

Spectral Analysis for Randomly Excited Articulated vehicles

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Abstract

The spectral density analysis of the articulated vehicles is a subject of great importance and it is widely applied in the area of vehicles design and safety. So the vehicles when ride over rough roads, respond dynamically and inadequate road holding may arise. This paper examines the ride safety of an articulated vehicle over irregular roads through an analytical study. The vehicle responses to road surface undulations are studied using spectral density approach which is reviewed and then applied on a vehicle model to obtain the dynamic wheel loads and the results are drawn from the classical system and conclusions are drawn as to the applicability of this approach. For loaded and unloaded articulated vehicles, it may be concluded that for unloaded vehicle the danger of the departure of the wheel from the road is higher than with the loaded vehicles.

Keywords: frequency analysis; random loading processes

التحليل بطريقة الكثافة الطيفية للعربات المفصلية المعرضة لاحمال عشوائية

الخلاصة

تعتبر طريقة التحليل بطريقة الكثافة الطيفية للعربات المفصلية من المواضيع المهمة التي تأخذ مساحة واسعة في التطبيقات الخاصة بالتصميم والامان. حيث يؤدي سير المركبات الارضية على الطرق المتموجة الأسطح إلى تأثيرها ديناميكياً، الأمر الذي ينتج عنه عدم كفاية مقدار الثبات على الطرق. ويقدم هذا البحث دراسة تحليلية لمدى الأمان الذي ينتج عن سير العربات المفصلية على الطرق المتعرجة الأسطح. وتنتهج الدراسة في تقدير مثل هذه العربات بتعرجات الطرق منهج الكثافة الطيفية وقد تم شرح هذا المنهج ثم طبق على نموذج لمركبة لتعيين الاحمال الديناميكية المؤثرة على عجلاتها. وينتهي البحث بنتائج تطبيقية على نظام تقليدي والاستنتاجات الخاصة بإمكانية تطبيق هذا المنهج. ومن النتائج المهمة نلاحظ بان خطر الانحراف في المركبات الغير محملة يكون اكبر مما إذا كانت المركبات محملة.

Nomenclature

a_d	Vertical position of the driver, [m]
a_j	Vehicle measurement, [m]
b_d	Horizontal position of the driver, [m]
b_i	Vehicle dimension, [m]
C	Damping matrix, [kNs/m]
C_f	Excitation matrix, [kNs/m]
C_j	Suspension damping rate, [kNs/m]
C_{tj}	Tire damping rate, [kNs/m]
E	Expectation operator

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$H_{jk}(\omega)$	Complex frequency response
I_s, I_t	Trailer and tractor pitch moments of inertia, respectively, [kg m ²]
i	$\sqrt{-1}$
K	Stiffness matrix, [kN/m]
K_f	Excitation matrix, [kN/m]
k_j	Suspension stiffness, [kN/m]
k_{tj}	Tire stiffness, [kN/m]
M	Mass matrix, [kg]
M_j	Unsprung mass, [kg]
M_s, M_t	Trailer and tractor sprung mass respectively, [kg]
P_j	Tire dynamic load, [kN]
P_s	Tire static load, [kN]
R_{xj}	Correlation function
$S_{p_i}(\omega)$	Tire power spectral density, [m ² .sec]
$S_{uu}(\omega)$	Road input spectral density matrix, [m ² .sec]
$S_{xx}(\omega)$	Output spectral density matrix, [m ² .sec]
$S_o(\omega)$	Spatial spectral density function, [m ² .sec]
t	Time, [sec]
τ	Time delay, [sec]
u	Vector of displacement excitation, [m]
v	Vehicle speed, [Km/h]
χ	Vector of generalized coordinate, [m]
X_s, X_t	Horizontal motion coordinates of the centres of gravity of the trailer and tractor, respectively, [m]
Y_j	Vertical motion coordinates of vehicle axles (j=1,2,3), [m]
Y_s, Y_t	Vertical motion coordinates of the centers of gravity of the trailer and tractor, respectively, [m]
ω	Angular frequency, [rad/s]
θ_s, θ_t	Rotational coordinate about the centers of gravity of the trailer and tractor, respectively, [m/s]
δ_u	static deflection, [m]
σ	standard deviation of the random input, [m]
σ^2	variance of the random input, [m ²]
PSD's	power spectral densities, [m ² .sec]
$P(q)$	probability of the input

Introduction

The dynamic response of an articulated vehicle to road irregularities has been a subject of great interest to automotive designers who are responsible of geometry of the design as well as maintenance and road engineers who are responsible for the material and

manufacturing routes. This interest is primarily motivated by a desire to improve ride quality to reduce wear to vehicle components, to prevent loading damage, and most of all to ensure safe operation. With increased loading and operation at higher speeds, it is very desirable to

improve the performance of vehicle components and its suspension system. As a vehicle rides over roads which are irregular in nature, it responds dynamically and imparts dynamic loads, which vary in their static wheel loads, to the roads. For directional control of vehicles, it is necessary to maximize the force available at the road, in order to resist skidding and transmit deceleration and acceleration forces from the tires to the road. Therefore, reduction in the force available at the road as a result of dynamic wheel loads may lead to loss of road-to-wheel-contact and consequently impairs vehicle safety.

Since all roads have some surface irregularities which are random, statistical representations of these roads are required. Many mathematical models of varying complexity have been developed to analyze the dynamic behavior of articulated vehicles in response to road irregularities, analytical methods of solution have been employed. Analytical tools such as eigenvalues/vectors and transfer function can be used for preliminary design and evaluation of vehicle performance. Spectral analysis technique can be employed for an articulated vehicle model to determine the random characteristics of vehicles (displacement, velocities, acceleration, dynamic loads,...) items of interest include the peak values, rms values, probability of exceeding a particular level or range dominating frequencies, and further study of fatigue failure of vehicle components and roads [1]. Although the theory often yields correct qualitative results, it cannot include the often critical effects of dry friction, suspension stops, and

wheel hops, digital simulation can overcome this difficulty and can handle these nonlinear effects [2] however, this technique is suitable as a design tool since it is not costly and the results are easy to interpret.

The power spectral density technique can be adopted as an analytical tool for vehicle design [3]. This technique has the advantage of allowing the vehicle designer to interpret the vehicle response in terms of natural frequencies, damping ratios, and modes of vibration.

Temporal variations in the loading conditions can cause irreversible changes in the characteristics of the structure systems that may significantly affect their performance. These changes, referred to as degrading (or deterioration) phenomena, are usually not taken into account in the conventional analysis of vibratory systems. Such analysis concentrates on the characterization of the response under various excitations assuming that the system properties are fixed. As far as the response of dynamical systems to random excitation is concerned, the methods elaborated allow to characterize a stochastic response process in a variety of important situations and in the same time they provide the information about the reliability estimation [4].

This paper examines the ride safety of an articulated vehicle over irregular road through an analytical study, the articulated vehicle responses to road surface undulations are studied using spectral density approach. Fig.(1) shows the schematic diagram of the articulated vehicle input/output problem.

Vehicle mathematical models the articulated vehicle models used in

this study are six-degree of freedom plane linear and nonlinear models, as shown in Fig (2). The generalized articulated vehicle coordinates are the tractor vertical displacement (Y_t), tractor pitching motion (θ_t), trailer vertical displacement (Y_s), trailer pitching motion (θ_s), tractor front and rear wheel vertical displacements (Y_1 and Y_2) respectively, and trailer wheel vertical displacement (Y_3). The vehicle models are excited by vertical displacement at the front and rear wheels of the tractor and the trailer wheel. These displacements are assumed to be known stochastic processes resulting from the randomly profiled road spectral density approach.

In order to study the vehicle road interaction, it is essential that the road should be quantitatively described in a manner useful to the vehicle designer. The dynamic response of the vehicle depends greatly on the road profile in the vertical direction. The articulated vehicle is subjected to three vertically imposed displacements, one at each wheel. The description of the road geometry should be complete enough to describe adequately the displacement imposed at each wheel and the correlation between them. Based on previous measurement of road way evaluation [5] the road irregularities are assumed stationary ergodic random input, processes with zero mean, and the amplitude probability density is Gaussian.

Random variable $q(t)$, the mean (average) or expected value is:

$$E[q(t)] = \int_{-\infty}^{\infty} q P(q) dq \quad (1)$$

The stationary random function is

$$E[q(t)] = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T q(t) dt \quad (2)$$

The magnitude of the random variable is the mean-square value $E[q(t)]$ of the input (q) and probability of the input $P(q)$ product given by:

$$E[q^2(t)] = \int_{-\infty}^{\infty} q^2 \rho(q) dq \quad (3)$$

which for stationary random function is given by

$$E[q^2(t)] = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T q^2(t) dt \quad (4)$$

The variance σ^2 is the ensemble average of the square of the deviation from the mean or defined as the mean-square value about the mean, or

$$\sigma^2 = E[q^2] - (E[q])^2 \quad (5)$$

$$\sigma^2 = \int (q - E[q])^2 \rho[q] dq \quad (6)$$

For zero mean value the variance is

$$\sigma_q^2 = E[q^2(t)] \quad (7)$$

In terms of the standard deviation, the Gaussian distribution is given by the equations:

$$p(q) = \frac{1}{s\sqrt{2\pi}} \exp\left(-q^2/2s^2\right) \quad (8)$$

In the road spectrum $S(\omega)$ as a function of spatial frequency (ω) is the angular frequency in rad/sc. The input spectral density matrix is defined as $[S_{ij}(\omega)]$ where the element $[S_{ij, uk}(\omega)]$ denotes cross-spectral density between (u_j) and (u_k) inputs ($j \neq k$) and direct spectral density of (u_j) when $j=k$ ($j, k=1, 2, 3$). Then the direct spectral density is

$$S_{ij}(\omega) = (1/2\pi v) S_o(\omega) \quad (9)$$

where $S_o(\omega)$ is the spatial spectral density of road roughness, v vehicle speed

The equations of motion of the vehicle can be written in the form $M\ddot{\chi} + C\dot{\chi} + K\chi = C_f\dot{u} + K_f u$ (10) where $(M, C, \text{ and } K)$ are 6×6 matrices (each matrix consists of six elements k^s, m^s, c^s as shown in Table 1) representing the mass, damping, and stiffness matrices of the system, respectively. C_f and K_f are the forced damping and stiffness matrices, respectively. (χ) and (u) are column vectors of the generalized coordinates and road inputs, respectively. Define $H_{jk}(\omega)$ as the complex frequency response function for the (χ_j) output due to a harmonically varying imposed displacement of unit amplitude at the (u_j) input

Let

$$u = \int_{-\infty}^{\infty} u(i\omega) \text{Exp}(i\omega t) d\omega$$

$$\dot{u} = \int_{-\infty}^{\infty} i\omega u(i\omega) \text{Exp}(i\omega t) d\omega$$

$$c = \int_{-\infty}^{\infty} \chi(i\omega) \text{Exp}(i\omega t) d\omega$$

$$\dot{c} = \int_{-\infty}^{\infty} i\omega c(i\omega) \text{Exp}(i\omega t) d\omega$$

$$\ddot{c} = \int_{-\infty}^{\infty} -\omega^2 c(i\omega) \text{Exp}(i\omega t) d\omega$$

by substituting in Eq.(2) and canceling $\text{Exp}(i\omega t)$ terms yields

$$H(i\omega) = \text{Response/Excitation}$$

$$H(i\omega) = [k - \omega^2 M + i\omega C]^{-1} [K_f + i\omega C_f] \quad (11)$$

Then

$$\chi(i\omega) = H(i\omega)u(i\omega) \quad (12)$$

The power spectral density of the generalized displacement response (χ) can be expressed in terms of the input spectral densities and the system response function as follows:

Using Fourier transformations [6-10] yields,

$$S_{\chi_j}(\omega) = |H(i\omega)|^2 S_{u_j}(\omega) \quad (13)$$

$$\text{or } S_{\chi_j}(\omega) = H(\omega) S_{u_j}(\omega) H^*(\omega) \quad (14)$$

where $(*)$ denotes matrix conjugate transposition. $S_{\chi_j}(\omega)$ is 6×6 matrix containing the spectral densities of the (χ) vector along the diagonal and the cross spectral densities on off-diagonal elements.

To compute the standard deviation (σ) of the variables in the (χ) vector, the diagonal terms of $S_{\chi_j}(\omega)$ may be integrated over the effective frequency band of the random input variables, i.e.

$$E[\chi_j^2] = \int_{\omega_1}^{\omega_2} S_{\chi_j}(\omega) d\omega, j=1, \dots, 6 \quad (15)$$

And residue integration yields,

$$E[\chi_j^2] = (S_o(\omega)\omega_j)/(8\zeta_j K_j^2) \quad (16)$$

To compute the power spectral density of the generalized velocities, let the autocorrelation function of the generalized displacement χ_j be given by [11]

$$R_{\chi_j}(\tau) = E[\chi_j(t)\chi_j(t+\tau)] \quad (17)$$

Differentiating equation (9) twice yields

$$\begin{aligned} \frac{d^2}{d\tau^2} R_{\chi_j}(\tau) &= E[\ddot{\chi}_j(t-\tau)\ddot{\chi}_j(t)] \\ &= -R_{\chi_j}(\tau) \end{aligned} \quad (18)$$

The relation between the autocorrelation function and the spectral density function is given by [12]

$$R_{cj}(t) = \int_{-\infty}^{\infty} S_{cj}(w) \text{Exp}(iwt) dw \quad (19)$$

The second derivative of this Fourier integral yields

$$\frac{d^2}{dt^2} R_{cj}(t) = - \int_{-\infty}^{\infty} w^2 S_{cj}(w) \text{Exp}(iwt) dw \quad (20)$$

Combining equations (10) and (12), the power spectral density of the generalized velocity $\dot{\chi}_j$ is given by

$$S_{\dot{\chi}_j}(\omega) = \omega^2 S_{\chi_j}(\omega) \quad (21)$$

The calculation of the power spectral density of each wheel dynamic load imparted to the pavement is as follows:

From virtual work principle the static load is given by [13]

$$P_s = k\delta_u \quad (22)$$

where δ_u is the static deflection
And the dynamic load is given by

$$P_j = K_{ij}(Y_j - u_j) + C_{ij}(\dot{y}_j - \dot{u}_j), j=1,2,3 \quad (23)$$

The mean square value of the dynamic wheel load (variance) σ^2 is

$$E[P_j^2] = K_{ij}^2 E[\chi_j^2] + C_{ij}^2 E[\dot{\chi}_j^2] \quad (24)$$

$$\begin{aligned} & \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega S_{\dot{\chi}_j}(\omega) \text{Exp}(i\omega\tau) d\omega \\ E[P_j^2] &= s^2 p_j = K_{ij}^2 \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{cj}(w) \text{Exp}(iwt) dw + C_{ij}^2 \\ E[P_j^2] &= (K_{ij}^2 + C_{ij}^2) S_{p_j}(\omega) \phi^2 \quad (25) \end{aligned}$$

and the corresponding power spectral density is:

$$S_{p_j}(\omega) = (k_{ij} + \omega^2 c_{ij})(S_{y_j} + S_{u_j} - 2\text{Real}S_{u_j, y_j}) \quad (27)$$

where the cross-spectral density S_{u_j, y_j} can be calculated from the following matrix equation which gives the relation between the inputs and outputs,

$$S_{u_j, y_j}(\omega) = S_{u_j}(\omega) H^T(\omega) \quad (28)$$

where $H^T(\omega)$ is the transpose of $H(\omega)$. The probability that the tire leaves the road can be calculated from the following equation:

$$P_r = \frac{1}{2} \{1 - (P_s / \sqrt{2E[P_j^2]})\} \quad (29)$$

where P_s and P_j are the static and dynamic wheel loads, respectively, and $E[P_j^2]$ is the expected mean square value of the dynamic wheel load.

Result and discussion

The vehicle parameters are given in Table(1). The computer algorithm used in this study for determining the vehicle nonlinear response to random excitation using spectral density approach is shown in Fig(3) [14].

To study the influence of vehicle parameters on the dynamic wheel loads, which the vehicle imposes on

the surfaces, several values of suspension springs and dampings and the tire characteristics have been considered based on the nature of the articulated vehicles. Figs. (4) and (5) show the time history and the corresponding Gaussian probability distribution used in this paper. The power spectral densities of the dynamic wheel load given in Fig.(6) describes the influence of the articulated vehicle suspension, k_1, k_2 and k_3 , on the wheel dynamic load in the resonance regions, the wheel dynamic load becomes smaller as the suspension becomes softer. However, in the resonance region, decreasing suspension spring stiffness produces a slightly larger peak dynamic load. The manner in which the dynamic wheel loads of the articulated vehicle varies with the damping parameter, c_1, c_2 and c_3 . It can be seen that, with the damping in the suspension, the dynamic loads are affected appreciably near resonance regions. The figure indicates that, increasing the damping decreases the (PSD) of the wheel load first, but with high damping the (PSD) value increases again, because high damping stiffens the suspension and transmits more load to then road.

Fig. (7) shows that the stiffness of the tires appreciably affects the (RMS) spectra of the dynamic load between the wheel and road at resonance. The higher peak value at resonance for higher tire stiffness results in the higher (RMS) wheel Figs.(8) shows the PSD's velocity for loaded and unloaded articulated vehicles, respectively. From the plots shown in Figs.(8) it may be concluded that for unloaded vehicle the danger of the departure of the wheel from the road is higher than with the loaded vehicle. The

increased safety of the loaded vehicle is principally due to the increase in static load. The unloaded vehicle is the least satisfactory. Some amount of frictional forces at both the tractor front and rear suspension are necessary to reduce the dynamic wheel load fluctuations for both loaded and unloaded vehicles. However, for the optimum design the loading conditions must take into account (i.e. for loaded and unloaded vehicles, the optimum values of the frictional forces are different for the two loading condition) the road is given by Fig. (9). It may be concluded that the probability of the departure of the wheel is high, some frictional forces may be beneficial in reducing the fluctuations of the wheel dynamic loads and consequently in reducing the probability of departure of the wheel from the road.

The tire deformation is given in Fig.(10) which shows the influence of the road input on the elastic deformation sectors of the tire. Fig. (11) gives the total translation of the tire with respect to the frequency of the road and indicates the fluctuations of the dynamics wheel load with the randomly road inputs. The mode shapes of the articulated vehicle for the first sixth modes shown in Fig.(12) which is useful to compute the eigenvalues for different modes of vibration. Fig. (13) shows the articulated vehicle modal shape simulation using Nastran program which is easy to determine the amplitude spectra.

Conclusion

The problem of the dynamic interaction between the articulated vehicle and the road surfaces undulations is investigated in depth

by means of spectral analysis. The effects of the suspension stiffness and damping parameters generated in the laminated springs on road safety are discussed and evaluated. The articulated vehicle dynamic analysis using spectral density approach can provide a fundamental understanding of articulated vehicle stochastic response and riding quality.

Understanding the influence of the imported nonlinearities, such as suspension dry friction, inertia forces, will give a new insight to the problem of vehicle/road dynamic interaction. The spectral density technique in this paper is capable of including such nonlinear effects. The technique is useful to a vehicle designer since it allows him to interpret a complex nonlinear system like a vehicle in terms of the frequency domain, mode shapes, and damping ratios.

For loaded and unloaded articulated vehicles, it may be concluded that for unloaded vehicle the danger of the departure of the wheel from the road is higher than with the loaded vehicle. The increased safety of the loaded vehicle is principally due to the increase in static load. The unloaded vehicle is the least satisfactory. Some amount of frictional forces at both the tractor front and rear suspension are necessary to reduce the dynamic wheel load fluctuations for both loaded and unloaded vehicles. However, for the optimum design the loading conditions must be taken into account.

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Fig(1) Block diagram of the excitation – response problem

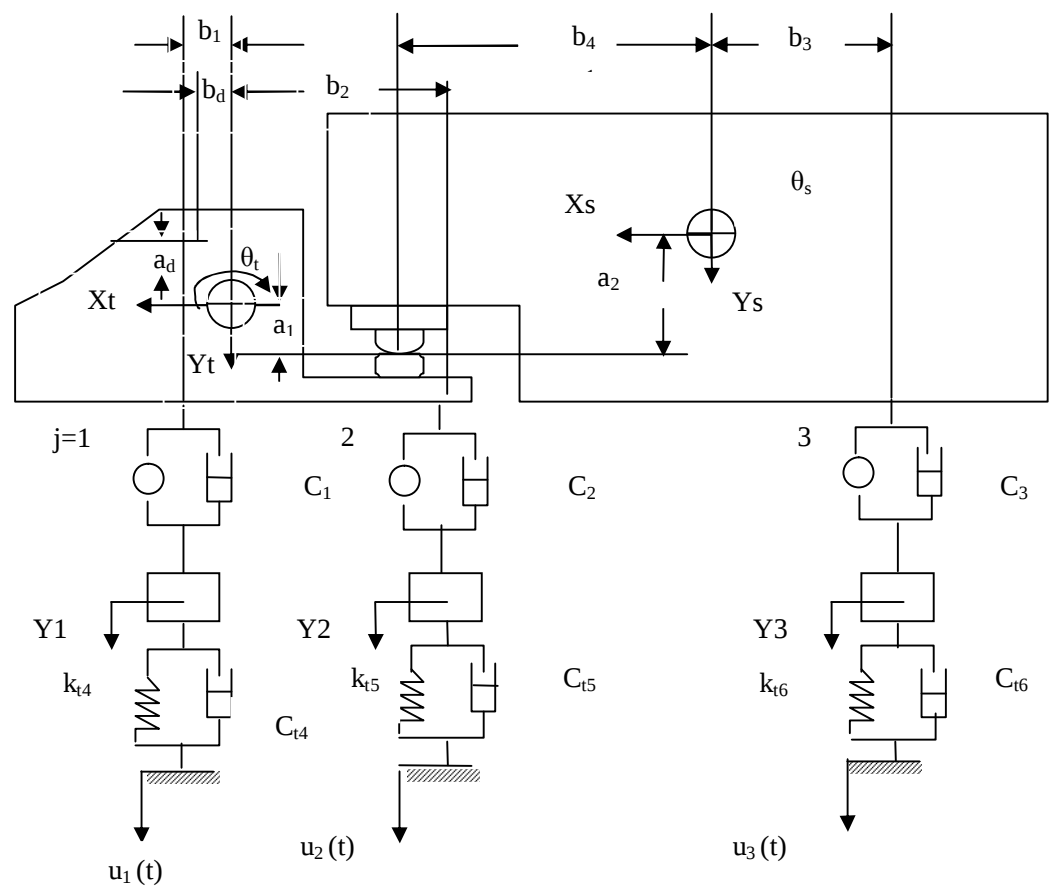


Fig. (2) Tractor –trailer vehicle model

Table (1) Parameters for articulated vehicle model

Dimensions			
b ₁	1.37 m	b ₅	1.5 m
b ₂	3.01 m	a ₁	-0.4 m
b ₃	4.6 m (loaded)	a ₂	1.04 m (loaded)
b ₃	4.1 m (unloaded)	a ₂	-0.33 m (unloaded)
b ₄	5.6 m (loaded)	a _d	0.99 m
b ₄	5.5 m (unloaded)	b _d	0.63 m
Mass and Inertia			
M _t	tractor sprung mass		7550 kg
I _t	tractor pitch moment of inertia		4480 kg m ²
M _s	trailer sprung mass		16190 kg (loaded)
M _s	trailer sprung mass		4330 kg (unloaded)
I _s	trailer pitch moment of inertia		162000 kg m ² (loaded)
I _s	trailer pitch moment of inertia		54150 kg m ² (unloaded)
M ₁	tractor front unsprung mass		370 kg
M ₂	tractor rear unsprung mass		1560 kg
M ₃	trailer front unsprung mass		1560 kg
Stiffness and damping			
K ₁	385 kN/m	C ₁	12.4 kNs/m
K ₂	640 kN/m	C ₂	30 kNs/m
K ₃	640 kN/m	C ₃	30 kNs/m
K _{t4}	1660 kN/m	C _{t4}	0.8 kNs/m
K _{t5}	6260 kN/m	C _{t5}	1.3 kNs/m
K _{t6}	6260 kN/m	C _{t6}	1.3 kNs/m
Spatial spectral density of road roughness S _o (ω) =15×10 ⁻⁶ m ² .sec			
Vehicle speed=90 Km/h			

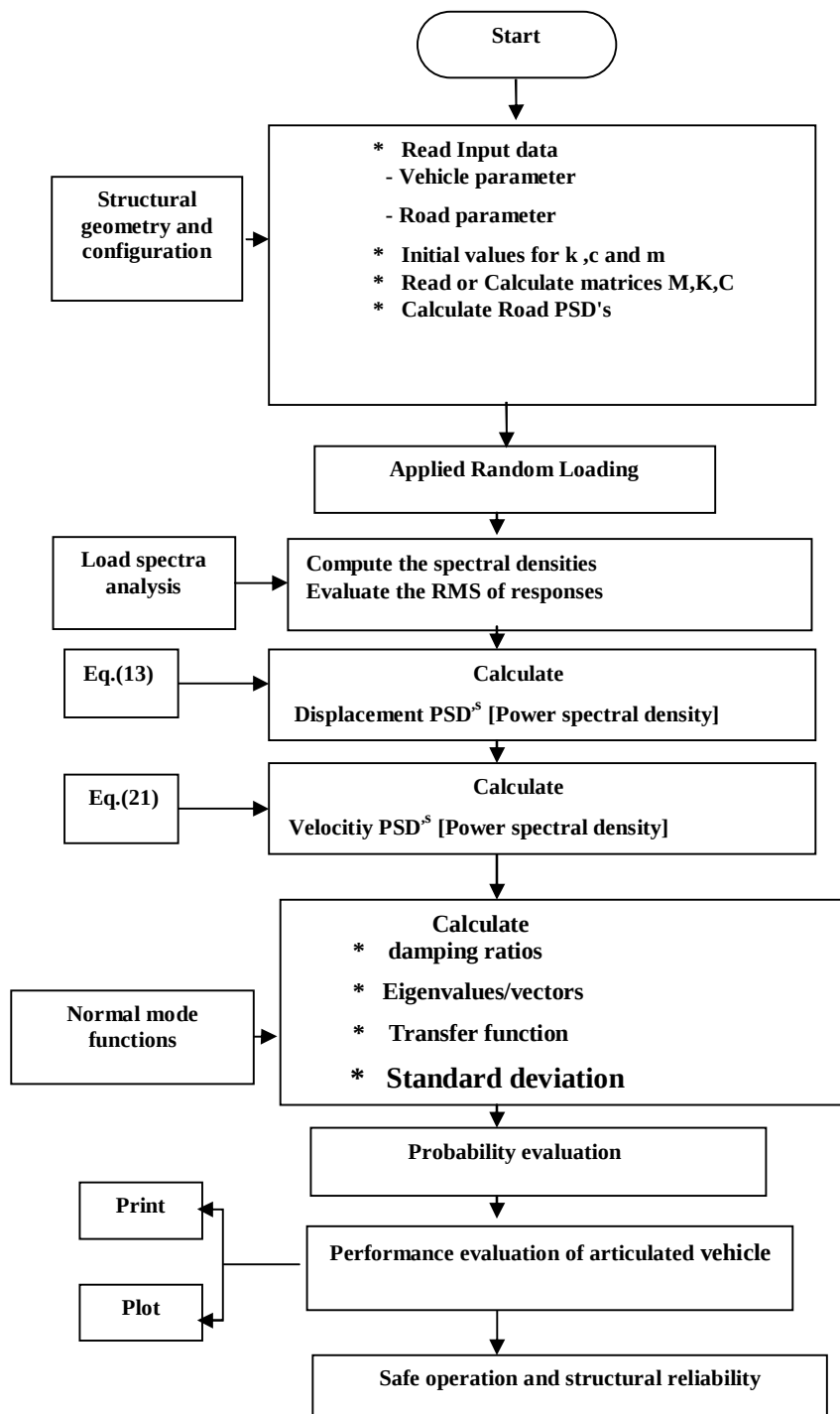


Fig. (3) Flow diagram for numerical algorithm

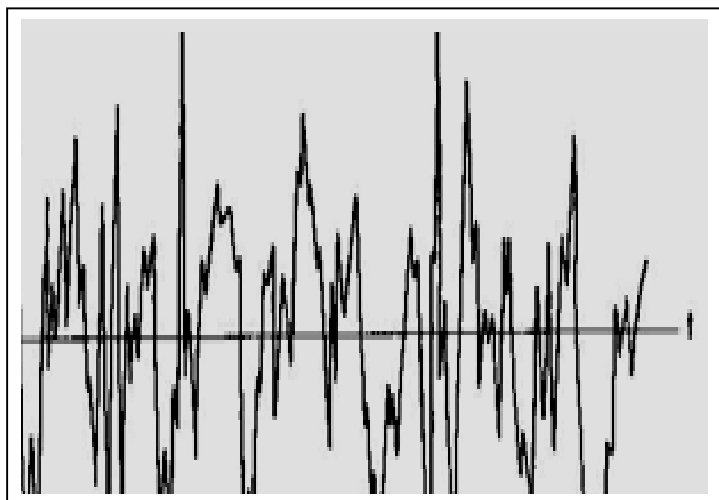


Fig. (4) Time history of road random input

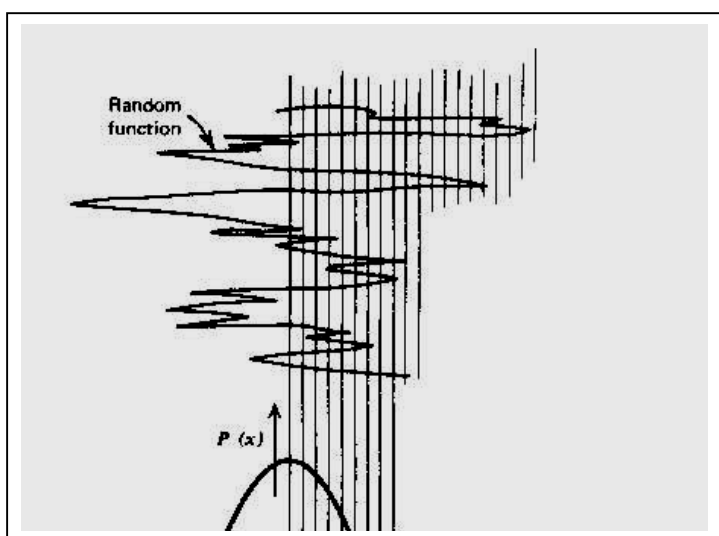


Fig. (5) Gaussian distribution of road random input

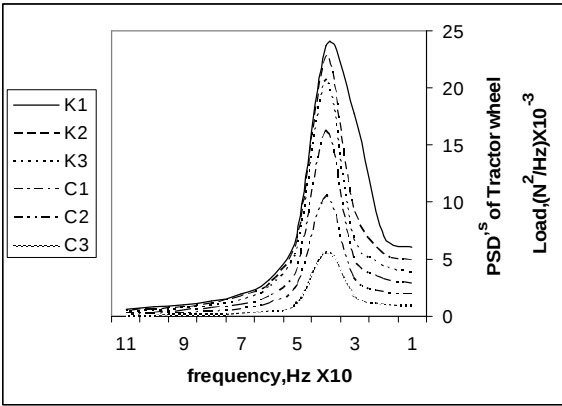


Fig.(6) Effect of tractor-trailer suspensions stiffness on wheel load spectra

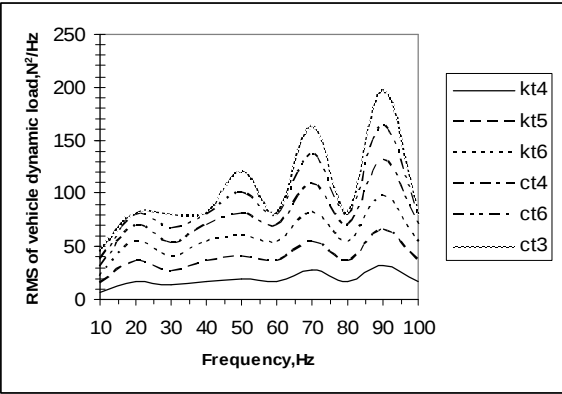


Fig. (7) Effect of tire stiffness and damping on vehicle wheel load RMS

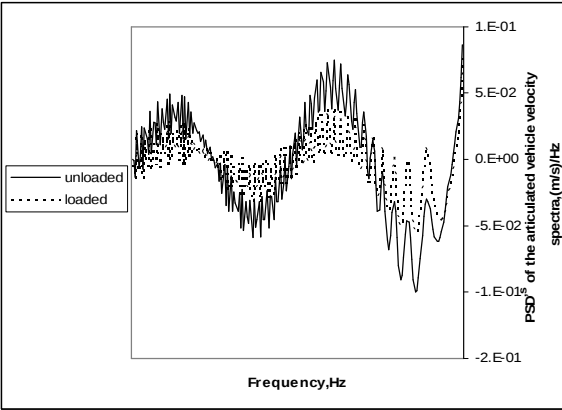


Fig. (8) Effect of articulated loaded-unloaded on articulated vehicle velocity spectrum

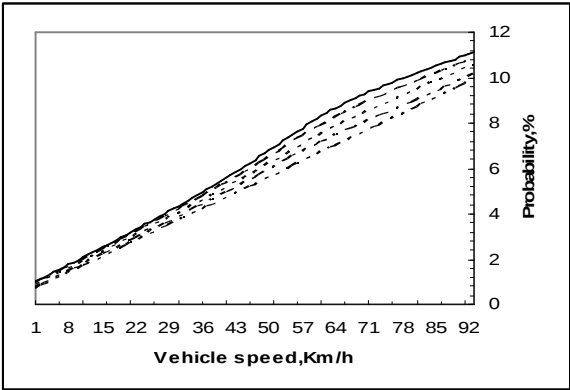


Fig. (9) Effect of speed on trailer off the road (Probability of departure of the wheel from the road)

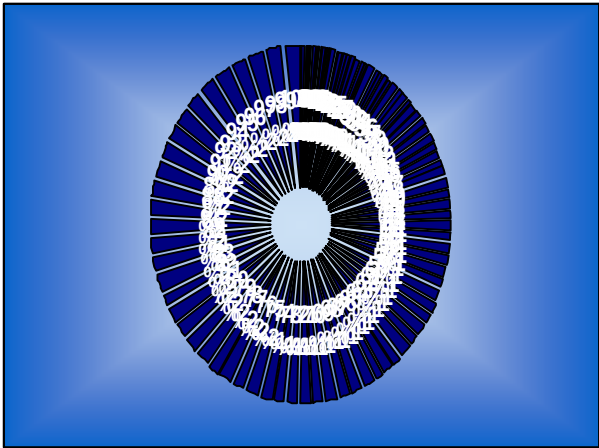


Fig. (10) Tire deformation

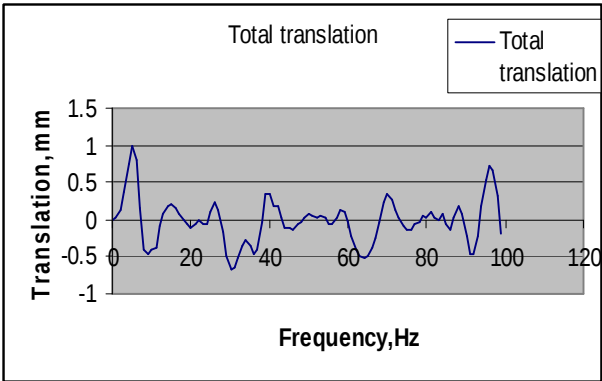


Fig. (11) Tire total translation

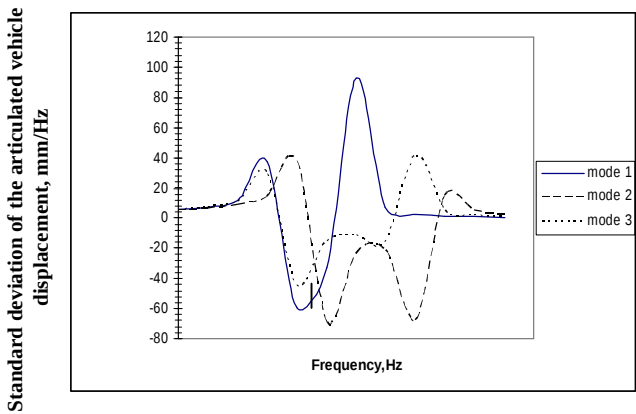


Fig. (12) The mode shapes of the articulated vehicle for the sixth mode of vibration

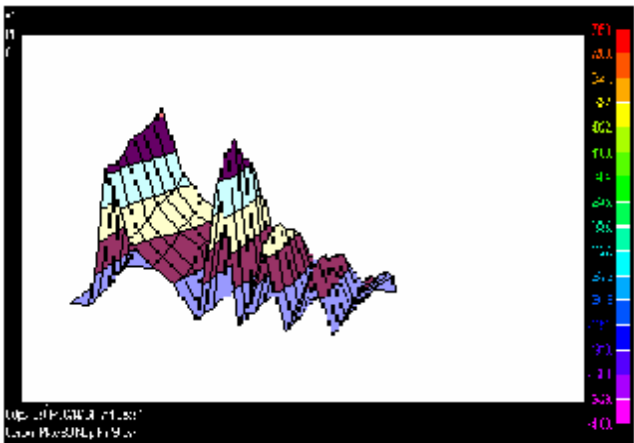


Fig. (13) Modal shape of the articulated vehicle (Tractor –trailer vehicle model)