Eng. & Technology, Vol.25, No.5, 2007

Image Denoising Using Hybrid Transforms

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Received on: 24/7/2005
Accepted on: 20/5/2007

Abstract

In this paper a new family of transformation for image denoising is presented, Multiridgelet and Walidlet transforms, which have been proposed as alternatives to Discrete Wavelet and Multiwavelet transforms.

Walidlet transform is an intelligent tool for solving image processing problems such as image denoising with straight edges, the general algorithm of image denoising using discrete multiwavelet transform is introduced, then followed by the general algorithm of image denoising using Walidlet transform that is proposed here in order to achieve better results than that for Walidlet thresholding, finally, a comparative study is presented to show the differences between a mentioned algorithms.

Keywords: 2-D Discrete Multiridgelet Transform, 2-D Discrete Walidlet Transform.

ازالة الضوضاء من الصورة باستخدام التحويلات الهجينة

الخلاصة:

تتناول هذه المقالة مجموعة جديدة من التحويل لغرض ازالة الضوضاء من الصورة وهي تحويلات (ملتيرجليت Multiridgelet ووليدليت Walidlet), اللذين تم اقتراحهما كبدائل لتحويل الموجة المتقطع Wavelet وتحويل المتعدد المويجات Multiwavelet.

ان تحويل وليدليت هو اداة ذكية لحل مشاكل معالجة الصورة مثل ازالة الضوضاء من الصورة مع تقوية حواف الصورة , حيث تم عرض الخوارزمية العامة لازالة الضوضاء من الصورة باستخدام تحويل متعدد المويجات وتحويل وليدليت المقترح مع تطبيق عملية العتبة (Thresholding) للحصول على افضل النتائج الممكنة, واخيرا دراسة المقارنة بين الخوارزميتين المذكورتين اعلاه واظهار الفرق في نتائج هاتين الخوارزميتين.

1. Introduction

The transformation is the process that processes the spatial domain of the signal and translates it to another domain [1], this processing isolates the approximation information (which represents the intelligent information) from the details (which contain the noise), such as discrete Fourier transform, discrete Wavelet transform, discrete Multiwavelet transform, discrete Ridgelet transform and so on.

Denoising of images is an important task in image processing and analysis, and it plays a significant role in modern applications in different fields,

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http s://doi.org/10.30684/etj.25.5.7
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including medical imaging and preprocessing for computer vision. Denoising goal is to remove that noise, resulting in minimal damage to the image. Since in most cases the result consumer is human, the criterion for denoising fidelity would be the human visual perception of the result, rather than any of known mathematical criteria [2].

With the fastest growing areas of applying these transforms in multiple domains, here it is necessary to introduce a new transform with high performance and intelligent properties to improve the performance of the previous transforms. Therefore, we proposed a new hybrid transform for image denoising.

The main structure of the proposed transform contains two fundamental transforms, these are: two dimensions discrete multiwavelet transform (2-D DMWT) and a new proposed local transform named multiridgelet transform (2-D DMRT) , in other words, hybrid the properties of the (2-D DMWT with the 2-D DMRT) in one transform named Walidlet transform.

2. Multiridgelet Transform

To improve the performance of the discrete Multiwavelet and Rideglet transforms and to overcome the weakness points of these transforms, a new hybrid transform is proposed, named the Multiridgelet transform.

The main idea of the Ridgelet transform is to map a line sampling scheme into a point sampling scheme using the Radon transform,then the Wavelet transform can be used to effectively handle the point sampling scheme in the Radon domain[3]. On the other hand, the main idea of Multiridgelet transform depends on the Ridgelet transform with changing the second part of this transform with Multiwavelet transform to improve the performance and output quality of the Ridgelet transform.

In fact, the Multiridgelet transform leads to a large family of orthonormal and directional bases for digital images, including adaptive schemas, However, the Multiridgelet transform overcomes the weakness point of the wavelet and Ridgelet transforms in higher dimensions, since, the wavelet transform in two dimensions is obtained by a tensor-product of one dimensional wavelets and they are thus good at isolating the discontinuity across an edge, but will not see the smoothness along edge.

The geometrical structure of the Multiridgelet transform consists of two fundamental parts, these are :

a- The Radon Transform .
b- The One Dimensional Mltiwavelet Transform .

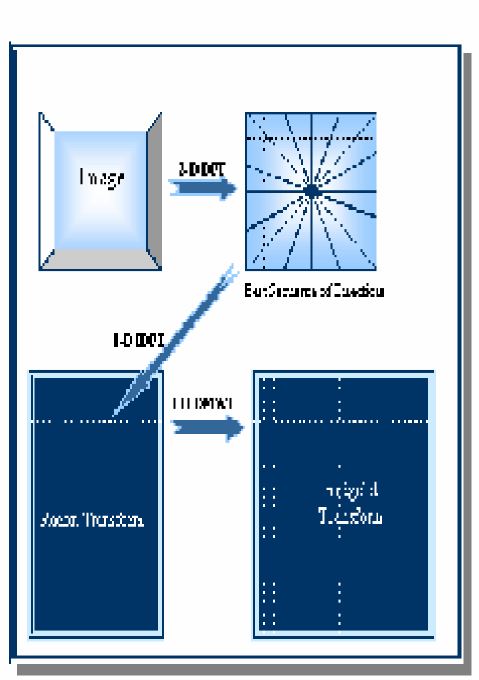

Figure (1) : Multiridgelet Transform

A. The Radon Transform

The radon function in the image processing computes projections of an image matrix along specified directions. A projection of a two-dimensional function f(x,y) is a set of line integrals. The radon function computes the line integrals from multiple sources along parallel paths, or beams, in a certain direction. The beams are spaced 1 pixel unit apart. To represent an image, the radon function takes multiple, parallel-beam projections of the image from different angles by rotating the source around the center of the image. The following figure shows a single projection at a specified rotation angle.

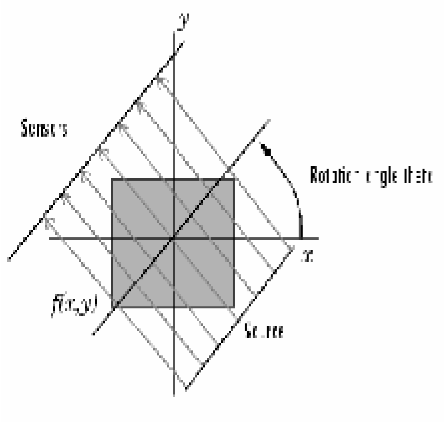

Figure (2): Parallel-Beam Projection at Angle Theta

For example, the line integral of f(x,y) in the vertical direction is the projection of f(x,y) onto the x-axis; the line integral in the horizontal direction is the projection of f(x,y) onto the y-axis [4], as shown in figure (3).

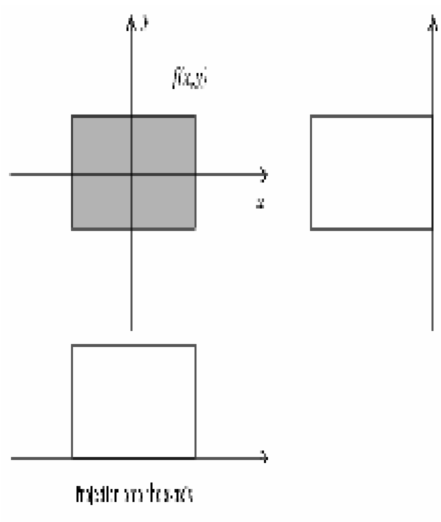

Figure (3): Horizontal and Vertical Projections of Simple Function

B. The One Dimensional Discrete Multiwavelet Transform (1-D DMWT)

Wavelet is a useful tool for signal processing applications such as image processing. Until recently, only scalar wavelets were known: wavelets generated by one scaling function. However, one can imagine a situation when there is more than one scaling function. This leads to the notion of muliwavelets, which have several advantages in comparison to scalar wavelets.

Such features such as short support, orthogonality, symmetry, and vanishing moments are known to be important in signal processing. A scalar wavelet cannot possess all these properties at the same time.

On the other hand, a multiwavelet system can have all of them simultaneously and provide good reconstruction while preserving length (orthogonality) good performance at the boundaries (via linear-phase symmetry), and a high order of approximation (vanishing moments).

Thus, multiwavelets offer the possibility of superior performance for image processing applications, compared with scalar wavelets [5].

Therefore the 1-D DMWT was applied on each slice of the Radon transform, in the other word the 1-D DMWT was applied on each row on the Radon transform coefficients, to get the Multiridgelet coefficients.

3. Walidlet transform

To improve the performance of the multiridgelet transform and make it very effective in the multiple purposes, such as image enhancement via (denoising, sharpening, or smoothing), image reconstruction, image compression, image or signal encryption and so on, it is necessary to create a new transform, which is named, Walidlet transform [6]. It is the hybrid transform, since, the geometrical structure of this transform consists of two fundamental local transforms, these are:

a. *Two Dimensions Discrete Multiwavelet Transform (2-D DMWT).*
b. *Two Dimensions Discrete Multiridgelet Transform (2-D DMRT).*

The intelligent idea behind Walidlet transform is grouping the good properties of the two local transforms in one hybrid transform to obtain a new transform with strong and intelligent properties.

The performance of this transform is very well in many domains, and we expect it becomes very useful tool in different cases.

The algorithms of the new hybrid transforms (Walidlet, local Multiridgelet transforms and best sequence of directions) are described in the next paragraphs.

4. A General Algorithm For Computing Multiridgelet Transform

To compute Multiridgelet transform, the next steps should be followed :

1. **Resizing :** Here it is necessary to check for image dimensions, image matrix should be a square matrix, N*N matrix, where N must be the power of two. If the image is not a square matrix, a zero padding operation should be performed to the image (adding rows or columns of zeros to get a square matrix and they must be a prime number).
2. **Go to 2-D DFT :** Computing two dimensions Fast Fourier Transform.
3. **Save DC component :** Store DC component and replace its position with zero.
4. **Computing the best sequence of directions :** described in sec .5 .
5. **Compute the Fourier slices :** Finding the Fourier slices that must be taken.
6. **Rearranging the coefficients matrix :** Sorting the result matrix, which results from 2_D DFT according to the new positions of computing the best sequence of directions [6].
7. **Back to the spatial domain :** Applying 1-D IDFT to each row. The result is (Radon Transform).
8. **Resizing :** Image matrix should be a square matrix, as mentioned before.
9. **Applying 1_D DMWT :** Here it is required to apply the one dimensional discrete multiwaelet transform (1_D DMWT) for each row. The result matrix is Multiridgelet Transform coefficients.

Figure (4) exhibits the flowchart of proposed local transform (Multiridgelet Transform).

5. A General Algorithm For Computing The Best Sequence of Directions

To compute the best sequence of directions, the following steps should be followed:

1. Resizing, image matrix should be a square matrix, N*N matrix, and image dimensions should be a prime numbers, as mentioned before.
2. Construct the matrix of points (K, L) that belongs to slices, don't include point (K , L) =(0 , 0) .
3. Add horizontal and vertical slices.
4. Centralize these slices mod (the size of the transform).
5. Polarization, converting from cartesian to polar coordinate .
6. Find the pair (K, L) for each slices that has minimum max(abs(K) , abs(L)) and positive angle [6] :
   a. Construct the max (abs (K), abs(L)) and put it in

a new matrix, such as D matrix .

b. Eliminate points with negative angles .

c. Construct the minimum value of matrix D and store the result in [Y,I].

Y and I values are very important because the end result depends on them, where:

Y : The lower value of each column in the D matrix .

I : The position of this lower value Y, in which row it locate .

7. Retrieve those points and put them in a set of directions .
8. Add horizontal and vertical slices .
9. Rearrange or sort those directions in ascending order of angles, the result is the best sequence of directions.

Figure (4.a) exhibits the subroutine of computing the best sequence of directions.

6. A General Algorithm For Computing Walidlet Transform

This transform is proposed here for image enhancement, it has intelligent mathematical properties. The next steps exhibit the sequence of this algorithm :

1. Resizing**,** as mentioned before.

2. Applying 2-D DMWT : Here it is required to apply the two dimensions discrete multiwavelet transform (2_D DMWT), repeated row method of compution to each image.

3. Subband decomposition, as mentioned before.

4. The Multiridgelet transform : Each subband is analyzed via the discrete multiridgelet transform, described previously.

5. Integration : Grouping the multiridgelet coefficients of each subband in the result matrix according to their positions (LL**,** LH**;** HL**,** HH)**.** The result matrix is called the Walidlet coefficients.

Figure(5) exhibits the flow diagram of the hybrid transform (Discrete Walidlet Transform).

7. A General Algorithm For Computing Inverse Multiridgelet Transform

To compute the Inverse Multiridgelet transform, the next steps should be followed:

1. **Apply 1_D IDMWT :** Here it is required to apply the one dimension inverse discrete multiwaelet transform (1_D IDMWT) for each row.
2. **Inverse resizing :** Check coefficients matrix length, image length should be a prime number. If the coefficients matrix length is not a prime number, a zero padding operations should be performed to the coefficients matrix size, such as removing rows and columns from the coefficients matrix (which

are added in the forward steps).

3. **Apply 1-D DFT :** Here it is required to apply the one dimensional discrete Fourier transform .
4. **Compute the best sequence of directions:** The same geometrical algorithm of computing the best sequence of directions was applying in the forward steps and backward steps, described previously.
5. **Compute the Fourier slices:** Find the Fourier slices that must be taken.
6. **Rearrang Fourier slices:** Sort the Fourier slices according to the new positions, getting from compute the best sequence of directions is obtained.
7. **Assign back DC component :** Here it is necessary to assign back DC component to its original position.
8. **Inverse resizing :** Checking coefficients matrix length, image length should be a prime number. If the coefficients matrix length is not a prime number, a zero padding operations should be performed to the coefficients matrix size, such as removing rows and columns from the coefficients matrix (which are added in the forward steps).
9. **Apply 2_D DFT :** Here it is required to apply the two dimensions discrete Fourier transform (2-D DFT).

8. A General Algorithm For Computing Inverse Walidlet Transform

To reconstruct the original 2-D signal (N * N matrix) from the discrete Walidlets transformed 2-D signal (N * N matrix), the Inverse Discrete Walidlets Transform (IDWT) should be used. The next steps exhibit the sequence of this algorithm :

1. **Subband decomposition,** as mentioned before.
2. **Inverse Multiridgelet Transform for Each subband.**
3. **Integration :** Grouping the resultant subands from previous step in one matrix.
4. **Applying 2_D IDMWT :** Here it is required to apply the inverse two dimensions discrete multiwavelet transform (2-D IDMWT), using inverse repeated row method of computation to the result coefficients from the previous step, to have an N * N original reconstructed two dimensions signal matrix.

9. Denoising Algorithm Using Walidlet Transform

The algorithm of the proposed method is as follows :

1. Obtain the Walidlet Transform coefficients of the observed noisy image:

$$g^i_j = DWLT\ x \quad \ldots(1.1)$$

where :
DWLT: Discrete Walidlet Transform .
x :original image .
g^i_j :Walid Transform coefficients.
i , j : image dimensions .

2. Select the threshold type. If the selection is of soft thershold type, then filter the Walidlet transform coefficients using these equations:

$$G^i_j = T_h\left(g^i_j, Thv\right) = \begin{cases} sign\left(g^i_j\right)^* \left(\left|g^i_j\right| - Thv\right), \left|g^i_j\right| > Thv \\ 0 \qquad , \qquad Otherwise; \end{cases} \quad \ldots(1.2)$$

Or in other form [7]:

$$G^i_j = T_h\left(g^i_j, Thv\right) = \left\{ sign\left(g^i_j\right)^* \left(\left|g^i_j\right| - Thv\right), \left(\left|g^i_j\right| - Thv\right) \right. \quad \ldots(1.3)$$

where:

$$sign(g^i_j) = \begin{cases} +1 & , \text{ if } \quad g^i_j > 0 \\ 0 & , \text{ if } \quad g^i_j = 0 \\ -1 & , \text{ if } \quad g^i_j < 0 \end{cases}$$

where: Thv is the threshold value, ($Thv = \sqrt{2s_z^2 \log_e(N)}$ is used).

Hence, soft-thresholding is a mean of translating all coefficients towards zero by a certain amount defined by Thv.

3. If the selection is hard thershold "kill and keep" strategy [8] or "gating" [9] type, then filter the Walidlet transform coefficients using this equation:

$$G^i_j = T_h\left(g^i_j, Thv\right) = \begin{cases} g^i_j, \left|g^i_j\right| > Thv; \\ 0 \ , \ Otherwise; \end{cases} \quad \ldots(1.4)$$

4. Inverse the Walidlet Transform thresholded coefficients to get the denoised (reconstructed) image using the following equation :

$$\hat{X} = \left(DWLT\right)^{-1} \hat{G}^i_j \quad \ldots(1.5)$$

where:
$\hat{X}$: estimated image .
$(DWLT)^{-1}$: inverse Walidlet Transform .
z^i_k : thresholded Walidlet Transform coefficients .
i,j : image dimensions .

10. Multidenoising Algorithm using Walidlet Transform

The Walidlet transform is a new method proposed for image enhancement, it has intelligent mathematical properties, the next steps exhibit the sequence of this algorithm:

1. **Resizing,** as mentioned before.

2. **Applying 2-D DMWT :** Here it is required to apply the two dimensions discrete multiwavelet transform (2_D DMWT), repeated row method of compution to each

image with applying thresholding operation (soft or hard threshold).

3. **Subband decomposition :** The result matrix of step two is decomposed into four subbands.
4. **Applying Multiridgelet transform :** Each subband is analyzed via the discrete multiridgelet transform. described previously.
5. **Integration :** Grouping the resultant subbands from step (4) into one matrix.
6. **Applying thresholding operation :** Here it is necessary to applyy soft-thresholding or hard-thresholding operation to the result matrix (Walidlet coefficients) for image enhancement.
7. **Applying (IDWLT):** Here the inverse Walidlet transform must be applied to the thresholded coefficients to get the enhancement image (reconstructed image).

11. Conclusions

This paper, presents a strategy for digitally implementing a new hybrid transforms (Multiridgelet and Walidlet transforms) for image denoising. The resulting implementations have the exact reconstruction property, and give stable reconstruction under perturbations of the coefficients.

There are, of course, many compating strategies to translate the theoretical results on Multiridgelets and Walidlets into digital representations.
There are several innovative choices which we now highlight:

1. Walidlet transform is important technique in image denoising applications due to eliminate the noise. It takes the advantages of multiple transforms.

2. The level of decomposition of Walidlet transform and the thresholding scheme (e.g. soft or hard) are two other important parameters in Multiwavelet thresholding denoising algorithm. Number of decomposition level of Walidlet transform depends on the size of the observed data. However, as a rule of thumb, the number of decomposition levels of Walidlet transform is taken when the corresponding approximate Walidlet coefficients band at that scale is noiseless. For thresholding scheme, its choice depends on the application, considering if oversmoothness or MSE is tolerated more.

3. The performance of Multidenoising algorithm (using Walidlet transform) is compared with the denoising algorithm using Walidlet transform. The results of the

comparison can be listed in the following items:

a. The proposed algorithm of multidenoising using Walidlet transform gives better result than proposed algorithm of denoising using Walidlet transform in term of PSNR at least for moderate and large values of noise. It gives reconstructions which exhibit higher perceptual quality than denoising algorithm based reconstructions.

b. Computational complexity (time) of the Multidenoising Walidlet algorithm for image denoising is more than that of denoising Walidlet algorithm depending on number of using thresholding operation and number of decomposition levels .

4. As a result, image denoising algorithms introduced in this paper can be ranked according to its visual quality of performance as shown in table (1).

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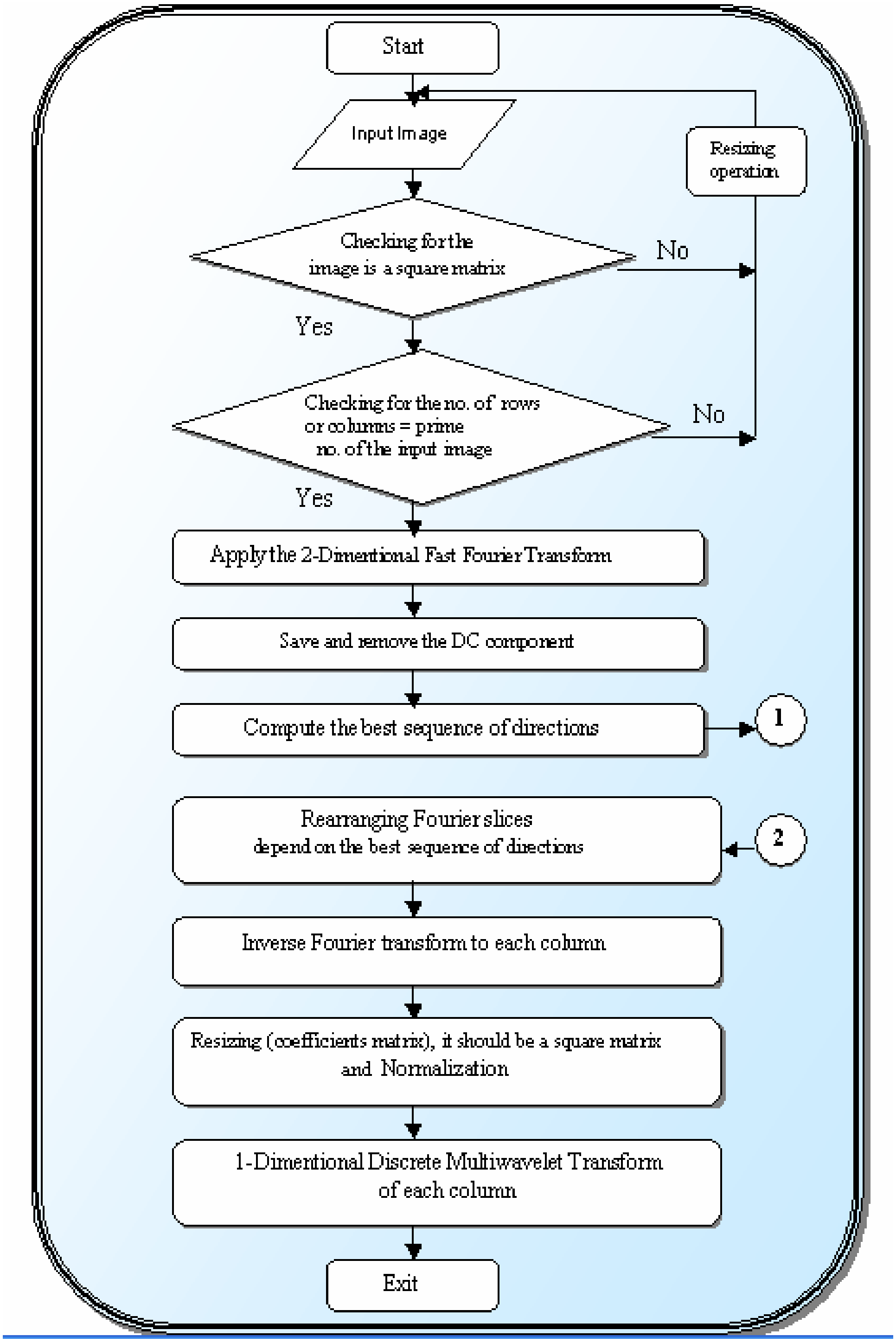
Table (1): MSE, PSNR, and SNR results of the filtered image with Multiwavelet transform (denoising algorithm), Walidlet transform (denoising algorithm) and Walidlet transform (multidenoising algorithm).

Noisy Image			
Image	MSE dB	PSNR dB	SNR dB
1	0.0107	26.6746	10.9746
2	0.0093	24.4635	8.5764
3	0.0081	19.5895	5.0287
4	0.0072	14.8490	2.5958
Denoising Image by Using Multiwavelet Transform			
Image	MSE dB	PSNR dB	SNR dB
1	0.0062	54.6452	23.9474
2	0.0040	49.4645	20.3736
3	0.0038	40.4698	18.9831
4	0.0031	33.4746	16.8469
Denoising Image by Using Walidlet Transform			
Image	MSE dB	PSNR dB	SNR dB
1	0.0057	63.8534	29.9845
2	0.0032	58.6790	27.0029
3	0.0029	46.5323	22.9759
4	0.0025	39.4986	19.9354
Multidenoising Image by Using Walidlet Transform			
Image	MSE dB	PSNR dB	SNR dB
1	0.0051	67.2479	32.0987
2	0.0028	61.8542	30.7839
3	0.0024	50.7905	26.0972
4	0.0020	41.9986	22.9948



Figure(4): Flowchart of proposed local transform (Multiridgelet Transform).

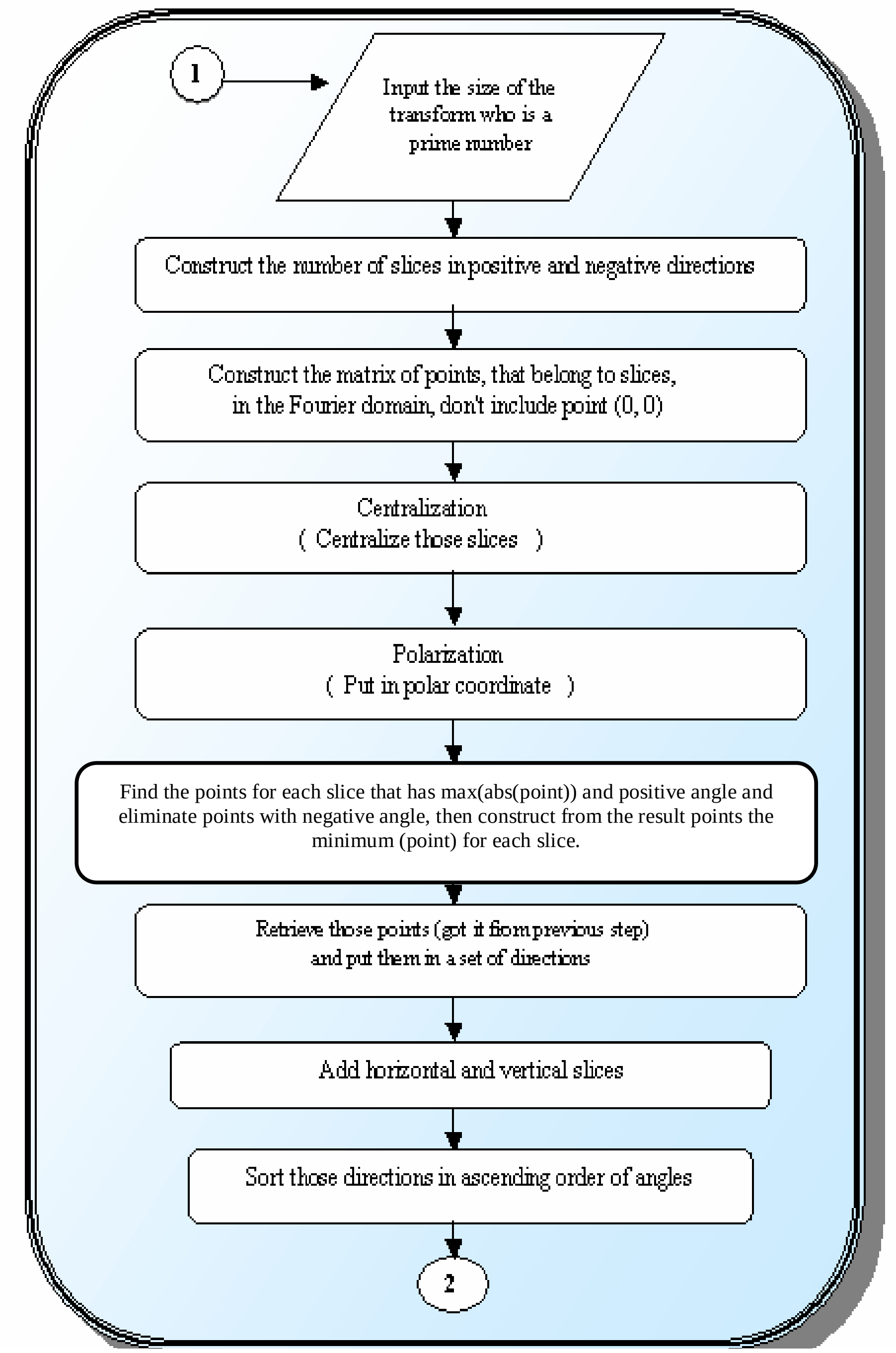


Figure (4.a): The flowchart which represent the subroutine of computing the best sequence of directions.

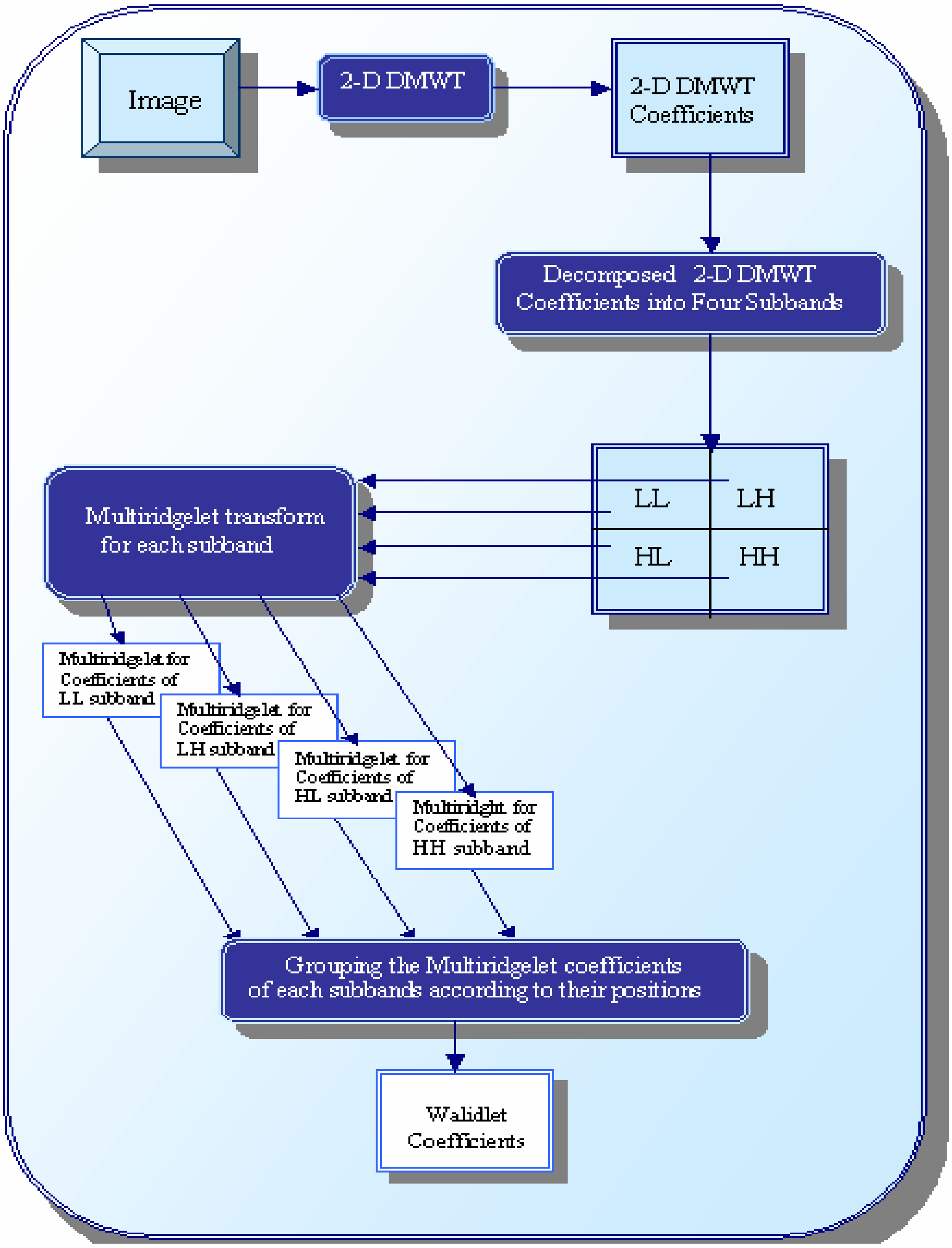


Figure (5): Flow diagram of the hybrid transform (Discrete Walidlet Transform).