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Assessing The Dead Loads Effect on the Dynamic Behavior of Plates by Using Finite Elements Method

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Abstract

In this research, a rectangular plate element for the finite elements method, which takes into consideration the effect of dead loads, is proposed. The element stiffness matrix that includes the effect of dead loads is derived. It is shown that the stiffness of plate increases when the effect of dead loads is included in the calculations. The validity of the proposed method is confirmed by numerical example and the results show a good agreement when compared with that obtained from the closed-form solutions. The proposed method based on a finite elements formulation is more easily applied to plate's structures under different support conditions and various types of dead loads.

Keywords: Dead load, shape function, stiffness matrix, strain energy.

مصفوفة الجساءة للاعضاء ذات النهايات
المثخنة متضمنة ادخال تأثير تشوهات القص العرضي

الخلاصة

في هذا البحث تم اقتراح عنصر صفائحي مستطيل لطريقة العناصر المحددة والذي ياخذ بنظر الاعتبار تاثير الاحمال الميتة . ان مصفوفة الجساءة لهذا العنصر مع ادخال تاثير الاحمال الميتة قد تم اشتقاقها. وقد لوحظ بان جساءة الصفيحة تزداد عند اخذ تاثير الاحمال الميتة في الحسابات. وقد جرى اختبار الطريقة المقترحة بامثلة عددية وقد اظهرت النتائج تطابقا جيدا مع تلك المستحصلة من صيغة الحل المغلق. ان الطريقة المقترحة وباستخدام صيغة العناصر المحددة تكون سهلة التطبيق للمنشات الصفائحية تحت مختلف حالات الاسناد وتحت مختلف الاحمال الميتة.

List of Symbols

a,b	Plate element dimensions.
[B]	Element strain matrix.
$c_o,c_1,$ c_2,c_3	Coefficients.
D	Flexural rigidity of the plate. Elastic matrix.
[D]	Element nodal force vector.
{F}	Element equivalent concentrated force vector.
$\{F_{eq}\}$	Thickness of the plate.
h	Dead load stiffness matrix of the element.
$[\overline{K}]$	Element elastic stiffness matrix of the plate element.
$[K_e]$	Geometric stiffness matrix.

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$[K_g]$	Consistent mass matrix.
$[M]$	Shape function.
$[N]$	Live load.
p	Reference dead load.
$\bar{p}_o$	Concentrated load.
P	Reference concentrated dead load.
$\bar{P}_o$	Strain energy produced by live loads.
U	Strain energy produced by live loads.
U^*	Element strain energy.
$\bar{U}$	Additional strain energy resulting from the initial bending stresses produced by the dead loads.
w	Dynamic deflection produced by live load.
$\bar{w}$	Deflection produced by the dead load.
$[\alpha]$	Constant.
$\{\delta\}$	Element displacement vector.
υ	Poisson's ratio.
ω_n	The nth natural frequency including the effect of dead loads.
ω_{on}	The nth natural frequency excluding the effect of dead loads.

1. Introduction

Plates or shells, like any structure, are always subjected to dead loads. When plates are subjected to live loads in addition to dead loads, the plate deflects from a reference state caused by the initial dead loads. The deflection should include the effect of the initial bending moments. This effect of dead loads is more significant in plates because of the smaller stiffness of plates. And a better understanding of the dead load effect will lead to more accurate estimate of the effects of live loads. Takabatake (1991) studied the effect of dead loads on the natural frequencies of beam and proposed a governing equation of beam including the effect of dead load. Takabatake (1992) extended the elementary plate theory and analyzed the effect of dead loads in the dynamic analysis for plates with limited geometry and support conditions.

In this research the finite elements formulation for plates, which takes into account the effect of dead load is proposed. The element stiffness matrix including the effect of dead loads is derived and the validity of the proposed method is confirmed by numerical examples.

2. Finite Element Formulation

For an element, and by using Hamilton's principal, the following dynamic equation which takes into consideration the effect of dead loads can be expressed as (Clough and Penzien 1989):

$$\{F\}=[M]\{\ddot{\delta}\}+\left([K_e]+[\bar{K}]-[K_g]\right)\{\delta\}-\{F_{eq}\} \quad (1)$$

in which $\{\delta\}$ is the element displacement vector, $\{F\}$ is the element nodal force vector , [M] is the consistent mass matrix, [Ke] is the element elastic matrix, [Kg] is the geometric stiffness matrix, $[\bar{K}]$ is the dead load stiffness matrix and {feq.}

is the element equivalent concentrated force vector.

For plate element, it is assumed that the external forces are transverse loads only and the axial forces are neglected. Deflections $\overline{w}$ are produced by the dead loads $\overline{p}$ is considered as the reference state. The dynamic deflections w by live load p are measured from the reference state. The element strain energy U* can be expressed as:

$$U^* = \overline{U} + U \quad (2)$$

where U is strain energy produced by live loads p and $\overline{U}$ is the additional strain energy resulting from the initial bending stresses produced by the dead loads p.

Different types of strain – displacement relation for U and $\overline{U}$ are used to include the effect of dead loads. For strain energy U, the linear strain – displacement relations are used; for the strain energy $\overline{U}$, the nonlinear strain – displacement relations are used. Thus, the strain energies U and $\overline{U}$ can be written as:

$$U = \frac{D}{2}\iint \left\{ \begin{matrix} (w_{xx} + w_{yy})^2 + 2(1-\upsilon) \\ \left[(w_{xy})^2 - w_{xx} w_{yy}\right] \end{matrix} \right\} dx.dy \quad (3)$$

$$\overline{U} = D\iint \left[\begin{matrix} \overline{w}_{xx} w_{xx} + \overline{w}_{yy} w_{yy} + \\ 2\overline{w}_{xy} w_{xy} \\ +\nu\left(\begin{matrix} \overline{w}_{yy} w_{xy} + \overline{w}_{xx} w_{yy} - \\ 2\overline{w}_{xy} w_{xy} \end{matrix} \right) \end{matrix} \right] dx.dy$$

$$+\frac{Eh}{4(1-\upsilon^2)}\iint \left\{ \begin{matrix} (\overline{w}_x)^2 (w_x)^2 + \\ (\overline{w}_y)^2 (w_y)^2 \\ +2\overline{w}_x \overline{w}_y w_x w_y + \\ \nu\left[\begin{matrix} (\overline{w}_y)^2 (w_x)^2 + \\ (\overline{w}_x)^2 (w_y)^2 \\ -2\overline{w}_x \overline{w}_y w_x w_y \end{matrix} \right] \end{matrix} \right\} dx > dy \quad (4)$$

Equations (3) and (4) can be expressed as:

$$U = \frac{1}{2}\{\delta\}T\ [Ke]\ \{\delta\} \quad (5)$$

$$\overline{U} = \{\overline{\delta}\}T[Ke]\{\delta\} + \{\delta\}T\left[\overline{K}\right]\{\delta\} \quad (6)$$

where

$$\{\delta\}^T [K_e]\{\delta\} = \frac{D}{2}\iint \left\{ \begin{matrix} (w_{xx} + w_{yy})^2 + \\ 2(1-\upsilon)\left[(w_{xy})^2 - w_{xx} w_{yy}\right] \end{matrix} \right\} dx.dy \quad (7)$$

$$\{d\}^T \left[\overline{K}\right]\{d\} = \frac{Eh}{4(1-u^2)}\iint \left\{ \begin{matrix} (\overline{w}_x)^2 (w_x)^2 + \\ (\overline{w}_y)^2 (w_y)^2 \\ +2\overline{w}_x \overline{w}_y w_x w_y + \\ u\left[\begin{matrix} (\overline{w}_y)^2 (w_x)^2 \\ +(\overline{w}_x)^2 (w_y)^2 - \\ 2\overline{w}_x \overline{w}_y w_x w_y \end{matrix} \right] \end{matrix} \right\} dx.dy \quad (8)$$

in which $[K_e]$ is the element elastic stiffness matrix of the plate element and $\left[\overline{K}\right]$ is the dead load stiffness matrix of the element.

Suppose the element displacements are expressed as follows:

$$w = [N]\ \{\delta\} \quad (9a)$$

$$\overline{w} = [N]\left\{\overline{\delta}\right\} \quad (9b)$$

The element stiffness matrix can be written as:

$$[Ke] = \iint\ [B]T[D]\ [B]\ dx\ dy \quad (10)$$

in which [B] is the element strain matrix and [D] is the elastic matrix.

Therefore, the dead load stiffness matrix is:

$$[\overline{K}]=\frac{Eh}{4(1-\upsilon^2)}\iint\left\{\begin{matrix}\left(\begin{matrix}\{\overline{\delta}\}^T[N_x]^T[N_x]\{\overline{\delta}\}+\\ \upsilon\{\overline{\delta}\}[N_y]^T[N_y]\{\overline{\delta}\}\end{matrix}\right)\\ [N_x]^T[N_x]\\ +\left(\begin{matrix}\{\overline{\delta}\}^T[N_y]^T[N_y]\{\overline{\delta}\}+\\ \upsilon\{\overline{\delta}\}^T[N_x]^T[N_x]\{\overline{\delta}\}\end{matrix}\right)\\ [N_y]^T[N_y]\\ +(1-\upsilon)\{\overline{\delta}\}^T[N_x]^T[N_y]\{\overline{\delta}\}\\ ([N_x]^T[N_y]+[N_y]^T[N_x])\end{matrix}\right\}dx.dy \quad (11)$$

where [N] is the shape function.

Figure (1) shows a 4- nodded plate element. The element nodal displacement vector {δ} and element nodal force vector {F} can be defined as:

$$\{\delta\}=[w_1,\theta_{x1},\theta_{y1},w_2,\theta_{x2},\theta_{y2},w_3,\theta_{x3},\theta_{y3},w_4,\theta_{x4},\theta_{y4}]^T \quad (12)$$

$$\{F = [Q_1,M_{x1},M_{y1},Q_2,M_{x2},M_{y2},Q_3,M_{x3},M_{y3},Q_4,M_{x4},M_{y4}]^T \quad (13)$$

Suppose

$$w = [H]\,[\alpha] \quad (14)$$

where

$$[H]=[1,x,y,x^2,xy,x^3,x^2\ ,x^3,x^2y,xy^2,y^3,x^3y,xy^3] \quad (15)$$

and

$$[\alpha] = [\alpha_1,\alpha_2,\ \ldots\ldots\alpha_{12}]^T \quad (16)$$

Applying the boundary conditions, the constant [α] can be expressed as:

$$[\alpha] = [C]^{-1}\{\delta\} \quad (17)$$

Thus the element shape function [N] and the element elastic stiffness matrix $[k_e]$ can easily be obtained. In calculating the dead load stiffness matrix $[\overline{K}]$, the dead load displacement function is simply selected as:

$$\overline{w} = c_o + c_1x + c_2y + c_3xy \quad (18)$$

Applying the boundary conditions, we have:

$$c_o = \overline{w}_1$$

$$c_1 = \frac{\overline{w}_2-\overline{w}_1}{a}$$

$$c_2 = \frac{\overline{w}_4-\overline{w}_1}{b} \quad (19)$$

$$c_3 = \frac{\overline{w}_1+\overline{w}_3-\overline{w}_2-\overline{w}_4}{ab}$$

From eq. (18)

$$\overline{w}_x = c_1x + c_3y$$

$$\overline{w}_y = c_2 + c_3x \quad (20)$$

Equation (11) can be simply expressed as:

$$[\overline{K}] = \frac{Eh}{2(1-\upsilon^2)}([C]^{-1})^T[S][C]^{-1} \quad (21)$$

in which [S] can be expressed as:

$$[S]=\iint\{[(c_1+c_{3y})^2+\upsilon(c_2+c_{3x})^2][H_x]^T[H_x]+[(c_2+c_{3x})^2+\upsilon(c_1+c_{3y})^2][H_y]^T[H_y]+(1-\upsilon)(c_1+c_{3y})(c_2+c_{3x})[H_x]^T[H_y]+[H_y]^T[H_x])\}\,dx\,dy \quad (22)$$

By evaluating the integrals in eq. (22) and from eq. (21), we obtain $[\overline{K}]$.

3. Example Analysis

In the following analysis, the calculate values obtained from the proposed element are compared with the closed form solutions. The natural frequencies of a simply supported plate with the following reference properties (Takabatake 1992) are calculated.

Lx = Ly = 5 m, h = 0.05 m, E = 20.59 x 1010 N/m_2, υ = 0.3. A reference dead load p_o is assumed to be po = 3825 N/m_2. In fig. (2) and (3), Δ = [(ωn – ω_{on}) / ω_{on}] x 100%, where ω_n is the nth natural frequency including the effect of dead loads and ω_{on} is the nth natural frequency excluding the effect of dead loads. Two types of meshes, (5x5 and 8x8) elements, were selected.

Fig. (2) shows the closed form solution given by Takabatake (1992) for the three lowest frequencies and the finite elements solutions from eq. (21). From fig. (2), we can see that the results obtained using the proposed method is in a good agreement with those given by Takabatake

Fig. (3) shows the effect of the concentrated load $\bar{P}$ at the center of the plate on the natural frequencies of the simply supported plate. The reference concentrated dead load $\bar{P}_o$ is assumed to be $\bar{P}_o$ = 5000 N, while $\bar{p}_o$ = 3825 N/m^2 = constant. It is shown that the effect of the dead load is more obvious for the first natural frequencies and it can be neglected for frequencies higher than the first five natural frequencies. The larger dead load is the more view effect.

4. Conclusions

The effect of dead load on the dynamic behavior for rectangular plate element has been derived and the validity of the proposed procedure is confirmed by the numerical examples. Although the obtained results from the finite elements method are in a good agreement with those given by approximate closed-form solution, the proposed method based on a finite elements formulation is more easily applied to practical structures under various support conditions and various types of dead loads. The calculated results also show that the flexural stiffness of plates is increased when the effect of dead loads is considered. This effect of dead loads on flexural stiffness should be included in the design of plates or shell structures for which the geometric nonlinearity is included.

5. References

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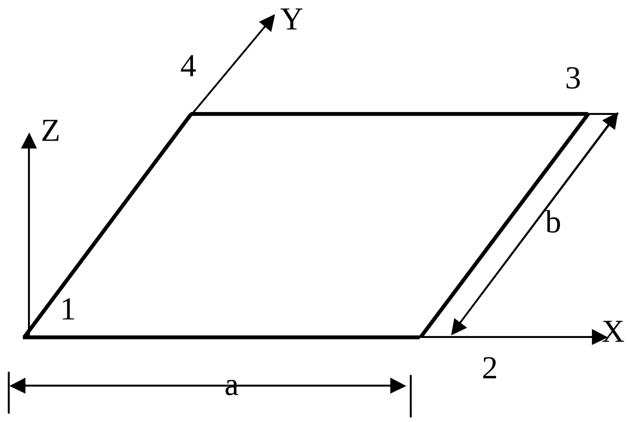

Fig. (1) A 4-node element

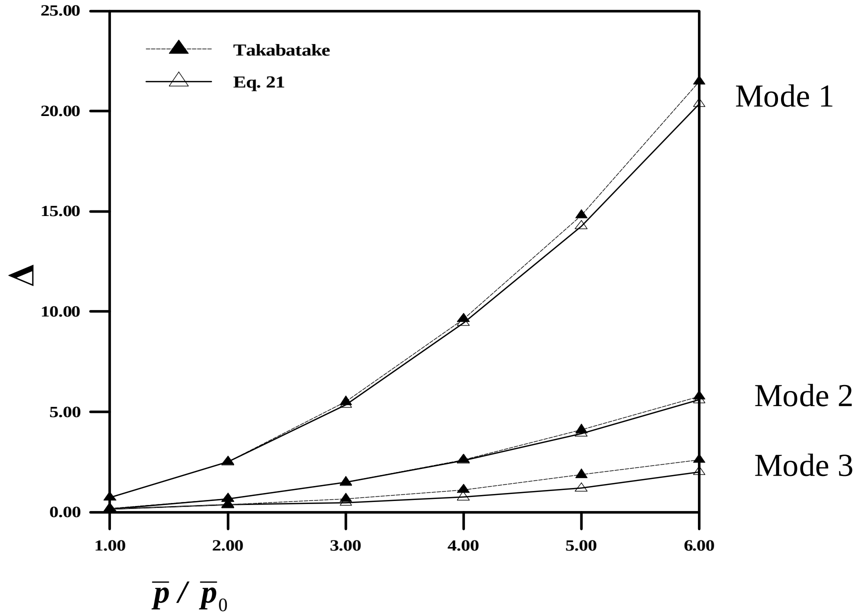

Fig. (2) Comparison of calculated results (simply supported plate)
(Lx=Ly= 5m, h= 0.05 m, E = 20.59 x 10^{10} N/m^2, υ = 0.3, $\bar{p}_o$ = 3825 N/m^2 N/m^2)

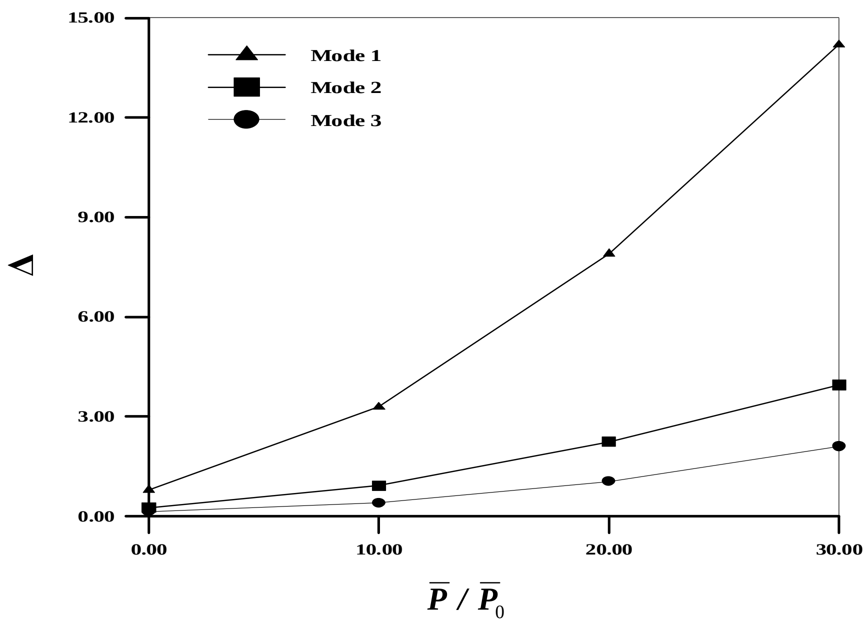

Fig. (3) Relation between Δ and $\bar{P} / \bar{P}_o$ (simply supported plate)
(Lx=Ly= 5m, h= 0.05 m, E = 20.59 x 10^{10} N/m^2, υ = 0.3, $\bar{p}_o$ = 3825 N/m^2, $\bar{P}_o$ = 5000N)