[Al-Qadisiyah Journal of Pure Science](https://qjps.researchcommons.org/home)

Volume 29 | Number 2 Article 12

12-20-2024

Weakly and Strongly Forms of Fibrewise Fuzzy ω-Topological Spaces

M. A. Hussain
Department of Mathematics, College of Education for Pure Science (Ibn Al-haitham), University of Baghdad, Baghdad-Iraq

Y.Y. Yousif
Department of Mathematics, College of Education for Pure Science (Ibn Al-haitham), University of Baghdad, Baghdad-Iraq, Hwe1ggh597@gmail.com

Recommended Citation

Hussain, M. A. and Yousif, Y.Y. (2024) "Weakly and Strongly Forms of Fibrewise Fuzzy ω-Topological Spaces," *Al-Qadisiyah Journal of Pure Science*: Vol. 29 : No. 2 , Article 12.
Available at: https://doi.org/10.29350/2411-3514.1290

ORIGINAL STUDY

Weakly and Strongly Forms of Fibrewise Fuzzy ω-Topological Spaces

Mohammed A. Hussein*, Yousif Y. Yousif

Department of Mathematics, College of Education for Pure Science (Ibn Al-haitham), University of Baghdad, Baghdad, Iraq

Abstract

This paper is devoted to introduce weak and strong forms of fibrewise fuzzy ω-topological spaces, namely the fibrewise fuzzy θ-ω-topological spaces, weakly fibrewise fuzzy θ-ω-topological spaces and strongly fibrewise fuzzy θ-ω-topological spaces. Also, Several characterizations and properties of this class are also given as well. Finally, we focused on studying the relationship between weakly fibrewise fuzzy θ-ω-topological spaces and strongly fibrewise fuzzy θ-ω-topological spaces.

Keywords: Weakly fibrewise fuzzy θ-ω-topological spaces, strongly fibrewise fuzzy θ-ω-topological spaces

1. Introduction and preliminaries

In order to began the category in the classification of fibrewise (shortly., *fw*) sets on a given set, named the base set, which say B. A *fw* set on B consist of a set M with a function $p: M \to B$ that is named the projection (shortly., proj.). The fibre over b for every point b of B is the subset $M_b = p^{-1}(b)$ of M. Perhaps, fibre will be empty because we do not require p is surjective, also, for every subset B^* of B we considered $M_{B^*} = p^{-1}(B^*)$ as a *fw* set over B^* with the projection determined by p. The concept of fuzzy sets was introduced by Zadeh [18]. The idea of fuzzy topological spaces was introduced by Chang [6]. The concept of fuzzyω-continuity, fuzzy almost ω-continuous and fuzzy weakly ω-continuous in topological spaces was introduced by Gazwan [1]. In this paper, we introduce and study seven weak and strong forms of fibrewise fuzzy topological spaces, called fibrewise fuzzy ω-topological spaces, fibrewise fuzzy almost ω-topological spaces, fibrewise fuzzy almost weakly ω-topological spaces, fibrewise fuzzy weakly θ-ω-topological spaces, fibrewise fuzzy θ-ω-topological spaces, fibrewise fuzzy strongly θ-ω- topological spaces and fibrewise fuzzy almost strongly ω-topological spaces, we study their basic properties and we shall discuss relationships between weakly fibrewise fuzzy θ-ω-topological spaces and strongly fibrewise fuzzy θ-ω-topological spaces, we built on some of the result in Refs. [3,4,15–17] (see Fig. 1).

Definition 1.1. [7,8] A mapping $\vartheta: M \to N$, where M and N are FW sets over B, with proj.'s $p_M: M \to B$ and $p_N: N \to B$, is said to be FW mapping (written as FW-M) if $p_N \circ \vartheta = p_M$, or $\vartheta(M_b) \subseteq N_b$, for all point $b \in B$.

Observe that a FW-M $\vartheta: M \to N$ over B limited by restriction, a FW-M $\vartheta: M_{B^*} \to N_{B^*}$ over B^* for all subset $B^* \subseteq B$.

Definition 1.2. [8] The fibrewise topology (written as FWT) on a FW set M over a topological space (B, σ) signify any topology on M for which the proj. p is continuous (written as FWTS).

Definition 1.3. [8] Let M and N be FWTS's over B, the FW-M $\vartheta: M \to N$ is said to be:

(a) continuous if $b \in B$ and for all point $m \in M_b$, the pre image of all open set of $\vartheta(m)$ is an open set of m.
(b) open if $b \in B$ and for all point $m \in M_b$, the image of all open set of m is an open set of $\vartheta(m)$.

Received 1 October 2022; accepted 30 December 2022.
Available online 18 April 2025

* Corresponding author.
E-mail address: Hwe1ggh597@gmail.com (M.A. Hussein).

https://doi.org/10.29350/2411-3514.1290

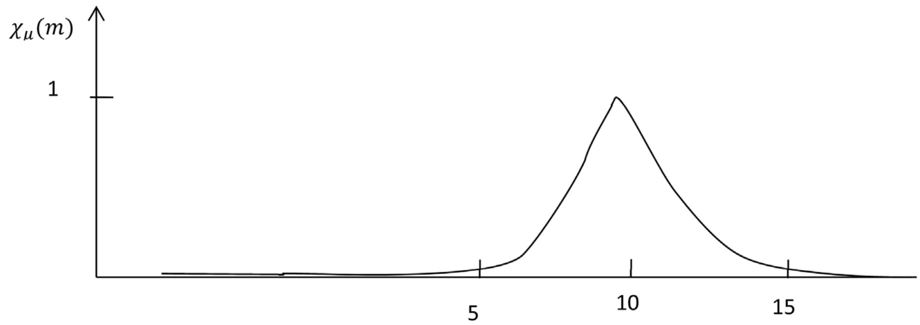

Fig. 1. Relationships between weakly fibrewise fuzzy q-u-topological spaces and strongly fibrewise fuzzy q-u-topological spaces.

Definition 1.4. [8] The FWTS (M,τ) over (B,σ) is said to be:

(a) FW closed (written as FWC) if the proj. p is closed mapping.
(b) FW open (written as FWO) if the proj. p is open mapping.

Definition 1.5. [18] assume that M is a nonempty set, a fuzzy set μ in M is a mapping $\chi_\mu : M \rightarrow I$ where I is the closed unite interval $[0,1]$ which is written as:

$$\mu = \{(m, \chi_\mu(m)) : m \in M, 0 \le \chi_\mu(m) \le 1\},$$

The family of each fuzzy subsets in M will be symbol by I^M thus is $I^M = \{\mu : \mu \text{ is fuzzy subset of } \mu\}$ and χ_μ is called the membership function.

Example 1.6. [12] We will suppose a possible membership function for the fuzzy set of real numbers close to zero as follows, $\chi_\mu : \mathbb{R} \longrightarrow [0,1]$, where

$$\chi_\mu(m) = \frac{1}{1 + (m - 10^2)}, \forall\, m \in \mathbb{R}$$

Definition 1.7. [18] A fuzzy set in M is empty denoted by $\overline{0}_\mu$, if its membership function is identically the zero function, i.e.,
$\overline{0}_\mu : M \rightarrow [0,1]$ s.t $\overline{0}_\mu(m) = 0\ \forall m \in M$.

Definition 1.8. [18] A universal fuzzy set in M, denoted by $\overline{1}_\mu$, is a fuzzy set defined as $\overline{1}_\mu(m) = 1\ \forall\, m \in M$.

Definition 1.9. [18] Let $\mu, \lambda \in I^M$. A fuzzy set μ is a subset of an fuzzy set λ, denoted by $\mu \le \lambda$ iff $\mu(m) \le \lambda(m)$, $\forall m \in M$.

Two fuzzy sets μ and λ are said to be equal $(\lambda = \mu)$ if $\lambda(m) = \mu(m)$, $\forall m \in M$.

Definition 1.10. [18] Let λ and μ be fuzzy sets in M. Then, for all $m \in M$,
$\psi = \lambda \vee \mu \Leftrightarrow \psi(m) = max\,\{\lambda(m),\ \mu(m)\}$,
$\delta = \lambda \wedge \mu \Leftrightarrow \delta(m) = min\,\{\lambda(m), \mu(m)\}$,
$\eta = \lambda^c \Leftrightarrow \eta(m) = 1 - \lambda(m)$.

More generally, for a family $\Lambda = \{\lambda_i \mid i \in I\}$ of fuzzy sets in M, the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined by
$\psi(x) = sup_i\,\{\lambda_i(m) \mid m \in M\}$,
$\delta(x) = inf_i\,\{\delta_i(m) \mid m \in M\}$.

Definition 1.11. [6] A fuzzy topology is a family τ of fuzzy sets in M, which satisfies the following conditions:

(a) $\overline{0}, \overline{1} \in \tau$;
(b) If $\lambda, \mu \in \tau$, thus $\lambda \wedge \mu \in \tau$;
(c) If $\lambda_i \in \tau$ for all $i \in I$, thus $\vee\, i\ \lambda_i \in \tau$.

(M, τ) is said to be fuzzy topological spaces and each member of τ is named fuzzy open set on M and its complement is fuzzy closed set.

Definition 1.12. [10] A fuzzy set on M is named a fuzzy point iff it takes the value 0 for each $y \in M$ except one, say, $m \in M$. If its value at m is r $(0 < r \le 1)$ we denote thus fuzzy point by m_r, when the point m is named its support.

Definition 1.13. [6,14] Let μ be a fuzzy set and let (M,τ) be a fuzzy topological space. μ is a fuzzy neighborhood of a fuzzy point m_r if there exist a fuzzy open set ν since $r \le \nu(m) \le \mu(m)$, $\forall\ m \in M$.

Definition 1.14. [6] assume that (M, τ) is a fuzzy topological space as well $\mu \in I^M$. The fuzzy closure (fuzzy interior) of A is symbol by *cl(μ)* (*int(μ)*) is defined by:
$cl(\mu) = \wedge\,\{\lambda^c \in \tau,\ \mu \le \lambda\}$
$int(\mu) = \vee\,\{\xi \in \tau;\ \xi \le \mu\}$.

Evidently, *cl((μ)* (resp., *int(μ))* is the smallest fuzzy closed (resp., largest fuzzy open) subset of M which contains (resp., contained in) μ. Note that μ is fuzzy closed (fuzzy open) iff $\mu = cl(\mu)$ (resp., $int(\mu)$).

Definition 1.15. [6] assume that f: $M \to N$ is a mapping. For a fuzzy set β in N and membership function β(n). The inverse image of β under f is the fuzzy set f^{-1} (β) in M with membership function is denoted by the rule:

$$f^{-1}(\beta)\,(m) = \beta(f\,(m)),\ \forall m \in M. \tag{1}$$

For a fuzzy set λ in M, the image of λ under f is the fuzzy set $f(\lambda)$ in B with membership function $f(\lambda)(n)$, $n \in N$ is given by

$$f\,(\lambda)\,(n) = \begin{cases} \sup\limits_{m \in f^{-1}(n)} \{(\lambda(m)\}, if\, f^{-1}(n) & \text{nonempty} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Definition 1.16. [2] Assume that m_r is a fuzzy point and μ a fuzzy set in M. Then m_r is said to be in μ or (belong to μ) or (m_r content in μ) denoted $m_r \in \mu$ if and only if $r \le \mu$(m), for all m $\in$ M

Definition 1.17. [9,19] The set $\{m : m \in M, \mu(m) > 0\}$ is called the support of μ and is denoted by Supp(μ)

Definition 1.18. [10] A fuzzy point m_r is said to be quasi-coincident with μ denoted by $m_r q \mu$ if there exist $m \in M$ such that $r + \mu(m) > 1$, if m_r is not quasi coincident with μ, then $r + \mu(m) \le 1$ $\forall m \in M$ and denoted by $m_r\ \tilde{q}\mu$.

Definition 1.19. [10] A fuzzy set μ in (M, τ) is called a "Q-neighborhood of m_λ " iff $\exists \nu \in \tau$ such that $m_\lambda q \nu < \mu$.

The family of all Q-nbhd's of m_λ is called the system of Q-nbhd's of m_λ.

Definition 1.20. [2] Fuzzy regular space if for each fuzzy point m_r in M and each fuzzy closed set F with $m_r \tilde{q} F$ there exists fuzzy open μ, λ in M such that $r \le \mu$(m) , F(m) $\le$ λ(m) ∀ m ∈ M and $\tilde{q}\lambda$.

Definition 1.21. [11] A fuzzy set μ is fuzzy θ-closed if $\mu = cl_\theta\,(\mu) = \{m_r$ fuzzy point in (M, τ): $(cl\ (v))\ q\ \mu$, U is fuzzy open q-nbd. of $m_r\}$. The complement of fuzzy θ-closed called fuzzy θ-open set.

Definition 1.22. [14] Let μ be a fuzzy set in a fuzzy topological space (M, τ) is named a fuzzy uncountable iff supp(μ) is an uncountable subset of M.

Definition 1.23. [1] A fuzzy point m_r of a fuzzy topological space (M, τ) is named a fuzzy condensation point of μ on M if min{μ(m), λ(m)}is fuzzy uncountable for each fuzzy open set λ containing m_r. And the set of all fuzzy condensation point of μ is denoted by Cond (μ)

Definition 1.24. [1] A fuzzy subset μ in a fuzzy topological space (M, τ) is called a fuzzy ω-closed set if it contains each its fuzzy condensation point. The complement fuzzy ω-closed sets are called fuzzy ω-open sets. And the family of all fuzzy ω-open (resp.fuzzy ω-closed) sets in a fuzzy topological space (M, τ) will be denoted by f. ω-open (resp. f.ω-closed).

Definition 1.25. [1] Assume that μ is a fuzzy set of a fuzzy topological space (M, τ) then The ω-closure of μ is symbol by $cl^\omega(\mu)$ and known that by $cl^\omega\mu$ (m) $= inf\{\,F(m) : F\, i\ \ s$ a fuzzy ω-closed set in, $(m) \le F(m)\}$.

Definition 1.26. [1] For a fuzzy topological space (M, τ) is named a fuzzy ω-regular space when all fuzzy ω-closed subset μ in M so well a fuzzy point m_r in M so that $m_r \widehat{q}\ \mu$, there exists two fuzzy ω-open sets λ and ν such that $r \le \lambda$(m), μ(m) $\le \nu$(m) and $\lambda \widehat{q}\ \nu$

Definition 1.27. [6] A mapping $\phi : (M, \tau) \to (N, \Lambda)$ is said to be

(a) fuzzy continuous (briefly f. continuous) if the inverse image of every fuzzy open set of N is a fuzzy open set in M.
(b) fuzzy open (briefly f. open) map if the image of every fuzzy open set of M is a fuzzy open set in N.
(c) fuzzy close (briefly f. close) map if the image of every fuzzy close set of M is a f. close set in N.

Definition 1.28. [13] A mapping $\phi : (M, \tau) \to (N, \Lambda)$ is said to be Fuzzy θ-continuous (f. θ.continuous, for short) if for each fuzzy point m in (M, τ) and each fuzzy open q-nbd. ν of $\phi(m)$, there exists fuzzy open q-nbd. u of m so that $p(cl(u)) \le cl(\nu)$.

Definition 1.29. [1] A mapping $\phi : (M, \tau) \to (N, \Lambda)$ is said to be

(a) Fuzzy ω-continuous at a fuzzy point $m \in M$ when all fuzzy open subset λ in N contains ϕ (m) there exists a fuzzy ω-open subset μ on M which contains m so that $\phi(\mu) \le \lambda$ so well ϕ is called fuzzy ω-continuous if it is fuzzy ω-continuous at every fuzzy point.
(b) Fuzzy almost ω-continuous at a fuzzy point $m \in M$ when all fuzzy open subset λ in N contains ϕ

(m) there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(\mu) \leq int(cl(\lambda))$ so well ϕ is named fuzzy almost ω-continuous if it is fuzzy almost ω-continuous at every fuzzy point.

(c) Fuzzy weakly ω-continuous at a fuzzy point $m \in M$ when all fuzzy open subset λ in N contains ϕ (m) there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(\mu) \leq cl(\lambda)$ so well ϕ is named fuzzy ω-continuous if it is fuzzy ω-continuous at every fuzzy point.

2. Weakly fibrewise fuzzy θ-ω-topological spaces

In this section, we study the weakly fibrewise fuzzy θ-ω-topological spaces and some theorems concerning them.

First, we introduced the following definition.

Definition 2.1. A mapping $\phi : (M,\tau) \to (N,\Lambda)$ is said to be fuzzy almost weakly θ-ω-continuous (briefly, f. almost weakly θ-ω-continuous) if in a fuzzy point $m \in M$ when all fuzzy open subset λ in N contains ϕ (m) there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(\mu) \leq cl(\lambda)$ so well ϕ is named fuzzy almost weakly ω-continuous if its fuzzy almost weakly ω-continuous at every fuzzy point.

Definition 2.2. A mapping $\phi : (M,\tau) \to (N,\Lambda)$ is said to be fuzzy θ-ω-continuous (briefly, f. θ-ω-continuous) at a fuzzy point $m \in M$ when all fuzzy open subset λ in N contains ϕ (m) there exists a fuzzy ω-open subset μ of M which contains m so that $\phi(cl^{\omega}(\mu)) \leq cl(\lambda))$ as well ϕ is named fuzzy θ-ω-continuous if its fuzzy θ-ω-continuous at every fuzzy point.

Definition 2.3. A fuzzy set A is fuzzy θ-ω-closed if $A = cl^{\omega}_{\theta}$ (A) = {p fuzzy point in (X,τ): $(cl^{\omega}$ $(U))$ q A, U is fuzzy ω-open q-nbd. of p}. The complement of fuzzy θ-ω-closed called fuzzy θ-ω-open set.

Definition 2.4. A mapping $\phi : (M,\tau) \to (N,\Lambda)$ is said to be fuzzy weakly θ-ω-continuous (briefly, f. weakly θ-ω-continuous) if in a fuzzy point $m \in M$ when all fuzzy open subset λ in N contains ϕ (m) there exists a fuzzy θ-ω-open subset μ of M which contains m so that $\phi(\mu) \leq cl(\lambda)$ so well ϕ is named fuzzy weakly θ-ω-continuous if its fuzzy weakly θ-ω-continuous at every fuzzy point.

Definition 2.5. Let (B,σ) be a fuzzy topological space the fibrewise fuzzy ω-topological spaces, fibrewise fuzzy almost weakly ω-topological spaces, fibrewise fuzzy almost ω-topological spaces, fibrewise fuzzy weakly θ-ω-topological spaces and fibrewise fuzzy θ-ω-topological spaces (briefly, FWF ω-top. sp., FWF almost weakly ω-top. sp., FWF almost ω-top. sp., FWF weakly θ-ω-top. sp. and FWF θ-ω-top. sp.) on a fibrewise set M over B mean any fuzzy topology on M which of them the projection function p are fuzzy ω-continuous, fuzzy almost weakly ω-continuous, fuzzy almost ω-continuous, fuzzy weakly θ-ω-continuous and fuzzy θ-ω-continuous (briefly, f. ω-continuous, f. almost weakly ω-continuous, f. almost ω-continuous, f. weakly θ-ω-continuous and f. θ-ω-continuous).

Theorem 2.6. The FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp., then it is FWF almost ω-top. sp.

Proof. Assume that (M,τ) is a FWF ω-top. sp. over (B,σ), then the proj. $p : (M,\tau) \to (B,\sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. almost ω-continuous. Assume that $m \in M_b$; $b \in B$ and, μ is a fuzzy open set contains $p(m)$ in B. Since p is f. ω-continuous, there is a f. ω-open set λ containing m so that $p(\lambda) \leq \mu$. Thus, $int(\mu) \leq \mu$ and $\mu \leq cl(\mu)$. Then, $int(\mu) \leq cl(\mu)$ and $int(int(\mu) \leq int(\ cl(\mu))$. It follows that, $p\ (\lambda) \leq int(\ cl(\mu))$. Therefore $p(\lambda) \leq int(\ cl(\mu))$. So, p is f. almost ω-continuous. Hence (M,τ) is FWF almost ω-top. sp.

We can prove the same way by used property of fuzzy interior and fuzzy closure set.

The converses does not hold as we show by the following examples:

Example 2.7. Let $M = \{a,b,c\}$, $B = \{x,y,z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2, \mu_3\}$ where
$\mu_1 = \{(a, 0.1)\}$
$\mu_2 = \{(b, 0.2)\}$
$\mu_3 = \{(a, 0.1), (b, 0.2)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(z, 1)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M,\tau) \to (B,\sigma)$ be the fuzzy function as $p(a) = p(b) = p(c) = z$. Let $\lambda = \{(a, 0.1)\}$ fuzzy open in M and $\nu = \{(b, 0.2)\}$. Then, $p(cl\{(b, 0.2)\}) \leq cl\{(a, 0.1)\}$ but $p(\{(b, 0.2)\}) \nleq int(cl(\{(a, 0.1)\}$. Then, $(M,\ \tau)$ is FWF θ-ω-top. sp. but not FWF almost ω-top. sp.

Example 2.8. Let $M = \{a,b,c\}$, $B = \{x,y,z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a, 0.3), (b, 0), (c, 0.5)\}$
$\mu_2 = \{(a, 0.7), (b, 1), (c, 0.5)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0), (y, 0.3), (z, 1)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M,\tau) \to (B,\sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Let $\eta = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$ fuzzy θ-ω-open in M and $\nu = \{(a, 0), (b, 0.3), (c, 0.5)\}$ is fuzzy open in B.

Then, $p(\eta) \leq cl(\nu)$ but $p(cl(\eta)) \nleqslant cl(\nu)$. Then, (M,τ) is FWF weakly θ-ω-top. sp. but not FWF θ-ω-top. sp.

Example 2.9. In Example 2.6, (M,τ) over (B,σ) is a FWF weakly θ-ω-top. sp., but is not FWF ω-top. sp. Moreover, (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp., but is not FWF almost ω-top. sp. Moreover, (M,τ) over (B,σ) is FWF almost weakly ω-top. sp., but is not FWF θ-ω-top. sp., and not FWF ω-top. sp.

Example 2.10. Let $=\{a,b\}$, $B=\{x, y\}$, $\tau=\{\overline{0},\overline{1},\mu_1,\mu_2,\mu_3\}$ where
$\mu_1=\{(a,0.9),(b,0.7)\}$
$\mu_2=\{(a,1),(b,0.9)\}$
$\mu_3=\{(a,0.11),(b,0.31)\}$
And let $\sigma=\{\overline{0},\overline{1},\lambda\}$, where $\lambda=\{(x,0.11),(y,0.31)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p:(M,\tau)\rightarrow(B,\sigma)$ be the fuzzy function as $p(a)=x$, $p(b)=y$. Let $\eta=\{(a,0.7),(b,0.4)\}$ fuzzy ω-open in M and $\nu=\{(a,0.11),(b,0.31)\}$ is fuzzy open in B. Then, $p(\eta)\leq int(cl(\nu))$ but $p(\eta))\nleq\nu$. Then, (M,τ) is FWF almost ω-top. sp. but not FWF ω-top. sp. Moreover, (M,τ) is FWF almost weakly ω-top. sp. but not FWF almost ω-top. sp.

Lemma 2.11. [1] A fuzzy topological space (M,τ) is fuzzy ω-regular if and only if for all fuzzy point m in M and all fuzzy ω-open μ containing m, there exists fuzzy ω-open set λ such that $m\in\lambda\leq cl^{\omega}(\lambda)\leq\mu$.

Theorem 2.12. Let (M,τ) be a fuzzy ω-regular space. The FWF topological space (M,τ) over (B,σ) is FWF almost weakly ω-top. sp., then it is FWF θ-ω-top. sp.

Proof. Let (M,τ) be a FWF almost weakly ω-top. sp. over (B,σ), then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. almost weakly ω-continuous. It suffices to demonstrate that p is f. θ-ω-continuous. Assume that $m\in M_b$; $b\in B$ so well, μ is a fuzzy open set containing $p(m)$ in B. Since p is f. almost weakly ω-continuous, there exists is a f. ω-open set λ containing m such that $p(\lambda)\leq cl(\mu)$. Because (M,τ) is a fuzzy ω-regular space, by Lemma 2.12, there is η fuzzy ω-open in M_b, $b\in B$ so that $m\in\eta\leq cl^{\omega}(\eta)\leq\lambda$. Therefore, $p(cl^{\omega}(\eta))\leq cl(\mu)$. Then, p is f. θ-ω-continuous. Then (M,τ) is FWF θ-ω-top. sp.

Corollary 2.13. Let (M,τ) be an fuzzy ω-regular space. The FWF topological space (M,τ) over (B,σ) is FWF almost weakly ω-top. sp. if and only if it is FWF θ-ω-top. sp.

Theorem 2.14. Assume that $\phi:(M,\tau)\rightarrow(N,\Lambda)$ is a f. ω-continuous fibrewise surjection function, when (M,τ) so well (N,Λ) are FWF topological spaces on (B,σ). If (N,Λ) is a FWF almost weakly ω-top. sp., then (M,τ) is so.

Proof. Assume that $m\in M_b$, $b\in B$ and λ be a fuzzy open set containing $p_M(m)$ in B, since p_N is f. almost weakly ω-continuous, there exists is a fuzzy open set μ containing $\phi(m)$ in N_b, $b\in B$ such that $p_N(\mu)\leq cl(\lambda)$. Since ϕ is f. ω-continuous, then for each $m\in M_b$, $b\in B$ and each fuzzy open set μ of $\phi(m)=n\in N_b$ in N, there exists a f. ω-open η of m in M_b, $b\in B$ such that $\phi(\eta)\leq\mu$. Thus, $p_N(\phi(\eta))\leq p_N(\mu)$. And, $p_M=(p_N\circ\phi)_\eta\leq p_N(\eta)$. Then, $p_M(p_N\circ\phi)_\eta\leq cl(\lambda)$. Thus, p_M f. almost weakly ω-continuous. Hence, (M,τ) is FWF almost weakly ω-top. sp.

Theorem 2.15. Let (M,τ) be a fuzzy ω-regular space. The FWF topological space (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp., then it is FWF ω-top. sp.

Proof. Assume that (M,τ) is a FWF weakly θ-ω-top. sp. over (B, σ), then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. weakly θ-ω-continuous. It suffices to demonstrate that p is f. ω-continuous. Let $m\in M_b$; $b\in B$ and, μ be a fuzzy open set containing $p(m)\in B$. Where M is a f. ω-regular space there is a fuzzy open set $\mu_1\in M_b$ so that $p(m)\in\mu_1$. And, $cl(\mu_1)\leq\mu$ where p is f. weakly θ-ω-continuous, there is an f. ω-open set λ containing m such that $p(\lambda)\leq cl(\mu_1)$. It follows that, $p(\lambda)\leq\mu$. Therefore, p is f. ω-continuous. Thus, (M,τ) is FWF ω-top. sp.

Corollary 2.16. Assume that (M,τ) is a fuzzy ω-regular space. The FWF topological space (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp. if and only if it is FWF ω-top. sp.

Theorem 2.17. The FWF topological space (M,τ) over (B,σ) is fuzzy ω-regular space. If (M,τ) is FWF θ-ω-top. sp., then it is FWF almost ω-top. sp.

Proof. Assume that (M,τ) is a FWF θ-ω-top. sp. over (B, σ), then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. θ-ω-continuous. It suffices to demonstrate that p is f. almost ω-continuous. Let $m\in M_b$; $b\in B$ and, μ be a fuzzy open set containing $p(m)$ in B. Because p is θ-ω-continuous, there exists is a f. ω-open set η containing m such that $p(cl^{\omega}(\eta))\leq cl(\mu)$. Because $int(cl(\mu)\leq cl(\mu)$, then $p(cl^{\omega}(\eta))\leq int(cl(\mu))\leq cl(\mu)$, then $p(cl^{\omega}(\eta))\leq cl(\mu)$. Also (M,τ) is f. ω-regular space, there exists is a f. ω-open set η_1 in M_b such that $m<\eta_1$. Also, $cl(\eta_1)\leq\eta$. Thus, $p(cl^{\omega}(\eta_1))\leq p(\eta)$ and $int(cl(\mu)\leq cl(\mu)$. It follows, $p(\eta)\leq int(cl(\mu))$. So, p is f.

almost ω-continuous. Thus (M,τ) is FWF almost ω-top. sp.

Corollary 2.18. The FWF topological space (M,τ) over (B,σ) is fuzzy ω-regular space. Then (M,τ) is FWF θ-ω-top. sp. if and if it is FWF almost ω-top. sp.

Theorem 2.19. Assume that (B,σ) is a fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost weakly ω-top. sp., then it is FWF ω-top. sp.

Proof. Assume that (M,τ) is a FWF almost weakly ω-top. sp. over (B,σ), then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. almost weakly ω-continuous. It suffices to demonstrate that p is f. ω-continuous. Let $m\in M_b$; $b\in B$ and, μ be a fuzzy open set containing $p\,(m)\in B$. Where B is a f. ω-regular space there is a fuzzy open set μ_1 in B so that $p\,(m)\in\mu_1$. So well, $cl(\mu_1)\leq\mu$ since p is f. almost weakly ω-continuous, there exists is a f. ω-open set λ containing m such that $p(\lambda)\leq cl(\mu_1)$. It follows that, $p(\lambda)\leq\mu$. Therefore, p is f. ω-continuous. Then (M,τ) is FWF ω-top. sp.

Corollary 2.20. Assume that (B,σ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost weakly ω-top. sp. if and only if it is FWF ω-top. sp.

Theorem 2.21. Assume that (M,τ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp., then it is FWF θ-ω-top. sp.

Proof. Assume that (M,τ) is a FWF weakly θ-ω-top. sp. over $(B,\ \sigma)$, then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. weakly θ-ω-continuous. It suffices to demonstrate that p is f. θ-ω-continuous. Assume that $m\in M_b$; $b\in B$ and, μ is an fuzzy open set containing $p\,(m)\in B$. Where M is an f. ω-regular space there is a fuzzy open set $\mu_1\in M_b$ so that $p\,(m)\in\mu_1$. And, $cl(\mu_1)\leq\mu$ where p is f. weakly θ-ω-continuous, there is f. ω-open set λ containing m such that $p(\lambda)\leq cl(\mu)$. It follows that, $p(cl^{\omega}(\lambda))\leq cl(\ \mu)$. Therefore, p is f. θ-ω-continuous. Thus, (M,τ) is FWF θ-ω-top. sp.

Corollary 2.22. Assume that (M,τ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp. if and only if it is FWF θ-ω-top. sp.

Theorem 2.23. Assume that (B,σ) is a fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost ω-top. sp., then it is FWF ω-top. sp.

Proof. Assume that (M,τ) is a FWF almost ω-top. sp. over (B,σ), then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. almost ω-continuous. It suffices to demonstrate that p is f. ω-continuous. Assume that $m\in M_b$; $b\in B$ and, μ is an fuzzy open set containing $p(m)\in B$. Since p is f. almost ω-continuous, there exists is an f. ω-open set λ contains m such that $p(\lambda)\leq int(\ cl(\mu))$. Because $int(cl(\mu))\leq cl(\mu)$. Then $(\lambda)\leq int(cl(\mu))\ \leq cl(\mu)$. Thus, $p(\lambda)\leq cl(\mu)$, and B is a f. ω-regular space there exists is a f. ω-open set λ_1 in M_b such that $m\in\lambda_1$. And, $cl(\mu_1)\leq\mu$. Therefore, $p(\lambda)\leq cl(\mu_1)\leq\mu$. It follows that, $p(\lambda)\leq\mu$. Thus, p is f. ω-continuous. Then (M,τ) is FWF ω-top. sp.

corollary 2.24. Assume that (B,σ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF almost ω-top. sp. if and only if it is FWF ω-top. sp.

Theorem 2.25. Assume that (B,σ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp., then it is FWF almost ω-top. sp.

Proof. Assume that (M,τ) is a FWF weakly θ-ω-top. sp. over $(B,\ \sigma)$, then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. weakly θ-ω-continuous. It suffices to demonstrate that p is f. almost ω-continuous. Let $m\in M_b$; $b\in B$ and, μ be a fuzzy open set containing $p\,(m)\in B$. Where B is f. ω-regular space, there is a fuzzy open set $\mu_1\in B$ so that $\lambda(m)\in\mu_1$ and $cl(\mu_1)\leq\mu$. Because p is weakly θ-ω-continuous, there is a f. ω-open set λ contains m so that $p\,(\lambda)\leq cl(\mu_1)$. Where, $int(cl(\mu)\leq cl(\ \mu)$, then $p\,(\lambda)\leq int(cl(\mu))\leq cl(\mu)$, Therefore $p(\lambda)\leq int(cl(\mu))$. So, p is f. almost ω-continuous on M. Then (M,τ) is FWF almost ω-top. sp.

Corollary 2.26. Assume that (B,σ) is an fuzzy ω-regular space. For a FWF topological space (M,τ) over (B,σ) is FWF weakly θ-ω-top. sp. if and only if it is FWF almost ω-top. sp.

Theorem 2.27. The FWF topological space (M,τ) over (B,σ) is FWF θ-ω-top. sp. Iff the graph fuzzy mapping $g:(M,\ \tau)\rightarrow(M,\ \tau)\times\ (B,\ \sigma)$, knowledge before $g(m)=(m\ \ ,\ p(m))$, for every $m\in M$ is a f. θ-ω-continuous

Proof. Necessity. Let g be an f. θ-ω-continuous. It suffices to demonstrate that (M,τ) is a FWF θ-ω-top. sp. over (B,σ), i.e. the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ is f. θ-ω-continuous. Let $m\in M_b$, $b\in B$ and λ be a fuzzy open set containment $p\,(m)$. Thus, $M\times\lambda$ is an fuzzy open set of $M\times B$ containing $g(m)$. Because g is θ-ω-continuous, there is f. ω-open set η contains m so that $g\,(cl^{\omega}(\eta))\leq cl(M\times\lambda)=M\times\ cl(\lambda)$. Therefore, $p(cl^{\omega}(\eta))\subseteq cl(\lambda)$. Then, p is f. θ-ω-continuous. Then, (M,τ) is FWF θ-ω-top. sp.

Sufficiency. Assume that (M,τ) is a FWF θ-ω-top. sp. over $(B,\ \sigma)$, then the proj. $p:(M,\tau)\rightarrow(B,\sigma)$ f. θ-ω-continuous. It suffices to demonstrate that g is f. θ-ω-

continuous. Let $m \in M_b$; $b \in B$ and μ be a fuzzy open set of $M \times B$ containing $g\,(m)$, there exists a fuzzy open sets $\eta_1 \leq M$. And, $\lambda \leq B$ such that $g\,(m) = (m, p(m)) < \eta_1 \times \lambda \leq \mu$. Because p is f. θ-ω-continuous, there is f. ω-open η_2 so that $p(cl^{\omega}(\eta_2)) \leq cl(\mu)$. Assume $\eta = \eta_1 \wedge \eta_2$. Then, η is f. ω-open in M. Therefore, $g(cl^{\omega}(\eta)) \leq cl(\eta_1) \times \; p(cl^{\omega}(\eta_2)) \leq cl(\eta_1) \times cl(\lambda) \leq cl(\mu)$. Then , g is θ-ω-continuous.

Theorem 2.28. Assume that (M,τ) is a FWF topological space over (B,σ) so well (B,σ) is a fuzzy ω-regular space. The following properties are equivalent:

(a) FWF weakly θ-ω-top. sp.
(b) FWF ω-top. sp.
(c) FWF almost ω-top. sp.
(d) FWF θ-ω-top. sp.
(e) FWF almost ω-top. sp.

proof. The proof follows directory from by Theorems 2.15, 2.6, 2.17, and 2.25.

Remark 2.29. The relation between FWF weakly ω-top. sp.is given by the following figure (see Fig. 2):

3. Strongly fibrewise fuzzy θ-ω-topological spaces

In this part, we study the strongly fibrewise θ-ω-continuous topological spaces and some theorems concerning them.

Definition 3.1. A function $\phi : (M,\tau) \to (N,\Lambda)$ is called fuzzy almost strongly ω-continuous (shortly., f. almost strongly ω-continuous) when if every $m \in M$ so well all fuzzy open set μ in B contains $\phi(m)$, there is a fuzzy ω-open subset λ so that $\phi(cl(\lambda)) \leq int(cl(\mu))$.

Definition 3.2. A function $\phi : (M,\tau) \to (N,\Lambda)$ is named fuzzy strongly θ-ω-continuous (briefly f. strongly θ-ω-continuous) when if every $m \in M$ so well all fuzzy open set μ in B contains $\phi(m)$, there is a fuzzy ω-open subset λ such that $\phi(cl^{\omega}(\lambda)) \leq \mu$.

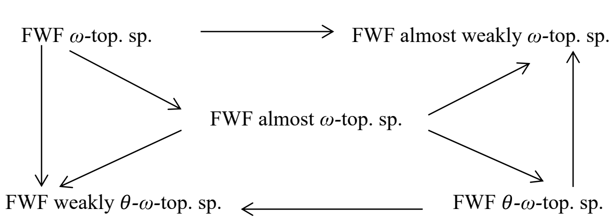

Fig. 2. The relation between FWF weakly w-top. sp.

Definition 3.3. The FWF topological space (M,τ) over (B,σ) is named a FWF strongly θ-ω-top. sp. (resp., FWF almost strongly ω-top. sp.) if the proj. function p is f. strongly θ-ω-continuous mapping (resp., f. almost strongly ω-continuous) mapping.

The converses does not hold as we show by next examples:

Example 3.4. Assume $M = \{a,b,c\}$, $B = \{x,y,z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a,0.3),(b,0),(c,0.5)\}$
$\mu_2 = \{(a,0.7),(b,0.7),(c,0.5)\}$
So well assume that $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x,0.6),(y,0.7),(z,0.5)\}$ is the fuzzy topologies on set M and B respectively and let the projection function $p : (M,\tau) \to (B,\sigma)$ be the fuzzy function as $(a) = p(b) = y$, $p(c) = z$. Let $\eta = \{(a,0.5),(b,0.5),(c,0.5)\}$ fuzzy ω-open of M so well $\nu = \{(a,0.6),(b,0.7),(c,0.5)\}$ is fuzzy open of B. Thus, $p(\eta) \leq (\nu)$ but $p(cl^{\omega}(\eta)) \nleqslant (\nu)$. Then, (M,τ) is FWF ω-top. sp. but not FWF strongly θ-ω-top. sp.

Theorem 3.5. Assume that (B,σ) is a fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp., then it is FWF strongly θ-ω-top. sp.

Proof. Assume that (M,τ) is a FWF ω-top. sp. over (B, σ), then the proj. $p : (M,\tau) \to (B,\sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ and, λ is a fuzzy open set containing $p\,(m) \in B$. Since B is a fuzzy regular space there is a fuzzy open set μ, such that $p(m) \in \mu \leq cl(\mu) \leq \lambda$ since p is f. ω-continuous. Thus, M_{μ} is a f. ω-open set so well, $M_{cl(\mu)}$ is a f. ω-closed. Assume $\xi = M_{\mu}$. Then, $m \in M_{\mu} \leq M_{cl(\mu)}$, ξ is a f. ω-open. Also $cl^{\omega}(\xi) \leq M_{cl(\mu)}$, we have $p(cl^{\omega}(\xi)) \leq cl(\mu) \leq \lambda$. Therefore, p is f. strongly θ-ω-continuous. Then, (M,τ) is FWF strongly θ-ω-top. sp.

Corollary 3.6. Assume that (B,σ) is a fuzzy regular space. For a FWF topological space (M,τ) over (B,σ) is FWF ω-top. sp. if and only if it is FWF strongly θ-ω-top. sp.

Example 3.7. Let $M = \{a,b,c\}$, $B = \{x,y,z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a,0.1),(b,0.2),(c,0.5)\}$
$\mu_2 = \{(a,0.4),(b,0.3),(c,0.5)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x,0.3),(y,0.4),(z,0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M,\tau) \to (B,\sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Let $\eta = \{(a,0.2),(b,0.1),(c,0.3)\}$

fuzzy ω-open of M so well $\nu = \{(x, 0,3), (y, 0.4), (z, 0.5)\}$ is fuzzy open of B. Thus, $p(\eta) \leq (\nu)$ but $p(cl(\eta)) \nleq int(cl(\nu))$. Then, (M, τ) is FWF ω-top. sp. but not FWF almost strongly ω-top. sp.

Theorem 3.8. Assume that (M, τ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF ω-top. sp., then it is FWF almost strongly ω-top. sp.

Proof. Assume that (M, τ) is a FWF ω-top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. almost strongly ω-continuous. Assume that $m \in M_b$; $b \in B$ and, λ is a fuzzy open set contains $p\ (m)$ in B. Since p is f. ω-continuous, there is a f. ω-open set μ contains m in M so that $p(\mu) \leq \lambda$. And, $\lambda \leq cl(\lambda)$. Thus, $p(\mu) \leq cl(\lambda)$. Since M is f. ω-regular, there is a f. ω-open set $\mu_1 \in M$ so that $m \in \mu_1$ and, $cl(\mu_1) \leq \mu$. Thus, $p(cl(\mu_1)) \leq p(\mu)$. And, $p(\mu) \leq cl(\lambda)$ then, $int(cl((\lambda)) \leq cl(\lambda)$. It follows that, $(cl(\mu_1)) \leq int(cl(\lambda))$. Therefore, p is f. almost strongly ω-continuous. Thus, (M, τ) is FWF almost strongly ω-top. sp.

Corollary 3.9. Assume that (M, τ) is a fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF ω-top. sp. if and only if it is FWF almost strongly ω-top. sp.

Example 3.10. Let $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a, 0.3), (b, 0, 4), (c, 0.5)\}$
$\mu_2 = \{(a, 0.2), (b, 0.2), (c, 0.5)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.2), (y, 0.2), (z, 0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Then, (M, τ) is FWF θ-ω-top. sp. but not FWF strongly θ-ω-top. sp.

Theorem 3.11. Assume that (B, σ) is a fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF θ-ω-top. sp., then it is FWF strongly θ-ω-top. sp.

Proof. Assume that (M, τ) is a FWF θ-ω-top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. θ-ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set contains $p\ (m)$ in B. Because p is f. θ-ω-continuous, there is a f. ω-open set μ contains m in M so that $p(cl^{\omega}(\mu)) \leq cl(\lambda)$ since B is f. regular, there exists is a fuzzy open set $\aleph$ such that $(m) \in \aleph \leq cl(\aleph) \leq \lambda$. Then, $(cl^{\omega}(\mu)) \leq cl(\aleph) \leq \lambda$. Therefore, $(cl(\mu)) \leq \lambda$. Thus, p is f. strongly θ-ω-continuous. Then (M, τ) is FWF strongly θ-ω-top. sp.

Corollary 3.12. Let (B, σ) be a fuzzy regular space. The FWF topological space (M, τ) over (B, σ) is FWF θ-ω-top. sp. if and only if it is FWF strongly θ-ω-top. sp.

Example 3.13. Let $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a, 0.3), (b, 0, 4), (c, 0.5)\}$
$\mu_2 = \{(a, 0.3), (b, 0.3), (c, 0.5)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.3), (y, 0.3), (z, 0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Then, (M, τ) is FWF θ-ω-top. sp. but not FWF almost strongly ω-top. sp.

Theorem 3.14. Assume that (B, σ) is a fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF θ-ω-top. sp., then it is FWF almost strongly ω-top. sp.

Proof. Assume that (M, τ) is a FWF θ-ω-top. sp. over (B, σ), then the projection $p : (M, \tau) \rightarrow (B, \sigma)$ f. θ-ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ and, λ is a fuzzy open set contains $p\ (m) \in B$. Since p is f. θ-ω-continuous, there exists is an f. ω-open set μ contains $m \in M$ such that $p(cl^{\omega}(\mu)) \leq cl(\lambda)$. Since B is f. ω-regular, there exists is a fuzzy open set λ_1 in B such that $p(m) \in \lambda_1$ so well $cl(\lambda_1) \leq \lambda$. Thus, $(cl(\lambda_1)) \leq cl(\lambda_1)$. It follows that, $p(cl(\mu)) \leq int(cl(\lambda_1))$. Then, p is f. almost strongly ω-continuous. Then (M, τ) is FWF strongly θ-ω-top. sp.

Example 3.15. Let $M = \{a, b\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2, \mu_3\}$ where
$\mu_1 = \{(a, 0.6), (b, 0, 7)\}$
$\mu_2 = \{(a, 1), (b, 0.9)\}$
$\mu_2 = \{(a, 0.2), (b, 0.3)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.2), (y, 0.3)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$, let $\eta = \{(a, 0.5), (b, 0.5)\}$ fuzzy ω-open in M. Then, (M, τ) is FWF almost ω-top. sp. but not FWF almost strongly ω-top. sp.

Theorem 3.16. Assume that (M, τ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp., then it is FWF almost strongly ω-top. sp.

Proof. Assume that (M, τ) is a FWF almost ω-top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f.

almost ω-continuous. It suffices to demonstrate that p is f. almost strongly ω-continuous. Assume that $m \in M_b$; $b \in B$ so well, λ is a fuzzy open set containing $p\ (m) \in B$. Since p is f. almost ω-continuous. There is a f. ω-open set μ containing m of M so that $p(\mu) \leq int(cl(\lambda))$. Since M is fuzzy ω-regular. There is a f. ω-open set $\mu_1 \in M$ so that $m \in \mu_1$ so well, $cl(\mu_1) \leq \mu$. Thus, $(cl(\mu_1)) \leq p(\mu)$. where, $p(cl(\mu_1)) \leq p(\mu) \leq int(cl(\lambda))$. It follows that, $(cl(\mu_1)) \leq int(cl(\lambda))$. Therefore, p is f. almost strongly ω-continuous. Then (M, τ) is FWF almost strongly ω-top. sp.

Corollary 3.17. Assume that (M, τ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp. if and only if it is FWF almost strongly ω-top. sp.

Lemma 3.18. Assume that $\phi : (M, \tau) \rightarrow (N, \Lambda)$ is a f. strongly θ-ω-continuous fibrewise surjection function, since (M, τ) so well (N, Λ) are FWF topological spaces over (B, σ). Just as (N, Λ) is a FWF top. sp., so (M, τ) is FWF strongly θ-ω-top. sp.

Theorem 3.19. The FWF topological space (M, τ) over (B, σ) is FWF strongly θ-ω-top. sp. and (M, τ) is a fuzzy ω-regular iff the graph fuzzy mapping $g : (M, \tau) \rightarrow (M, \tau) \times (B, \sigma)$, knowledge before $g(m) = (m, p(m))$, for all $m \in M$ is a f. strongly θ-ω-continuous.

Proof. By Lemma 3.17. Then, (M, τ) is FWF strongly θ-ω-top. sp. if the graph mapping g is f. strongly θ-ω-continuous. It follows that, M is fuzzy regular. To prove conversely. Assume that (M, τ) is a FWF strongly θ-ω-top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. strongly θ-ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $g\ (m)$ in $M \times B$, there exists fuzzy open sets μ_1 in M also ν in B such that $g\ (m) = (m, p(g)) \in \mu_1 \times \nu \leq \lambda$. Because p is f. strongly θ-ω-continuous, there is μ_2 is f. ω-open so that $p(cl^{\omega}(\mu_2)) \leq \lambda$. Because M is a f. ω-regular and, $\mu_1 \wedge \mu_2$ is f. ω-open, there is μ f. ω-open such that $m \in \mu \leq cl^{\omega}(\mu) \leq \mu_1 \wedge \mu_2$ by Lemma 2.12. Therefore, $g(cl^{\omega}(\mu)) \leq \mu_1 \times p\ (cl^{\omega}(\mu_2)) \leq \mu_1 \times \nu \leq \lambda$. Then, g is f. strongly θ-ω-continuous.

Example 3.20. In Example 3.14. Then, (M, τ) is FWF almost ω-top. sp. but not FWF strongly θ-ω-top. sp.

Theorem 3.21. Assume that (M, τ) is a fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp., then it is FWF strongly θ-ω-top. sp.

Proof. Assume that (M, τ) is a FWF almost ω-top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. almost ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m) \in B$. where p is f. almost ω-continuous. There is a f. ω-open set μ containing $m \in M$ so that $p(\mu) \leq int(cl(\lambda))$. Where M is fuzzy ω-regular. There is a f. ω-open set $\mu_1 \in M$ such that $m \in \mu_1$ so well, $cl(\mu_1) \leq \mu$. Thus, $(cl(\mu_1)) \leq p(\mu)$. Then, $int(cl(\lambda)) \leq cl(\lambda))$. It follows that, $p(cl(\mu_1)) \leq \lambda$. Therefore, p is f. strongly θ-ω-continuous. Then (M, τ) is FWF strongly θ-ω-top. sp.

Corollary 3.22. Assume that (M, τ) is an fuzzy ω-regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp., if and only if it is FWF strongly θ-ω- top. sp.

Theorem 3.23. Assume that (M, τ) is an FWF topological space over (B, σ) so well (B, σ) is a fuzzy ω-regular space. The following properties are equivalent:

(a) FWF almost strongly θ-ω-top. sp.
(b) FWF ω-top. sp.
(c) FWF almost ω-top. sp.
(d) FWF θ-ω-top. sp.

proof. The proof follows directory from by Theorems 3.4, 3.6, 3.10, 3.12 and 3.13.

Remark 3.24. The relation between FWF strongly ω-top. sp. is given by the following figure (see Fig. 3):

4. Relationship between weak and strong forms of fibrewise fuzzy ω-topological spaces

In this section, we study the relation between FWF weakly θ-ω-top. sp. and FWF strongly θ-ω-top. sp. and the some theorems concerning them.

Definition 4.1. A mapping $\phi : (M, \tau) \rightarrow (N, \Lambda)$ are said to be fuzzy almost weakly (resp., fuzzy almost strongly) continuous (briefly, f. almost weakly and f. almost strongly) continuous if for each $m \in M$ and each fuzzy open neighborhood (resp., fuzzy open set) λ of N containing $\phi(m)$, there exists a f. ω-open

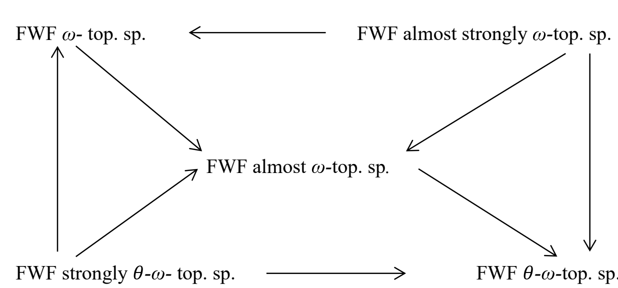

Fig. 3. The relation between FWF strongly w-top. sp.

neighborhood (resp., f. ω-open set) μ of M so that $\phi\ (int\ (cl(\mu)) \leq \lambda$ (resp., $\phi(\mu) \leq cl(\lambda)$, $\phi(cl(\mu)) \leq \lambda$.

Definition 4.2. A mapping $\phi : (M, \tau) \rightarrow (N, \Lambda)$ is said to be f. super (resp., f. weakly, f. strongly) ω-continuous if for each $m \in M$ and each fuzzy open (resp., fuzzy regular open) set λ of N containing $\phi(m)$, there is a fuzzy open set μ of M so that $\phi\ (\mu) \leq cl(\lambda)$ (resp., $\phi(cl(\mu)) \leq \lambda$).

Definition 4.3. A mapping $\phi : (M, \tau) \rightarrow (N, \Lambda)$ is called fuzzy weakly θ-continuous (briefly, f. weakly θ-continuous) if for each $m \in M$ and each fuzzy open λ of B containing $p(m)$, there exists a fuzzy open set μ of M such that $p\ (\mu) \leq cl(\lambda)$.

Definition 4.4. The FWF topological space (M, τ) over (B, σ) is named a FWF super ω-top. sp. (resp., FWF weakly ω-top. sp., FWF strongly ω-top. sp., FWF almost strongly ω-top. sp., FWF almost weakly ω-top. sp., FWF weakly θ-top. sp.) if the projection function p is fuzzy super ω-continuous mapping (resp., f. weakly ω-continuous, f. strongly ω-continuous, f. almost strongly ω-continuous, f. almost weakly ω-continuous, f. weakly θ-continuous) mapping.

The relation between FWF weakly and FWF strongly ω-top. sp. given by the following figure (see Fig. 4). The following examples show that these implications are not reversible:

Example 4.5. Assume $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2, \mu_3\}$ where
$\mu_1 = \{(a, 0.3), (b, 0, 4), (c, 0.5)\}$
$\mu_2 = \{(a, 0.2), (b, 0.2), (c, 0.5)\}$
$\mu_3 = \{(a, 0.5), (b, 0.6), (c, 0.5)\}$
So well assume that $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.5), (y, 0.6), (z, 0.5)\}$ is the fuzzy topologies on set M and B respectively and let the projection function $p : (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$, $p(c) = z$. let $\eta = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$ fuzzy ω-open in M. Then, (M, τ) is FWF super ω-top. sp. but not FWF strongly θ-ω-top. sp.

Theorem 4.6. Assume that (M, τ) is a fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF super ω-top. sp., then it is FWF strongly θ-ω-top. sp.

Proof. Assume that (M, τ) is a FWF super ω-top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. super

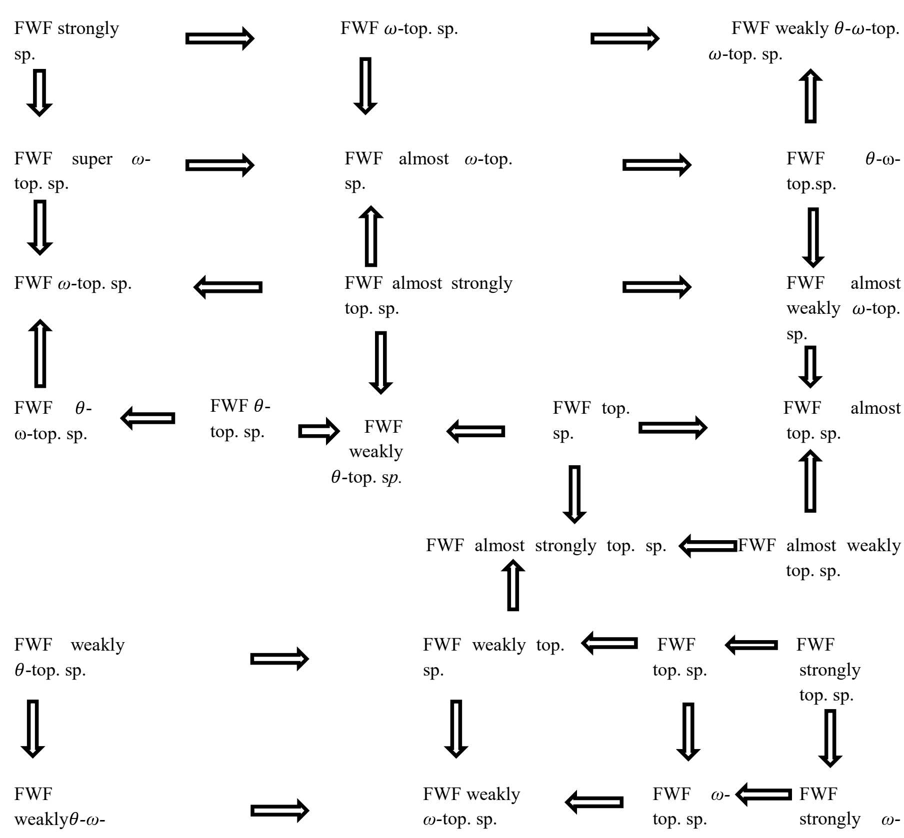

Fig. 4. The relation between FWF weakly and FWF strongly w-top. sp.

ω-continuous. It suffices to demonstrate that p is f. strongly θ-ω-continuous. Assume that $m \in M_b$; $b \in B$ so well, λ is a fuzzy open set containing $p\ (m) \in B$. Because of p is a f. super ω-continuous, there exists is a fuzzy regular open set μ containing m, such that $p(\mu) \leq \lambda$. Because $int(cl(\lambda)) \leq cl(\lambda)$, then $p(\mu) \leq int(cl(\lambda)) \leq cl(\lambda)$. Then, $p\ (\mu) \leq cl(\lambda)$. And, M is a fuzzy regular space, there is an fuzzy open set ν so that $m \in \nu \leq cl(\nu) \leq \mu$. since, $p(cl(\nu)) \leq \lambda$. Therefore, p is f. strongly θ-ω-continuous. Then (M, τ) is FWF strongly θ-ω-top. sp.

Corollary 4.7. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF super ω-top. sp. if and only if it is FWF strongly θ-ω-top. sp.

Example 4.8. Let $M = \{a, b\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a, 0.7), (b, 0, 6)\}$
$\mu_2 = \{(a, 0.7), (b, 0.9)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.7), (y, 0.6)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$, let $\eta = \{(a, 0.5), (b, 0.5)\}$ fuzzy ω-open in M. Then, (M, τ) is FWF ω-top. sp. but not FWF super ω-top. sp.

Theorem 4.9. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF ω-top. sp., then it is FWF super ω-top. sp.

Proof. Assume that (M, τ) is a FWF ω-top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. ω-continuous. It suffices to demonstrate that p is f. super ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m)$ in B. Because of p is a f. ω-continuous, there is a fuzzy ω-open set μ contains m, so that $p(\mu) \leq \lambda$, also $int(\ cl(\mu)) \leq cl(\mu)$. Then, $p(int(\ cl(\mu)) \leq p\ (cl(\mu))$. And, M is a fuzzy regular space. There is an fuzzy open set μ_1 such that $\in \mu_1 \leq cl(\mu_1) \leq \mu$. Thus, $p(int(cl(\mu)) \leq p\ (cl(\mu_1))$ so well, $p(\mu) \leq \lambda$. Then, $p(int(cl(\mu)) \leq \lambda$. It follow that, p is f. super ω-continuous. Then (M, τ) is FWF super ω-top. sp.

Corollary 4.10. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF ω-top. sp. if and only if it is FWF super ω-top. sp.

Example 4.11. For an fuzzy topological space $(M, \tau) = (B, \sigma)$ Let $\sigma = \tau = \{\overline{0}, \overline{1}, \mu : \frac{1}{3} \leq \mu(m) \leq \frac{2}{3}$, for some fixed element m of M and $\mu(m) = 0$, otherwise$\}$. Assume that (M, τ) is a FWF topological space over (B, σ) also assume that the projection function $p : (M, \tau) \rightarrow (B, \sigma)$ is the fuzzy function as the identity maps. Then, (M, τ) is FWF top. sp. but not FWF strongly top. sp.

Theorem 4.12. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF top. sp. , then it is FWF strongly top. sp.

Proof. Assume that (M, τ) is a FWF top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. continuous. It suffices to demonstrate that p is f. strongly continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m)$ in B. Because of p is a f. continuous, there is a fuzzy open set μ contains m, so that $p(\mu) \leq \lambda$, where M is fuzzy regular space, there is a fuzzy open set $\mu_1 \in M$ such that $m \in \mu_1$ also, $cl(\mu_1) \leq \mu$. Thus, $p(cl(\mu_1)) \leq p(\mu)$. Then, $p(cl(\mu_1)) \leq \lambda$. Therefore, p is f. strongly continuous. Then (M, τ) is FWF strongly compact.

Corollary 4.13. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF ω-top. sp., if and only if it is FWF strongly compact.

Theorem 4.14. Let (B, σ) be a fuzzy regular space. The FWF topological space (M, τ) over (B, σ) is FWF weakly top. sp., then it is FWF top. sp.

Proof. Assume that (M, τ) is a FWF weakly top. sp. over (B, σ), then the proj. $p : (M, \tau) \rightarrow (B, \sigma)$ f. weakly continuous. It suffices to demonstrate that p is f. continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p\ (m) \in B$. Where B is fuzzy regular, there is a fuzzy open set $\lambda_1 \in B$ so that $p(m) \in \lambda_1$ also, $cl(\lambda_1) \leq \lambda$. Because p is weakly continuous, there exists is a fuzzy open set μ containing m in M so that $p(\mu) \leq cl(\lambda_1)$. Thus, $p(\mu) \leq \lambda$. It follows that, p is f. continuous. Then, (M, τ) is FWF top. sp.

Corollary 4.15. Assume that (B, σ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF weakly top. sp. if and only if it is FWF top. sp.

Example 4.16. Let $= \{a, b\}$, $B = \{x, y\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2, \mu_3\}$ where
$\mu_1 = \{(a, 0.60), (b, 0.60)\}$
$\mu_2 = \{(a, 1), (b, 0.9)\}$
$\mu_3 = \{(a, 0.\ 11), (b, 0.31)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.11), (y, 0.31)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M, \tau) \rightarrow (B, \sigma)$ be the fuzzy function as $p(a) = x$, $p(b) = y$. Let $\eta = \{(a, 0.7), (b, 0.4)\}$ fuzzy ω-open in M also $\nu = \{(a, 0.11), (b, 0.31)\}$ be an fuzzy open of B. Thus, $p(\eta) \leq int(cl(\nu))$ but $(int\ \ cl(\eta)) \nleq \nu$. Then, (M, τ) is FWF almost ω-top. sp. but not FWF super ω-top. sp.

Definition 4.17. [5] A fuzzy topological space (M, τ) is called a fuzzy semi-regular space iff the collection of all fuzzy regular open sets of M forms a base for fuzzy topology τ.

Theorem 4.18. assume that (M, τ) and (B, σ) are an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp., then it is FWF super ω-top. sp.

Proof. Assume that (M, τ) is a FWF almost ω-top. sp. over (B, σ), then the projection $p : (M, \tau) \to (B, \sigma)$ f. almost ω-continuous. It suffices to demonstrate that p is f. super ω-continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p(m)$ in B. Because of p is f. almost ω-continuous, there exists is a f. ω-open set μ containing m. For each fuzzy regular open set λ of B contains $p(m)$ so that $p(\mu) \leq \lambda$. Thus, $(\mu) \leq (int(cl(\lambda))$. Because the space M is fuzzy semi-regular space, There exists is a fuzzy open set $\mu_1 \in M$ so that $m \in \mu_1$ also, $\lambda \leq int(cl(\lambda)) \leq \mu$. Thus, $(\lambda) \leq p(int(\ cl(\lambda))) \leq p(\mu)$. Also, $p(\mu) \leq int(cl(\ \mu))$. Thus, $(int(cl(\lambda))) \leq p(\mu) \leq int(cl(\lambda))$. So well, the space B is fuzzy semi-regular space, there exists is a fuzzy open set λ_1 in B such that $p(m) \in \lambda_1$ then, $\mu \leq int(cl(\mu)) \leq \lambda$. Thus, $p(\mu) \leq p(int(cl(\mu)))$. It follows that, $p(int(cl(\mu))) \leq \lambda$. Then, p is f. super ω-continuous. Hence (M, τ) is FWF super ω-top. sp.

corollary 4.19. Let (M, τ) and (B, σ) be a fuzzy regular space. The FWF topological space (M, τ) over (B, σ) is FWF almost ω-top. sp. if and only if it is FWF super ω-top. sp.

Example 4.20. Let $M = \{a, b, c\}$, $B = \{x, y, z\}$, $\tau = \{\overline{0}, \overline{1}, \mu_1, \mu_2\}$ where
$\mu_1 = \{(a, 0.1), (b, 0.2), (c, 0.5)\}$
$\mu_2 = \{(a, 0.4), (b, 0.3), (c, 0.5)\}$
And let $\sigma = \{\overline{0}, \overline{1}, \lambda\}$, where $\lambda = \{(x, 0.3), (y, 0.4), (z, 0.5)\}$ be the fuzzy topologies on set M and B respectively and let the projection function $p : (M, \tau) \to (B, \sigma)$ be the fuzzy function as $p(a) = y$, $p(b) = x$, $p(c) = z$. Let $\nu = \{(x, 0, 3), (y, 0.4), (z, 0.5)\}$ is fuzzy open in B. Then, $p(\mu_1) \leq cl(\nu)$ but $p(cl(\mu_1)) \nleqslant int(cl(\nu))$. Then, (M, τ) is FWF almost weakly top. sp. but not FWF almost strongly top. sp.

Theorem 4.21. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost weakly ω-top. sp., then it is FWF almost strongly top. sp.

Proof. Assume that (M, τ) is a FWF almost weakly top. sp. over (B, σ), then the proj. $p : (M, \tau) \to (B, \sigma)$ f. almost weakly continuous. It suffices to demonstrate that p is f. almost strongly continuous. Let $m \in M_b$; $b \in B$ and, λ be a fuzzy open set containing $p(m)$ in B. Because of p is f. almost weakly continuous, $m \in M_b$, $b \in B$ for each open set λ of B containing $p(m)$ there is a fuzzy open set μ contains m so that $p(\mu) \leq cl(\lambda)$. Because the space M is a fuzzy regular space, there is a fuzzy open set $\mu_1 \in M$ such that $m \in \mu_1$ also $cl(\mu_1) \leq \mu$, so $p(cl(\mu_1)) \leq p(\mu)$. Also, $p(\mu) \leq cl(\lambda)$. Then, $p(cl(\mu_1)) \leq cl(\lambda)$ also, $int(cl(\lambda_1)) \leq cl(\lambda_1)$. Then, $(cl(\lambda_1) \ ) \leq int(cl(\lambda_1))$. It follows that, p is f. almost strongly continuous. Hence (M, τ) is FWF almost strongly top. sp.

Corollary 4.22. Assume that (M, τ) is an fuzzy regular space. For a FWF topological space (M, τ) over (B, σ) is FWF almost weakly ω-top. sp. if and only if it is FWF almost strongly top. sp.

Theorem 4.23. Assume that (M, τ) is a FWF topological space over (B, σ) also (B, σ) is a fuzzy regular space. The following properties are equivalent:

(a) FWF strongly top. sp.
(b) FWF top. sp.
(c) FWF weakly top. sp.

proof. The proof follows directory from by Theorems 4.12, 2.16.

Definition 4.24. [4] Let M and B be an fuzzy spaces are called fuzzy homeomorphic denoted by $M \cong B$ if there exists a fuzzy homeomorphism on M to B.

Theorem 4.25. The FWF topological space (M, τ) over (B, σ) is FWF strongly top. sp. Also (M, τ) is a fuzzy regular, so the graph fuzzy function : $(M, \tau) \to (M, \tau) \times (\ B, \sigma)$, defined by $g(m) = (m, p(m))$, for each $m \in M$ is a f. strongly continuous.

Proof. Assume that (M, τ) is a FWF strongly top. sp. over (B, σ), then the proj. $p : (M, \tau) \to (B, \sigma)$ f. strongly continuous mapping. Let $m \in M_b$, $b \in B$ and μ be a fuzzy open set of $M \times B$ containing $p(m)$. There exists fuzzy open sets $\xi_1 \in I^M$ and $\lambda \in I^B$ so that $g(m) = (m, \ p(m)) < \xi_1 \times \ \lambda \leq \mu$. Where p is f. strongly continuous also, M is fuzzy regular space, there is an fuzzy open set ξ containing m in M so that $cl(\xi) \leq \xi_1$ also $p(cl(\xi)) \leq \lambda$. Therefore, $p(cl(\xi)) \leq \xi_1 \times \ \lambda \leq \mu$. Then, p is f. strongly continuous. Thus, the mapping $g = id_M \triangle p : (M, \tau) \to (M, \tau) \times (B, \sigma)$ maps fuzzy homeomorphically onto the graph $g(m)$ which is fuzzy closed subset of $M \times B$, so p is f. continuous and because M is an fuzzy regular, then $M \times B$ is fuzzy regular, by Theorem 4.24. Hence, $g : M \rightarrow M \times B$ is f. strongly continuous mapping.

Theorem 4.26. Assume that (M, τ) is a FWF topological space over (B, σ) also (B, σ) is a fuzzy regular space. The following properties are equivalent:

(a) FWF almost strongly θ-ω-top. sp.
(b) FWF ω-top. sp.
(c) FWF almost ω-top. sp.
(d) FWF θ-ω-top. sp.
(e) FWF almost weakly ω-top. sp.

proof. The proof follows directory from by Theorems 3.6, 2.15, 3.16.

Funding

Self-funding.

References

[1] Abdul Husein GH. Some types of fuzzy covering dimension on fuzzy topological spaces. Ph.D. Thesis. College of Education , Al-Mustansiriyah University; 2021.
[2] Al-khafaji MA. On separation axioms of fuzzy topological spaces. College of Education , Al-Mustansiriyah University; 2006. Ph.D. Thesis.
[3] Ashaea GS, Yousif YY. Some Type of Mapping in bitopological space baghdad. Sci J 2021;18(1):149−55.
[4] Ashaea GS, Yousif YY. Weakly and strongly forms of ω-perfect mappings. Iraqi J Sci 2020;(Special Issue):45−55.
[5] Azad KK. On fuzzy semicontinuity, Fuzzy almost continuity and fuzzy weakly continuity. J Math Anal Appl 1981;82:14−32.
[6] Chang CL. Fuzz topological spaces. J Math Anal Appl 1968; 24:182−90.
[7] Hussain MA, Yousif YY. Some Type of fibrewise Fuzzy topological spaces international. J Nonlinear Anal Appl 2021; (2):751−765, 12.
[8] James IM. Fibrewise topology. London: Cambridge University Press; 1989.
[9] Lou SP, Pan SH. Fuzzy structure. J Math Analy Appl 1980;76: 631−42.
[10] Ming PP, Mong LY. Fuzzy topology I. Neighborhood structure of a fuzzy point and moor-smith convergence. J Math Anal Appl 1980;76:571−99.
[11] Mukherjee MN, Sinha SP. On some near − fuzzy continuous function between fuzzy topological spaces. J Math Anal Appl 1990;34:245−54.
[12] Negoita CV, Ralescu DA. Application of fuzzy sets to systems analysis, Basel, Stuttgart. 1975.
[13] Park JH, Lee BY, Choi JR. Fuzzy θ-Connectedness. J Fuzzy Set Syst 1993;59:237−44.
[14] Wang X, Ruan D, Kerre EE. Mathematics of fuzziness − basic issues, vol. 245; 2009. p. 185−6.
[15] Yousif YY. Fibrewise fuzzy topological spaces ibn Al haitham. J Pure Appl Sci (Ankara) 2021;34(3).
[16] Yousif YY, Hussain MA. Fibrewise soft near separation axioms. In: The 23th science conference of college of education. Al-Mustansiriyah University; 26-27 April 2017. p. 400−14.
[17] Yousif YY, Khalil B. Feeble regular and feeble normal spaces in α-topological spaces using graph. Int J Nonlinear Anal 2021;12(2, 32):415−23.
[18] Zadeh LA. Fuzzy sets. Inf Control 1965;8:338−53.
[19] Zougdani H. Covering dimension of fuzzy spaces. Mat Vesn 1984;36:104−18.