The Open Mapping in Fuzzy 𝕂-proximity Space

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Abstract

In this paper, we define fuzzy 𝕂 -proximally open mapping. The properties of this mapping and its relationship with other types of maps were also studied. It was also proven through fuzzy 𝕂 -proximally open mapping that a relation $\mathcal{S}$ on the corresponding space is fuzzy 𝕂 -proximity.

1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [1]. In 1974, Chang [2] defined fuzzy topology. In 1979, Katsaras [3] introduced the nation of fuzzy proximities, on the base of the axioms suggested by Efremovic [4].

2. Preliminaries

In this paper $\mathbb{W}$ denote a nonempty set and $I^{\mathbb{W}}$ is the collection of all fuzzy set. $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ (or $f: \mathbb{W} \to \mathbb{M}$) means a mapping f from a fuzzy 𝕂-proximity space $\mathbb{W}$ to a fuzzy 𝕂-proximity space $\mathbb{M}$. For any two fuzzy sets $\mathbb{m}$ and $\mathbb{mm}$ in $\mathbb{W}$, $\mathbb{m} \vee \mathbb{mm}$ and $\mathbb{m} \wedge \mathbb{mm}$ defined as followings: For any $a \in \mathbb{W}$, $(\mathbb{m} \vee \mathbb{mm})(x) = \sup\{\mathbb{m}(a), \mathbb{mm}(a)\}$ and $(\mathbb{m} \wedge \mathbb{mm})(a) = \inf\{\mathbb{m}(a), \mathbb{mm}(a)\}$, respectively. And $\mathbb{m} \leq \mathbb{mm}$ if for each $a \in \mathbb{W}$, $\mathbb{m}(a) \leq \mathbb{mm}(a)$. $\mathbb{m}^c$ is the complement of a fuzzy set $\mathbb{m}$ in $\mathbb{W}$ defined by $\mathbb{m}^c = 1 - \mathbb{m}$. The constant maps all of $\mathbb{W}$ to 0 and 1 denoted by 0 and 1, respectively. A A fuzzy point a_β in $\mathbb{W}$ is a fuzzy set defined by $a_\beta = 0$ for all $b \in \mathbb{W}$ except one, called $a \in \mathbb{W}$ such that $a_\beta = \beta$, where $0 < \beta < 1$, we say a_β belong to $\mathbb{m}$ denoted by $a_\beta \in \mathbb{m}$, if $\beta \leq \mathbb{m}(a)$, for any $a \in \mathbb{W}$. Evidently, every fuzzy set $\mathbb{m}$ can be expressed as the union of all the fuzzy points which belong to $\mathbb{m}$. This paper was an attempt to give the most important properties of the fuzzy 𝕂-proximally open mapping.

Definition 2.1 [2]: A subset $\mathbb{S}$ of $I^{\mathbb{W}}$ is called fuzzy topology on $\mathbb{W}$ if the following statements are complete:

- $0,1 \in \mathbb{S}$;
- If $\mathbb{m}, \mathbb{mm} \in \mathbb{S}$, then $\mathbb{m} \wedge \mathbb{mm} \in \mathbb{S}$;
- If $\mathbb{m}_i \in \mathbb{S}$, then $sup_{i \in \Delta} \mathbb{m}_i \in \mathbb{S}$, for each $i \in \Delta$.

We say the pair $(\mathbb{W}, \mathbb{S})$is a fuzzy topological space, of its for short.

Definition 2.2 [3]: A relation $\mathcal{S}$ on $I^{\mathbb{W}}$ is called a fuzzy proximity if the following statements are complete:

C1- If $\mathbb{m}\mathcal{S}\mathbb{mm}$ then $\mathbb{mm}\mathcal{S}\mathbb{m}$;

C2- $(\mathbb{m} \vee \mathbb{mm})\mathcal{S}\mathbb{k}$ if and only if $\mathbb{m}\mathcal{S}\mathbb{k}$ or $\mathbb{mm}\mathcal{S}\mathbb{k}$;

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C3- If $\mathbb{m}\mathcal{S}\mathbb{mm}$ then $\mathbb{m} \neq 0$ and $\mathbb{mm} \neq 0$;

C4- If $\mathbb{m}\bar{\mathcal{S}}\mathbb{mm}$, then $\exists \mathbb{k} \in I^{\mathbb{W}} \ni \mathbb{m}\bar{\mathcal{S}}\mathbb{k}$ and $(1-\mathbb{k})\bar{\mathcal{S}}\mathbb{mm}$;

C5- If $\mathbb{m} \wedge \mathbb{mm} \neq 0$ then $\mathbb{m}\mathcal{S}\mathbb{mm}$.

We say the pair $(\mathbb{W}, \mathcal{S})$is a fuzzy proximity space.

Definition 2.3 [6]: A relation $\mathcal{S}$ on $I^{\mathbb{W}}$ is called a fuzzy $\mathbb{K}$-proximity if the following statements are complete:

S1- $a_\beta \mathcal{S}(\mathbb{m} \vee \mathbb{mm})$ if and only if $a_\beta \mathcal{S}\mathbb{m}$ or $a_\beta \mathcal{S}\mathbb{mm}$;

S2- $a_\beta \bar{\mathcal{S}} 0$ for all a_β;

S3- If $a_\beta \in \mathbb{m}$ then $a_\beta \mathcal{S}\mathbb{m}$;

S4- If $a_\beta \bar{\mathcal{S}}\mathbb{m}$ then $\exists \mathbb{k} \in I^{\mathbb{W}} \ni a_\beta \bar{\mathcal{S}}\mathbb{k}$ and $b_\beta \bar{\mathcal{S}}\mathbb{m}$ for all $b_\beta \in (1-\mathbb{k})$.

We say the pair $(\mathbb{W}, \mathcal{S})$ is a fuzzy $\mathbb{K}$-proximity space.

Notes:

1- Clear that the fuzzy proximity on $I^{\mathbb{W}}$ implies the fuzzy $\mathbb{K}$-proximity on $I^{\mathbb{W}}$.

2- The pairs $(\mathbb{W}, \mathcal{S})$and $(\mathbb{M}, \mathcal{S})$ we mean in the next stage of this paper is the fuzzy $\mathbb{K}$-proximity space.

Definition 2.4 [6]: A fuzzy $\mathbb{K}$-proximity $\mathcal{S}$ on $I^{\mathbb{W}}$ is called discrete $\mathbb{K}$-proximity, if we define $a_\beta \mathcal{S}\mathbb{m}$ if and only if $a_\beta \wedge \mathbb{m} \neq 0$.

Definition 2.5: Let $(\mathbb{W}, \mathcal{S})$ be a fuzzy proximity space. A fuzzy set $\mathbb{m} \in I^{\mathbb{W}}$ is called F$\mathcal{S}$-closed if $a_\beta \mathcal{S}\mathbb{m} \rightarrow a_\beta \in \mathbb{m}$.

Definition 2.6: Let $(\mathbb{W}, \mathcal{S})$ be a fuzzy proximity space, then the family $\tau_{\mathcal{S}} = \{\mathbb{m} \in I^{\mathbb{W}}: (1-\mathbb{m})$ is F$\mathcal{S}$-closed $\}$ is called fuzzy topology induced by $\mathcal{S}$.

Definition 2.7: Let $(\mathbb{W}, \mathcal{S})$ be a $\mathbb{K}$- proximity space and $\mathbb{m} \in I^{\mathbb{W}}$, a $\mathcal{S}$-closure of $\mathbb{m}$ (briefly $\bar{\mathbb{m}}^{\mathcal{S}}$)and $\mathcal{S}$-interior of $\mathbb{m}$ (briefly $\mathbb{m}^{o\mathcal{S}}$) are defined as:

$$\bar{\mathbb{m}}^{\mathcal{S}} = \wedge \{\mathbb{mm}: \mathbb{m} \leq \mathbb{mm}, \mathbb{mm}\ is\ \text{F}\mathcal{S}\text{-closed}\}$$

$$\mathbb{m}^{o\mathcal{S}} = \vee \{\mathbb{mm}: \mathbb{mm} \leq \mathbb{m}, (1-\mathbb{mm}) is\ \text{F}\mathcal{S}\text{-closed}\}$$

Theorem 2.7 [7]: Let $(\mathbb{W}, \mathcal{T})$ be a fuzzy topological space and $\mathcal{S}$ is a binary relation defined by $a_\beta \mathcal{S}_{\mathbb{W}}\mathbb{m}$ if and only if $a_\beta \in \bar{\mathbb{m}}^{\mathcal{S}}$, then $\mathcal{S}$ is a fuzzy $\mathbb{K}$-proximity on $I^{\mathbb{W}}$ and the fuzzy topology $\tau_{\mathcal{S}}$ induced by $\mathcal{S}$ is the given topology $\mathcal{T}$.

Definition 2.8 [7]: Let $\mathbb{m}$ and $\mathbb{mm}$ be a fuzzy set in $\mathbb{K}$-proximity space $(\mathbb{W}, \mathcal{S})$. Then we say that $\mathbb{m}$ and $\mathbb{mm}$ are in the relation $\ll$ and write $\mathbb{m} \ll \mathbb{mm}$ if $\mathbb{m}\,\bar{\mathcal{S}}\,(1-\mathbb{mm})$.

Proposition 2.9: Let $(\mathbb{W}, \mathcal{S}_{\mathbb{W}})$ be a fuzzy $\mathbb{K}$-proximity space. If $\mathbb{mm} \ll \mathbb{m}$, then $\exists \mathbb{k} \in I^{\mathbb{W}}$such that $\mathbb{mm} \ll \mathbb{k} \ll \mathbb{m}$.

Proof: Let $\mathbb{mm} \ll \mathbb{m}$, then $\mathbb{mm}\bar{\mathcal{S}}_{\mathbb{W}}(1-\mathbb{m})$. So, $\exists \mathbb{k} \in I^{\mathbb{W}}$ such that $\mathbb{mm}\bar{\mathcal{S}}_{\mathbb{W}}(1-\mathbb{k})$ and $\mathbb{k}\bar{\mathcal{S}}_{\mathbb{W}}(1-\mathbb{m})$. So, $\mathbb{mm} \ll \mathbb{k}$ and $\mathbb{k} \ll \mathbb{m}$.

Definition 2.10 [7]: A mapping $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is said to be fuzzy proximally mapping (F$\mathcal{S}$-continuous) if $a_\beta \mathcal{S}_{\mathbb{W}} \mathbb{m}$ implies $f(a_\beta)\mathcal{S}_{\mathbb{M}} f(\mathbb{m})$.
Equivalently, $f: \mathbb{W} \to \mathbb{M}$ is said to be F$\mathcal{S}$-continuous if $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$ implies $f^{-1}(b_\beta)\bar{\mathcal{S}}_{\mathbb{W}} f^{-1}(\mathbb{mm})$ or $b_\beta \ll \mathbb{mm}$ implies $f^{-1}(b_\beta) \ll f^{-1}(\mathbb{mm})$.

Definition 2.11 [7]: A fuzzy set $\mathbb{m}$ in $\mathbb{K}$-proximity space $(\mathbb{W}, \mathcal{S})$ is called $\mathcal{S}$-neighborhood of a fuzzy point a_β (in symbols $a_\beta \ll \mathbb{m}$) if $a_\beta\, \overline{\mathcal{S}}\, (1-\mathbb{m})$.

Definition 2.12 [7]: Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two $\mathbb{K}$-proximity on $\mathbb{W}$, then we define $\mathcal{S}_2 > \mathcal{S}_1$ if $a_\beta \mathcal{S}_1 \mathbb{m}$ implies $a_\beta \mathcal{S}_2 \mathbb{m}$ (we say that $\mathcal{S}_1$is finer than $\mathcal{S}_2$ or $\mathcal{S}_2$is coarser than $\mathcal{S}_1$).

3. Fuzzy $\mathbb{K}$-proximities Open Mapping

Definition 3.1: A mapping f from $(\mathbb{W}, \mathcal{S}_{\mathbb{W}})$ into $(\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is said to be fuzzy $\mathbb{K}$-proximity open (F$\mathcal{S}$-open) if $b_\beta \mathcal{S}_{\mathbb{M}} \mathbb{mm}$ implies $a_\beta \mathcal{S}_{\mathbb{W}} \mathbb{m}$, where $b_\beta = f(a_\beta)$, $\mathbb{mm} = 1 - f(1-\mathbb{m})$ and $\mathbb{m} \neq 0$.
Equivalently, $f: \mathbb{W} \to \mathbb{M}$ is said to be F$\mathcal{S}$-open if $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} \mathbb{m}$ implies $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$ or $a_\beta \ll 1-\mathbb{m}$ implies $b_\beta \ll 1 - \mathbb{mm}$. where $b_\beta = f(a_\beta)$ and $\mathbb{mm} = 1 - f(1-\mathbb{m})$ and $\mathbb{m} \neq 0$.

Proposition 3.2: Let $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ be $F\mathcal{S}$-open and injective mapping, then f is a fuzzy open with respect to the fuzzy topologies $\tau_{\mathcal{S}_{\mathbb{W}}}$and $\tau_{\mathcal{S}_{\mathbb{M}}}$.
Proof: Let $b_\beta \in f((\mathbb{m})^{o\mathcal{S}}) = f(1-(\overline{1-\mathbb{m}}^{\mathcal{S}}))$, where $b_\beta = f(a_\beta)$ then $b_\beta \in 1 - f(\overline{1-\mathbb{m}}^{\mathcal{S}})$ i.e. $b_\beta \notin f(\overline{1-\mathbb{m}}^{\mathcal{S}})$, thus $a_\beta \notin (\overline{1-\mathbb{m}}^{\mathcal{S}})$ and $a_\beta \bar{\mathcal{S}}_{\mathbb{W}}(1-\mathbb{m})$. Since f is $F\delta$-open, then $f(a_\beta)\bar{\mathcal{S}}_{\mathbb{M}} 1 - f(\mathbb{m})$, that is $f(a_\beta) \notin (\overline{1-f(\mathbb{m})}^{\mathcal{S}})$ and $f(a_\beta) \in 1 - (\overline{1-f(\mathbb{m})}^{\mathcal{S}})$ implies $f(a_\beta) \in (f(\mathbb{m}))^{o\mathcal{S}}$, this prove $f((\mathbb{m})^{o\mathcal{S}}) \subseteq (f(\mathbb{m}))^{o\mathcal{S}}$. Hence the mapping f is fuzzy open.

Proposition 3.3: The composition of $F\mathcal{S}$-open maps is $F\mathcal{S}$-open.
Proof: Clear

Proposition 3.4: Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two $\mathbb{K}$-proximity on $\mathbb{W}$, then the identity mapping $i: (\mathbb{W}, \mathcal{S}_1) \to (\mathbb{W}, \mathcal{S}_2)$ is $F\mathcal{S}$-open if and only if $\mathcal{S}_1 > \mathcal{S}_2$.
Proof: Let $a_\beta \mathcal{S}_2 \mathbb{mm}$, then $a_\beta \mathcal{S}_1 \mathbb{m}$, from definition of fuzzy identity mapping $\mathbb{m} = 1 - \left(1 - f^{-1}(\mathbb{mm})\right) = \mathbb{mm}$ so, $a_\beta \delta_1 \mathbb{mm}$. Hence, $\mathcal{S}_1 > \mathcal{S}_2$.

Proposition 3.5: A bijective mapping $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is $F\mathcal{S}$-open if and only if $f^{-1}: (\mathbb{M}, \mathcal{S}_{\mathbb{M}}) \to (\mathbb{W}, \mathcal{S}_{\mathbb{W}})$ is $F\mathcal{S}$-continuous.
Proof: Let $b_\beta \mathcal{S}_{\mathbb{M}} \mathbb{mm}$, then $a_\beta \mathcal{S}_{\mathbb{W}} \mathbb{m}$ where $b_\beta = f(a_\beta)$ and $\mathbb{mm} = 1 - f(1-\mathbb{m})$, since f is injective then $\mathbb{mm} = f(\mathbb{m})$ and $\mathbb{m} = f^{-1}(\mathbb{mm})$, that is $f^{-1}(b_\beta)\mathcal{S}_{\mathbb{W}} f^{-1}(\mathbb{mm})$.
Conversely, Let $b_\beta \mathcal{S}_{\mathbb{M}} \mathbb{mm}$, then $f^{-1}(b_\beta)\mathcal{S}_{\mathbb{W}} f^{-1}(\mathbb{mm})$. Since f is bijective, then there exists a_β and $\mathbb{m}$ in $I^{\mathbb{W}}$ s.t $x_\gamma = f^{-1}(b_\beta)$ and $\mathbb{m} = f^{-1}(\mathbb{mm})$. Thus, $a_\beta \mathcal{S}_{\mathbb{W}} \mathbb{m}$, where $b_\beta = f(a_\beta)$ and $\mathbb{mm} = f(\mathbb{m}) = 1 - f(1-\mathbb{m})$.

Definition 3.6: Let $(\mathbb{W}, \mathcal{S}_D)$be a fuzzy proximity space. A proximity $\mathcal{S}_D$is called discrete if we define $\mathbb{m}\mathcal{S}_D \mathbb{mm}$ if and only if $\mathbb{m} \wedge \mathbb{mm} \neq 0$.

Proposition 3.7: An injective mapping $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is $F\mathcal{S}$-open if $\mathcal{S}_{\mathbb{M}}$ is a discrete $\mathbb{K}$-proximity on $I^{\mathbb{M}}$.

Proof: Let $b_\beta \mathcal{S}_{\mathbb{M}} \mathbb{mm}$ then $f(a_\beta) \wedge 1 - f(1-\mathbb{m}) \neq 0$ where $b_\beta = f(a_\beta)$ and $\mathbb{mm} = 1 - f(1-\mathbb{m})$, but f is injective then $f(\mathbb{m}) = 1 - f(1-\mathbb{m})$ and $a_\beta \wedge \mathbb{m} \neq 0$, thus $a_\beta \mathcal{S} \mathbb{m}$. Hence f is $F\mathcal{S}$-open.

Definition 3.8: A mapping f from $(\mathbb{W}, \mathcal{S}_{\mathbb{W}})$ into $(\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is said to be fuzzy $\mathbb{K}$-proximity closed (F$\mathcal{S}$-closed) if $b_\beta \mathcal{S}_{\mathbb{M}} \mathbb{mm}$ implies $a_\beta \mathcal{S}_{\mathbb{W}} \mathbb{m}$, where $b_\beta = f(a_\beta)$, $\mathbb{mm} = f(\mathbb{m})$ and $\mathbb{m} \neq 0$.

Equivalently, $f: \mathbb{W} \to \mathbb{M}$ is said to be F$\mathcal{S}$-closed if $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} \mathbb{m}$ implies $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$ or $a_\beta \ll 1 - \mathbb{m}$ implies $b_\beta \ll 1 - \mathbb{mm}$ where $b_\beta = f(a_\beta)$, $\mathbb{mm} = f(\mathbb{m})$ and $\mathbb{m} \neq 0$.

Proposition 3.9: An injective mapping $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is $F\mathcal{S}$-closed if and only if it is $\mathcal{S}_{\mathbb{M}}$ is a fuzzy $\mathbb{K}$-proximity on $I^{\mathbb{M}}$.

Proof: Let $b_\beta \mathcal{S}_{\mathbb{M}} \mathbb{mm}$, then $a_\beta \mathcal{S}_{\mathbb{W}} \mathbb{m}$ where $b_\beta = f(a_\beta)$ and $\mathbb{mm} = 1 - f(1-\mathbb{m})$, since f is injective then $\mathbb{mm} = f(\mathbb{m})$, that is $F\mathcal{S}$-closed.

Theorem 3.10: Let f be a injective mapping from a fuzzy $\mathbb{K}$-proximity space $(\mathbb{W}, \mathcal{S}_{\mathbb{W}})$ into Y, the coarsest fuzzy $\mathbb{K}$-proximity $\mathcal{S}_{\mathbb{M}}$ which be assigned to $\mathbb{M}$ in order that f be fuzzy $\mathbb{K}$-proximally open is defined by $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$ if and only if there exists $\mathbb{m}^* \in I^{\mathbb{W}}$ s.t $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*)$ and $f(\mathbb{m}^*) \in 1 - \mathbb{mm}$, where $b_\beta = f(a_\beta)$, $\mathbb{mm} = f(\mathbb{m})$.

Proof: First, we must prove that $\mathcal{S}_{\mathbb{M}}$ is a fuzzy $\mathbb{K}$-proximity on $\mathbb{M}$.

K1- Let $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} (\mathbb{mm}_1 \vee \mathbb{mm}_2)$, then there exists $\mathbb{m}^* \in I^X$ s.t. $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*)$ and $f(\mathbb{m}^*) \in 1 - \mathbb{mm}_1 \vee \mathbb{mm}_2)$, where $b_\beta = f(a_\beta)$, $\mathbb{mm}_1 \vee \mathbb{mm}_2 = 1 - f(1 - \mathbb{m}\ )$ from which $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}_1$ and $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}_2$ follow. If $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}_1$ and $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}_2$, then there exists $\mathbb{m}^*{}_1, \mathbb{m}^*{}_2 \in I^{\mathbb{W}}$ s.t. $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*{}_1)$, $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*{}_2)$ and $f(\mathbb{m}^*{}_1) \in 1 - \mathbb{mm}_1$, $f(\mathbb{m}^*{}_2) \in 1 - \mathbb{mm}_2$, then $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} (1 - (\mathbb{m}^*{}_1 \vee \mathbb{m}^*{}_2))$ and $f(\mathbb{m}^*{}_1 \vee \mathbb{m}^*{}_2) \in 1 - (\mathbb{mm}_1 \vee \mathbb{mm}_2)$, that is $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} (\mathbb{mm}_1 \vee \mathbb{mm}_2)$.

K2- $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} 0$ is clear for all b_β.

K3- If $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$, then there exists $\mathbb{m}^* \in I^{\mathbb{W}}$ s.t. $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*)$ and $f(\mathbb{m}^*) \in 1 - \mathbb{mm}$. Thus $a_\beta \notin (1 - \mathbb{m}^*)$ and then $b_\beta \notin 1 - f(\mathbb{m}^*)$ also $b_\beta \in 1 - \mathbb{mm}$, this prove that $b_\beta \notin \mathbb{mm}$.

K4- If $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$, then there exists $\mathbb{m}^* \in I^{\mathbb{W}}$ s.t. $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*)$ and $f(\mathbb{m}^*) \in 1 - \mathbb{mm}$. Since $\mathcal{S}_{\mathbb{W}}$ satisfies condition (K4), then there exists $\mathbb{m} \in I^{\mathbb{W}}$ s.t. $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} \mathbb{m}$ and $a_\beta{}^* \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*)$ for all $a_\beta{}^* \in 1 - \mathbb{m}$. Assume $f(\mathbb{m}\ ) = \rho$ then $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$, since $a_\beta{}^* \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}\ ^*)$ and $\mathbb{m}^* \in f^{-1}(f(\mathbb{m}^*)) \in 1 - f^{-1}(\mathbb{mm})$, then $(1 - \mathbb{m}\ ) \bar{\mathcal{S}}_{\mathbb{W}} f^{-1}(\mathbb{mm})$ which implies $(1 - f(\mathbb{m}\ )) \bar{\delta}_Y \mathbb{mm}$ i.e. $b_\beta{}^* \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$ for all $b_\beta{}^* \in (1 - \rho)$.

Now, to show that $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is fuzzy $\mathbb{K}$-proximally open. Suppose that $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} \mathbb{m}$ i.e. $a_\beta \ll 1 - \mathbb{m}$ by proposition (2.9) then there exists $\mathbb{m}^* \in I^{\mathbb{W}}$ s.t $a_\beta \ll \mathbb{m}^* \ll 1 - \mathbb{m}$. Thus, $a_\beta \bar{\mathcal{S}}_{\mathbb{W}} \mathbb{m}^*$ and $\mathbb{m}^* \bar{\mathcal{S}}_{\mathbb{W}} \mathbb{m}$ that is $\mathbb{m}^* \in 1 - \mathbb{m}$, if $\mathbb{mm} = f(\mathbb{m})$ then $f(\mathbb{m}^*) \in 1 - \mathbb{mm}$, by assumption $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$ and this prove that f is F$\mathcal{S}$-open.

It remains to show that $\mathcal{S}_{\mathbb{M}}$ coarsest fuzzy $\mathbb{K}$-proximity. Let $\acute{\mathcal{S}}_{\mathbb{M}}$ be any fuzzy $\mathbb{K}$-proximally on $\mathbb{M}$ s.t $f: (\mathbb{W}, \mathcal{S}_{\mathbb{W}}) \to (\mathbb{M}, \mathcal{S}_{\mathbb{M}})$ is fuzzy $\mathbb{K}$-proximally open mapping. If $b_\beta \bar{\mathcal{S}}_{\mathbb{M}} \mathbb{mm}$, then there exists $\mathbb{m}^* \in I^{\mathbb{W}}$ s.t. $a_\beta{}^* \bar{\mathcal{S}}_{\mathbb{W}} (1 - \mathbb{m}^*)$ and $f(\mathbb{m}^*) \in 1 - \mathbb{mm}$. Since f is F$\mathcal{S}$-open then $b_\beta \bar{\acute{\mathcal{S}}}_{\mathbb{M}} (1 - f(\mathbb{m}^*))$ but $\mathbb{mm} \in 1 - f(\mathbb{m}^*)$, then $b_\beta \bar{\acute{\mathcal{S}}}_{\mathbb{M}} \mathbb{mm}$, hence $\mathcal{S}_{\mathbb{M}} < \acute{\mathcal{S}}_{\mathbb{M}}$.

4. Conclusions and future studies

The result of this paper initiated was the definition of the fuzzy 𝕂-proximity open mapping and relationship with fuzzy 𝕂-proximity continuous. In addition, new fuzzy a 𝕂-proximity relation was defined on the basis of this mapping. In the future it is possible to link this mapping to the fuzzy 𝕂-proximity compact.

References

[1] L.A. Zadah, Fuzzy sets, Informs. Contr.8 (1965), 333-353.
[2] C.L. Chang, Fuzzy topological space, J. Math. Anal.Appl. 24 (1968), 182-190.
[3] A. K. Katsaras, Fuzzy proximity spaces, J. Math. Anal.Appl. 68 (1979), 100-110.
[4] P-M. Pu- and Y-M. Lin, Fuzzy topology 1, J. Math. Anal. Appl. 76 (1980), 571-599.
[5] C. K. Wong, Fuzzy points and Local properties of fuzzy topology, J. Math. Anal. Appl. 46 (1974), 316-328.
[6] K. D. Park, On the Fuzzy 𝕂-proximities, J. Nat.Sci. Res. Inst. Dongguk Univ. 14 (1994), 19-24.
[7] K. D. Park, Fuzzy 𝕂-proximities Mapping, Kangweon-Kyungki Math Jour. 14 (2006), 7-11.