Solving Differential Equations Using MATLAB

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ABSTRACT

The present paper demonstrates the route used for solving differential equations. Their applications are important in our daily lives, and the most important applications of differential equations are in science and engineering, where scientists and engineers can, through the use of differential equations, describe all the phenomena of the world. In short, it helps us describe all the phenomena in which social changes occur, and describe the way this change works, such as the spread of diseases, the rate of population growth, electric current, and others. Devoted to presenting the applications of some differential equations and their solution in the MATLAB program, and studying the following applications: engineering applications, physical applications, electrical applications, and others.

1. Introduction

A differential equation, In science of mathematics is an equation that correlate one or more unknown functions and their derivatives.[1][14] In applications, the functions generally explain quantities of physical, the derivatives demonstrate the equation that determines a relationship among the two. the Relations of this kind are commonplace; thence, Differential equations are indispensable for modeling dynamics in physics, biology, economics, and engineering. [15]

The studying of differential equations be composed mainly of the learning of their solutions (Each equation is satisfied by this set of functions), and of the properties of their solutions. [18]. just the simplest differential equations are solvable by formulas considering explicit; therefore, there are a many of properties for solutions of introduce differential equation may be determined without computing them exactly. [22] [23].

MATLAB is strongly recommended that you must acquire competency in a minimum of one software package from this list. This software automates and streamlines routine processes to calculations in calculus and linear algebra [16] [17].

The differential equation has many types, and therefore it constitutes ambiguity in how to solve it and how to distinguish between its types. Because the differential equation has several forms according to the order and degree determined by the variable itself, we resort to using a program in which we can find solutions to fulfill the initial condition given in the question [18].

In this paper, the aim is to present differential equations and some of their applications in science as follows: Definitions and basic concepts of differential equations [19]. Study on a computer system for solving differential equations. Studying differential equations and their applications with the rest of the sciences [20].

2. Research Terminology

- Paper method

in science of mathematics, the Equations consider are a mathematical formula that represent the equality of two expressions, by connecting them with the equals sign. [2, 3]. Differential equation (DE) define a mathematical equation that attach several function of one or more variable with its derivatives. And MATLAB can be introduced as powerful computing tools for curing the calculations

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included in scientific engineering problems. [4]. in addition, the order of a differential equation can be defines as the order of the greatest derivative in the equation. And the degree of a differential equation is the greater power of the highest – order for the derivative in the equation.

Examples:-

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0 \text{ ……… (1)}$$

the degree of these equation here is 1,the order of these equation here is 2.

$$\frac{dy}{dx} = 3x + 2 \text{ ………(2)}$$

the degree of the equation here is 1, the order of these equation here is 1. General solution:- A general solution of nth order differential equations is one that involves n necessary arbitrary constants.

In the Particular solution, the solution gained from the solution of general by assigning fixed value to the arbitrary constant.

Where did the importance of differential equations come from? Consider the differential equations are significant because for much systems of physical, perhaps, can be formulate a differential equation that describes how the changes of system in time. Therefore, the Understanding the solutions of the differential equation is then of paramount interest.

- Why are MATLAB important?

MATLAB provides tools for testing the accuracy of equation solutions, Integrations, derivative and to enhanced the results in styling problems that involve multiple differing parameters. MATLAB can be employed in laboratory sessions to analyze experimental datasets and to use for plots to visualize experiments and extract conclusions.

- First Orders Differential Equation

The Popular introduction for first order differential equation is a relation of the form

$$F(x, y, y') = 0, \text{…………(3)}$$

where F is defined as function on a set D in E. The differential function

$y = F(x)$ is said to differential equation solution of the if

$$F(x, y, y') = 0; \text{………..(4)}$$

x and y are name the independent and the dependent variables correspondingly. The third type of equations for a rise in differential fields. [7].

- Separable Variables

In the differential equation, a first order define to have separable variables, if the function $F(x,y)$ is a product of a continuous of x and a continuous function of y that is ,if it has the form [8]

$$\frac{dy}{dx} = g(x)h(y). \text{ ………… (5)}$$

To solve it we separate the variables x and y by putting the equation into the form

$$\frac{dy}{h(y)} = g(x)dx. \text{ ……… (6)}$$

Example (1): Find the general solution of the differential equation.

$$\frac{dy}{dx} = \frac{x-1}{y} \quad \text{.........} \ (7)$$

$$ydy = (x-1)dx \qquad \text{(by integral).}$$

$$\frac{y^2}{2} = \frac{x^2}{2} - x + c \qquad \text{(by multiplying in 2)}$$

$$y^2 = x^2 - 2x + 2c \qquad \text{(with the root of the both side)}$$

$$y = \mp\sqrt{x^2 - 2x + c1} \qquad (\,when\ c_1 = 2c)$$

Example (2): Detected the solution's general for the differential equation

$$dy = \sin x \ \cos^2 y \ dx \qquad \cos y \neq 0, y = (2n+1)\frac{\pi}{2}$$

$$\frac{dy}{\cos^2 y} = \sin x \, dx$$

$$\sec^2 y \, dy = \sin x \ dx$$

$$\tan y = -\cos x + c$$

$$y = \tan^{-1}(-\cos x + c)$$

- Homogeneous Differential Equation

Differential equation $Mdx + Ndy = 0$, consider homogeneous if it can be designed in the form [9]

$$\frac{dy}{dx} = \frac{\varphi(x,y)}{\omega(x,y)} \ \text{.......}(8)$$

Where $\varphi(x,y)$ and $\omega(x,y)$ are homogeneous functions of the same degree. Taking x^n common both from the number and the denominator of Eq. (8).

$$\frac{dy}{dx} = \frac{x^n \varphi(x,y)}{x^n \omega(x,y)} = F\left(\frac{y}{x}\right) \text{.......}(9)$$

Such types of equation can be solved by substitution $y = vx.$

Equation (9) thus become $v + x\frac{dv}{dx} = F(v)$ Or $\frac{dv}{F(v)-v} = \frac{dx}{x}.$

The variable have now been separated and the solution is

$$\int \frac{dv}{F(v)-v} = \ln x + c$$

After the integration v should be replaced by $\frac{y}{x}$ to get the required solution.

Example (3): Solve $(x^2 - y^2)dx + 2xy\,dy = 0$

Solution: The given equation can be written as $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$.......(10)

Put $y = vx \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ putting in (1)

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x^2\,v} \quad \rightarrow \quad v + x\frac{dv}{dx} = \frac{x^2(v^2-1)}{x^2\,2v}$$

$$\rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v} - v$$

$$x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v} \quad \rightarrow x\frac{dv}{dx} = \frac{-(v^2+1)}{2v}$$

$$\rightarrow \frac{2v\,dv}{(v^2+1)} = \frac{-\,dx}{x} \qquad \textit{be integral}$$

$$\ln(v^2+1) = -\ln x + \ln c$$

$$\rightarrow \ln((\frac{y}{x})^2 + 1) = \ln\frac{c}{x}$$

$$\rightarrow (\frac{y}{x})^2 + 1 = \frac{c}{x}$$

$$\left(\frac{y}{x}\right)^2 = \frac{c}{x} - 1 \quad \rightarrow \frac{y}{x} = \pm\sqrt{\frac{c}{x} - 1}$$

$$\rightarrow y = \pm x\sqrt{\frac{c}{x} - 1}$$

Example (4): Solve $xdy - ydx = \sqrt{x^2} + y^2\ dx$

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2} \quad ...(1)$$

$$y = xv$$

$$\rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v \qquad \textit{putting in } (1)$$

$$x\left(x\frac{dv}{dx}+v\right)-xv=\sqrt{(x^2+x^2v^2)}$$

$$\rightarrow x^2\frac{dv}{dx}+xv-xv=\sqrt{x^2(1+v^2)}$$

$$\rightarrow\ x^2\ \frac{dv}{dx}=x\sqrt{(1+v^2)}$$

$$\rightarrow\frac{dv}{\sqrt{1+v^2}}=\frac{dx}{x}\quad be\ integral$$

$$\rightarrow\ \sinh^{-1}v=lnx+c$$

$$\rightarrow\sinh^{-1}\frac{y}{x}\quad=\ln x+c$$

$$\rightarrow y\quad=xsinh\,(\ln x+c)$$

- Exact Equations

First –order equation $y'=F(x,y)$ always can be written as

$$M(x,y)dx\ +N(x,y)dy\ =0\qquad\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Where $M(x,y)$ and $N(x,y)$ are some function [10]. If the left –hand side of (12) is total differential of some function $F(x,y)$

$$F(x,y)=M(x,y)dx\ +N(x,y)d\ y\qquad\ldots\ldots\ldots\ldots(13)$$

Then equation (13) is written as

$$dF(x,y)=0\ \ldots\ldots\ (14)$$

Such equation is called exact. Solution of (14) is

$$F(x,y)=C.\ \ldots\ldots\ldots(15)$$

Recall from calculus that in order for $M(x,y)dx\ +N(x,y)\ dy$ be a total differential, it is necessary and sufficient that

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}\qquad\ldots\ldots\ldots\ldots(16)$$

Example (5): Solve: $\frac{y}{x^2}dx-\frac{1}{x}\ dy=0$

$M(x,y)=\ \frac{y}{x^2}\ ,N(x,y)=\ -\frac{1}{x}$

$$\rightarrow\quad\frac{\partial M}{\partial y}=\frac{1}{x^2}\quad,\frac{\partial N}{\partial x}=\frac{1}{x^2}$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}\quad Exact$$

$$\int M(x,y)dx=\frac{-y}{x}$$

$$\int N(x,y)dy=\frac{-y}{x}$$

$$\frac{\partial}{\partial y}\int M(x,y)dx = -\frac{1}{x}$$

$$\rightarrow \int [\frac{\partial}{\partial y}\int M(x,y)dx]dy = \frac{-y}{x}$$

$$\int M(x,y)dx + \int N(x,y)dy - \int [\frac{\partial}{\partial y}\int M(x,y)dx]dy = C$$

$$\frac{-y}{x} + \frac{y}{x^2} + \frac{y}{x} = c$$

$$\frac{-y}{x} = c$$

- Linear Equations

Given a first order, ordinary linear equation of the general form [11]

$$\frac{dy}{dx} + p(x)y = Q(x) \text{............}(17)$$

Multiply both sides by an integral factor

$$u(x) = e^{\int p(x)\,dx} \text{...........}(18)$$

To get $y(u(x)) = \int u(x)Q(x)dx + c$

The general solution for (18) is then

$$y = \frac{1}{u(x)}\int u(x)Q(x)dx + c \text{ ....}(19)$$

Example (6): Solve: $\frac{dy}{dx} + \frac{1-2x}{x^2}\ y = 1$

$$u(x) = e^{\int \frac{1-2x}{x^2}dx}$$

$$= e^{\int \frac{1}{x^2} - \frac{2}{x^2}}\ dx$$

$$= e^{\frac{-1}{x} - 2\ln x}$$

$$u(x) = e^{\frac{-1}{x}} e^{\ln x^{-2}}$$

$$u(x) = \frac{1}{x^2} e^{\frac{-1}{x}}$$

$$u(x)y = \int \frac{1}{x^2} e^{\frac{-1}{x}}\ dx$$

$$\frac{1}{x^2} e^{\frac{-1}{x}}\ y = e^{\frac{-1}{x}} + c \qquad \textit{mulitiplied by } x^2$$

$$y e^{\frac{-1}{x}} = x^2 e^{\frac{-1}{x}} + cx^2 \qquad \textit{mulitiplied by } e^{\frac{1}{x}}$$

$$y = x^2 + cx^2 e^{\frac{1}{x}}$$

3. Using MATLAB To Solve First-Order Differential Equation

MATLAB provides a powerful environment for solving first-order differential equations using built-in numerical solvers like ode45, ode23, and others. These solvers use numerical integration techniques, such as Runge-Kutta methods, to approximate solutions efficiently. To solve a first-order differential equation:

dy/dt=f(t,y)dy/dt = f(t, y)dy/dt=f(t,y)

users define the function in a separate script or as an anonymous function, specify initial conditions, and call an appropriate solver. MATLAB then computes and visualizes the solution, making it an essential tool for engineers, scientists, and researchers dealing with differential equations.

- Problem 1:

Detect the solution's general for the following differential equations kind a first order (DEs).

$$y' \cos x - (y+1)\sin x = 0\,; \quad y(0) = 0.$$

This MATLAB script solves the differential equation from type first order using the ode45 solver, which is well-suited for non-stiff differential equations. The equation is rewritten in the standard form dy/dt=f(t,y)dy/dt = f(t, y)dy/dt=f(t,y) and defined as an anonymous function. The solver takes an initial condition y(0)=y0y(0) = y_0y(0)=y0 and integrates the equation over a given time span. Finally, the script plots the numerical solution for visualization.

The solution in MATALB shown in the following: and the figure 1: shown the result in MATLAB

$$y = \frac{1}{\cos x} - 1$$

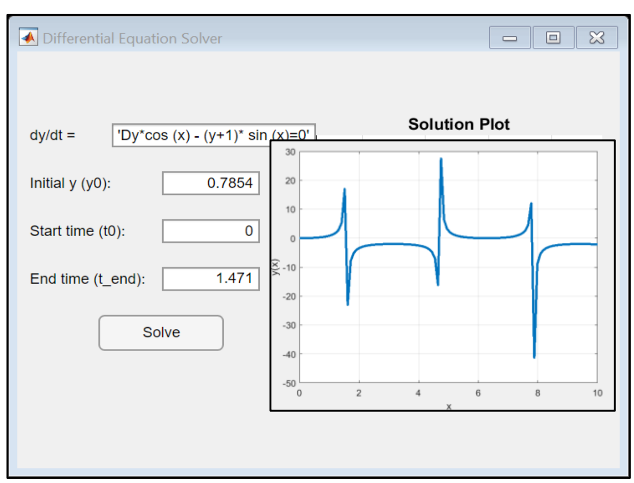

Figure (1)

- Problem 2:

1. Detect the solution's general of ODE

$$Dy + y = 3.$$

2. by crossing from (0, 2), can be Detected a particular solution and Determined the value of $y(\infty)$.

3. By a set of different solutions of this ODE with the initial conditions can be Plot $(0, k),\ k \in [-2,11]$ (dsolv2.m).

The solution :

a) The solution's general are $y(t) = Ae^{-t} + 3$.

b) For the solution in a particular,

$$y(0) = 2 = Ae^{0} + 3 = A + 3 \rightarrow A = -1$$

$$\rightarrow y(t) = -e^{-t} + 3\,.$$

Substituting $t = \infty$ in the equation yields $y(\infty) = 3$.

a) For this ODE there are group of the various solutions with the initial condition (0,k), $k \in [-2,11]$ *is shown* in Figure: 2, particular solution is be seen in red color.

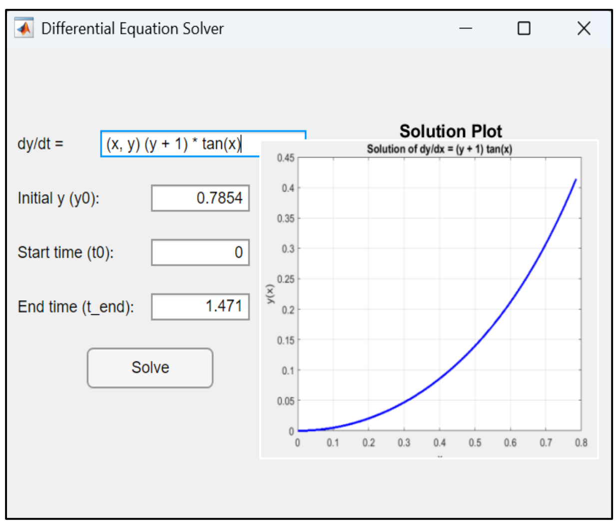

Figure (2)

- Problem 3:

Detect the solution's general for the following differential equations from type first order (DEs).

y'cos〖x-(y+1)〗 sin〖x=0 ; y(0)=0.〗

a) Rearrange the equation to express dy/dxdy/dxdy/dx in terms of xxx and yyy.

b) Define the function in MATLAB as an anonymous function.

c) the initial condition is set as y(0)=0y(0) = 0y(0)=0.

d) Use ode45 to solve in MATLAB the differential equation numerically.

e) Plot a solution that visualize y(x)y(x)y(x) over a defined range. Shown in Figure: (3)

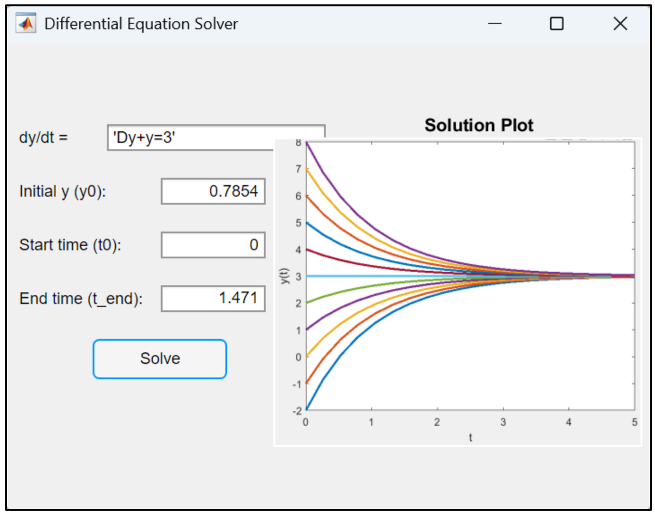

Figure (3)

- Problem 4:

Detect the solution's general for the following differential equations from type first order (DEs).

Dy+te^(y+t)=0

with the initial condition: y(1)=0

a) Rearrange the equation to express: dy/dtdy/dtdy/dt in terms of ttt and yyy.
b) Define the function in MATLAB using an anonymous function.
c) Set the initial condition y(1)=0y(1) = 0y(1)=0.
d) Use ode45 to solve the equation numerically over the interval [1,5][1, 5][1,5].
e) Plot the solution to visualize y(t)y(t)y(t). shown in Figure: (4)

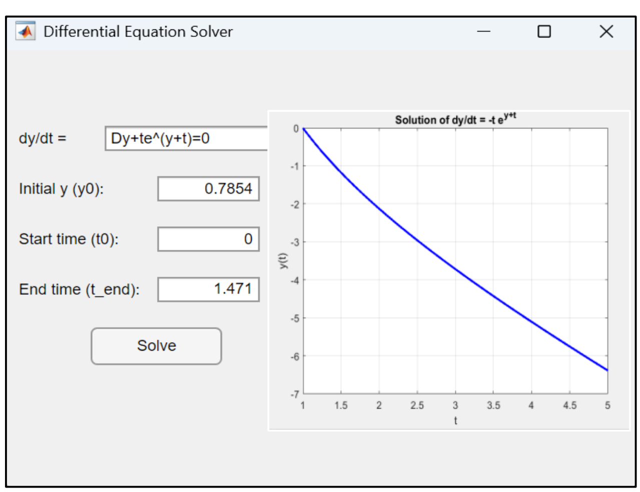

Figure (4)

4. Conclusion:

- Through research, we arrive at some points that can be summarized as follows:

- Presenting the equations in advance simplify the choice of the type and method of different differential solutions, as it begins with order reduction and separation and ends with the common equations.
- Enjoying the importance of differential equations from sitting in our daily lives in several fields, most notably in the field of computers, engineering, physics, and medicine. There are issues that are created by the second differential equations.
- The alternative solution to solve the problem of constructing how to choose the solution to solve differential equations.

5. Recommendations

Recommendations: Through the study of the research, we explored some things that must be taken into account regarding differential equations, which are:

- Expansion of higher order differential equations, an explanation of the importance of differential equations.
- Show that solving differential equations varies from easiest to most difficult.
- Explaining the applications in which differential equations are used to clarify the importance and place of mathematics, especially differential equations.

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