Evaluation Of The Method Of Stress Characteristics For Estimation Of The Soil Bearing Capacity

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Abstract

The classical bearing capacity theories rely on the superposition of three separate bearing capacities – a technique that is inherently conservative – but they also rely on tabulated or curve-fitted values of the bearing capacity factor, $N\gamma$, which may be unconservative. Further approximations are introduced if the footing is circular (multiplicative shape factors are used to modify the plane strain values of , Nc, Nq and $N\gamma$) or if the soil is non-homogeneous (calculations must then be based on some representative strength). By contrast, the method of stress characteristics constructs a numerical solution from first principles, without resorting to superposition, shape factors or any other form of approximation.

In this paper, the validation of the method of stress characteristics is tested by solving a wide range of bearing capacity problems. The results are compared with classical bearing capacity theories; namely, Terzaghi, Myerhof, Hansen and Vesic methods.

It was concluded that **t**he bearing capacity predicted by the method of stress characteristics for the case of a circular footing in clay ranges between (3.7 – 4.0) greater than Terzaghi, Meyerhof, and Vesic methods. This means that the method is not conservative for this case and can be dependent for economic design of foundations. The bearing capacity predicted by this method increases linearly with (D/B).

For all values of the angle of friction, ϕ, the method reveals bearing capacity values for smooth footings greater than Terzaghi and Hansen and smaller than Meyerhof and Vesic theories. Considering the foundation to be rough, the method gives bearing capacity values greater than all other methods. The difference increases as the angle of internal friction (ϕ) increases. This makes the method unreliable for rough foundations.

Keywords: Bearing capacity, Stress characteristics

تقييم طريقة خصائص الإجهاد في تخمين قابلية تحمل التربة

الخلاصة

إن النظريات التقليدية لحساب قابلية تحمل التربة تعتمد على تجميع ثلاث قابليات تحمل مختلفة (و هذه التقنية تعتبر ضمنيا تقليدية) و لكنها تعتمد أيضا على قيم معامل قابلية تحمل التربة, N□ التي إما أن تكون مرتبة في جداول أو قيم مقربة من المنحنيات وهذه القيم قد تكون غير تقليدية أو مبالغ فيها. و هناك تقريبات إضافية تدخل إذا كان الأساس دائريا (حيث تستعمل معاملات الشكل لتغيير قيم الانفعال المستوي للمعاملاتNc و □q و N□) أو عندما تكون التربة غير متجانسة (حيث في هذه الحالة يجب أن تكون الحسابات مبنية على قيم ممثلة للمقاومة). و بالمقارنة, تقوم طريقة خصائص الإجهاد بإنشاء حل عددي من المبادئ الأساسية دون الرجوع إلى التجميع أو معاملات الشكل أو أي شكل من أشكال التقريب.

في هذا البحث تم اختبار صلاحية طريقة خصائص الإجهاد بحل مدى واسع من مسائل قابلية التحمل. و قورنت النتائج مع النظريات التقليدية لقابلية التحمل مثل طرق ترزاكي و ميرهوف و هانسن و فسك. و قد وجد أن قابلية التحمل المحسوبة بطريقة خصائص الإجهاد لحالة أساس دائري في الطين أكبر بحوالي (3.7 - 4.0) مرة من طرق ترزاكي و ميرهوف و فسك. و هذه يعني أن الطريقة ليست تقليدية لهذه الحالة و يمكن اعتمادها لتصميم اقتصادي للأسس. كما أن قابلية التحمل المحسوبة بهذه الطريقة تزداد خطيا مع نسبة العمق إلى عرض الأساس (D/B). و لجميع قيم زاوية الاحتكاك الداخلي (□) تعطي هذه الطريقة قيما لقابلية التحمل للأسس الملساء أعلى من نظريات ترزاكي و هانسن و أقل من نظريات ميرهوف و فسك. و عند اعتبار الأساس خشنا, تعطي الطريقة قيما لقابلية التحمل أعلى من جميع الطرق الأخرى, و يزداد الفرق مع زيادة زاوية الاحتكاك الداخلي (□) و هذا يجعل الطريقة غير معتمدة للأسس الخشنة.

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Introduction

During the last fifty years, several bearing capacity theories were proposed for estimating the ultimate bearing capacity of shallow foundations.

In 1948, Terzaghi proposed a well-conceived theory to determine the ultimate bearing capacity of a shallow rough rigid *continuous* (strip) foundation supported by a homogeneous soil layer extending to a great depth. Hansen (1970) proposed approximate relationships for the bearing capacity coefficients, Nc, $N\gamma$ and Nq.

In 1951, Meyerhof published a bearing capacity theory which could be applied to rough shallow and deep foundations.

Vesic (1973) suggested a better mode to obtain ϕ' for estimating the bearing capacity coefficients *Nc´* and *Nq´* for foundations on sand in terms of relative density.

It was previously suggested that the plane strain soil friction angle ϕ_p instead of ϕ_r be used to estimate the bearing capacity. To that effect, Vesic (1973) raised the issue that this type of assumption might help explain the differences between the theoretical and experimental results for long rectangular foundations. However, it does not help to interpret results of tests with square or circular foundations.

The theory uses the method of stress characteristics (also known as the slip line method) to solve the classical geotechnical bearing capacity problem of a rigid foundation, resting on a cohesive-frictional soil mass, loaded to failure by a central vertical force. Figure (1) shows the terminology used in this method.

Soil

- The soil is modelled as a rigid–perfectly plastic Mohr-Coulomb material, assumed to be isotropic and of semi-infinite extent.
- The cohesion c can vary linearly with depth ($c = c_o + k.z$).
- The friction angle ϕ and unit weight γ are taken to be constant.

Footing

- Plane strain (strip footing) and axially symmetric (circular footing) analyses can be performed.
- The soil–footing interface can be modelled as smooth or rough.
- A uniform surcharge pressure q can be applied to the soil adjacent to the footing.

Solution

- The mesh or 'net' of characteristics is constructed in an interactive environment.
- A sequence of increasingly accurate calculations, each one involving a finer mesh, can be used to obtain a converged solution.
- A variety of automated strategies, including mesh adaptivity, maintain the accuracy and efficiency of the method when solving difficult problems.

Results

- The bearing capacity is reported as a force Q and an average pressure, q_u.
- The mesh of characteristics can be annotated to show the footing tractions (as used to obtain the bearing capacity), and/or the principal stresses in the soil.

Status of Solutions

In the terminology of limit analysis, a converged solution obtained using the computer program ABC (developed by Martin in 2004) is classified as a 'partial' or 'incomplete' lower-bound collapse load. This is because only part of the stress field at collapse, namely the part needed to compute the bearing capacity, is constructed. For many problems in both plane strain and axial symmetry it has been shown that the bearing capacity obtained in this manner is identical to the exact collapse load, but various additional calculations (not currently performed by ABC) are needed to establish this formally for a particular combination of parameters. Specifically, these calculations involve proving that the partial lower-bound stress field can:

- be extended throughout the rest of the soil mass without violating equilibrium or yield;
- be associated with a velocity field that gives a coincident upper-bound collapse load.

It is important to realize that the bound and uniqueness theorems of limit analysis are only valid for ideal materials that exhibit perfect plasticity, i.e. no post-yield hardening or softening, and an associated flow rule. The latter requirement means that, in the case of a Mohr-Coulomb soil, the theorems are only applicable if the dilation angle ψ is equal to the friction angle ϕ. Even though real soils only exhibit such behaviour in the special case of undrained shearing (ϕ_u = ψ_u = 0), calculations based on associativity remain an important point of reference, e.g. the bearing capacity design methods in codes and standards are invariably based on factors Nc, Nq and N_γ that pertain to soil with an associated flow rule. Investigations into the effect of non-associativity on the bearing capacity of shallow foundations are ongoing.

Possible applications

Rigorous checks of traditional Nc, Nq and N_γ calculations, such as those prescribed in design codes:

These methods rely on the superposition of three separate bearing capacities – a technique that is inherently conservative – but they also rely on tabulated or curve-fitted values of the bearing capacity factor N_γ, which may be unconservative. Further approximations are introduced if the footing is circular (multiplicative shape factors are used to modify the plane strain values of Nc, Nq and N_γ) or if the soil is non-homogeneous (calculations must then be based on some representative strength). By contrast, the method of stress characteristics constructs a numerical solution from first principles, without resorting to superposition, shape factors or any other form of approximation.

Establishing benchmarks for the validation of other calculation methods:

To validate the performance of, say, a commercial finite element package that is to be used for bearing capacity calculations, a series of test problems could be specified and solved using the program ABC written by Martin (2004). These problems could then be analysed using the FE package, ensuring that the settings adopted were consistent with ABC (Mohr-Coulomb yield criterion, associated flow rule, etc.).

Bases and Criteria of the Method:

Introduction

If it is assumed a priori that the soil is at yield, the two-dimensional stress state at a point can be fully specified in terms of two auxiliary variables, namely the mean stress σ and the orientation θ of the major principal stress, together with a function that defines the radius of Mohr's circle of stress (and thus the strength of the soil). The sign conventions adopted for x, z, σ and θ are indicated in Figure (2), which also shows the yield criterion used in the program ABC.

$$\boldsymbol{R = c.\cos\phi + \sigma.\sin\phi} \qquad (1)$$

This is the two-dimensional form of the Mohr-Coulomb criterion. By definition, a Mohr-Coulomb soil is isotropic, because at a given point the strength does not depend on the orientation of the principal stresses – equation (1) is independent of θ. If the strength parameters c and ϕ are constant, the soil is described as homogeneous; if c and/or ϕ vary with position, the soil is non-homogeneous (though still isotropic, as defined above). For simplicity, ABC only allows vertical non-homogeneity in c, via the linear equation.

$$\boldsymbol{c = c_o + k.z} \qquad (2)$$

The friction angle ϕ is taken to be constant throughout the soil.

When the stresses-at-yield of Figure (2) are combined with the equations of equilibrium, a pair of coupled partial differential equations is obtained (spatial variables x and z, field variables σ and θ). Standard techniques can be used to show that this equation system is hyperbolic, and hence there are two distinct characteristic directions – here denoted α and β – along which the partial differential equations reduce to (coupled) ordinary differential equations. The relevant equations for plane strain and axial symmetry are summarised in the next section.

Governing equations

In the method, ε (= π/4 - ϕ/2) denotes the angle between the direction of the major principal stress and the directions of the α and β characteristics (Figure 2a). Note that the characteristics coincide with the planes on which the Mohr-Coulomb criterion is satisfied (Figure 2b).

Underside of footing

The soil directly beneath the footing is (by assumption) in a state of active failure. The orientation θ of the major principal stress depends on the roughness of the underside of the footing.

If it is smooth, the major principal stress is vertical:

. □$_{footing}$ = 0 (3)

If it is rough (and full roughness is mobilised, then the Mohr-Coulomb criterion is satisfied on the plane of the interface, hence:

. □$_{footing}$ = -□/4 - □/2 (4)

Mohr's circles for the two cases are shown in Figure (3). In the smooth case, the interface shear stress is zero, and the α and β characteristics are inclined at to the vertical. In the rough case the footing exerts an inward shear stress on the soil, with the β characteristic tangential to the interface.

4. Finite difference formulation

4.1 Plane strain

Stresses in terms of auxiliary variables:

$$\left.\begin{aligned} \sigma_{xx} &= \sigma - R.\cos 2\theta \\ \sigma_{zz} &= \sigma + R.\cos 2\theta \\ \tau_{xz} &= R.\sin\theta \end{aligned}\right\} \qquad (5)$$

Characteristics:

	Directions (6)	Ordinary differential equations (7)
α	$\frac{dx}{dz} = \tan(\theta + \varepsilon)$	$d\sigma + \frac{2R}{\cos\phi} d\theta = (\gamma_x - \gamma_z \tan\phi - k)\,dx + (\gamma_z + \gamma_x \tan\phi)\,dz$
β	$\frac{dx}{dz} = \tan(\theta - \varepsilon)$	$d\sigma - \frac{2R}{\cos\phi} d\theta = (\gamma_x + \gamma_z \tan\phi + k)\,dx + (\gamma_z - \gamma_x \tan\phi)\,dz$

In equations (7), □$_x$ and □$_z$ denote body forces per unit volume in the x and z directions.

The basis of the method of characteristics is as follows. Referring to Figure (4), if the solution is known at two points A (on an α characteristic) and B (on a β characteristic), it can be propagated to a new point C by integrating the governing equations simultaneously along the α segment AC and the β segment BC.

To solve the four nonlinear equations (6 α, β) and (7 α, β) for the solution at the new point, it is convenient and customary to adopt a midpoint finite difference scheme, in conjunction with a fixed-point iteration strategy. For details about the algorithm used in the program ABC see Martin (2004).

Clearly the computational burden in terms of floating-point operations is significantly higher in axial symmetry than it is in plane strain. This is another reason why it invariably takes ABC somewhat longer to solve a circular footing problem than a comparable strip footing problem, (Martin, 2004).

Stress field construction

Smooth footings (solution type 1)

Figure (5a) shows a completed mesh of characteristics for a smooth strip footing problem. For clarity, very coarse subdivision counts are used for both and the fan zone (5 and 10 respectively). This type of mesh, in which all of the α characteristics proceed to the footing, is referred to in ABC as solution type 1. Note that it is only necessary to calculate half of the stress field, which is symmetric about the z axis. Suppose that the mesh has been partially constructed to the stage shown in Figure (5b), and that a new α characteristic is to be added.

Rough footings (solution types 2 and 3)

When constructing the stress field for a rough footing problem, the symmetry requirement that when (major principal stress direction = vertical on z axis) means that the fully-mobilised roughness condition of Figure (3b) cannot apply over the whole of the soil–footing interface. Instead there are two possibilities, and these are shown schematically in Figure (7). In solution type 2, full roughness is not mobilised at any point of the interface: no α characteristics progress to the footing, so no β characteristics become tangential to it. In solution type 3, full roughness is mobilised on part, but not all, of the interface: there is a defined region (away from the axis of symmetry) where the α characteristics do progress to the footing, spawning β characteristics that originate tangentially, as per Figure (3b). In both of the rough footing solution types, the bearing capacity can be found without extending the stress field into the blank 'false head' region OC_1C_2.

The solution type that is applicable to a particular rough footing problem depends on the geometry (plane strain or axial symmetry), the friction angle □, and the dimensionless ratio

$$F = \frac{kB + \gamma B \tan\phi}{c_o + q\tan\phi} \qquad (8)$$

It should be pointed out that the naming of the solution types in ABC is quite arbitrary – there is no generally accepted terminology. For example Davis and Booker (1971, 1973) emphasise the role of the footing breadth B, the soil properties and surcharge being assumed given; they therefore refer to solution types 2 and 3 as "narrow rough footing" and "wide rough footing" solutions. Equation (8) confirms that, as expected, these terms are consistent with small and large values of F.

Comparison of the Method with Bearing Capacity Theories:

In this paper, the validation of the method of stress characteristics and the program ABC is tested by solving a wide range of bearing capacity problems. The results will be compared with classical bearing capacity theories; namely, Terzaghi, Myerhof, Hansen and Vesic methods.

- **Case 1: Circular Footing in Clay**

Figures (8) to (11) show the variation of bearing capacity with the foundation width to depth ratio (D/B) for circular foundations constructed in different clays.

It can be noticed that the results of ABC method are close to those of Hansen's method especially at D/B > 1.5. The bearing capacity predicted by ABC ranges between (3.7 – 4.0) greater than Terzaghi, Meyerhof, and Vesic methods. This means that the method is not conservative for this case and can be dependent for economic design of foundations.

- **Case 2: Circular Footing in Sand**

Figures (12) (14) show the variation of bearing capacity with (D/B) for circular foundations constructed in loose to dense sands.

As the case in clay, the bearing capacity predicted by ABC increases linearly with (D/B). For all values of ϕ, ABC reveals bearing capacity values for smooth footings greater than Terzaghi and Hansen and smaller than Meyerhof and Vesic theories.

Considering the foundation to be rough, the method gives bearing capacity values greater than all other methods. The difference increases as the angle of internal friction (ϕ) increases. This makes the method unreliable for rough foundations.

- **Case 3: Strip Footing in Clay**

Figures (15) to (18) show the variation of bearing capacity with (D/B) for strip foundations constructed in clays.

The same trend is obtained to that of circular foundations in clay but the ABC method here reveals bearing capacity values in the range (4 – 7) times those calculated by Terzaghi, Meyerhof and Vesic's methods.

- **Case 4: Strip Footing in Sand**

Figures (19) to (21) show the variation of bearing capacity with (D/B) for strip foundations constructed in loose to dense sands.

The bearing capacity in this case also increases linearly with (D/B). The bearing capacity predicted by ABC method for smooth footing is smaller than those predicted by all other approaches which means that ABC is conservative for the case of smooth strip foundations in sand.

On the other hand, the bearing capacity values predicted by ABC method for a rough strip foundation lies in the middle among other theories.

- **Case 5: Circular Footing in Clay (Strength increasing with depth)**

Figures (22) to (25) show the variation of bearing capacity with (D/B) for circular foundations constructed on non-homogeneous clays with strength increasing with depth according to eq. (2). In this case the constant k is considered to be 1.5 kN/m. It can be noticed that the bearing capacity predicted by ABC method is greater than those predicted by other theories except Hansen's method. This is true for (D/B) less than (1.0) and all clays. When (D/B) is greater than (1.0), Hansen's theory reveals bearing capacity values less than ABC for clays with undrained strength ($c \geq 60$ kPa).

Case 6: Strip Footing in Clay (Strength increasing with depth)

Figures (26) to (29) show the variation of bearing capacity with (D/B) for strip foundations constructed on nonhomogeneous clays with strength increasing with depth according to eq. (2). In this case the constant k is considered to be 1.5 kN/m. The same trend is noticed in this case as that of the case of circular footing (Case 5).

Conclusions:

1. The bearing capacity predicted by the method of stress characteristics for the case of a circular footing in clay ranges between (3.7 – 4.0) greater than Terzaghi, Meyerhof, and Vesic methods. This means that the method is not conservative for this case and can be dependent for economic design of foundations. The bearing capacity predicted by this method increases linearly with (D/B).

2. For all values of ϕ, the method of stress characteristics reveals bearing capacity values for smooth footings greater than Terzaghi and Hansen and smaller than Meyerhof and Vesic theories.

Considering the foundation to be rough, the method gives bearing capacity values greater than all other methods. The difference increases as the angle of internal friction (ϕ increases. This makes the method unreliable for rough foundations.

3. The method of stress characteristics for the case of a strip footing in clay reveals bearing capacity values in the range (4 – 7) times those calculated by Trzaghi, Meyerhof and Vesic methods.

4. The bearing capacity predicted by the method of stress characteristics for smooth strip footing in sand is smaller than those predicted by all other approaches which means that the method is conservative for the case of smooth strip foundations in sand. On the other hand, the bearing capacity values predicted by this method for a rough strip foundation lies in the middle among other theories.

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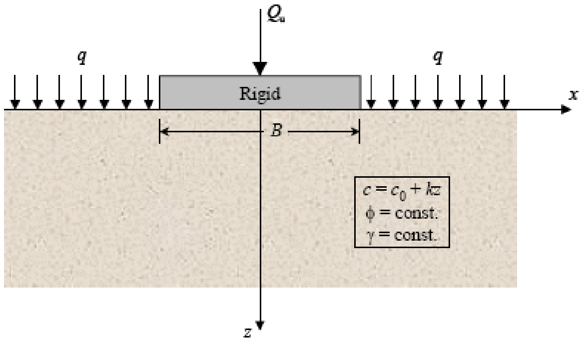
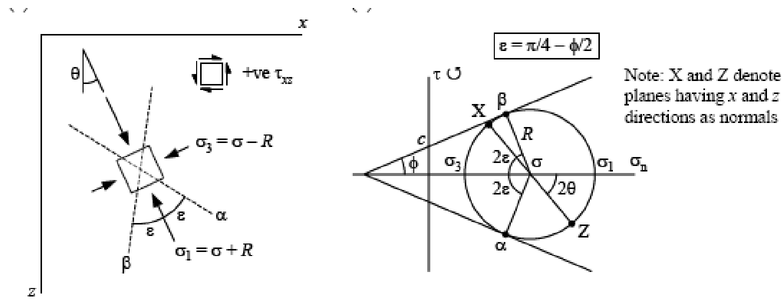


Fig. (1) – Problem definition.



(a) (b)

Fig. (2) – Notation and sign conventions.

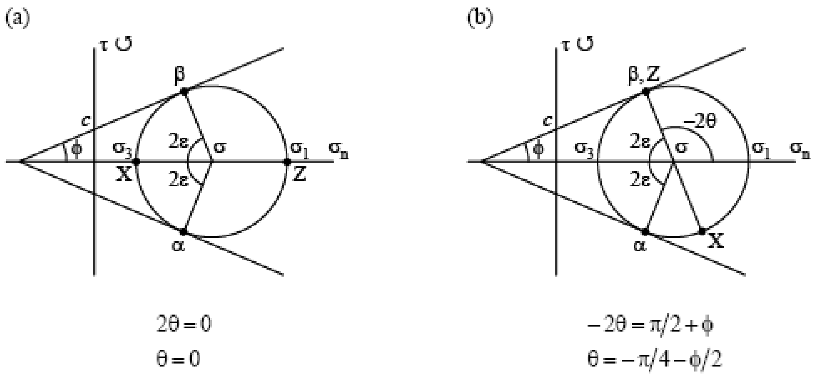


Fig. (3) - Mohr's circle for underside of footing: (a) smooth (b) rough.

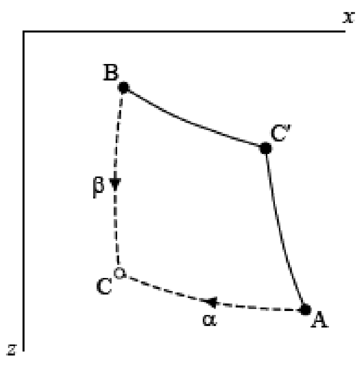
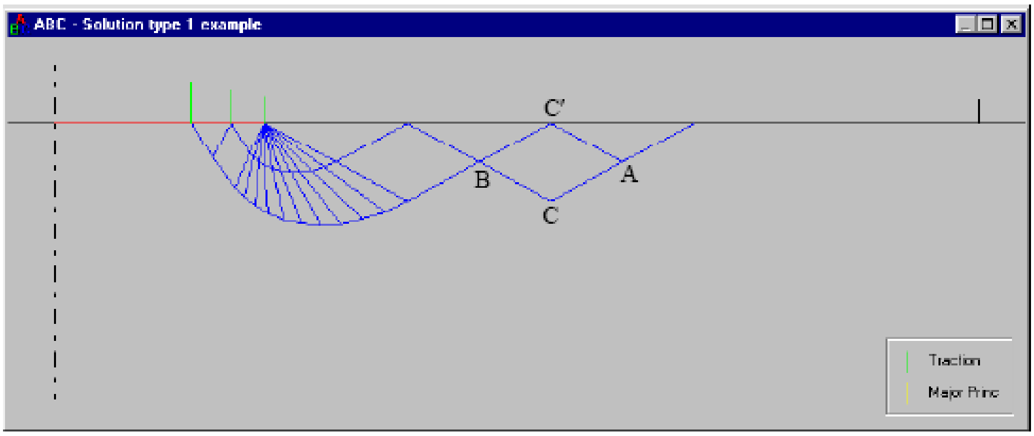
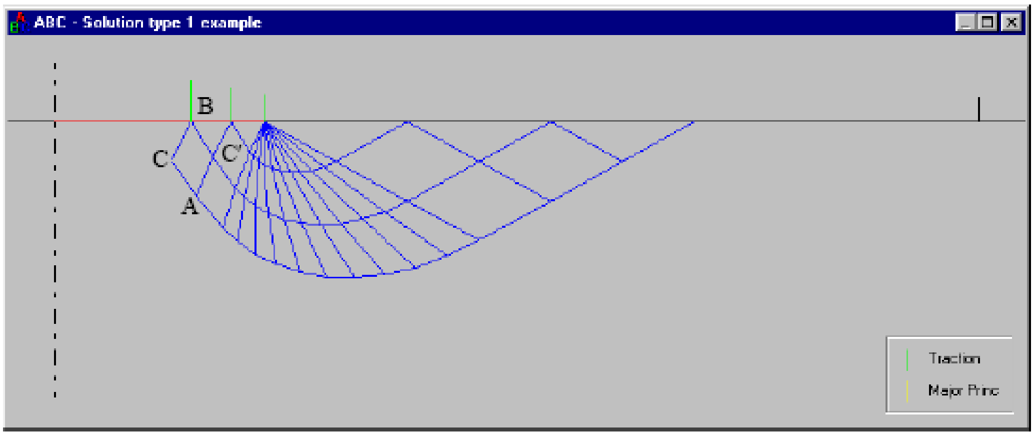


Fig. (4) - Calculation of new solution point in body of soil (CalcAB).

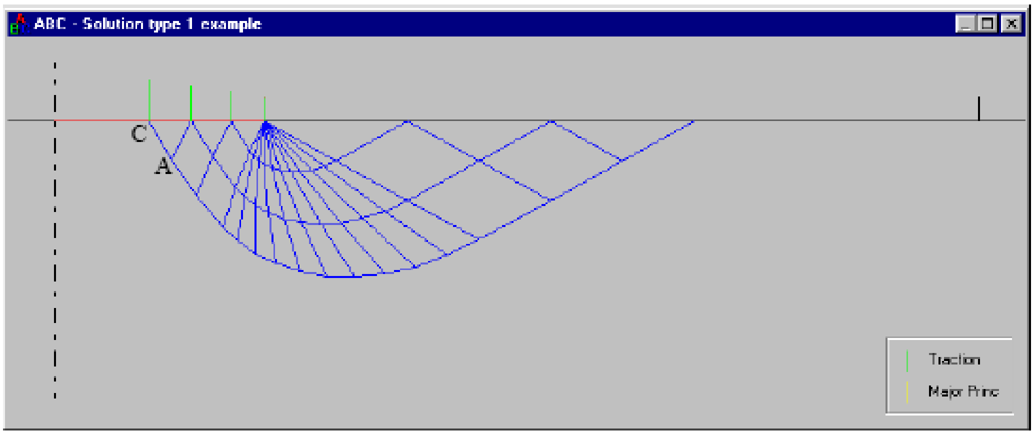
(d) CalcAB



(e) CalcAB



(f) CalcA



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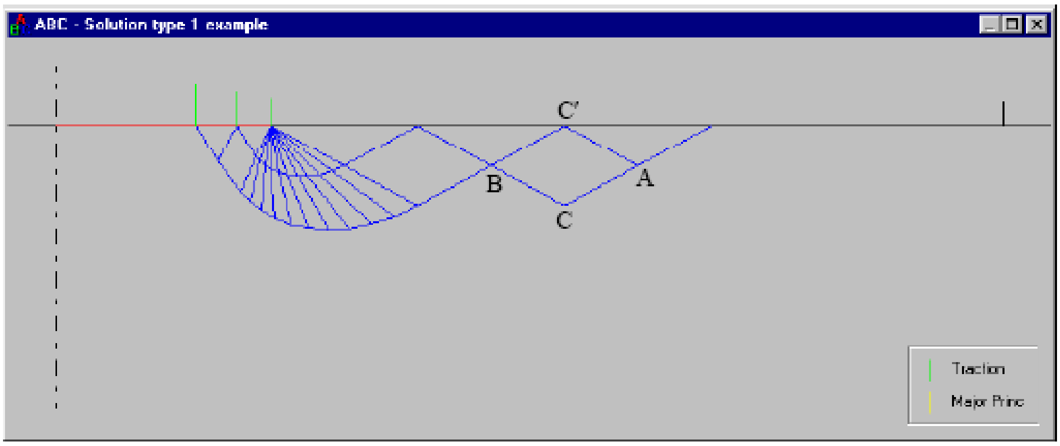
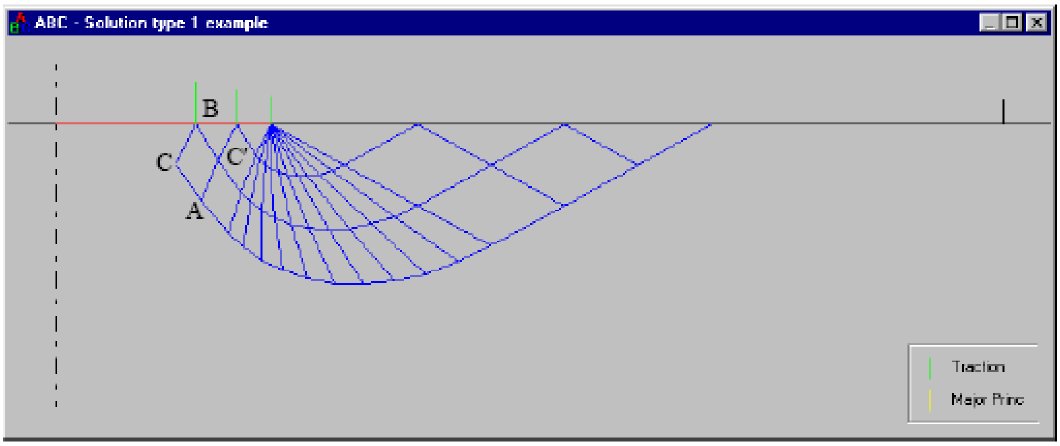


(e) CalcAB



(f) CalcA

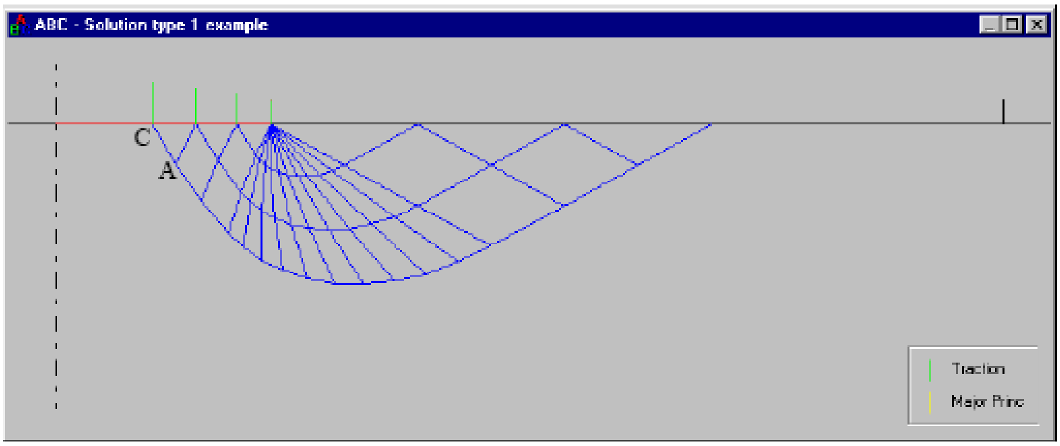
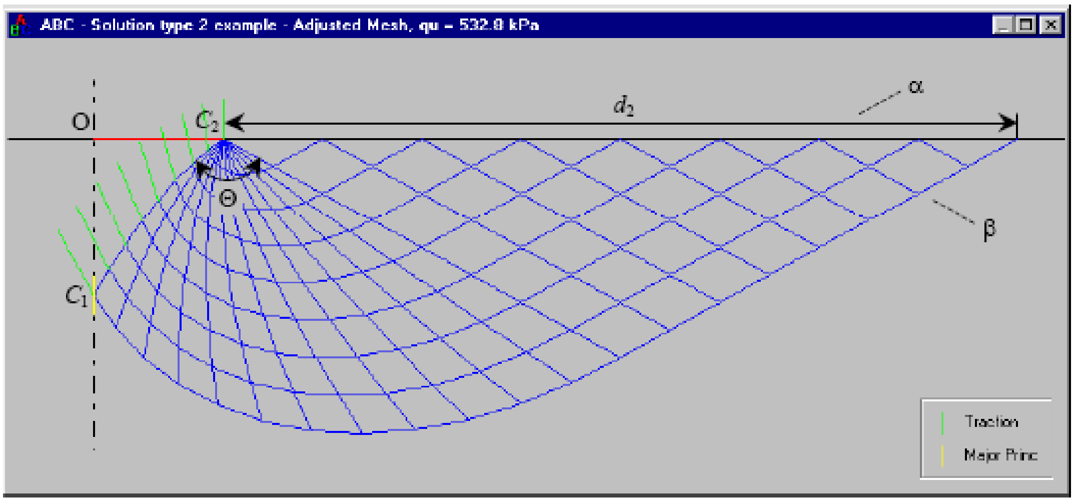


Fig. (5) - Solution type 1, showing (a) completed mesh; (b)-(f) construction procedure.

(a) Solution type 2 – no α characteristics progress to footing



(b) Solution type 3 – some α characteristics progress to footing

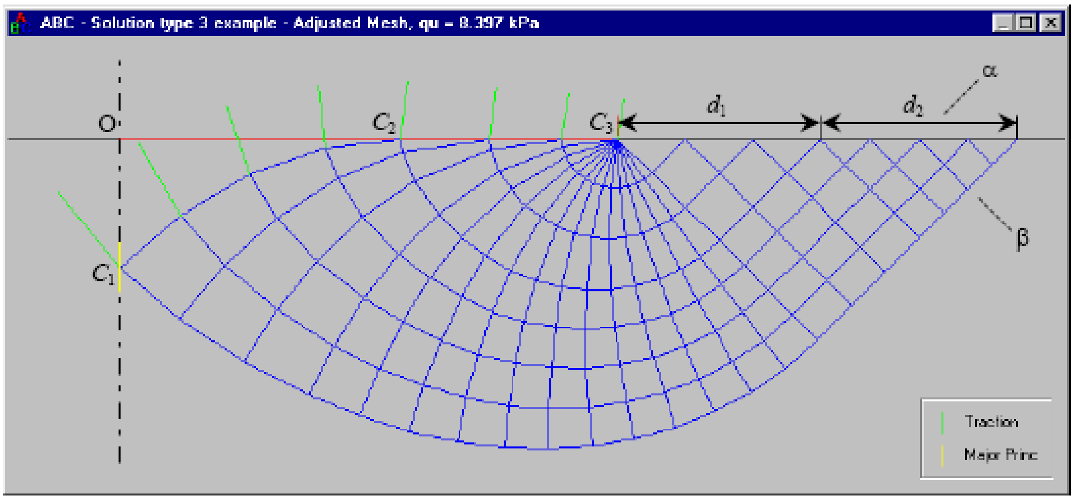


Fig. (7) - Solution types 2 and 3 .

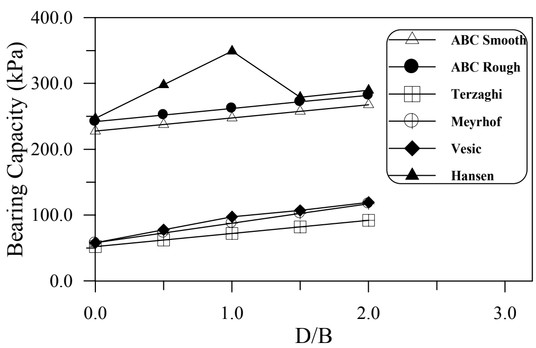


Fig. (8) – Bearing capacity for circular footing in clay (c = 40 kPa).

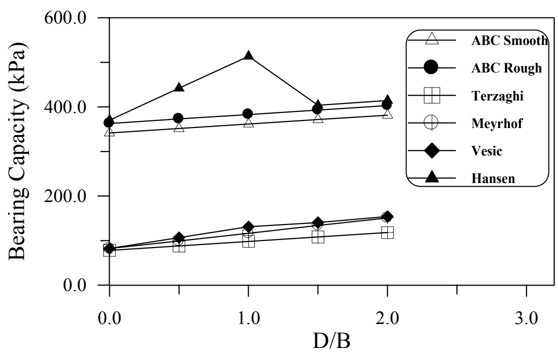


Fig. (9) – Bearing capacity for circular footing in clay (c = 60 kPa).

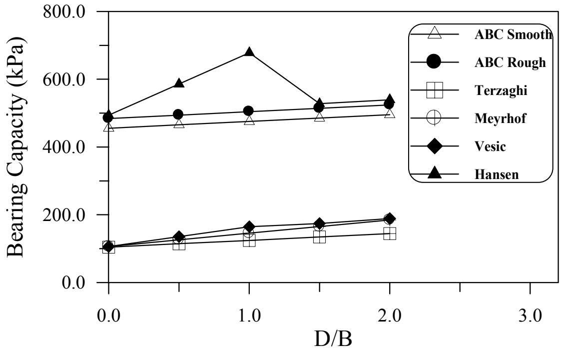


Fig. (10) – Bearing capacity for circular footing in clay (c = 80 kPa).

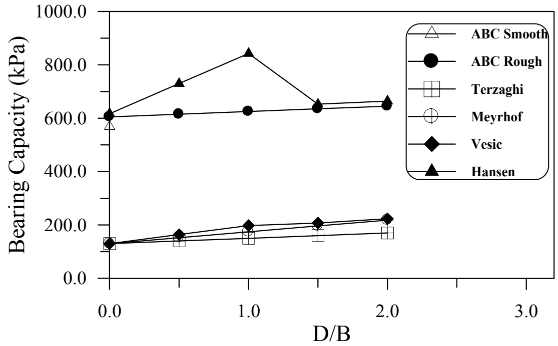


Fig. (11) – Bearing capacity for circular footing in clay (c = 100 kPa).

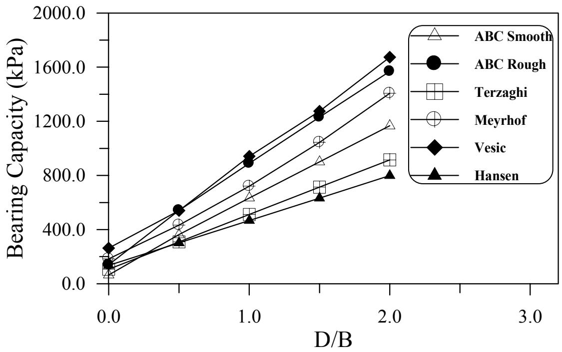


Fig. (12) – Bearing capacity for circular footing in sand ($\phi = 30^o$).

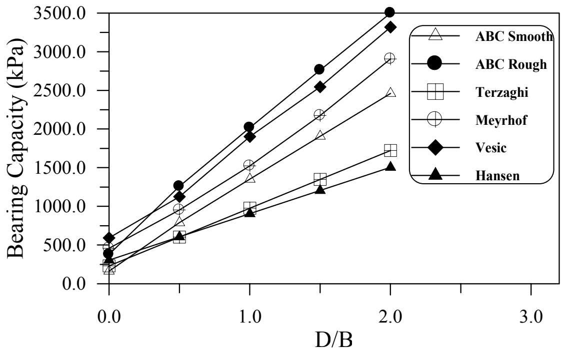


Fig. (13) – Bearing capacity for circular footing in sand ($\phi = 35^o$).

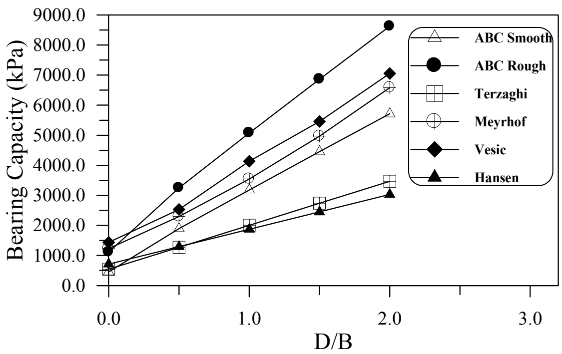


Fig. (14) – Bearing capacity for circular footing in sand ($\phi = 40^o$).

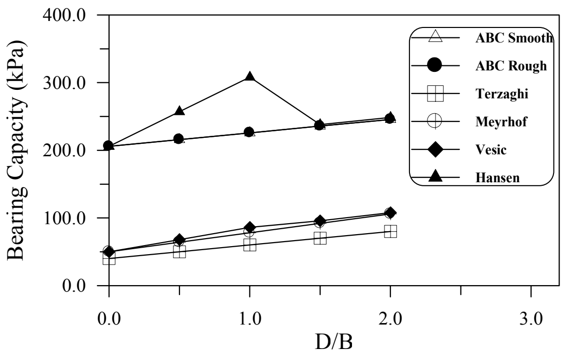


Fig. (15) – Bearing capacity for strip footing in clay (c = 40 kPa).

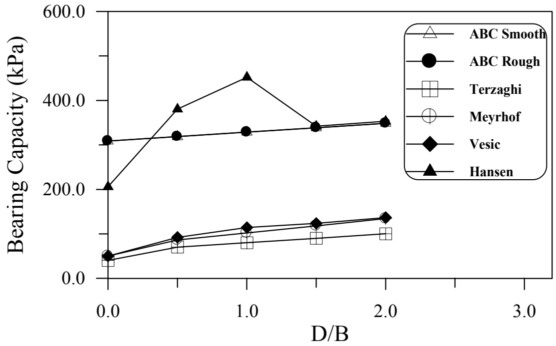


Fig. (16) – Bearing capacity for strip footing in clay (c = 60 kPa).

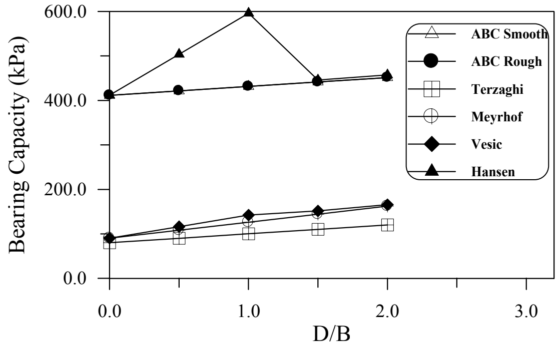


Fig. (17) – Bearing capacity for strip footing in clay (c = 80 kPa).

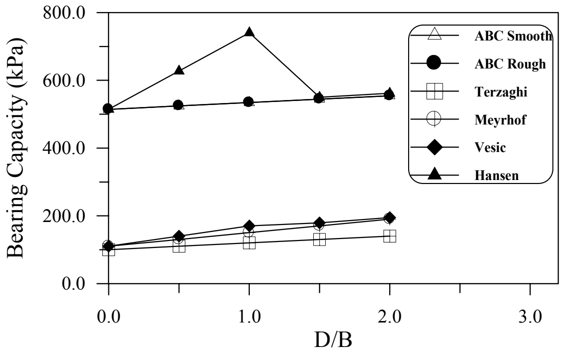


Fig. (18) – Bearing capacity for strip footing in clay (c = 100 kPa).

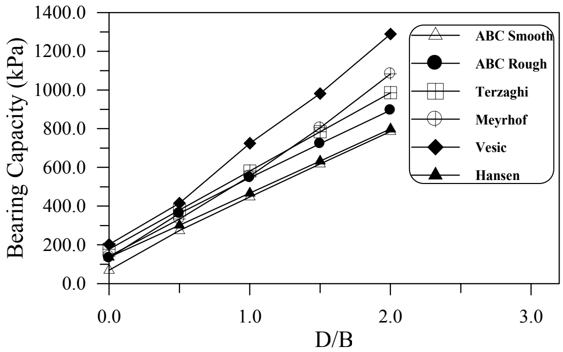


Fig. (19) – Bearing capacity for strip footing in sand ($\phi = 30^o$).

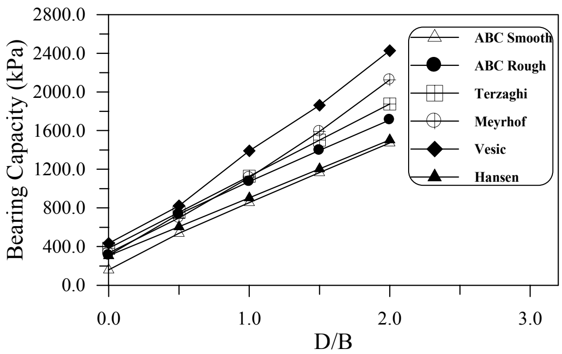


Fig. (20) – Bearing capacity for trip footing in sand ($\phi = 35^o$).

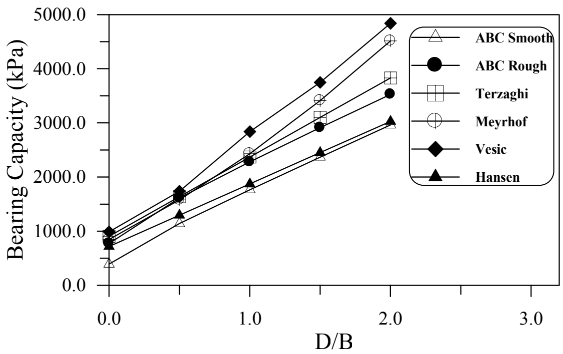


Fig. (21) – Bearing capacity for trip footing in sand ($\phi = 40^o$).

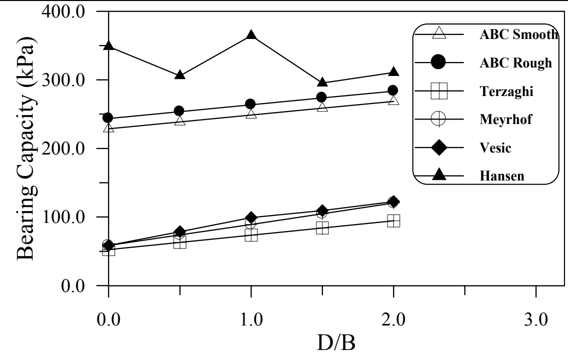


Fig. (22) – Bearing capacity for circular footing in clay (c = 40 kPa at surface).

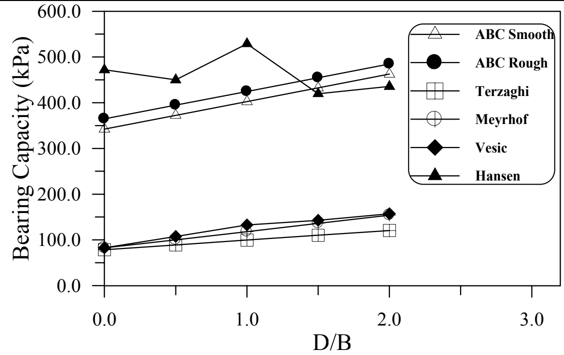


Fig. (23) – Bearing capacity for circular footing in clay (c = 60 kPa at surface).

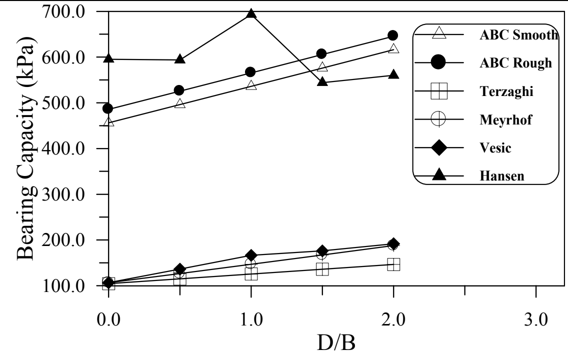


Fig. (24) – Bearing capacity for circular footing in clay (c = 80 kPa at surface).

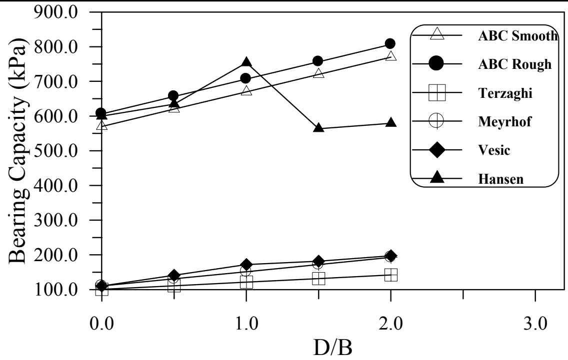


Fig. (25) – Bearing capacity for circular footing in clay (c = 100 kPa at surface).

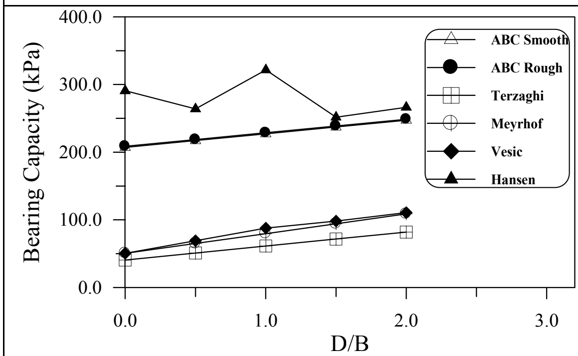


Fig. (26) – Bearing capacity for strip footing in clay (c = 40 kPa at surface).

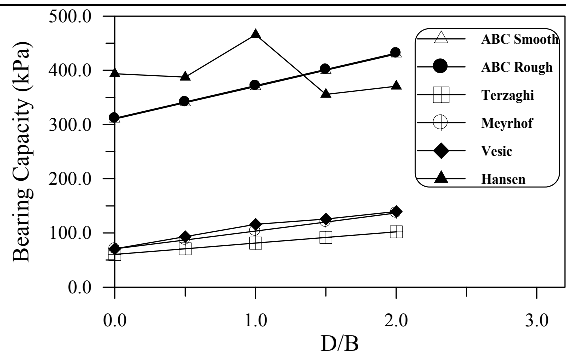


Fig. (27) – Bearing capacity for strip footing in clay (c = 60 kPa at surface).

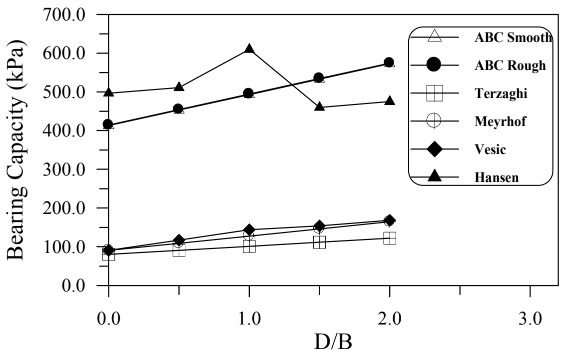


Fig. (28) – Bearing capacity for strip footing in clay (c = 80 kPa at surface).

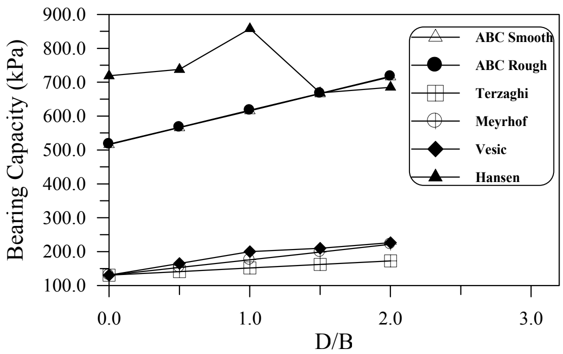


Fig. (29) – Bearing capacity for strip footing in clay (c = 100 kPa at surface).