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Estimating the Gini Index from Beta-Normal Distribution

Authors Names	**ABSTRACT**
Waleed Ahmed Hassen Al-Nuaami ***Publication data:*** 11/5/2024 ***Keywords****: Beta distribution; Cumulative distribution function; Gini coefficient; Normal distribution*	This paper introduces the idea for estimating the Gini index from the beta-normal distribution. The value of the Gini coefficient depends depend on the value of the parameters. By comparing the results obtained from the simulated Gini coefficient, it can be concluded that in general, as the amount of sample size increases, the Gini coefficient increases. The results revealed that, when the amount of the coefficient is close to zero, it approaches the balance between the distribution of wealth and income in the society.

1. Introduction

Various numerical indicators such as Solow's coefficient Soltow [1], Broun's coefficient of change [2], Theil's index [3], Atkinson's ratio [4], and Nelson's ratio [5] exist to express the inequality or variability of incomes among members of a given community. Indicators based on Lorenz curves, however, are among the most used indicators. The Gini coefficient is one of the most widely used indicators based on the Lorenz curve in measuring income inequality. The Gini coefficient is also used to study inequalities in health [6] and inequality over the lifetime of various age groups in tables [7] and [8]. Although some recent methods for estimation of variance have been developed, very common indicators such as the Gini coefficient are employed, for comparative purposes and trend analysis without taking the sampling changes into account, because point estimates can be easily obtained through these coefficients.

The Gini coefficient which can be achieved from the Gini Mean Difference [9] is widely regarded as one of the most well-known income inequality measurement methods. The Gini coefficient is also shown in the area between the 45° line and the Lorenz curve. The Lorenz curve proposed by Lorenzo [10] is another method of measuring the inequality. The Gini coefficient of zero (line 45 degrees) represents a perfect equality, so that all values are equal (for example, everyone has the same income), while the Gini coefficient of one (or 100) represents the maximum inequality between values (For example, among many people, only one is capitalist, while the rest do not have any income, in this case the Gini coefficient will be close to 1). The existence of widespread inequalities in income distribution leads to poverty and increases in its domestic level, as well as growing class gaps in societies. The income distribution describes the degree of inequality in individuals' income within a country. The phenomenon of income inequality is not only a major cause of poverty in developing countries, but also slows down the economic growth. Hence, discussions and judgments about the reciprocal effects of economic growth and income distribution along with the expansion of growth models are so important.

In statistics, there are many distributions that are used to model the distribution of random variables, but among all probability distributions, continues distributions with non-negative values that are generally skewed, are used to model income distributions. As few economic characteristics, such as income, take just non-negative values, it is necessary to use distributions which work with just positive values. In this

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regard, generalized distributions are used as families of such high-flexibility distributions to examine these economic characteristics. generalized beta distribution is located at the head of these distributions. Several papers have made valuable contributions to the understanding of income distribution and economic inequality. Sarabia and Castillo (2005) [11] focus on max-stable families and their applications to income distributions, highlighting their significance in modeling extreme events.

2. Gini Coefficient and beta-normal Distribution

*2.1 **Gini Coefficient***

The Gini coefficient is derived from the name Gini [19]–[21]. In the past years, the Gini coefficient has gradually become the main indicator of inequality in the economy. Statisticians use this index in many empirical studies and political research.

Anand [22] and Chakravarty [23] provided understandable research on the inequality measure that included the Gini coefficient. Other authors, such as Lambert [24], Silber [25], Atkinson and Bourguignon [26], provided understandable references to income inequality and poverty using a Gini coefficient as a measure of inequality. The Gini coefficient is used to measure the dispersion of income distribution, consumption distribution, distribution of wealth or distribution of other types of economic indicators.

Gini coefficient has different forms and interesting interpretations. For example, it can be expressed as the ratio of the two regions defined by the 45° line and the Lorenz curve in a single unit, as the different mean Gini function, as the covariance between earnings and their ranks, or as a particular type of matrix form.

The Gini index is one of the most common statistical indicators of diversity and inequality to measure the distribution of dispersion in Allison's sociology and social sciences. Furthermore, the Gini index is widely used in econometrics as a standard measure of inequality between individuals or between individuals in terms of income and wealth; Atkinson [4], Sen [27] and Anand [22].

The Gini coefficient is a complex inequality criterion that measures the inequality through the Lorenz curve. Figure (1) shows the Lorenz curve defined on (0,1) and continuously increases from (0, 0) to (1, 1).

To draw the Lorenz diagram, data are ranked from small to large, and then their cumulative distributions are calculated and finally are put against the cumulative ratio. In the Lorenz curve, the Gini coefficient (G) is the area between the Lorenz curve and the steady gradient.

$$G = \frac{A}{A+B} = 2A = 1 - 2B$$

It ranges from 0, in which all measurements are equal and represents perfect equality to $\frac{n-1}{n}$, with n representing a sample size and all measurements except one is 0. For a large sample, the final value of 1 represents the maximum possible inequality. Full reviews of the Gini coefficient can be found in Gini [21] and Bellu and Liberati [28].

The concept of the Gini index or "Gini coefficient" is closely related to the concept of the Lorenz curve. The Gini coefficient is a number between zero and one equal to the "enclosed area" between the Lorenz curve and the "perfect equality distribution line". Whenever resources and wealth of a society are distributed "quite fair" among all individuals, Lorenz curve reaches to "perfect equality line", and the Gini coefficient becomes zero. Conversely, in the case of an "absolutely unequal distribution of wealth" in a society or absolute monopoly (all the wealth of the community is handed to one person, while others have a zero-sum wealth), the Gini coefficient will be equal to one.

2.2 Beta-Normal Distribution

Eugene et al. [29] presented the beta-normal distribution by combining the beta distribution and the normal distribution. This distribution extends the normal distribution and offers more versatility in terms of its shapes, making it applicable to various scenarios. Subsequently, numerous authors have generalized similar distributions to the beta-normal.

Beta-normal distribution with parameters $\alpha > 0,\ \beta > 0,\ \mu \in \mathbb{R},\ \sigma > 0$, the probability density function (pdf) is given as follows:

$$f(x) = \frac{1}{B(\alpha,\beta)}\left[\Phi\left(\frac{x-\mu}{\sigma}\right)\right]^{\alpha-1}\left[1-\Phi\left(\frac{x-\mu}{\sigma}\right)\right]^{\beta-1}\sigma^{-1}\phi\left(\frac{x-\mu}{\sigma}\right), x \in \mathbb{R} \tag{1}$$

In this regard, $\Phi(.)$ and $\phi(.)$ are respectively cumulative distribution function and density function of the probability of standard normal distribution, and $B(.,.)$ is a Beta function. α and β are shape parameters that describe skewness, kurtosis, and bimodal state of equation (1). μ is a location parameter and σ is a scale parameter which indicates the amount of expansion and contraction of the shape. Beta-Normal distributions can be unimodal or bimodal to skew distributions.

A category of generalized Beta distribution were first introduced from its cumulative distribution function by [30]. Cumulative distribution function of a category of generalized beta distributions is defined using the following function:

$$F(x) = \frac{1}{B(\alpha,\beta)}\int_0^{G(x)} t^{\alpha-1}(1-t)^{\beta-1}dt, \alpha > 0, \beta > 0$$

where, $G(x)$ is a cumulative distribution function of a family of random variables, and $B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx$.

Barreto et al. [31], presented an important extension to beta Frechet (BF) distribution to conclude some general properties of this category. Suppose that, β is a real non integer number and $|x| < 1$. Consider the extension of power series as follows:

$$(1-x)^{\beta-1} = \sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(\beta)}{\Gamma(\beta-j)j!}x^j.\beta \tag{2}$$

As a result, the incomplete beta function can be expressed as follows:

$$B_x(\alpha, \beta) = x^{\alpha} \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(\beta) x^j}{\Gamma(\beta - j) j! (j + \alpha)} \tag{3}$$

In the above formula, $\beta - 1$ will be the upper limit of j, if β is defined as an integer. Defining the constant,

$$w_j(\alpha, \beta) = \frac{(-1)^j \Gamma(\beta)}{\Gamma(\beta - j) j! (j + \alpha)} \tag{4}$$

and using the relations (3) and (4), the cumulative distribution function of the β-Gamma distribution is expressed as follows:

$$F(x) = \frac{1}{B(\alpha, \beta)} \sum_{r=0}^{\infty} w_r(\alpha, \beta) G(x)^{\alpha + r} \tag{5}$$

Deriving (5), f(x) is extended as follows:

$$f(x) = \frac{1}{B(\alpha, \beta)} g(x) \sum_{r=0}^{\infty} (\alpha + r) w_r(\alpha, \beta) [G(x)]^{\alpha + r - 1} \tag{6}$$

Using the relation (2) twice, the expansion $[G(x)]^{\alpha}$ for non-integer α, is obtained as follows:

$$G(x)^{\alpha} = \left(1 - \left(1 - G(x)\right)\right)^{\alpha} = \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(\alpha + 1)}{\Gamma(\alpha - j + 1) j!} \left(1 - G(x)\right)^j$$

$$= \sum_{j=0}^{\infty} \sum_{r=0}^{j} \frac{(-1)^{j+r} \Gamma(\alpha + 1)}{\Gamma(\alpha - j + 1)(j - r)!\, r!} \left(G(x)\right)^r$$

$$= \sum_{j=0}^{\infty} \sum_{r=0}^{\infty} \frac{(-1)^{j+r} \Gamma(\alpha + 1)}{\Gamma(\alpha - j + 1)(j - r)!\, r!} \left(G(x)\right)^r \tag{7}$$

We assume that

$$S_r(\alpha) = \sum_{j=r}^{\infty} \frac{(-1)^{j+r} \Gamma(\alpha + 1)}{\Gamma(\alpha - j + 1)(j - r)!\, r!} \tag{8}$$

Consequently, using relation (8), the extension (7) is expressed as follows:

$$G(x)^{\alpha} = \sum_{r=0}^{\infty} S_r(a) \left(G(x)\right)^r \tag{9}$$

If α is not an integer, then the equation (9) of the distribution function (5) can be obtained as follows:

$$F(x) = \frac{1}{B(\alpha, \beta)} \sum_{r=0}^{\infty} t_r(\alpha, \beta) G(x)^r \tag{10}$$

where $t_r(\alpha,\beta) = \sum_{l=0}^{\infty} w_l(\alpha,\beta)s_r(\alpha+l)$. By a simple derivation of (10), an extension f(x) for a non-integer number, α is as follows:

$$f(x) = \frac{1}{B(\alpha,\beta)} g(x) \sum_{r=0}^{\infty} (r+1)t_{r+1}(\alpha,\beta)G(x)^r. \qquad (11)$$

The purpose of this period is to formulate a density expansion, when an integer α is unstable to obtain power series with G(x), which is expressed only for the correct powers. This proposition follows the main result of the paper [32].

In general, exacting first moments of a beta-normal distribution cannot be obtained. However, for some selected values of α and β, some exact first moments are obtained which are shown in the Table 2.1 [33].

Table 2.1: Exact first moments of a beta-normal for some values of α and β

β	α	μ_{BN}	σ_{BN}	Skewness	Kurtosis	Shape
0.05	0.05	0.0000	5.9484	0.0000	2.2091	Bimodal
	0.10	2.2859	4.8818	-0.0882	2.4224	Bimodal
	0.20	3.6896	4.0453	0.0025	2.7492	Unimodal
	0.50	4.6527	3.3957	0.2964	3.0046	Unimodal
	1.00	5.0344	3.1478	0.4778	3.1175	Unimodal
	10.00	5.7016	2.7983	0.7478	3.4176	Unimodal
	100.00	6.1625	2.6164	0.8726	3.6557	Unimodal
0.10	0.05	-2.2895	4.8818	0.0882	2.4224	Bimodal
	0.10	0.0000	4.0327	0.0000	2.3520	Bimodal
	0.20	1.6580	3.2951	0.0205-	2.5558	Unimodal
	0.50	2.7633	2.6144	0.1883	2.8898	Unimodal
	1.00	3.2639	2.3227	0.4011	3.0670	Unimodal
	10.0	4.1331	1.9151	0.7848	3.5467	Unimodal
	100.00	4.7070	1.7251	0.9561	3.9220	Unimodal
0.20	0.05	-3.6896	4.0453	-0.0025	2.7492	Unimodal
	0.10	-1.6580	3.2951	0.0205	2.5558	Unimodal
	0.20	0.0000	2.6797	0.0000	2.5582	Bimodal
	0.50	1.3407	2.0541	0.1081	2.8119	Unimodal
	1.00	1.9558	1.7481	0.3009	3.0181	Unimodal
	10.0	3.0485	1.3035	0.7769	3.6591	Unimodal
	100.00	3.7410	1.1165	0.9974	4.1825	Unimodal
0.50	0.05	-04.6527	3.3957	-0.2964	3.0046	Unimodal
	0.10	-2.7633	2.6144	-0.1883	2.8898	Unimodal
	0.20	-1.3407	2.0541	-0.1081	2.8119	Unimodal
	0.50	0.0000	1.5253	0.0000	2.8616	Unimodal
	1.00	0.7043	1.2479	0.1372	2.9831	Unimodal
	10.0	2.0809	0.8033	0.6172	3.5732	Unimodal
	100.00	2.9309	0.6317	0.8770	4.1548	Unimodal
1.00	0.05	-5.0344	3.1478	-0.4778	3.1175	Unimodal
	0.10	-3.3629	2.3227	-0.4011	3.0670	Unimodal
	0.20	-1.9558	1.7481	-0.3009	3.0181	Unimodal
	0.50	-0.7043	1.2479	-0.1372	2.9831	Unimodal
	1.00	0.0000	1.0000	0.0000	3.0000	Unimodal
	10.0	1.5388	0.5868	0.4099	3.3314	Unimodal
	100.00	2.0576	0.4294	0.6553	3.7652	Unimodal
10.0	0.05	-5.7016	2.7983	-0.7478	3.4176	Unimodal

		0.10	-4.1331	1.9151	-0.7848	3.5467	Unimodal
		0.20	-3.0485	1.3035	-0.7769	3.6591	Unimodal
		0.50	2.0809	0.8033	-0.6172	3.5732	Unimodal
		1.00	-1.5388	0.5868	-0.4099	3.3314	Unimodal
		10.0	0.0000	0.2842	0.0000	3.0130	Unimodal
		100.00	1.3541	0.1694	0.1303	3.0438	Unimodal
	100.0	0.05	-6.1625	2.6164	-0.8726	3.6557	Unimodal
		0.10	-4.7070	1.7251	-0.9561	4.1825	Unimodal
		0.50	-2.9309	0.6317	-0.8770	4.1548	Unimodal
		1.00	-2.5076	0.4294	-0.6553	3.7652	Unimodal
		10.0	-1.3541	0.1694	-0.1303	3.0438	Unimodal
		100.00	0.0000	0.0887	0.0000	3.0014	Unimodal

From Table 2.1, we can see that the shape of the beta-normal distribution would be left skewed, symmetric or right skewed, unimodal or bi-modal depends on the values of the parameters α and β. For example, if α < β, then the shape of beta-normal will be left skewed, if α = β, then the shape of beta-normal will be left symmetric, and if α > β, then the shape of beta-normal will be right skewed.

2.4 Calculate the Gini coefficient of the beta-normal distribution.

To compare income distribution inequality, various indices are used, many of which are derived from the Lorenz curve. However, the classic Gini coefficient and its variations are commonly used to measure income inequality. In this section, the Gini coefficient will be obtained for the beta-normal distribution. The Gini coefficient can be calculated directly from the cumulative distribution function F(x). While the F(x) is zero for all negative values and μ represents the mean distribution, so the Gini coefficient is equal to:

$$G = 1 - \frac{1}{\mu}\int_0^\infty \left(1 - F(x)\right)^2 dx = \frac{1}{\mu}\int_0^\infty F(x)\left(1 - F(x)\right)dx. \qquad (12)$$

to calculate the Gini coefficient for the beta-normal distribution with the cumulative distribution function (5) for integers α and β.

$$\int_0^\infty F(x)dx = \frac{1}{B(\alpha,\beta)}\sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(\beta)}{\Gamma(\beta-j)j!}\frac{1}{(\alpha+j)}\int_0^\infty G(x)^{\alpha+j}dx \qquad (13)$$

Since G(x) is a cumulative normal distribution function, we have,

$$\int_0^\infty G(x)^{\alpha+j}dx = \int_0^\infty \{\Phi(\frac{x-\mu}{\sigma})\}^{\alpha+j}dx. \qquad (14)$$

By placing $z = \frac{x-\mu}{\sigma}$, $dx = \sigma dz$ and applying binomial expansion we have,

$$\int_0^\infty G(x)^{\alpha+j} dx = \sigma \int_0^\infty \{\Phi(z)\}^{\alpha+j}\, dz = \sigma \sum_{k=0}^{\alpha+j} (-1)^k \binom{\alpha+j}{k} \int_0^\infty \{1-\Phi(z)\}^k dz$$

$$= \sigma \sum_{k=0}^{\alpha+j} (-1)^k \binom{\alpha+j}{k} \delta_k \qquad (15)$$

From the relations (13) and (14), the relation (12) is obtained as follows:

$$\int_0^\infty F(x)dx = \frac{\sigma}{B(\alpha,\beta)} \sum_{j=0}^{\beta-1} \frac{(-1)^j \Gamma(\beta)}{\Gamma(\beta-j)j!} \frac{1}{(\alpha+j)} \sum_{k=0}^{\alpha+j} (-1)^k \binom{\alpha+j}{k} \delta_k \qquad (18)$$

To get $\int F(x)^2\, dx$:

$$F(x)^2 = \left[\frac{1}{B(\alpha,\beta)} \sum_{j=0}^{\beta-1} \frac{(-1)^j \Gamma(\beta)}{\Gamma(\beta-j)j!} \frac{G(x)^{\alpha+j}}{(\alpha+j)}\right]\left[\frac{1}{B(\alpha,\beta)} \sum_{l=0}^{\beta-1} \frac{(-1)^l \Gamma(\beta)}{\Gamma(\beta-l)l!} \frac{G(x)^{\alpha+j}}{(\alpha+l)}\right]$$

$$= \frac{1}{B(\alpha,\beta)^2} \sum_{j=0}^{\beta-1}\sum_{l=0}^{\beta-1} \frac{(-1)^{j+l}\Gamma(\beta)^2}{\Gamma(\beta-j)\Gamma(\beta-l)j!\,l!} \frac{G(x)^{2\alpha+j+l}}{(\alpha+j)(\alpha+l)} \int_0^\infty F(x)^2 dx$$

$$= \frac{1}{B(\alpha,\beta)^2} \sum_{j=0}^{\beta-1}\sum_{l=0}^{\beta-1} \frac{(-1)^{j+l}\Gamma(\beta)^2}{\Gamma(\beta-j)\Gamma(\beta-l)j!\,l!} \frac{1}{(\alpha+j)(\alpha+l)} \int_0^\infty G(x)^{2\alpha+j+l}\, dx$$

$$= \frac{\sigma}{B(\alpha,\beta)^2} \sum_{j=0}^{\beta-1}\sum_{l=0}^{\beta-1} \frac{(-1)^{j+l}\Gamma(\beta)^2}{\Gamma(\beta-j)\Gamma(\beta-l)j!\,l!} \frac{1}{(\alpha+j)(\alpha+l)} \int_0^\infty \{\Phi(z)\}^{2\alpha+j+l}\, dz$$

$$= \frac{\sigma}{B(\alpha,\beta)^2} \sum_{j=0}^{\beta-1}\sum_{l=0}^{\beta-1} \frac{(-1)^{j+l}\Gamma(\beta)^2}{\Gamma(\beta-j)\Gamma(\beta-l)j!\,l!} \frac{1}{(\alpha+j)(\alpha+l)} \sum_{m=0}^{2\alpha+j+l} (-1)^m \binom{2\alpha+j+l}{m} \int_0^\infty \{1 - \Phi(z)\}^m\, dz$$

From the relation (13) we have,

$$\int_0^\infty \{1-\Phi(z)\}^m\, dz = \delta_m$$

so,

$$\int_0^\infty F(x)^2 dx$$

$$= \frac{\sigma}{B(\alpha,\beta)^2} \sum_{j=0}^{\beta-1}\sum_{l=0}^{\beta-1}\sum_{m=0}^{2\alpha+j+l} \frac{(-1)^{j+l+m}\Gamma(\beta)^2}{\Gamma(\beta-j)\Gamma(\beta-l)j!\,l!\,(\alpha+j)(\alpha+l)} \binom{2\alpha+j+l}{m} \delta_m. \qquad (19)$$

By placing the relationships (12) and (18) and (19) in (14), the Gini coefficient for the beta-normal distribution is obtained as follows,

$$G = \frac{1}{\mu}\int_0^{\infty} (F(x) - F(x)^2)dx = \frac{\frac{\sigma}{B(\alpha,\beta)} \sum_{j=0}^{\beta-1} \frac{(-1)^j \Gamma(\beta)}{\Gamma(\beta-j)j!} \frac{1}{(\alpha+j)} M}{\mu + \frac{\sigma}{B(\alpha,\beta)} T}$$

where

$$M = \sum_{k=0}^{\alpha+j} (-1)^k \binom{\alpha+j}{k} \delta_k - \frac{1}{B(\alpha,\beta)} \sum_{l=0}^{\beta-1} \sum_{m=0}^{2\alpha+j+l} \frac{(-1)^{l+m}\Gamma(\beta)}{\Gamma(\beta-l)l!\,(\alpha+l)} \binom{2\alpha+j+l}{m} \delta_m$$

and

$$T = \sum_{j=0}^{\beta-1} \left\{ \sum_{k=0}^{\alpha+j-2} \frac{(-1)^{j+k}}{(k+1)} \binom{\beta-1}{j} \binom{\alpha+j+l}{k} \delta_{k+1} + \frac{(-1)^{\alpha-1} - (-1)^j}{\alpha+j} \binom{\beta-1}{j} \delta_{\alpha+j} \right\}.$$

3. Simulation Study

Using software R, we have conducted a simulation study in this section. The Gini coefficient of the beta-normal distribution for various parameters, α, β, μ and σ for 10, 20, 30, 35, 50, and 100 samples with 100000 iterations, was calculated continuously, then it was compared to the actual Gini coefficient of the beta-normal distribution calculated in Maple software. The simulation results are summarized in the Table 3.1. In Table 3.1, we have two Gini coefficients, (i) the simulation of the beta-normal distribution in which the parameters of the beta-normal distribution have been estimated and (ii) the population Gini coefficient of the beta-normal distribution. By comparing the results obtained from the real and simulated Gini coefficients using R and Maple software respectively, it can be concluded that in general, as the amount of sample size increases, the Gini coefficient also increases, plus the value of the Gini coefficient depends on the value of the parameters. Table 3.1 points out that as the Gini coefficient is between 0 and 1, it cannot be calculated when the mean of the beta-normal is 0 or negative. As we mentioned above, when the amount of the coefficient is close to zero, it approaches the balance between the distribution of wealth and income in the society. These results can be obtained for any distribution of the combination.

Table 3.1: The actual Gini coefficient of the beta-normal distribution

α	β	μ	σ	n	Gini S	Mse	Bias	Gini R
1	0.5	0.7	1.25	10	0.26681	0.0000	0.0017	0.2685
				20	0.3546	0.0074	0.0861	
				30	0.3951	0.0160	0.1266	
				35	0.4242	0.0243	0.1558	
				50	0.5730	0.0927	0.3045	
				100	0.8563	0.3455	0.5878	
2	1	0	1	10	0.2597	0.0323	0.1796	0.4393
				20	0.3454	0.0088	0.0940	
				30	0.4358	0.0000	0.0035	
				35	0.5500	0.0122	0.1107	
				50	0.6502	0.0445	0.2109	
				100	0.8458	0.1652	0.4065	
4	1	0	1	10	0.1222	0.0527	0.2295	0.3517
				20	0.2693	0.0068	0.0823	
				30	0.3223	0.0009	0.0293	
				35	0.2119	0.0195	0.1398	
				50	0.3905	0.0015	0.0388	
				100	0.6593	0.0947	0.3077	
5	1	1	1	10	0.1619	0.0001	0.0118	0.1737
				20	0.1929	0.0004	0.0192	
				30	0.1579	0.0002	0.0158	
				35	0.2119	0.0015	0.0382	
				50	0.3369	0.0266	0.1632	
				10	0.3397	0.0276	0.1660	
1	2	1	1	20	0.2237	0.0450	0.2121	0.4358
				30	0.2617	0.0303	0.1741	
				35	0.5870	0.0229	0.1512	
				50	0.4123	0.0006	0.0235	
				100	0.5840	0.0219	0.1481	
				10	0.6687	0.0542	0.2329	
1	5	1	1	20	0.2094	0.0022	0.0470	0.2565
				30	0.2139	0.0018	0.0425	
				35	0.3528	0.0093	0.0964	
				50	0.3296	0.0053	0.0731	
				100	0.6683	0.1696	0.4118	
				10	0.7900	0.2846	0.5335	
3	2	0	1	20	0.2321	0.0327	0.1810	0.4131
				30	0.3368	0.0058	0.0763	
				35	0.2857	0.0162	0.1273	
				50	0.4711	0.0034	0.0581	
				100	0.5356	0.0150	0.1225	
				10	0.5399	0.0161	0.1268	
4	2	1	1	20	0.1460	0.0041	0.0638	0.2098
				30	0.1462	0.0040	0.0636	
				35	0.1203	0.0080	0.0895	
				50	0.1453	0.0042	0.0645	
				100	0.2055	0.0000	0.0043	
				10	0.2318	0.0005	0.0220	
2	3	1	1	20	0.1997	0.0320	0.1789	0.3786
				30	0.2308	0.0218	0.1478	
				35	0.2600	0.0141	0.1186	
				50	0.2792	0.0099	0.0994	
				100	0.4898	0.0124	0.1112	
					0.6976	0.1017	0.3190	

α	β	μ	σ	n	Gini S	Mse	Bias	Gini R
4	3	0	1.5	10	0.1988	0.0000	0.0028	0.2015
				20	0.2251	0.0006	0.0235	
				30	0.3103	0.0118	0.1088	
				35	0.4293	0.0519	0.2278	
				50	0.5985	0.1576	0.3969	
				100	0.6622	0.2122	0.4607	
2	4	1	1	10	0.0746	0.1120	0.3347	0.4093
				20	0.1740	0.0554	0.2353	
				30	0.2252	0.0339	0.1841	
				35	0.3363	0.0053	0.0730	
				50	0.4658	0.0032	0.0565	
				100	0.5741	0.0272	0.1648	
1	0.24	0	1	10	0.4037	0.0004	0.0208	0.4246
				20	0.5693	0.0209	0.1447	
				30	0.5496	0.0156	0.1251	
				35	0.6594	0.0552	0.2349	
				50	0.7079	0.0803	0.2833	
				100	0.7681	0.1180	0.3435	
0.5	1	0	1	10	0.5466	0.1613	0.4016	0.9482
				20	0.6001	0.1212	0.3481	
				30	0.6093	0.1148	0.3389	
				35	0.7154	0.0542	0.2328	
				50	1.0691	0.0146	0.1209	
1	0.5	0	1	10	0.3491	0.0071	0.0845	0.4336
				20	0.3771	0.0032	0.0565	
				30	0.5512	0.0138	0.1176	
				35	0.5910	0.0248	0.1574	
				50	0.7298	0.0878	0.2963	
				100	0.7635	0.1089	0.3300	
100	1	0	1	10	0.0509	0.0019	0.0441	0.0950
				20	0.0662	0.0008	0.0289	
				30	0.0709	0.0006	0.0241	
				35	0.0822	0.0002	0.0128	
				50	0.0836	0.0001	0.0114	
				100	0.0930	0.0000	0.0021	
10	0.5	0	1	10	0.1536	0.0037	0.0612	0.2149
				20	0.1837	0.0010	0.0311	
				30	0.2598	0.0020	0.0450	
				35	0.2864	0.0051	0.0715	
				50	0.3773	0.0264	0.1625	
				100	0.4800	0.0703	0.2652	
100	0.5	0	1	10	0.0506	0.0046	0.0679	0.1185
				20	0.0868	0.0010	0.0317	
				30	0.1415	0.0005	0.0231	
				35	0.1164	0.0000	0.0021	
				50	0.1131	0.0000	0.0053	
				100	0.1165	0.0000	0.0020	
1	0.2	0	1	10	0.3250	0.0078	-0.0884	0.4134
				20	0.3930	0.0004	-0.0204	
				30	0.4224	0.0001	0.0090	
				35	0.7870	0.1396	0.3736	
				50	0.8463	0.1875	0.4330	
				100	0.8652	0.2041	0.4518	

4. Some Concluding Remarks

The Gini index is one of the most common statistical indicators of diversity and inequality to measure the distribution of dispersion. There are many distributions in statistics that are used to model the income distributions. The four-parameter distribution is called beta-normal distribution, which is a generalization of normal distribution which is widely used to model the skewed distribution. This paper discusses the estimation of the Gini index from Beta-Normal distribution. To illustrate the findings of the paper, a simulation study has been conducted. From simulation results, it is concluded that in general, as the amount of sample size increases, the Gini coefficient increases. Moreover, the value of the Gini coefficient depends on the value of the parameters.

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