

NC Roughing Machining of Cavities Defined by Surface of Revolution Approach

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Abstract

In the NC roughing phase of cavities, process planar needs to find the machining layers as well as the feasible cutters for these layers. This Paper deals with the issues of Cavity Construction based on Surface of revolution technique using cubic Bezier curve as a generator curve. First the generator curve is constructed in x-z plane then allowing it to revolve through 360° about z-axis in order to generate the required surface cavity. A strategy of Maximum (critical cutter size) and Machining layers determination is also introduced in this paper. Numerical iteration is used for this purpose using MATLAB programming system to reduce the computational time. A comparison is made between different machining cavities critical cutter size and using cutters smaller than the critical size showing a wide reduction in the length of circular tool path(s) (CL) and the volume of removed metal (VRM). Numerical examples are presented to illustrate the proposed system.

Keywords: NC Roughing, VRM, CL, Cutter Selection, Inverse Point Interpolation.

التشغيل الخشن للتجاويف المعرفة بالاسطح الدورانية باستخدام المكنان المبرمجة.

الخلاصة

من الامور المهمة التي يحتاجها مخطط العملية الانتاجية خلال مرحلة القطع الخشن للتجاويف هي تحديد مستويات التشغيل والعدد الممكن استخدامها لقطع كل مستوي من مستويات التشغيل. في هذا البحث تم التطرق الى اسلوب توليد التجاويف المعرفة بالاسطح الدورانية باستخدام منحنى (بيزير) من الدرجة الثالثة كمنحنى مولد للسطح من خلال دورانه بزواوية مقدارها 360° حول محور ثابت. في هذا البحث ايضا تم استخدام طريقة التكرار البسيط لاجاد مستويات التشغيل وحساب قطر العدة الحرج اللازم لتشغيل كل مستوى من مستويات التشغيل. تم تطبيق الطريقة المعتمدة على مثالين مختلفين باستخدام قطر العدة الحرج وعدد ذات اقطار اقل من القطر الحرج, حيث اظهرت النتائج مقدار النقصان الكبير في حجم المعدن المزال وزيادة في طول مسار العدة الدائري في حالة استخدام عدد اصغر من القطر الحرج.

1.Introduction

It has been proven to be very difficult for manufacturing engineers to select cutters and determine machining layers [1-4]. The primary objective in NC machining is the efficiency of mainly two machining

stages namely roughing and finishing machining, since the reduction in roughing and finishing time can considerably increase productivity, which, in turn, leads to lower manufacturing cost [5,6]. Roughing is

to remove redundant material from a primary row stock, while finishing is to remove residual material along the surface after roughing is applied.

A surface of revolution is a three- dimension surface entity generated by the revolution of plane curve (generator curve), about an axis of revolution through a given angle [7]. The surfaces generated by this approach are symmetrical about the axis of rotation and have different industrial applications such as some of plastic extrusion blow molds, injection blow molds, symmetrical extrusion and deep drawing dies [8] (see Fig.1). In this research the Bezier curve is taken as the generator curve for defining the surface cavity. Since the curvature of surfaces generated by revolution of Bezier curve is highly variant, the surface of each boundary contour differs in shape from one to another along the depth of the cavity. Therefore, care must be taken in setting the depth of cut (see Fig.2). The cutter diameter must also be selected with care, as the gouging between the cutter and part surface needs to be controlled (see Fig.3). Increasing both cutter diameter and depth of cut will increase the roughing process efficiency [9,10].

Gouging is particularly pernicious problem in free form surface machining, and it is often encountered when the tool size is too large relative to the concave radius of curvature. Accordingly, both cutter selection and machining layers determination must be accurately selected in NC machining of freeform cavities. While the principle is simple, the practice is significantly more complex and number of common pitfalls can be identified.

2.Related Work

There are variety of methods, which have been done in the area of cutter selection. Yen-Hung Chem et al. [1] proposed an approach to handle the cutter selection and machining plane determination together for roughing and finishing process in NC machining of complex cavity.

Some techniques were presented by Hamdan W. [9] to determine the optimal cutter selection for freeform Bezier surface based on the minimum radius of curvature of the surface for finishing and based on the smaller polygon of characteristic of Bezier polyhedron which consists of nine polygons.

The two-dimensional pocket machining bounded by polygons and their cutter size determination based on circle fitting method was presented by Bala and Chang [11]. Lee and chang. [12] determined the cutter size for three and five axis machining of sculptured surfaces based on the cutting shapes of the part.

Linn and Mishra [13] determined the optimal machining sequence on planar surfaces based on integer programming (IP) approach.

Sinclair B. et al. [14] organized a method for describing a three dimensional irregular object by sectioning the object with a finite number of parallel planes separated by very small distance, though the author assumed that the object should be without holes.

Person H. [15] described an algorithm for machining arbitrarily shaped pockets using NC milling machine and throughout shrinking the pocket boundary in stepwise fashion.

However, none of the above work has addressed the issues of machining cavities defined by surface of revolution and non of them determined the relation between cutter size and machining depth of layer.

This work is dedicated to the development of a surface of revolution algorithm for construction three-dimensional cavity and addresses the inverse point interpolation approach to determine the relation between maximum cutter size and machining layer depth.

3. Generation a Surface of Revolution

Generation a surface, which involves purely rotation of a plane curve (which is called generation curve) in X-Y, X-Z or Y-Z plane about any axis of rotation, then we call the resultant surface will be referred to as a surface of revolution [16]. The generator curve adopted in this paper is a cubic Bezier curve.

3.1 Generation Bezier Curve

Bezier curve is one of the famous tools in CAGD [16]. To generate a Bezier curve, first the designer needs to choose four control points, two ends points of the curve (P1,P2) and two other points (P3,P4) which determine the characteristic polygon of curve being generated. Therefore, the designer can control the curve shape in predictable way by changing only a few simple control points as shown in Fig.4.

In matrix form the cubic Bezier curve can be written as follows [16] :

$$P(u)=U MB P^T \dots\dots\dots(1)$$

Where :

U is the independent parameter matrix:

$$U = [1 \ u \ u^2 \ u^3] \quad u \in [0,1]$$

MB is the cubic Bezier vector

$$MB = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ 1 & 3 & -3 & 1 \end{bmatrix}$$

P^T is the control point vector

$$P^T = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

The surface of revolution is constructed by revolving the generated Bezier curve (generator curve) about Z-axis in X-Z plane, where the Z-axis is assumed to be the depth of the cavity. Therefore, the vector equation of the generator curve is given by, (as shown in Figure 5) :

$$r(u) = [P_x(u) \ 0 \ P_z(u)] \dots\dots(2)$$

where P_x (u) and P_z (u) are the position vector of a general point on the generator BEZIER curve in X-Z plane which is given in Fig.(4):

$$P(u) = (1-u)^3 P_1 + 3u(1-u)^2 P_2 + 3u^2(1-u) P_3 + u^3 P_4 \dots\dots\dots(3)$$

Rewrite Equation (2) as a homogenous vector, we get :

$$P(u)=[P_x(u) \ 0 \ P_z(u) \ 1] \dots\dots\dots(4)$$

For rotation the generator curve through angle θ about the Z-axis, equation (4) will be extended from a curve P(u) of one parameter (u) to a surface S(u, θ) of two parameters (u) and (θ) as follows:

$$S(u, \theta) = [P_x(u) \ 0 \ P_z(u) \ 1] \cdot MR \dots(5)$$

where MR is the rotation matrix in clockwise direction about Z-axis, which is given by :

$$MR = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots (6)$$

Substituting MR in equation (5) yields :

$$S(u, \theta) = \begin{bmatrix} P_x(u)\cos\theta & P_x(u)\sin\theta \\ P_z(u) & 1 \end{bmatrix} \dots(7)$$

where $0 \leq u \leq 1$

and $0 \leq \theta \leq 2\pi$

Figure (6) depicts different surfaces obtained from applying Equation (7) to their generator curves.

4.Cutter Selection and Machining Layers Determination.

In this study the following useful constraints are suggested in order to facilitate the machining process:

- 1.The part surface of the raw material lies in the x-y plane.
- 2.The tool axis is parallel to Z-axis, therefore it is perpendicular to the machining or part surface.
- 3.The machining layers are the intersection of series of planes perpendicular to the Z-axis .
- 4.A series of machining layers M_{Li} are separated by a constant distance ΔZ between each adjacent layers in order to select a proper cutter size.
- 5.The machining layers are numbered from the top of the cavity to its

bottom sequentially, as indicated in Figure (7.a).

Since the surface of revolution approach is used to define the shape of the cavity, the contours of the intersection between machining layers and the surface cavity are always circular contours of different radii, but of the same centers (see Figure 7.b).

According to the constraint (4) above, below a simple rule for determining the maximum cutter size for each machining layer. First of all Z-coordinates of intersected points must be found (Z_i , Z_{i+1} , Z_{i+2} , Z_{i+3} , Z_{i+4} , Z_{min}) (see Figure 8) as follows:

$$Z_i = Z_{max} - (i-1) \cdot \Delta Z \dots\dots\dots(8)$$

where $i = 1, 2, 3, 4, 5, \dots, N$

where N is the number of machining layers which is given by :

$$N = \text{INT}(Z_{max} - Z_{min}) / \Delta Z \dots(9)$$

X-coordinates of intersected points (X_i , X_{i+1} , X_{i+2} , X_{i+3} , X_{i+4} , X_{min}) can not be calculated by the same method, since that BEZIER curve is a freeform curve, therefore ΔX is unknown, the values of the independent variables (u_i , u_{i+1} , u_{i+2} , u_{i+3} , u_{i+4} , u_{min}) are required to be found. To do that, the Inverse Point Interpolation technique may be a useful way to find these values by substituting a successive values of (Z_i , Z_{i+1} , Z_{i+2} , Z_{i+3} , Z_{i+4} , Z_{min}) in Equation (3) to obtain :

$$Z_i = (1-u_i)^3 P_{1z} + 3u_i(1-u_i)^2 P_{2z} + 3u_i^2(1-u_i) P_{3z} + u_i^3 P_{4z} \dots\dots\dots(10)$$

Using the Simple Iteration Method on Equation (10) over different calculated values of Z_i , we shall find the values ($u_i, u_{i+1}, u_{i+2}, u_{i+3}, u_{i+4}, \dots, u_{min}$) corresponding to ($Z_i, Z_{i+1}, Z_{i+2}, Z_{i+3}, Z_{i+4}, \dots, Z_{min}$). Substituting these values in Equation (3) again, yields :

$$X_i = (1-u_i)^3 P1x + 3u_i (1-u_i)^2 P2x + 3u_i^2 (1-u_i) P3x + u_i^3 P4x \dots\dots\dots(11)$$

Now refer to Fig.(8), the maximum cutter radius ($R_i \max$) of a given machining layer is the distance from Z-axis to the X- coordinate of the intersected points of that machining layer i.e.,

$$R_i \max = INT (X_i) \dots\dots\dots(12)$$

When the technologist use the maximum cutter radius, each layer requires one machining pass, since the maximum cutter radius is equal to the radius of the machining layer as shown in Fig.(9), the maximum Volume of Removed Metal (VRM) will be removed and minimum length of circular path (LC) will result, consequently, productivity increases as the machining time decreases in this case the length of circular path represent the circumferential of the cutter, where :

$$VRM_i = 2\pi R_i \max. \Delta Z_i \dots(13)$$

$$LC_i = 2\pi R_i \max \dots\dots\dots(14)$$

where:

VRM_i : the Volume of Removed Metal of (i) layer .

The total (VRM) and (LC) of the cavity (all machining layers) is given by :

$$i=N$$

$$i=1$$

$$i=N$$

$$i=1$$

$$(VRM)_T = \sum 2\pi R_i \max. \Delta Z_i \dots(15)$$

$$(LC)_T = \sum 2\pi R_i \max. \dots\dots(16)$$

When the cutters of existing NC machine tool magazine are smaller than the maximum cutter size, the technologist is obliged to select these cutters, in this case each machining layer requires more than one machining pass as shown in Figure (9), consequently the machining time will increase since :

Ø The (VRM) for each machining layer will be removed over a different number of passes.

Ø The total length of circular tool path will increase.

Referring to Fig.(10), the (VRM) and (LC) of each pass can be calculated as follows:

$$VRM/pass = \pi. \Delta Z_i. C_i. \zeta \dots\dots(17)$$

$$LC/pass = \pi. C_i. i \dots\dots\dots(18)$$

where:

C_i : the cutter diameter

$$\zeta = 1, 2, 4, 6, 8, \dots, 2(\xi - 1)$$

where:

ξ : the number of passes which is given by :

$$\xi = (R_e / C_i) + 0.5 \dots\dots(19)$$

$$R_e = 0.2 R_i \dots\dots\dots(20)$$

where R_e is the effective diameter to avoid the interlacing between the circular paths.

5. Illustrative Examples and Comparison

Two experimental examples illustrate the feasibility of proposed strategy for practical use.

Example I

The first example is shown in Fig.(11). Fig.(11.a), shows the generator Bezier curve. Revolving of this curve about Z-axis through an angle of rotation ranging from $[0 \text{ to } 2\pi]$, gives the surface of the intended cavity as shown in Fig.(11.b).

A series of ten machining layers ML_i ($i= 1, \dots, 11$) are used to intersect the part design to find the maximum cutter size on each machining layer. The number of machining layers (NML) is calculated with a depth of cut $\Delta Z = 0.5$, where :

$$NML = ((Z_{max} - Z_{min}) / \Delta Z) + 1 \dots\dots\dots(21)$$

The intersection contours of the ten machining layers in X-Y plane and on 3D space system are shown in Figs. (11.c) and (11.d) respectively.

The X-coordinates, maximum cutter size, length of circular path(s) and volume of removed metal for each machining layer are listed in Table (1).

Since the maximum cutter size for each machining layer is not always available, it is very convenient to show the effect of different cutter sizes on the volume of removed metal. Assuming that the NC machine tool magazine contains the following cutters, $T = [T_1 T_2 T_3 T_4 T_5]$ having the diameters, $C = [2 \ 5 \ 8 \ 10 \ 12]$ respectively.

Table (2) shows the feasible and unfeasible cutters for specified machining layer and the number of required passes for each machining layer corresponding to each cutter.

Table (3) shows the (VRM) per pass for feasible cutters, optimum cutters for each machining layer and the total (LC) for each feasible cutter at a given machining layer.

Example II

Fig.(12.a) and (12.b) illustrates the generated Bezier curve with its control points and corresponding revolved surface cavity respectively. In this example a series of fifteen machining layers with ($\Delta Z = 0.6$) are used to intersect the part design in order to find the feasible cutter size for each machining layer.

The intersection contour of some machining layers on 2 and 3 dimensional space system are shown in Fig.(12.c) and (12.d) respectively. The X-coordinates, maximum cutter size and volume of removed metal of the sixteen machining layers are calculated and arranged in Table (4) below. In this example the suggested NC machine tool magazine contain the five cutters shown in Table (5) with their corresponding results to be feasible or unfeasible cutter. Table (6) shows the (VRM) per pass for feasible cutters, optimum cutters for each machining layer and the total (LC) for each feasible cutter at a given machining layer.

6. Concluding Remarks

In this paper, a strategy of generation cavities based on surface of revolution approach is presented. The generator curve is a cubic Bezier curve, therefore the user has the capability of estimating the shape of the generator curve due to the power of the control Bezier polygon for reflecting its shape. A methodology of critical cutter size and machining layers determination are also presented in this work using the simple iteration method as a powerful numerical analysis tool. Both strategies were coded in MATLAB,

result in a significant reduction in computation time.

In particular, the following results can be deduced confidently:

The volume of the removed metal is inversely proportional to the machining layers as shown in Fig.(13), since that cutters radii are decreased from the top of the cavity to its bottom. Therefore, Fig.(14) shows that the volume of removed metal is directly proportional to the cutter radii.

According to the results obtained from Tables 2, and 6 the relation between cutter diameter versus number of passes is plotted in Fig.(15), which exhibits that as the cutter diameter increases, a wide reduction in the number of passes is obviously noticed, due to the increase of surface area of the cutter.

Since the length of circular tool path (LC) is dependent on the cutter diameter and number of passes, (LC) is plotted in 3D space system as a function of two main variables as shown in Fig.(16). From this Figure it is observed that as the cutter diameter increases, the number of passes decreases, but the length of circular tool path (LC) is increased since the centre of the cutter is far away from the centre of the cavity, resulting in a wide increase in the circumferential length of the tool path.

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Fig. (1) Some of Surface of Revolution Approach's Application [8].

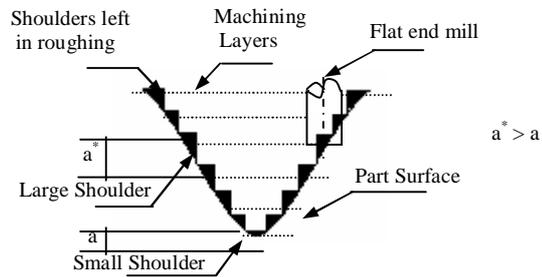


Fig.(2) The Effect of Cutting Depth (a) on the Shoulders Left by Roughing.

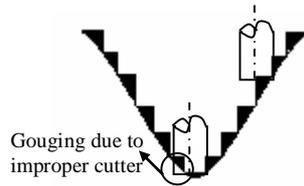


Fig.(3) Interference Between Cutter and Part Surface.

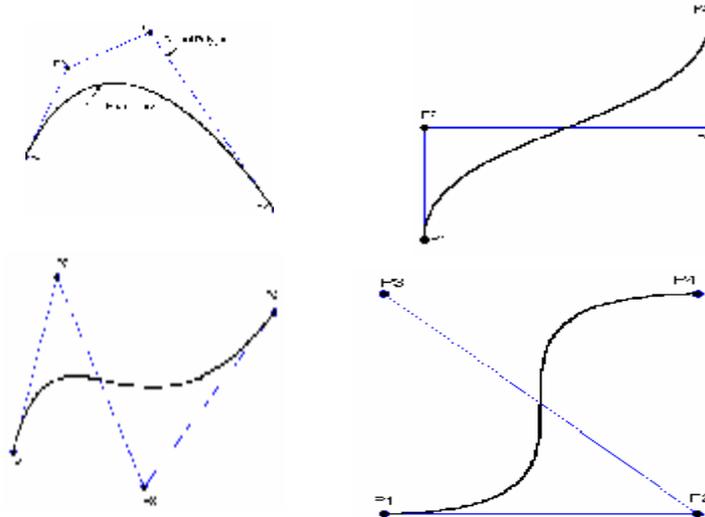


Fig.(4) The Shape of the Curve can be Changed by Changing the Position of Control Points.

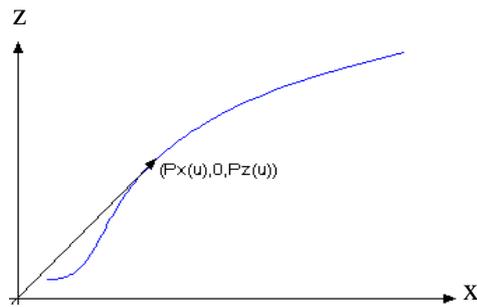


Fig.(5) The Position Vector of any Point on Bezier Curve.

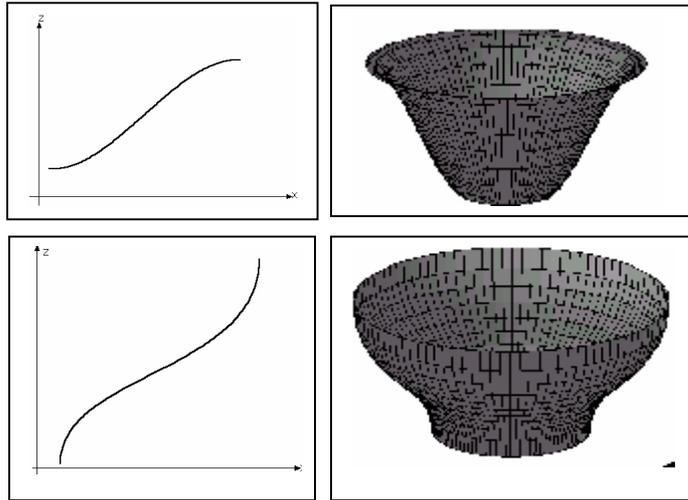


Fig. (6) The Surface of Revolution Approach of Different Generated Bezier Curve.

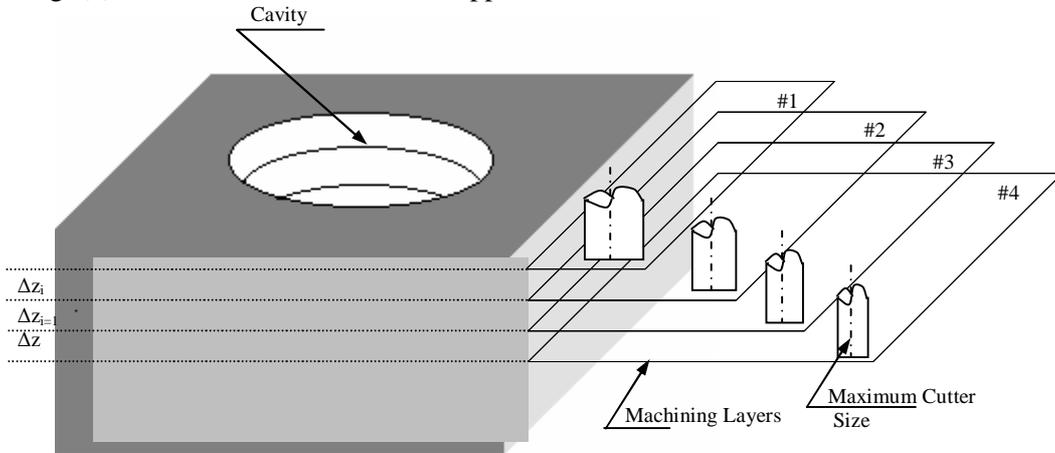


Fig. (7) : (a) Intersection Between Raw Material and Different Machining Layers at a Constant Depth of Cut.

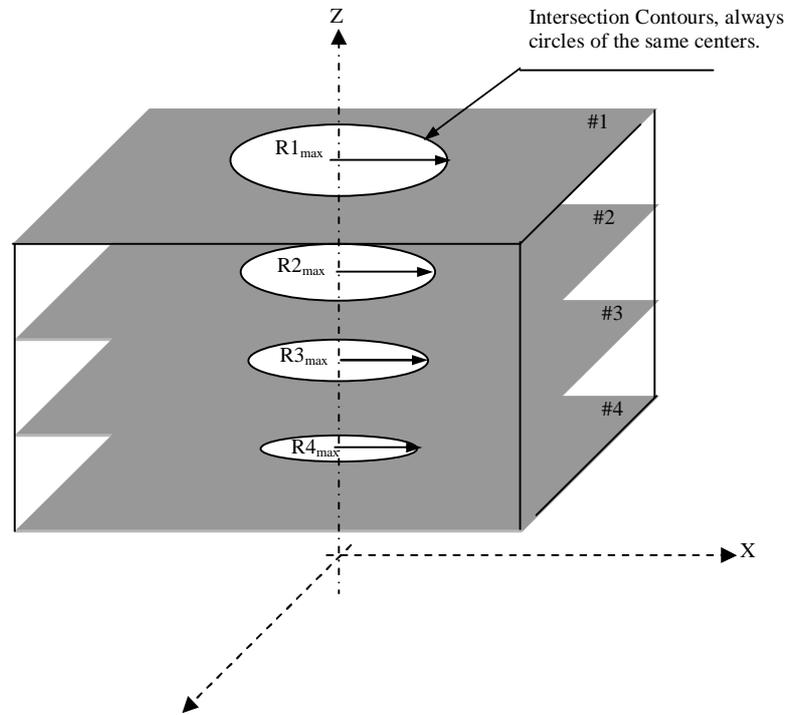


Fig. (7) : (b) Intersection of Contours are Always Circles of Different radii.

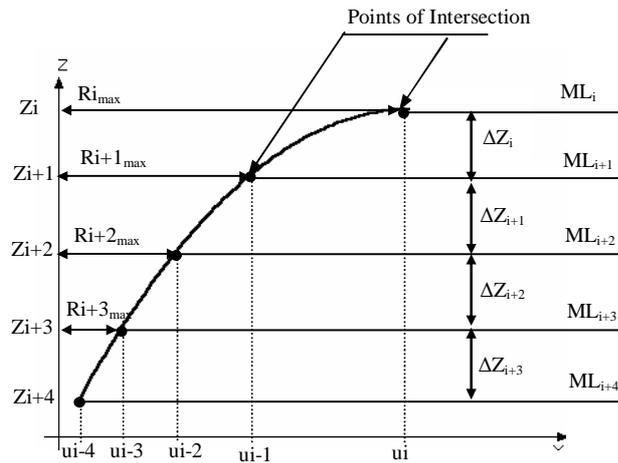


Fig.(8) The Procedure of Determining The Maximum Cutter Size of Each Machining Layer.

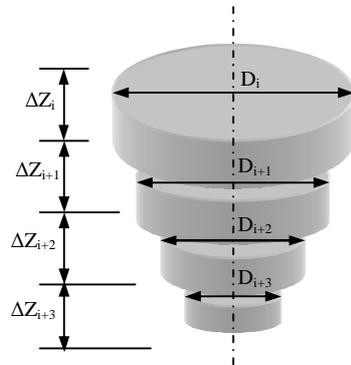


Fig. (9) The Relation Between Cutter Diameter and the Diameter of Machining Layer.

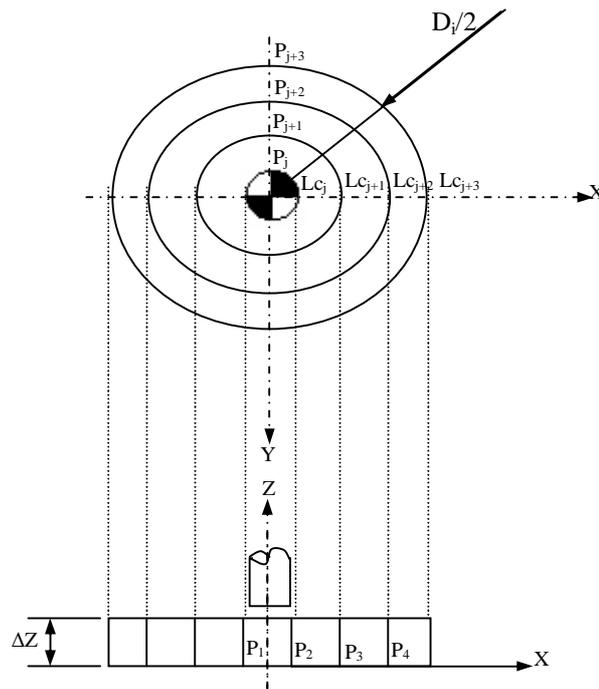


Fig.(10) The Relation Between Cutter Diameter and Number of Passes.

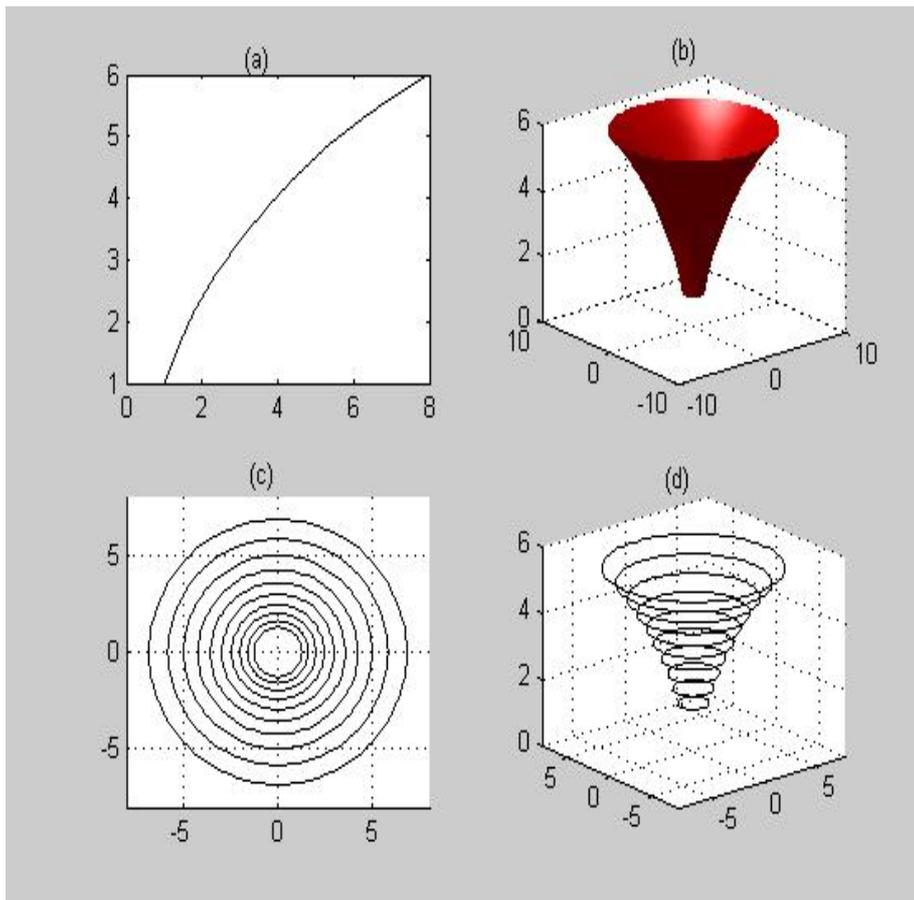


Fig. (11) (a) The Generated Bezier Curve. (b) The Surface Cavity
(c) The 2D Contours of Machining Layers. (d) The 3D Contours of the Machining
Layers.

Table 1 Machining Layers, Max. Cutter sizes, VRM and Length of
Circumferential Cut of Example I.

Machining Layers (ML)	X-coordinates	Maximum Cutter Size	Volume of Removed Metal (VRM) [cubic unit]	Length of Circumferential Cut
ML 1-2	7.102	14.0	11.155	22.311
ML 2-3	6.216	12.0	9.764	19.528
ML 3-4	5.670	11.0	8.906	17.812
ML 4-5	4.528	9.0	7.112	14.225
ML 5-6	3.750	7.0	5.854	11.780
ML 6-7	3.032	6.0	4.762	9.525
ML 7-8	2.386	4.0	3.747	7.495
ML 8-9	1.824	3.0	2.865	5.730
ML 9-10	1.358	2.0	2.133	4.266
ML 10-11	1.000	2.0	1.570	3.141

Table 2 The Proposed Tool Magazine Vs. No. of Passes of Example I

Diameter	2	5	8	10	12	2	5	8	10	12
Machining Layers (ML)	Tool Magazine					No. of Passes ξ				
	T ₁	T ₂	T ₃	T ₄	T ₅	T ₁	T ₂	T ₃	T ₄	T ₅
ML 1-2	1	1	1	1	1	7	3	2	2	2
ML 2-3	1	1	1	1	1	6	3	2	2	2
ML 3-4	1	1	1	1	0	6	3	2	2	—
ML 4-5	1	1	1	0	0	5	2	2	—	—
ML 5-6	1	1	0	0	0	4	2	—	—	—
ML 6-7	1	1	0	0	0	3	2	—	—	—
ML 7-8	1	0	0	0	0	3	—	—	—	—
ML 8-9	1	0	0	0	0	2	—	—	—	—
ML 9-10	1	0	0	0	0	2	—	—	—	—
ML 10-11	1	0	0	0	0	2	—	—	—	—

* 1: Cutter T_i is Feasible to Machining Layer ML_i

0: Cutter T_i not Feasible

Table 3 The VRM Vs. Length of Tool Path of Each ML of Example I

Diameter Machining Layers (ML)	2		5		8		10		12		Total VRM
	VRM/pass					LC					
	T ₁	T ₂	T ₃	T ₄	T ₅	T ₁	T ₂	T ₃	T ₄	T ₅	
ML 1-2	3.140	7.850	12.565	15.670	18.835	12.204	9.204	6.204	4.204	2.204	22.311
ML 2-3	3.140	7.850	12.565	15.670	18.835	10.432	7.432	4.432	2.432	0.432	19.528
ML 3-4	3.140	7.850	12.565	15.670	0	9.340	6.340	3.340	1.340	—	17.812
ML 4-5	3.140	7.850	12.565	0	0	7.056	4.056	1.056	—	—	14.225
ML 5-6	3.140	7.850	0	0	0	5.500	2.500	—	—	—	11.780
ML 6-7	3.140	7.850	0	0	0	4.064	1.064	—	—	—	9.525

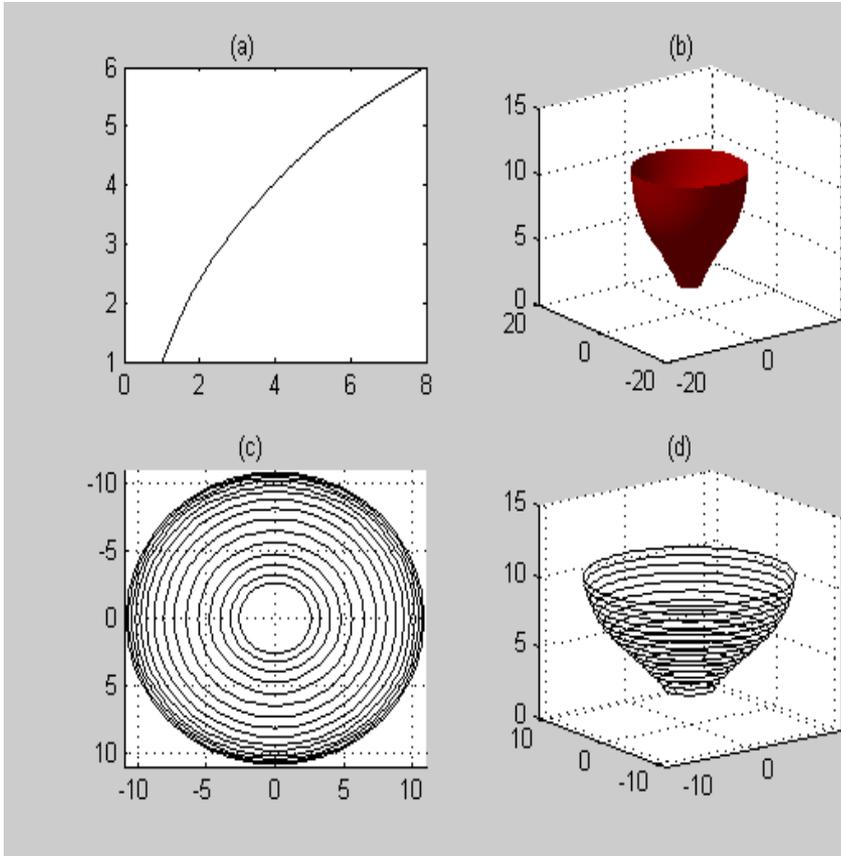


Figure (12) (a) The Generator Bezier Curve. (b) The Surface Cavity
(c) The 2D Contours of Machining Layers. (d) The 3D Contours of Machining
Layers.

Table 4 Machining Layers, Max. Cutter sizes, VRM and Length of
Circumferential Cut of Example II

Machining Layers (ML)	X- coordinates	Maximum Cutter Size	Volume of Removed Metal (VRM) [cubic unit]	Length of Circumferential Cut
ML 1-2	10.759	21.0	40.560	33.896
ML 2-3	10.443	20.0	39.369	32.791
ML 3-4	10.064	20.0	37.940	31.601
ML 4-5	9.614	19.0	36.243	30.188
ML 5-6	9.105	18.0	34.325	28.589
ML 6-7	8.552	17.0	32.240	28.853
ML 7-8	7.913	15.0	29.831	24.846
ML 8-9	7.288	14.0	27.475	22.884
ML 9-10	6.608	13.0	24.411	20.749
ML 10-11	5.885	11.0	22.185	18.479
ML 11-12	4.519	9.0	17.036	14.189
ML 12-13	4.376	8.0	16.497	13.74
ML 13-14	3.588	7.0	13.526	11.266
ML 14-15	2.791	5.0	10.521	8.763
ML 15-16	2.000	4.0	7.539	6.280

Table 5 The Proposed Tool Magazine Vs. No. of Passes of
Example II

Diameter	2	8	10	15	20	2	8	10	15	20
Machining Layers (ML)	Tool Magazine					No. of Passes ξ				
	T ₁	T ₂	T ₃	T ₄	T ₅	T ₁	T ₂	T ₃	T ₄	T ₅
ML 1-2	1	1	1	1	1	10	3	3	2	2
ML 2-3	1	1	1	1	1	9	3	3	2	2
ML 3-4	1	1	1	1	1	9	3	3	2	2
ML 4-5	1	1	1	1	0	9	3	3	2	—
ML 5-6	1	1	1	1	0	8	3	2	2	—
ML 6-7	1	1	1	1	0	8	3	2	2	—
ML 7-8	1	1	1	1	0	7	3	2	2	—
ML 8-9	1	1	1	1	0	7	2	2	2	—
ML 9-10	1	1	1	0	0	6	2	2	—	—
ML 10-11	1	1	1	0	0	6	2	2	—	—
ML 11-12	1	1	0	0	0	5	2	—	—	—
ML 12-13	1	1	0	0	0	4	2	—	—	—
ML 13-14	1	0	0	0	0	4	—	—	—	—
ML 14-15	1	0	0	0	0	3	—	—	—	—
ML 15-16	1	0	0	0	0	3	—	—	—	—

Table 6 The VRM Vs. Length of Tool Path of Each ML of Example 1

Diameter Machining Layers (ML)	VRM/pass					LC/pass					Total VRM
	2	8	10	15	20	2	8	10	15	20	
	T ₁	T ₂	T ₃	T ₄	T ₅	T ₁	T ₂	T ₃	T ₄	T ₅	
ML 1-2	3.141	12.566	15.708	23.561	31.415	19.590	13.590	11.590	6.590	1.590	65.973
ML 2-3	3.141	12.566	15.708	23.561	31.415	18.886	12.886	10.886	5.886	0.886	62.831
ML 3-4	3.141	12.566	15.708	23.561	31.415	18.128	12.128	10.128	5.128	0.128	62.831
ML 4-5	3.141	12.566	15.708	23.561	0	17.228	11.228	9.228	4.228	0	59.690
ML 5-6	3.141	12.566	15.708	23.561	0	16.21	10.210	8.210	3.210	0	56.548
ML 6-7	3.141	12.566	15.708	23.561	0	15.104	9.104	7.104	2.104	0	53.407
ML 7-8	3.141	12.566	15.708	23.561	0	13.826	7.826	5.826	0.826	0	47.123
ML 8-9	3.141	12.566	15.708	23.561	0	12.576	6.576	4.576	0	0	43.982
ML 9-10	3.141	12.566	15.708	0	0	11.216	5.216	3.216	0	0	40.840
ML 10-11	3.141	12.566	15.708	0	0	9.77	3.770	1.770	0	0	34.557
ML 11-12	3.141	12.566	0	0	0	7.038	1.038	0	0	0	28.274
ML 12-13	3.141	12.566	0	0	0	6.752	0.752	0	0	0	25.132
ML 13-14	3.141	0	0	0	0	5.176	0	0	0	0	21.991
ML 14-15	3.141	0	0	0	0	3.582	0	0	0	0	15.708
ML 15-16	3.141	0	0	0	0	2.000	0	0	0	0	12.566

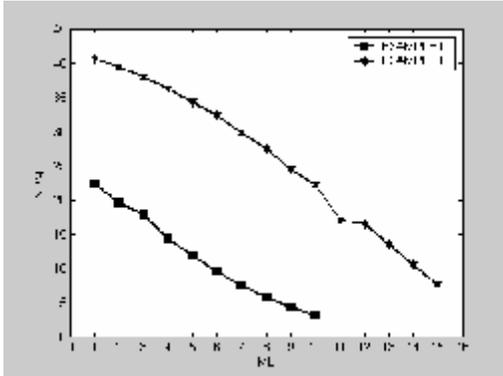


Fig. (13) Comparison of the Relation Between ML and VRM .

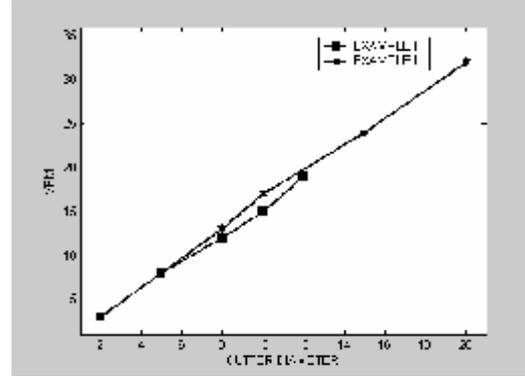


Fig.(14) Comparison of the Relation Between Cutter Diameter and VRM.

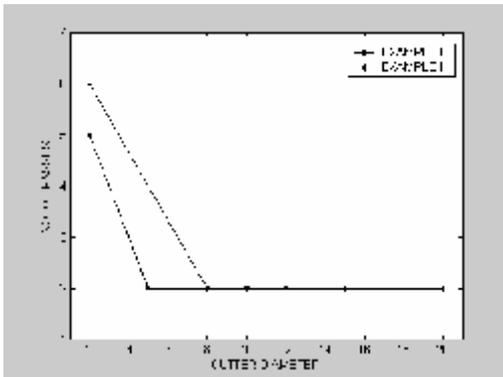


Fig. (15) Comparison of the Relation Between Cutter Diameter and No. of Passes.

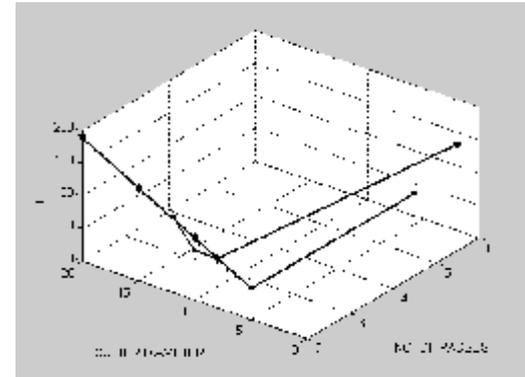


Fig. (16) Comparison of the Relation Between Cutter Diameter , No.of Passes and LC