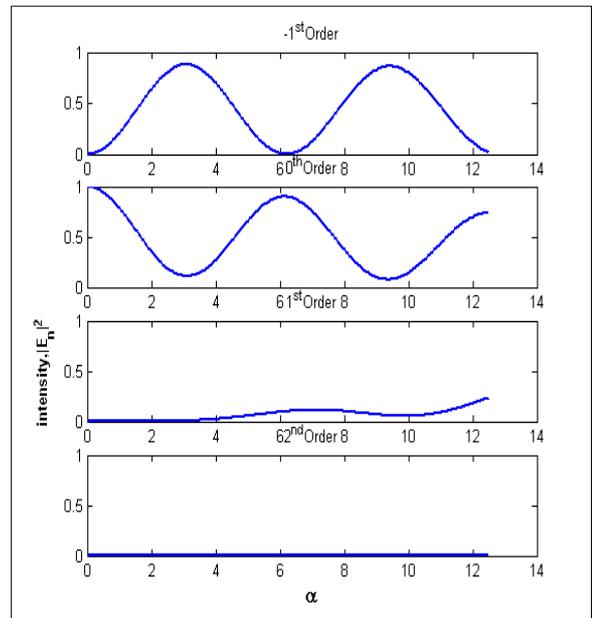
(a)



(b)



(c)



(d)

Figure (4): Solutions for four order diffraction when Q is equal to (a)2,, (b)8 ,(c)10 ,(d)14

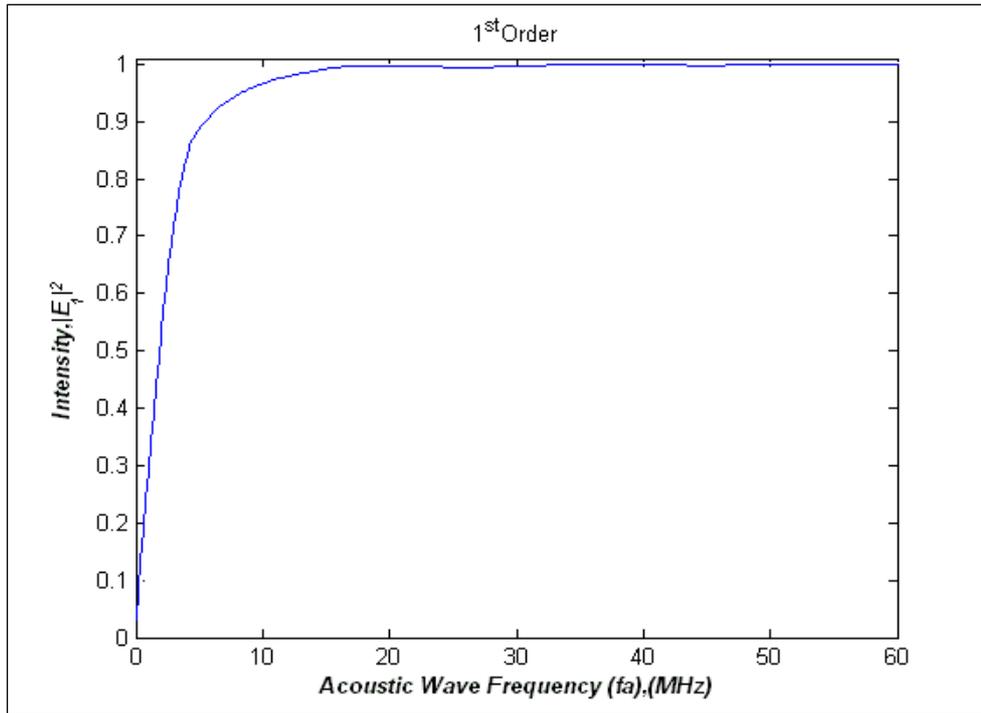


Figure (5): The frequency response of the Acousto-optic Cell

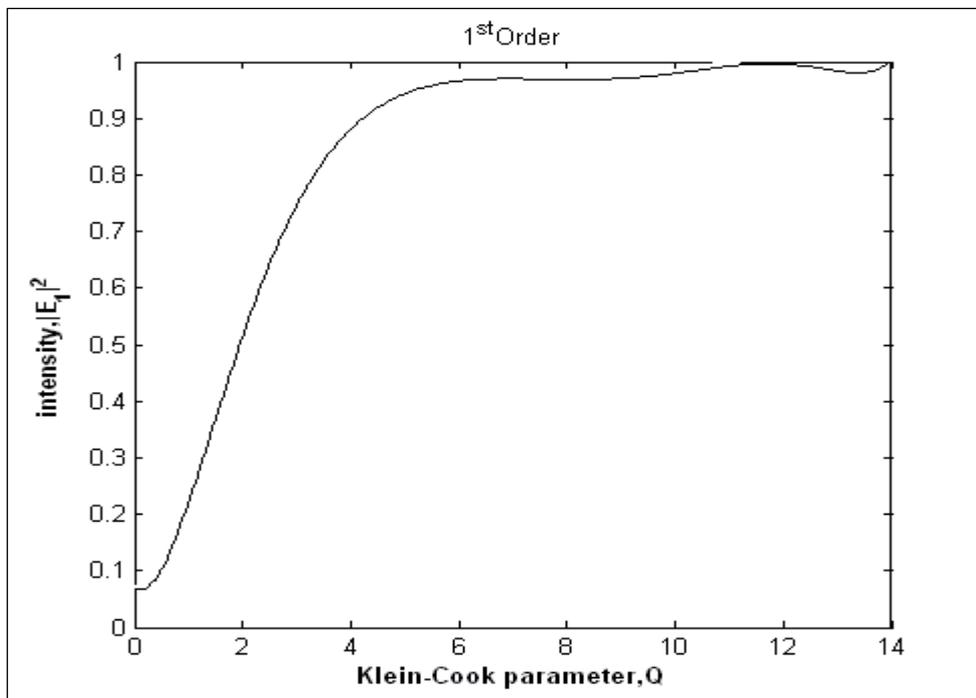


Figure (6): First order intensity versus the Klein-Cook parameter

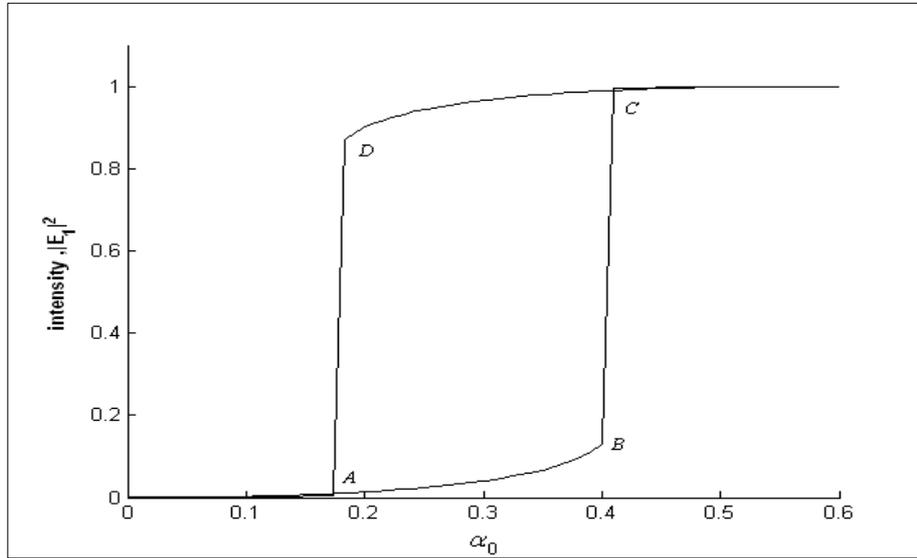


Figure (7) Simulation run for first order diffraction at feedback ( $\beta=2.6$ )

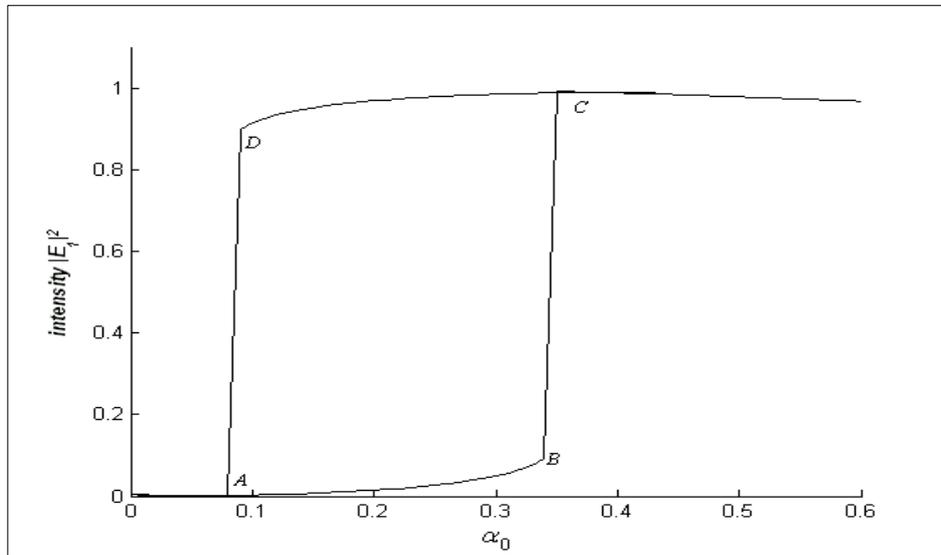
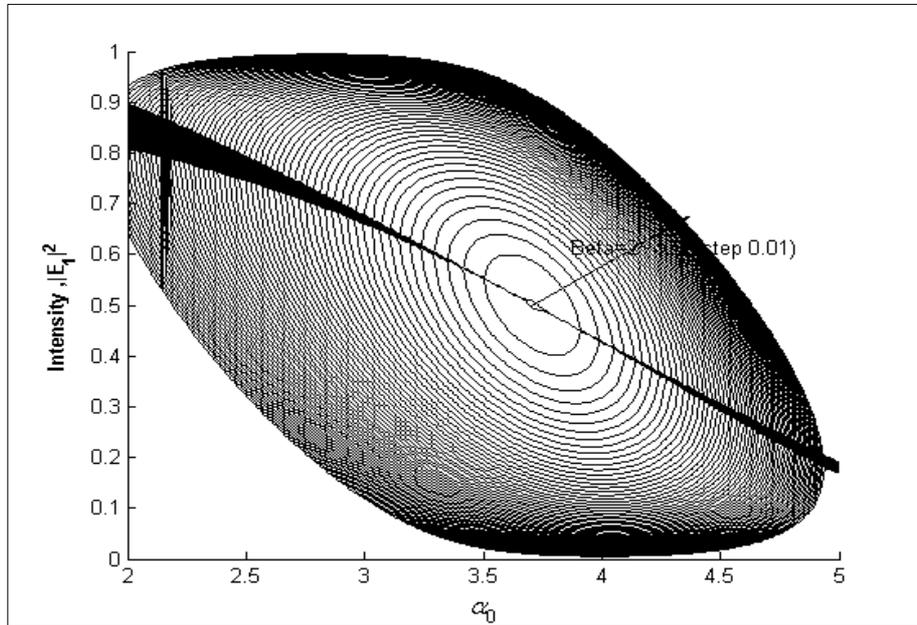
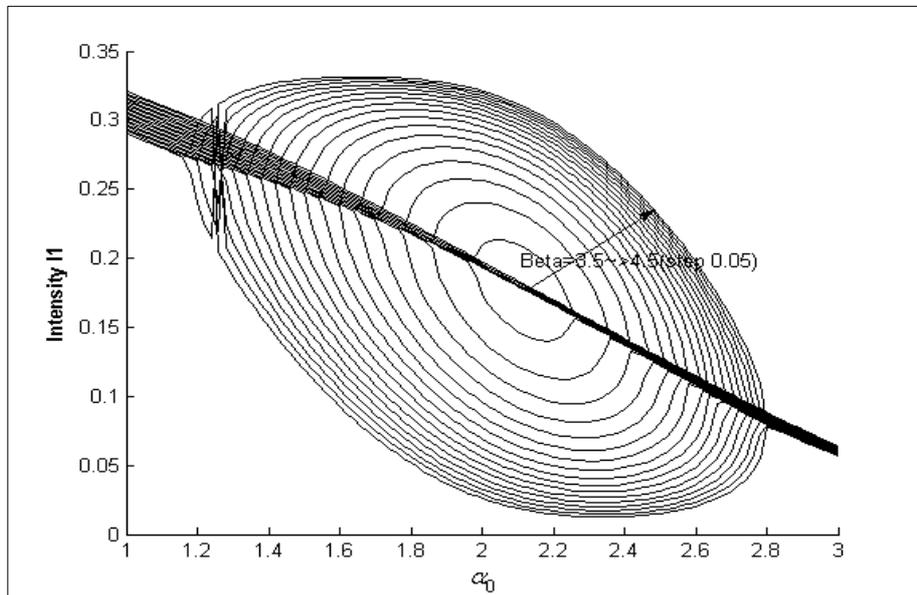


Figure (8) Simulation run for first order diffraction at feedback ( $\beta=3.5$ )



(a)



(b)

Figure (9): Simulation run for first order at different feedback (a)  $\beta=2-2.6$  and (b)  $\beta=3.5-4.5$