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ORIGINAL STUDY

Study the Point Spread Function and the Line Spread Function for Hexagonal Aperture

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Abstract

In this research, hexagonal aperture has been studied which consists of two trapezoidal shapes The diffraction pattern in the exit pupil formed by this aperture was studied when the object was a point using the point spread function (PSF), and also when the object is an infinite line using the line spread function (LSF). This study was done for a diffraction-limited system, and with the presence of some aberrations.

The effects of adding focus error (first-order aberrations) and spherical aberration (second-order aberration) were studied, with values of aberration factor equal) 0.25 λ, 0.5 λ, 0.75 λ, and 1 λ), which resulted in a larger central intensity (Strehl ratio) and a reduce in FWHM of the curves.

Keywords: Point spread function, Line spread function, Focal error, Spherical aberrations, Hexagonal aperture

1. Introduction

As its name submits, the line spread function is obtained with an infinitesimal slit in an opaque object. Measuring the line-spread function can reduce the technical difficulties associated with obtaining and measuring the point-spread function. The line-spread function (LSF) is a one-dimensional representation of the two-dimensional point-spread function (PSF). Since we assume that the PSF of the imaging system is space invariant, we can measure the shape of the line spread function (LSF) with a microdensitomer passing through the image of the slit, perpendicular to the slit length [1]. The spread function depends on diffraction produced by the lens aperture and the amount and type of aberrations in an optical system [2].

Many studies on the hexagonal aperture have been adopted, in 2006, V. N. Mahajan and G.ming Dai showed how the orthonormal coefficients are related to the corresponding Zernike coefficients for a hexagonal pupil [3]. In the next year, in 2007, G.ming Dai and V. N. Mahajan demonstrated the determination of the orthonormal hexagonal polynomials as an example of the matrix approach [4]. In 2013, J. A. Díazand V. N. Mahajan obtained a closed-form analytical expression for the aberration-free point-spread function (PSF) of a system with a hexagonal pupil [5]. Then in 2018, S. Itoh et al. derived a point spread function of hexagonally segmented telescopes by a new symmetrical formulation and by introducing three variables on a pupil plane, the Fourier transform of pupil functions is derived by a three-dimensional Fourier transform [6]. While in 2013, L. K. Abood et.al. used the modified Zernike polynomials for hexagonal aperture to describe the wavefront aberrations and to predict the initial state for the adaptive optics corrections [7]. In 2021, A.A. Kharnoob and A. F. Hassan introduced hexagonal synthetic aperture in an optical system and obtained a high-resolution image [8].

In this work, the PSF and LSF were studied for hexagonal aperture by synthesizing it into two trapezoidal figures which made the work easier.

2. The equation of point spread function (PSF)

The point spread function is the irradiance in the image of a point source in an optical system [9]. This

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function may be obtained by the Fourier transform of the pupil function, where the complex amplitude for the point located at the coordinates (u, v) in the image plane was given by [10].

$$F(u,v)=\frac{1}{A}\int_y\int_x f(x,y)e^{2\pi i(ux+vy)}dxdy \tag{1}$$

Where (u) represents complex amplitude in point (u, v), (A) is the exit pupil area, and $f(x,y)$ is the pupil function that can be expressed as [11].

$$f(x,y)=\tau(x,y)e^{ikw(x,y)} \tag{2}$$

$\tau(x,y)$: the real amplitude distribution in exit pupil and it is called "pupil transparency" often equals one unit if the illumination is uniform.

$eikw(x,y)$ is the wavefront of the aberration function (x) is the aberration factor.

Then the point spread function is then given by the complex square of the amplitude in the image [12].

$$G(u,v)=|G(u,v)|^2 \tag{3}$$

$$PSF=G(u,v)=n.f\left|\int_y\int_x f(x,y)e^{2\pi i(ux+vy)}dxdy\right|^2 \tag{4}$$

Where $(n.f)$: a normalization constant.

Assume that $z=2\pi u$ and $2\pi v=m$ equation (4) becomes:

$$PSF=G(z,m)=n.f\left|\int_y\int_x f(x,y)e^{i(zx+my)}dxdy\right|^2 \tag{5}$$

The intensity distribution on the two axes z and m are not symmetric; unlike the circular and square aperture.

3. The equation of line spread function (LSF)

Line object is defined as some points sources positioned side by side on a line that make the line object [13].

To calculate the complex amplitude of the line object, so that [1,14].

$$\mathrm{L(u,v)}=\int_{\mathrm{v}}\mathrm{G(u,v)dv} \tag{6}$$

From eq. (1) and eq. (7) we can obtain:

$$L(u,v)=\int_v\left|\int_y\int_x f(x,y)e^{2\pi i(ux+vy)}dxdy\right|^2 dv \tag{7}$$

we can obtain: $A*A=|A|^2$ by using

$$L(u,v)=\int_v\int_y\int_x\int_{y_1}\int_{x_1} f(x,y)f^*(x_1,y_1).e^{2\pi i(ux+vy)} .e^{-2\pi i(ux_1+vy_1)}dx_1dy_1dxdydv \tag{8}$$

$$L(u,v)=\int_y\int_x\int_{y_1}\int_{x_1} f(x,y)f^*(x_1,y_1).\mathrm{e}^{2\pi iux}.\mathrm{e}^{-2\pi iux_1} dx_1dy_1dxdy\int_v e^{2\pi iv(y-y_1)}dv \tag{9}$$

From (Dirac-delta function) [15]

$$\int_v e^{2\pi iv(y-y_1)}dv=\delta(y-y_1) \tag{10}$$

eq. (10) becomes

$$L(u)=\int_y\int_x\int_{x_1} f(x,y).\mathrm{e}^{2\pi iux}.\mathrm{e}^{2\pi iux_1}dx_1dxdy \int_{y_1} f^*(x_1,y_1)\delta(y-y_1)dy_1 \tag{11}$$

$$L(u)=\int_y\int_x\int_{x_1} f(x,y)f^*(x_1,y).\mathrm{e}^{2\pi iux}.\mathrm{e}^{2\pi iux_1}dx_1dxdy \tag{12}$$

Assume that $z=2\pi u$, equation (9) becomes

$$L(z)=\int_y\left[\int_x f(x,y).\mathrm{e}^{izx}dx\int_{x_1} f^*(x_1,y).\mathrm{e}^{izx_1}\mathrm{d}x_1\right]dy \tag{13}$$

normalized Line spread function will take the form: Then

$$LSF=L(z)=N\int_y\left|\int_x f(x,y).\mathrm{e}^{izx}dx\right|^2 dy \tag{14}$$

Where) N (: a normalization constant.

4. The aberration function

The aberration function appeared in eq. (2) could be any type of aberration, or may be more than one type. In this study, focus error and spherical aberration were taken to study their effect on point and line spread functions. The aberration equation can be defined as [16].

$$W(x,y) = W_{020}\,\rho^2 + W_{040}\,\rho^4 + W_{060}\,\rho^6 \tag{15}$$

Where W_{020} is defocus, W_{040} term is the third-order term (spherical aberration), and W_{060} is the fifth-order term.

In Cartesian coordinate (x, y):

$$W(x,y) = W_{020}(x^2+y^2) + W_{040}(x^2+y^2)^2 + \dots \tag{16}$$

5. Hexagonal aperture

Fig.(1) shows the hexagonal aperture divided into two trapezoidal shapes, ABEF and BCDE with limits of integration are: [0 to $\frac{\sqrt{3}}{2}$ for x axis and $(\frac{y}{\sqrt{3}})$-1 to $(\frac{-y}{\sqrt{3}})$+1 for y axis] and [$\frac{-\sqrt{3}}{2}$ to 0 for x axis and $(\frac{-y}{\sqrt{3}})$-1, $(\frac{y}{\sqrt{3}})$+1 for y axis], respectively.

For hexagonal aperture, equation (5) will be, as follows:

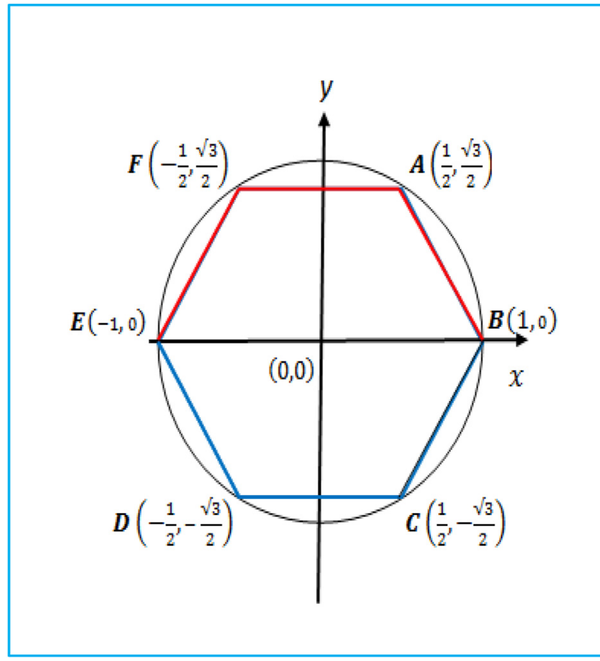

Fig. (1). Hexagonal aperture.

6. Results and discussion

A MATLAB (version 10) code and Mathcad were used in this work to solve the equations of PSF and LSF. The hexagonal aperture is used in the case of

$$PSF(z,m) = n.f\left| \int_0^{\frac{\sqrt{3}}{2}} \int_{\left(\frac{y}{\sqrt{3}}\right)-1}^{\left(\frac{-y}{\sqrt{3}}\right)+1} f(x,y)e^{i(zx+my)}dxdy + \int_{\frac{-\sqrt{3}}{2}}^{0} \int_{\left(\frac{-y}{\sqrt{3}}\right)-1}^{\left(\frac{y}{\sqrt{3}}\right)+1} f(x,y)e^{i(zx+my)}dxdy \right|^2 \tag{17}$$

If PSF written for u axis only (or z), it becomes,

$$\mathrm{PSF} = n.f\left| \int_0^{\frac{\sqrt{3}}{2}} \int_{\left(\frac{y}{\sqrt{3}}\right)-1}^{\left(\frac{-y}{\sqrt{3}},\right)+1} f(x,y)e^{i(zx)}dxdy + \int_{\frac{-\sqrt{3}}{2}}^{0} \int_{\left(\frac{-y}{\sqrt{3}},\right)-1}^{\left(\frac{y}{\sqrt{3}}\right)+1} f(x,y)e^{i(zx)}dxdy \right|^2 \tag{18}$$

And equation (14) or LSF will be as follows:

$$LSF = N\left[\int_0^{\frac{\sqrt{3}}{2}} \left| \int_{\left(\frac{y}{\sqrt{3}}\right)-1}^{\left(\frac{-y}{\sqrt{3}},\right)+1} f(x,y).\mathrm{e}^{\mathrm{i}zx}dx \right|^2 dy + \int_{\frac{-\sqrt{3}}{2}}^{0} \left| \int_{\left(\frac{-y}{\sqrt{3}},\right)-1}^{\left(\frac{y}{\sqrt{3}}\right)+1} f(x,y).\mathrm{e}^{\mathrm{i}zx}dx \right|^2 dy \right] \tag{19}$$

diffraction-limited system and with aberrations (focal error and spherical aberrations) with the aberration factor)W = 0.25λ, 0.5 λ, 0.75 λ and 1 λ), as in the following:

6.1. PSF and LSF for a perfect optical system

The results were shown in all figures, as in the central intensity (Strehl ratio = 1) of the point spread function and line spread function of the hexagonal aperture, because of the normalization function.

6.2. PSF with the presence of focal error and spherical aberration

Different values of focus aberration (W20 = 0.25, 0.5, 0.75, and 1) were taken to calculate values of point spread function, and they were shown in (Fig. 2). And central intensity is decreased from 1 for free of aberration to (0.861, 0.536, 0.215, and 0.037) respectively. The increase in focus aberration increases the secondary peaks, especially at (W20 = 0.75 and 1). Which effects negatively on the clarity image, while The effect of spherical aberration (W40), is less than the focus error (the values of W40 are taken as that of focus error) and the central intensity changed from 1 for free of aberration to (0.891, 0.629, 0.357, and 0.179). Spherical aberrations were preferable in increasing the value central intensity and decreasing the secondary peaks (Fig. 3).

Spherical aberration can be balanced with focus error by choosing the appropriate values of the two errors. To make this clear let (w20 = − w40) (Fig. 4). It is obvious that the effect of the error would be less than the effect of focus or the spherical errors separately. The central intensity is changed from 1 for free of aberration to (0.989, 0.956, 0.904, and 0.835). The balance increased central intensity (Strehl ratio) of the point spread function, the width of the function curve is decreased.

Figs. (5)–(7), show how the curve width of a point spread function decreased after using the two axes z and m for all values of W20, W40 and w20 = − w40.

6.3. LSF with the presence of focal error and spherical aberration

Fig. 8 describes the effect of focus aberration on a line spread function of a hexagonal aperture, it is noticed that the effect of focus aberration (0.25,0.5,0.75, and 1) on the central intensity value (Strehl ratio) will be (0.9, 0.663, 0.423, and 0.269) respectively. The curve width of a line spread function increases with the increasing of focus error value, leading to a decrease in resolution power, especially at (W20 = 0.5, 0.75 and 1), while in the presence of spherical aberrations (W40), the value of central intensity (Strehl ratio) will be (0.926, 0.744, 0.744, and 0.393) respectively, and also curve width is increased but less than that of focus error (Fig. 9).

When adding focus error to spherical aberration with (w20 = − w40), it is clear there is a balance happened as in (Fig. 10), it is noticed that the central intensity (Strehl ratio) will be (0.991, 0.964, 0.92and0.863) respectively.

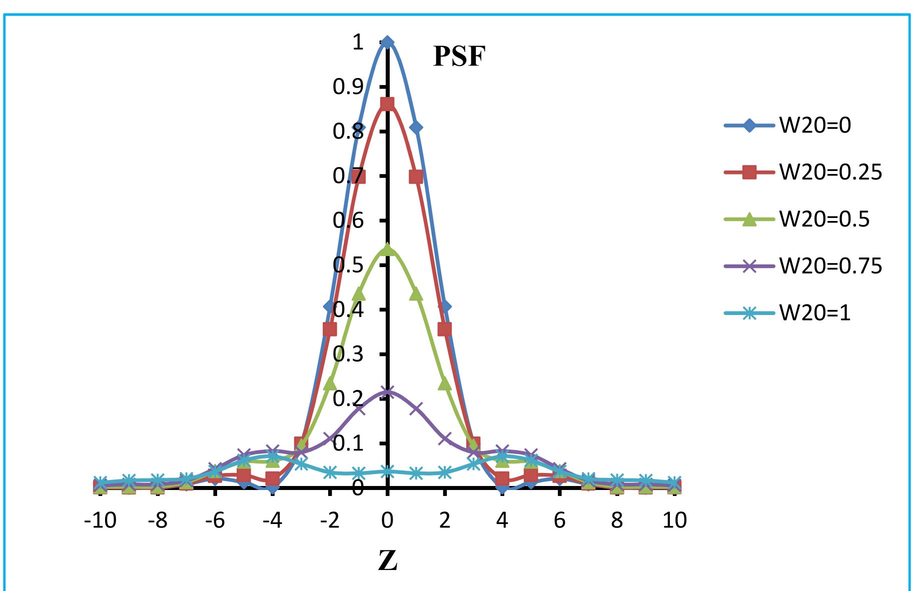

Fig. 2. PSF for hexagonal aperture of optical system with one axis (z), different values of W_{20} (focus error).

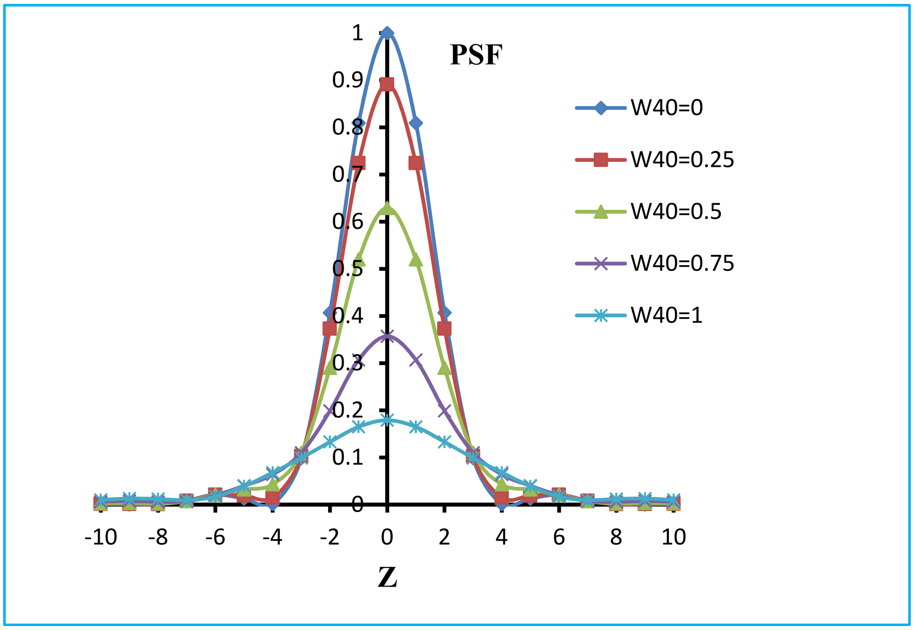

Fig. 3. PSF for hexagonal aperture of optical system with one axis (z), different values of W40 (spherical aberration).

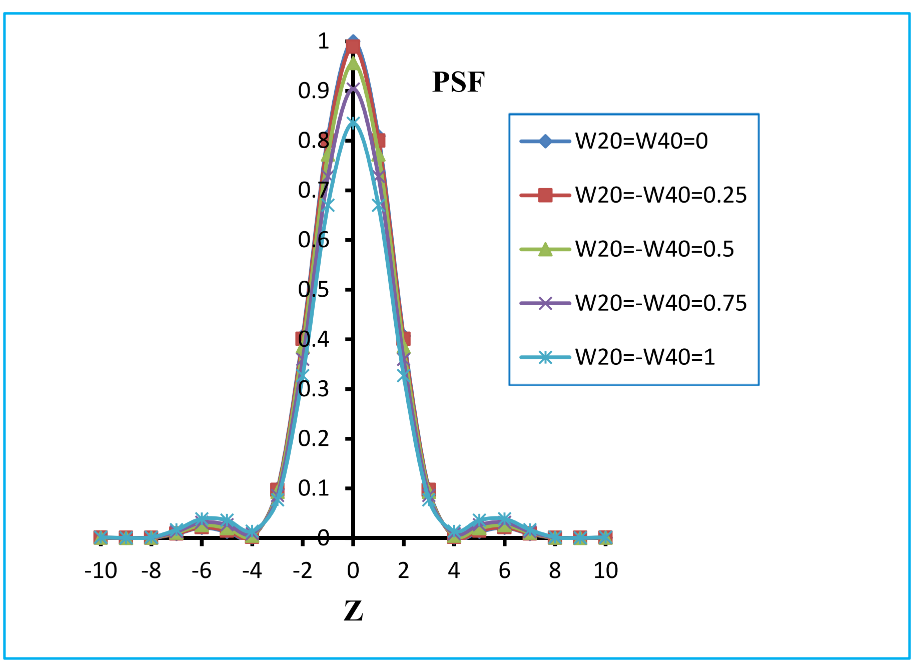

Fig. 4. PSF for hexagonal aperture of optical system with one axis (z), different values of balanced aberrations (W20 = −W40).

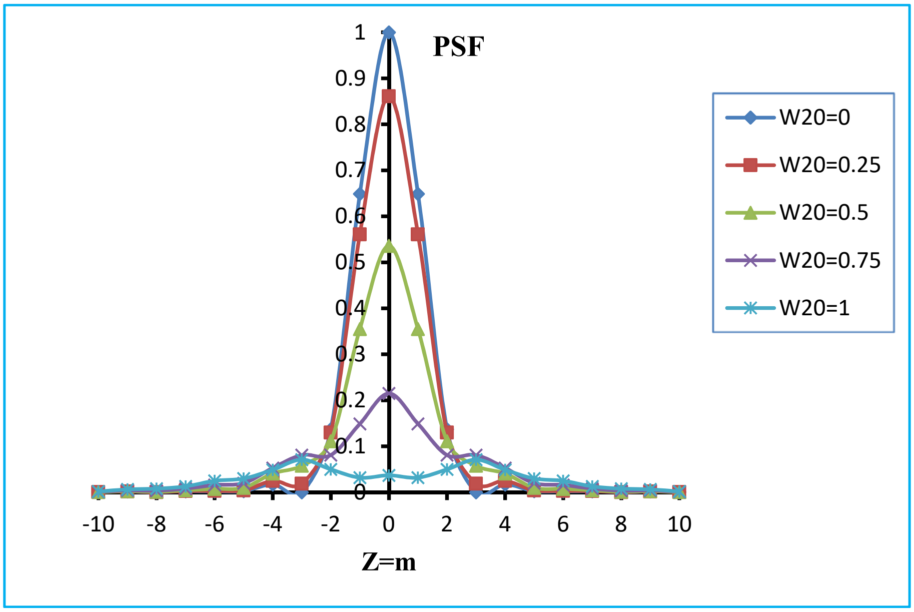

Fig. 5. PSF for hexagonal aperture of optical system with two axes z and m, different values of W_{20} (focus error).

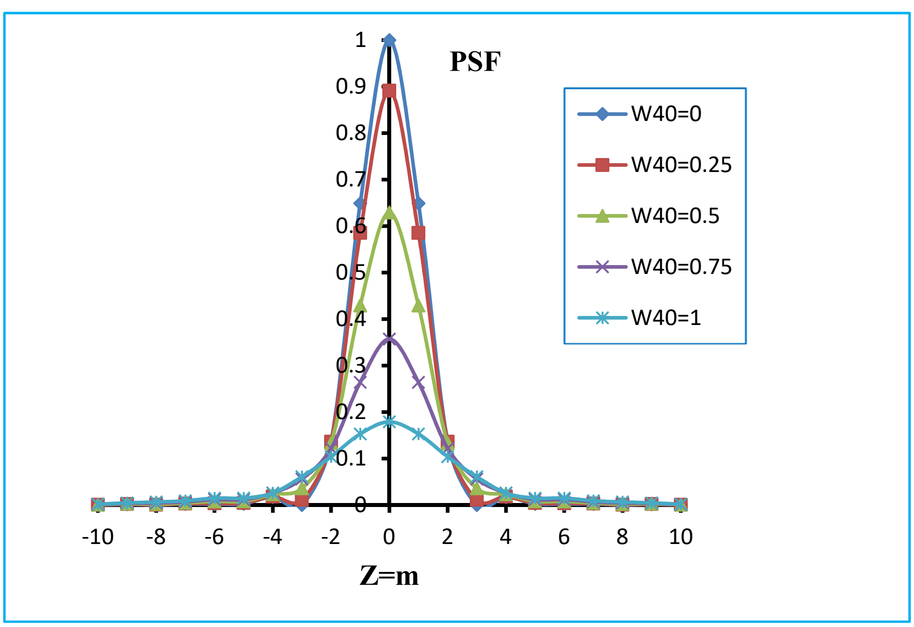

Fig. 6. PSF for hexagonal aperture of optical system with two axes z and m, different values of W40 (spherical aberration).

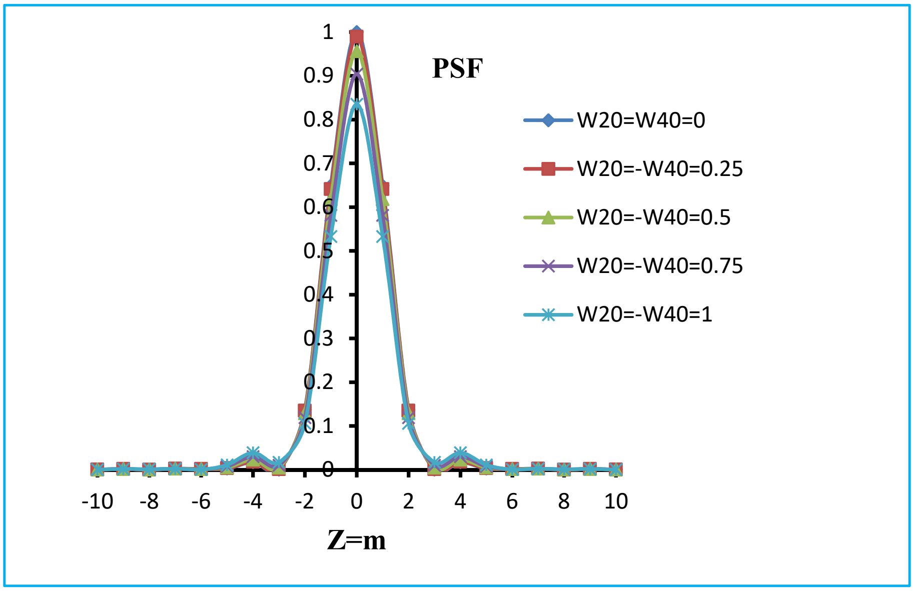

Fig. 7. PSF for hexagonal aperture of optical system with two axes z and m, different values of balanced aberrations (W20 = −W40).

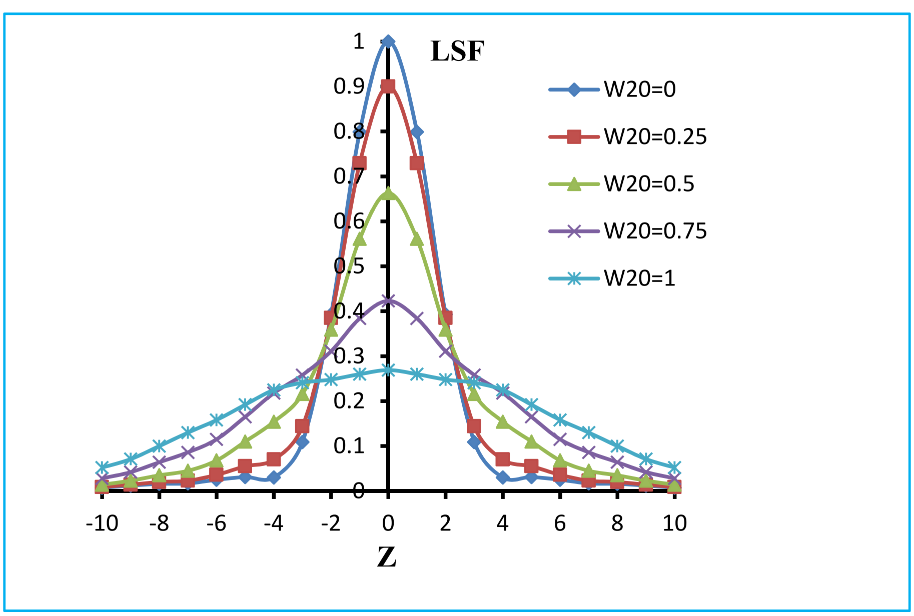

Fig. 8. LSF for hexagonal aperture of optical system, different values of W_{20} (focus error).

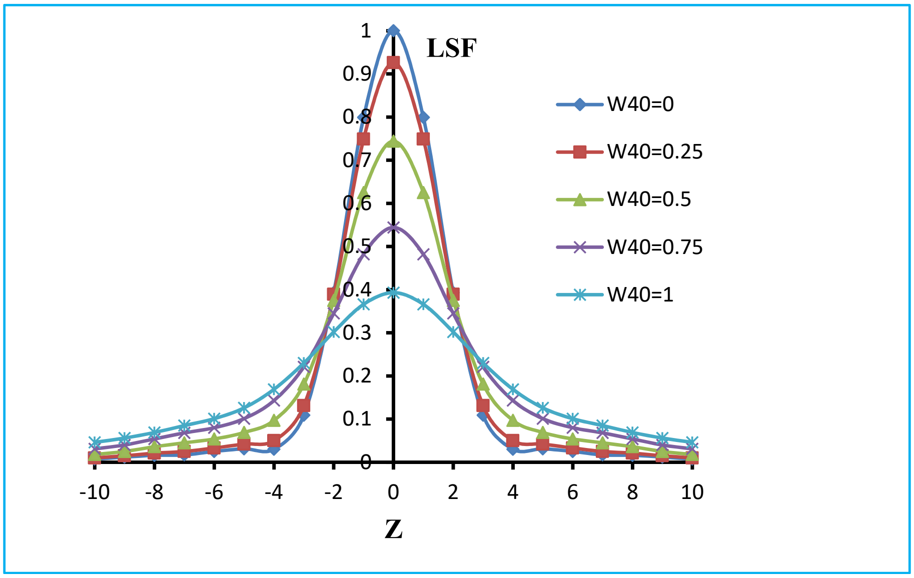

Fig. 9. LSF for hexagonal aperture of optical system, different values of W40 (spherical aberration).

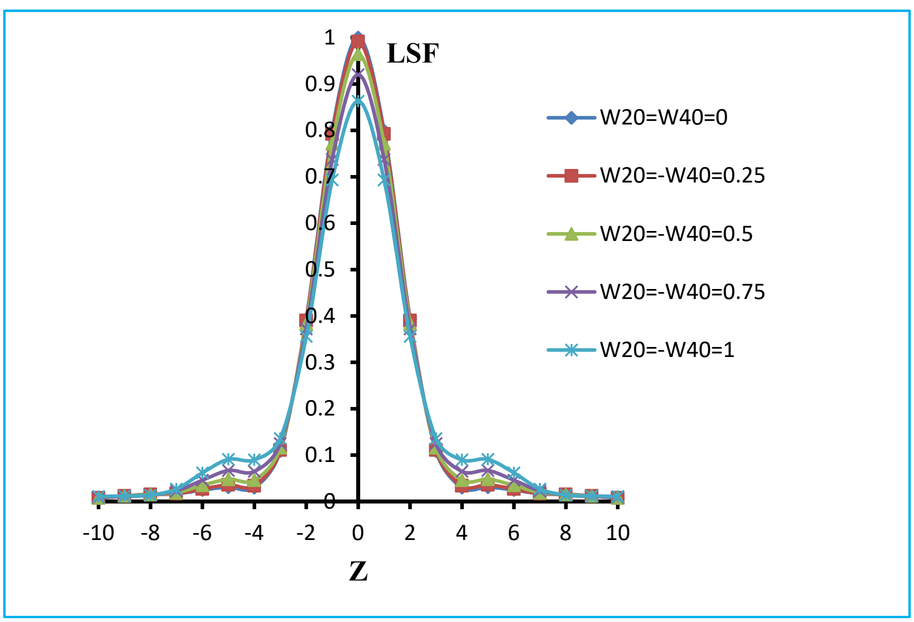

Fig. 10. LSF for hexagonal aperture of optical system, different values of balanced aberrations (W20 = −W40).

7. Conclusions

1- Strehl ratio is equal to one for an aberration-free system and decreased as aberration increases, and it depends on the type of aberration.
2- The secondary peaks and the widths of central maxima were increased with the presence of different aberrations.
3- The effect of focus error on Strehl ratio and the secondary maxima for focus error is more than that for spherical aberration for all values of aberrations.
4- Aberration balance could happen when focus error and spherical aberration present together with the same and opposite values, and this lead to good values of strehl ratio (close to 1) for all values of aberrations.

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