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ARTICLE

On Kolmogorove Space by Using Semi Feebly Open Set and its Closure and Boundary

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Abstract

In this paper, we introduce some new types of separation axioms called sf-T_0-space and sf-T_0'-space and sf-T_0''-space, which have been studied and some of their properties and relationships with each other and T_0-space which identified and we find some results that will be useful by using the set of type (sf-open).

Keywords: sf-T_0-space, sf-T_0'-space, T_0-space, sf-open, sf-closed

1. Introduction

We have studied in this work some new types in topological spaces namely sf-T_0- space and sf-T_0'- space and sf-T_0''- space and T_0- space and their relation with each other. In 1963 N. Levine [4] and Cameron [2] introduced the idea of a semi-open set. In Ref. [3] Crossly, S. G. and S. K. Hildebrand (1971) defined the meaning of semi closure. For this path S.N. Maheshwari and Tapi (1978) [6] introduced the conception of feebly open sets in Topological space which are closely related to semi-open sets in topological spaces in general. The complement of feebly open set is feebly closed set. We considered semi-feebly open sets with separation axioms in topological spaces [5]. Othman. R in (2020) the semi feebly simplification is given a semi feebly open set (sf-open set), semi feebly closed set (sf-closed set.), and we have to teach about their topological features. We will use $U^{\circ sf}$, $\overline{U}^{sf}$, $b_{sf}(A)$ to the sf-interior of U and sf-closure of U and sf-boundary of A, respectively.

2. Basics concepts

Definition (2.1). [4] Let (X, T) is space where $B \subseteq X$ is semi-open(s-open) if there is exists open set U in X which $U \subseteq B \subseteq \overline{U}$.

Definition (2.2). [2] Let $A \subseteq X$ which is topological space, we said that set A feebly open if there is open set U and $U \subset A \subset \overline{U}^{s}$.
A called feebly closed if the complement of A is feebly open, intersection of all feebly closed sets contain A is the feebly closure of A, symbolized ($\overline{A}^{f}$).

Remark (2.3). [2] Each open set a feebly open.

Remark (2.4). [3] Each feebly open set a semi-open.

Proposition (2.5). [1] Let $B \subseteq X$ which is topological space, B is feebly open iff $B \subset \overline{B^{\circ}}^{\circ}$.

Lemma (2.6). [1] Let A be a subset of topological space (X, T), then $\overline{A}^{s} = A \cup (\overline{A})^{c}$.

Definition (2.7). [5] Let $B \subseteq X$ which is topological space, B is said to be semi feebly open set; if for each semi open set U contain B we have $\overline{B}^{f} \subseteq U$, semi feebly closed set (sf-closed) is the complement of semi feebly open and $U \subseteq B^{\circ}$ in which U semi closed set in X. Collection of all sf-open set symbolized sf O(X) and collection of sf-closed subsets of X symbolized $sfcl$ (X).

Remark (2.8). [5]

i. Every f- closed set is sf-open set the opposite does not hold.

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ii. Each closed set is *sf*-open set the opposite does not hold.

Note (2.9). [5]

i. Each subset of discrete or indiscrete (X,T) space is *sf*- open.
ii. Each closed interval in usual topology is *sf*-open.

Definition (2.10). [6] Let B⊆X which is topological space, union of *sf*-open sets of X contains in B are called *sf*-Interior of B and symbolized $B^{\circ sf}$
$B^{sf} = \bigcup$ {A: A is sf−open in X and A⊆B}.

Definition (2.11). [5] Let B ⊆X which is topological space, intersection of *sf*-closed sets of X which contained B is called *sf*-closure of B and symbolized $\overline{B}^{sf}$, this means $\overline{B}^{sf} = \cap$ {F:F is *sf*-closed in X, B⊆F}.

Remark (2.12). [4]

1. Does not each open set is *sf*-open set.
2. Does not each *s*-open set is *sf*-open set.
3. Does not each *s*-closed set is *sf*-open set.
4. Does not each *f*-open set is *sf*-open set.

Definition (2.13). [5] Let B⊆X which is topological space, the *sf*-neighborhood of B is any subset in X that contain *sf*-open set containing B. The *sf*-neighborhood of a subset {x} is called *sf*-neighborhood of the point x.

Definition (2.14). [7] The space (X, T) is called T_0-space iff each pair of points $x, y \in$ X, x ≠y, there is an open set containing x but not containing y or open set containing y but not containing x. i.e.,
X is T_0 − space ⇔∀ $x, y \in X; x \neq y$ ∃ $U \in$ T; (x ∈ U ∧ y ∉ U) ∨ (x ∉ U ∧ y ∈ U)

Definition (2.15). [5] Let f:X⟶Y a mapping of X which is a topological space into a topological space (Y,T\`) then f called *sf**-continuous mapping if f^{-1} (B) is a *sf*-open set in X to each *sf*-open set B in Y.

Definition (2.16). [3] Let f: X⟶Y be a mapping of a space (X, T) into a space (Y,T\`), f called a (*sf*-open) mapping if f(B) a *sf*-open in Y for each *sf*-open set B in X.

Definition (2.17). [5] Let f: X⟶Y a mapping of (X, T) which is topological into a topological space (Y,T\`), f called a (*sf*-closed) mapping if f(E) is *sf*-closed in Y to each *sf*-open set E in X.

Definition (2.18). [5] Let f: X⟶Y a mapping of a topological space (X, T) in to a topological space (Y,T\`), f called *sf*-homeomorphism if :

i f a bijective.
ii f a *sf**-continuous.
iii f a *sf*-closed (*sf*-open).

Definition: (2.19). [5] Let (X,T) be a space and to be achieve conduct union if the union of *sf*-open sets of X is *sf*-open set.

Proposition: (2.20). [5] Let X a topological space, B ⊆A ⊆ X

i. $\overline{B}^{sf}$ is f-closed set.
ii. $B \subseteq \overline{B}^{sf}$.
iii. B is sf-closed set iff B ⊆ $\overline{B}^{sf}$.
iv. If B⊆A then $\overline{B}^{sf} \subseteq \overline{A}^{sf}$.

Proposition: (2.21). [5] Let (X,T) a conduct union space and B⊆A⊆X, then:

1. $B^{\circ sf}$ a *sf*-open set.
2. B is a *sf*-open set iff B = $B^{\circ sf}$.
3. $B^{\circ sf} = B^{\circ sf^{\circ sf}}$.
4. If B⊆A then $B^{\circ sf} \subseteq A^{\circ sf}$.

Proposition: (3.22). [5] Let (X,T) a space, B⊆A⊆X:

1. $\overline{B}^{sf}$ = B$\bigcup$ B'^{sf}.
2. B is a *sf*-closed set iff $B'^{sf} \subseteq B$.
3. $B'^{sf} \subseteq A'^{sf}$

3. Main results

We discus in this section definitions, theorems and examples about *sf*-T_0-space and *sf*-$T_0{}'$-space and *sf*-$T_0{}''$-space.

Definition: (3.1). The space (X, T) called *sf*-T_0-space iff every pair of points x, y∈X, x≠y there is a *sf*-open set containing x but not containing y or containing y but not containing x. i.e.
X a *sf*-T_0-space ⇔ ∀ x,y∈ X; x ≠ y ∃ V *sf*-open set in X; (x ∈ V ∧ y ∉V) ∨ (x ∉V ∧ y ∈V).

Lemma (3.2). [5] Let (X,T) a space and B ⊆X, x∈ $\overline{B}^{sf}$ iff for all *sf*-open sets U and $\in U, U \cap B \neq \emptyset$.

Theorem: (3.3). Let (X, T) a space is *sf*-T_0-space iff $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$ ∀ x, y ∈X; x ≠ y.

Proof. (⇒) Suppose that X is *sf*-T_0-space, ∀ $x,y \in$ X; x≠y
Then∃ U a *sf*-open set in X; ($x \in U \wedge y \notin$ U) ∨ (x ∉$U \wedge y \in U$)
If (x ∈ U ∧ y ∉ U) ∧ (x ∈ U ∧ y ∈ U^c)
Since U *sf*-open set ⇒ U^c *sf*-closed set and {y} ⊆ U^c ⇒ $\overline{\{y\}}^{sf} \subseteq \overline{U^c}^{sf} = U^c$ ⇒ {x} ⊈ U^c ⇒ $\overline{\{x\}}^{sf} \not\subseteq U^c \wedge$ x ∈ U ⇒ $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$

If $(x \notin U \wedge y \in U) \wedge (x \in U^c \wedge y \in U)$.
Since U sf-open set $\Rightarrow U^c$ sf-closed set and $\{x\} \subseteq U^c$
$\Rightarrow \overline{\{x\}}^{sf} \subseteq \overline{U^c}^{sf} = U^c$
Then $\{y\} \nsubseteq U^c \Rightarrow \overline{\{y\}}^{sf} \nsubseteq U^c \wedge y \in U \Rightarrow \overline{\{y\}}^{sf} \neq \overline{\{x\}}^{sf}$
($\Leftarrow$) Assume that $\overline{\{y\}}^{sf} \neq \overline{\{x\}}^{sf}$ then let $x,y \in X$, $x \neq y$
Let X is not sf-T_0-space
Then ($\exists\ x,y \in X$; $\forall\ U$ sf-open set of X; $x \in U \wedge y \in U$)
Let $z \in X \wedge z \in \overline{\{x\}}^{sf}$ (1)
Then $\forall\ U$ sf-open set; $z \in U \wedge U \cap \{x\} \neq \varnothing$ [lemma **(3.2)**]
But, $U \cap \{x\} \neq \varnothing \Rightarrow x \in U$
Then every set contain z must contains x. So, every sf-open set contain z must contain x and each sf-open set contain x must contains y.
Then every sf-open set contain z must contains y.
$\Rightarrow\ \forall\ U$ sf-open set; $z \in U \wedge U \cap \{y\} \neq \varnothing$
$\Rightarrow z \in \overline{\{y\}}^{sf}$ (2)
$\Rightarrow\ \forall\ z \in \overline{\{x\}}^{sf} \Rightarrow z \in \overline{\{y\}}^{sf} \Rightarrow \overline{\{x\}}^{sf} \subseteq \overline{\{y\}}^{sf}$
Let $z \in X$; $z \notin \overline{\{y\}}^{sf}$ (1)
$\forall\ U$ sf-open set; $z \in U \wedge U \cap \{y\} \neq \varnothing$.But, $U \cap \{y\} \neq \varnothing \Rightarrow y \in U$

Then every set contains z must contains y. So, every sf-open set contain z must containing y for each sf-open set containing y must containing x.
Then every sf-open set contains z must contains x.$\Rightarrow\ \forall\ U$ sf-open set; $z \in U \wedge U \cap \{x\} \neq \varnothing$
Then $z \in \overline{\{x\}}^{sf}$ ———————(2)
$\forall\ z \in \overline{\{y\}}^{sf} \Rightarrow z \in \overline{\{x\}}^{sf} \Rightarrow \overline{\{y\}}^{sf} \subseteq \overline{\{x\}}^{sf}$ Then $\overline{\{x\}}^{sf} = \overline{\{y\}}^{sf}$ which is contradiction
Therefore, X is sf-T_0-space.

Theorem: (3.4). The clopen subspace of sf-T_0-space is also sf-T_0-space.

Proof. Let (X, T) sf-T_0-space, (Y, T*) be clopen subspace of X
Let $a,b \in Y$, $a \neq b \Rightarrow a,b \in X$
Since X is sf-T_0-space $\Rightarrow \exists\ U$ sf-open set in X; ($a \in U$ and $b \notin U$) $\vee$($a \notin U$ and $b \in U$)
SinceY a clopen set in X then Y is sf-open set in X $\Rightarrow$ $U \cap Y$ is sf-open set in X
Then $U \cap Y$ is sf-open set in Y, and ($a \in U \cap Y \wedge b \notin U \cap Y$) $\vee$ ($a \notin U \cap Y \wedge b \in U \cap Y$)
Therefore, (Y, T*) is sf-T_0-space.

Theorem: (3.5). The property of sf-T_0-space a topological property.

Proof. Lets (X, T) $\cong$ (Y, T') and assume that X is sf-T_0-space
$\because$ (X, T) $\cong$ (Y, T'), then $\exists$ sf*-continuous mapping f: $X \to Y \ni f$ 1-1, f onto,
Let $y_1, y_2 \in Y \ni y_1 \neq y_2 \Rightarrow f^{-1}(y_1), f^{-1}(y_2) \in X$
Since f onto function $\Rightarrow f^{-1}(y_1) \neq \varnothing, f^{-1}(y_2) \neq \varnothing$
Since f 1-1 function $\Rightarrow \exists\ x_1 \in X \ni f^{-1}(y_1) = x_1$ and $\exists\ x_2 \in X \ni f^{-1}(y_2) = x_2$ and $x_1 \neq x_2$ and $x_1 \wedge x_2 \not\ni X$
Since X is sf-T_0-space$\Rightarrow\ \exists\ U sf$-open set in X $\ni$ ($x_1 \in U \wedge x_2 \notin U$) $\vee$ ($x_1 \notin U \wedge x_2 \in U$)
Since f sf-open then f (U) is sf-open in Y $\ni$(f (x_1) $\in$ f (U) $\wedge$ f(x_2) $\notin$ f (U)) $\vee$ (f (x_1) $\notin$ f (U) $\wedge$ f (x_2) $\in$ f (U))
Therefore Y is sf-T_0-space.

Theorem: (3.6). Let (X ,T), (Y, T') be two spaces. The product space X × Y is a sf-T_0-space iff each X and Y are sf-T_0-space.

Proof. ($\Rightarrow$) Assume that X × Y is sf-T_0-space, let a_1, $a_2 \in X$; $a_1 \neq a_2$ and $b_1, b_2 \in Y$; $b_1 \neq b_2$ then (x_1, y_1), (x_2, y_2) $\in X \times Y$; (a_1, b_1) $\neq$ (a_2, b_2)
Since X × Y is a sf-T_0-space then $\exists$ a sf-open set $U \times V$ in X × Y; ((a_1, b_1) $\in U \times V \wedge$ (a_2, b_2) $\notin U \times V$) or ((a_1, b_1) $\notin U \times V \wedge$ (a_2, b_2) $\in U \times V$) $\Rightarrow \exists\ U$ sf-open set in X; ($a_1 \in U \wedge a_2 \notin U$) $\vee$ ($a_1 \notin U \wedge a_2 \in U$) $\Rightarrow$X is a sf-T_0-space and $\exists\ V$ sf-open set in Y; ($b_1 \in V$ and $b_2 \notin V$) $\vee$ ($b_1 \notin V$ and $b_2 \in V$) $\Rightarrow$Y is a sf-T_0-space
($\Leftarrow$) Suppose that X, Y sf-T_0-space, let (a_1, b_1), (a_2, b_2) $\in X \times Y$; (a_1, b_1) $\neq$ (a_2, b_2) $\Rightarrow$ ($a_1, a_2 \in X \wedge a_1 \neq a_2$) and ($b_1, b_2 \in Y \wedge b_1 \neq b_2$)
Since X is a sf-T_0-space $\Rightarrow \exists\ U$ sf-open set; ($a_1 \in U$ and $a_2 \notin U$) $\vee$ ($a_1 \notin U$ and $a_2 \in U$)
Since Y is a sf-T_0-space$\Rightarrow \exists\ V$ sf-open set; ($b_1 \in V \wedge b_2 \notin V$) $\vee$ ($b_1 \notin V \wedge b_2 \in V$) then$\exists\ U \times V$ is a sf-open set; ((a_1, b_1) $\in U \times V \wedge$ (a_2, b_2) $\notin U \times V$) $\vee$ ((a_1, b_1) $\notin U \times V \wedge$ (a_2, b_2) $\in U \times V$)
Then X × Y is a sf-T_0-space.

Definition: (3.7). The topological space (X, T) called sf-$T_0{}'$-space iff for each pair of points $x, y \in X$ and $x \neq y$, then either the closure of a sf-open set containing x but not containing y or containing y but not containing x. i.e.
X a sf-$T_0{}'$-space $\Leftrightarrow\ \forall\ x, y \in X$; $x \neq y$ $\exists\ U$ sf-open set in X; ($x \in \overline{U}^{sf} \wedge y \notin \overline{U}^{sf}$) $\vee$ ($x \notin \overline{U}^{sf} \wedge y \in \overline{U}^{sf}$).

Remark: (3.8). Every T_0-space is sf-T_0-space. The converse does not hold.

Proof. Let X a T_0-space $\Rightarrow$,$y \in X$; $x \neq y$ $\exists\ U$ open set in X; ($x \in U \wedge y \notin U$) $\vee$ ($x \notin U \wedge y \in U$)
If ($x \in U \wedge y \notin U$) $\Rightarrow x \in U \wedge y \in U^c \Rightarrow x \notin U^c \wedge y \in U^c$
Since U is open set then U^c is closed set in X, then U^c is sf-open set in X and $x \notin U^c \wedge y \in U^c$
Then X is sf-T_0-space.
If ($x \notin U \wedge y \in U$) (similar way).
The following example proof the converse

Example (3.9). let X = {a, b, c}, T = {X, ∅, {a, b}}.
Closed set ={X, ∅, {c}}
s-open set = { X, ∅, {a,b}}

s-closed set = { X, ∅,{c}}
f-open set = {X, ∅, {a,b}}
f-closed set = {X, ∅,{c}}
sf-open sets={X, ∅, {c}}
a ≠ b ⇒ ∄ *sf*-open set containing a but not b or b but not a
∴(X, T) is not *sf*-T_0-space.

Remark (3.10). Not every *sf*-T_0-space. is *sf*-$T_0{}'$-space.

Example (3.11). Let X = {1,2,3} and T = {X, ∅ , {1},{2},{1,2},{2,3}}
Closed sets ={X, ∅, {2,3},{1,3},{3},{1}}
s-open sets = {X, ∅,{1},{2}, {1, 2}, {2,3}}
s-closed sets = {X, ∅, {2,3},{1,3},{3},{1}}
f-open sets ={X, ∅,{1},{2}, {1, 2}, {2,3}}
f-closed sets={X, ∅, {2,3},{1,3},{3},{1}}
sf-open sets={X, ∅, {2,3},{1,3},{3},{1}}
Sf-closed sets ={X, ∅, {1}, ,{2},{1,2},{2,3}}
1 ≠ 2 ⇒ ∃ *sf*-open set {1} in X; 1 ∈ {1} ∧ 2 ∉ {1}
1 ≠ 3 ⇒ ∃ *sf*-open set {1} in X; 1 ∈ {1} ∧ 3 ∉ {1}
2 ≠ 3 ⇒ ∃ *sf*-open set {2} in X; 2 ∈ {2} ∧ 3 ∉ {2}
∴ (X, T) is *sf*-T_0-space.
$\overline{\{1\}}^{sf} = \{1\}$, $\overline{\{3\}}^{sf} = \{2, 3\}$, $\overline{\{1,3\}}^{sf} = X$, $\overline{\{2,3\}}^{sf} = \{2, 3\}$
2≠3 ∄ *sf*-open set in X such that the closure of this set containing 2 but not 3 or 3 but not 2.
∴(X, T) is not *sf*-$T_0{}'$-space.

Definition: (3.12). [7] Let $A \subseteq X$ which is topological space. For each $a \in X$, a is to be *sf*-boundary point of A if every *sf*-neighborhood U_a of a, we have $U_a \bigcap A \neq \emptyset$ and $U_a \bigcap A^c \neq \emptyset$. The set of all *sf*-boundary point of A is symbolized b_{sf} (A).

Definition (3.13). The topological space (X, T) called *sf*-$T_0{}''$-space iff for every pair of points $x, y \in X$, x ≠y there exist a *sf*-open sets U, V such that U contain x and the *sf*-boundary of V contain x but not y or contain y but not x.i.e.
X is a *sf*-$T_0{}''$-space ⇔ ∀ $x, y \in X$; $x \neq y$ ∃ U, V *sf*-open sets; U containing x in X and (x ∈ b_{sf} (V) ∧ y ∉ b_{sf} (V)) ∨ (x ∉ b_{sf} (V) ∧ y ∈ b_{sf} (V)).

Remark (3.14). Not every *sf*-$T_0{}''$-space is *sf*-$T_0{}'$-space.

Example (3.15). X = {1, 2}, T={X, ∅, {1}}
Closed sets ={X, ∅, {2}}
s-open sets = {X,∅,{1}}
s-closed sets = { X ,∅,{2}}
f-open sets ={X, ∅, {1}}
f-closed sets ={X, ∅,{2}}
sf-open sets ={X, ∅, {2}}
sf-closed set = {X, ∅, {1}}
$\overline{\{2\}}^{sf} = X$
1≠2 ∄ *sf*-open set; the closure of this set containing 1 but not 2 or 2 but not 1.
∴ X is not *sf*-$T_0{}'$-space
Take U = {2} *sf*-open set ⇒$b_{sf}(\{2\}) = \overline{\{2\}}^{sf} - \{2\}^{\circ sf}$ = X-{2} = {1}
1≠2 and (1 ∈ b_{sf} ({2}) ∧ 2 ∉ b_{sf} ({2}))
∴ X is *sf*-$T_0{}''$-space.

Remark (3.16). Not every *sf*-T_0-space is *sf*-$T_0{}''$-space.

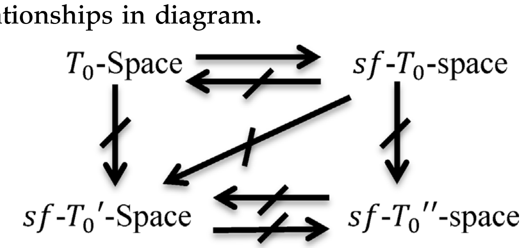
Example (3.17). Let X ={1, 2, 3}, T = {X, ∅, {2}}
Closed set ={X, ∅, {1,3}}
s-open set = {X, ∅, {2}, {1,2},{2,3}}
s-closed set = {X, ∅, {1,3}, {3},{1}}
f-open set ={X, ∅, {2}, {1,2},{2,3}}
f-closed set={X, ∅, {1,3}, {3},{1}}
sf-open set={X, ∅, {1, 3}, {3},{1}}
1 ≠ 2 ⇒ ∃ *sf*-open set {1} in X; 1 ∈ {1} ∧ 2 ∉ {1}
1 ≠ 3 ⇒ ∃ *sf*-open set {1} in X; 1 ∈ {1} ∧ 3 ∉ {1}
2 ≠ 3 ⇒ ∃ *sf*-open set {3} in X; 2 ∉ {3} ∧ 3 ∈ {3}
∴(X, T) is *sf*-T_0-space.
Take {1, 3} *sf*-open set
b_{sf} ({1, 3}) = $\overline{\{1,3\}}^{sf} - \{1,3\}^{\circ sf}$ = X-{1, 3} = {2}
1≠3 ∄ *sf*-open sets; 1 containing in *sf*-boundary of {1, 3} but not 3 or 3 but not 1
∴ X is not *sf*-$T_0{}''$-space.

Remark (3.18). Not every *sf*-$T_0{}'$-space is *sf*-$T_0{}''$-space. By example **(3.17)**: where X is not *sf*-$T_0{}''$-space
$\overline{\{1,3\}}^{sf} = X$, $\overline{\{3\}}^{sf} = \{2, 3\}$ $\overline{\{1\}}^{sf} = \{1, 2\}$
1 ≠ 2 ⇒ ∃ *sf*-open {3} in X; $\overline{\{3\}}^{sf} = \{2, 3\} \ni 2 \in \{2, 3\}$ ∧ 1∉ {2, 3}
1 ≠ 3 ⇒ ∃ *sf*-open {3} in X; $\overline{\{3\}}^{sf} = \{2, 3\} \ni 3 \in \{2, 3\}$ ∧ 1∉ {2, 3}
2 ≠ 3 ⇒ ∃ *sf*-open {1} in X; $\overline{\{1\}}^{sf} = \{1, 2\} \ni 2 \in \{1, 2\}$ ∧ 3∉ {1, 2}
Then (X, T) is *sf*-$T_0{}'$-space.

Remark (3.19). Not every T_0-space is *sf*-$T_0{}'$-space. By example **(3.15)** where X is T_0-space but not *sf*-$T_0{}'$-space.

3.1. Conclusions

The fourth separation axioms have the following relationships in diagram.



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