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Numerical Solution of Helmholtz Equation using Fuzzy Data

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ABSTRACT

In this paper, we have discussed fuzzification of partial differential equation Helmholtz Equation in two dimensions are considered . The interval of fuzzy interval used. The finite difference method applied of two different meshs using 5-points. first for initial values and the second for dissolve Helmholtz equation numerically.

MSC: Mathematical

1.Introduction

The Fuzzy differential equation was first presented by Chang Zadeh [8]. Dubois and Prade[1] has given extension principle. Raphel and Mhassin[10,2], used 5-points in mesh regular . Here implementing 5-points for finite difference method for dissolve Helmholtz equation in two variable numerically, then fuzzified.

2. *Definitions*

A triangular Fuzzy number □ *is defined by three real numbers with base as the interval [a, c] and b as the vertex of triangle. The membership function are defined as follows [2]*

$$\mu(X)(=)\left\{\frac{x-a}{b-a}\ ;\ where\ a \le x \le b\ \frac{x-c}{b-c}\ ;\ where\ b \le x \le c\ \ 0 \qquad ; \quad otherwise \right\}$$

where the α-cut is known by:.

$$\Delta_K(\ )(=)a+(b-a). \qquad and \qquad \Delta_P(\ )(=)\,c+(b-c).$$

A triangular Fuzzy number $_g$ *is defined by three real numbers with base as the interval* $[g_a\ ,g_c]$ *and* g_b *as the vertex of triangle. The membership function are defined as follows.*

$$_g(x)(=)\left\{\frac{x-g_a}{g_b-g_a};\ where\ \ g_a \le x \le g_b\ \frac{x-f_c}{g_b-g_c};where\ \ g_b \le x \le g_c\ 0 \qquad otherwise\right\}$$

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Where α-cut is defined by :. $\Delta_K(\ )(=)g_a + (g_b - g_a)$. *and* $\Delta_P(\ )(=)g_c + (g_b - g_c)$.

2.1 Finite difference using to solution Helmholtz equation

Helmholtz equation in two dimensional is difined by :

$$\left[v_{xx}(x,y) + v_{yy}(x,y)\right] + v(x,y) = g(x,y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

The region R in the x, y-plane . is a parameter,$v(x,y)$, is the function of the two independent variables (x,y), which is the required solution for and $g(x,y)$is represent forcing function.

Due to the relationship of the Helmholtz equation with the wave equation, it is used in solving physical problems related to the study of electromagnetic waves, seismology as well as the study of sound waves. Replacing v_{xx} and v_{yy} by the central difference equation the value of v(x_i, y_j) at any mesh point is the arithmetic mean of the values at four neighboring grid to the left, right, above and down, which is called standard five points Equation (SFPE) Fig.1, or in any mesh point is the Diagonally ,of The values at four neighboring grid to the left- above , right- above, right-down , left-down, we use the equation which is called five point Equation (DFPE) Fig.2, use for finding the initial data respectively as follows replacing v_{xx} and v_{yy} by finite difference method.

$$v_{xx} = \frac{v_{i-1,j} - 2v_{i,j} + v_{i+1,j}}{h^2}; \quad and \quad v_{yy} = \frac{v_{i,j-1} - 2v_{i,j} + v_{i,j+1}}{k^2}$$

So, (1) become

$$\frac{v_{i-1,j} - 2v_{i,j} + v_{i+1,j}}{h^2} + \frac{v_{i,j-1} - 2v_{i,j} + v_{i,j+1}}{k^2} + v_{i,j} = g_{i,j} \ldots\ldots\ldots\ldots.\ \ (2)$$

we can taking h = k in the square grid, the value of $v_{i,j}$at any point is the arithmetic mean of its values at the four neighboring mesh points to the left, right, above and down.

Then :

$$v_{i,j} = \frac{1}{4-h^2}\left[v_{i-1,j} + v_{i+1,j} + v_{i,j-1} + v_{i,j+1} - h^2 g_{i,j}\right] \quad \ldots.\ \ldots\ldots\ldots\ldots\ldots\ (3)$$

This is equation called Standard Five Points Equation(SFPE) as in Fig.1 or

$$v_{i,j} = \frac{1}{4-h^2}\left[v_{i+1,j+1} + v_{i+1,j-1} + v_{i-1,j+1} + v_{i-1,j-1} - 2h^2 g_{i,j}\right] \ldots\ldots\ldots\ldots..\ (4)$$

This is equation called Diagonally Five Points Equation(DFPE) as in Fig.2, The equation(4) wherever necessary.

3. Application of Fuzzy interval in Helmholtz Equation

In the table.1 ,From $s_1\ to\ s_{16}$ represents the boundary conditions of the square grid with fuzzy interval .

Table-1-

$s_1(=)[J_{1,1}, J_{1,2}, J_{1,3}]$	$s_2(=)[J_{2,1}, J_{2,2}, J_{2,3}]$	$s_3(=)[J_{3,1}, J_{3,2}, J_{3,3}]$	$s_4(=)[J_{4,1}, J_{4,2}, J_{4,3}]$
$s_5(=)[J_{5,1}, J_{5,2}, J_{5,3}]$	$s_6(=)[J_{6,1}, J_{6,2}, J_{6,3}]$	$s_7(=)[J_{7,1}, J_{7,2}, J_{7,3}]$	$s_8(=)[J_{8,1}, J_{8,2}, J_{8,3}]$
$s_9(=)[J_{9,1}, J_{9,2}, J_{9,3}]$	$s_{10}(=)[J_{10,1}, J_{10,2}, J_{10,3}]$	$s_{11}(=)[J_{11,1}, J_{11,2}, J_{11,3}]$	$s_{12}(=)[J_{12,1}, J_{12,2}, J_{12,3}]$
$s_{13}(=)[J_{13,1}, J_{13,2}, J_{13,3}]$	$s_{14}(=)[J_{14,1}, J_{14,2}, J_{14,3}]$	$s_{15}(=)[J_{15,1}, J_{15,2}, J_{15,3}]$	$s_{16}(=)[J_{16,1}, J_{16,2}, J_{16,3}]$

Fig.1

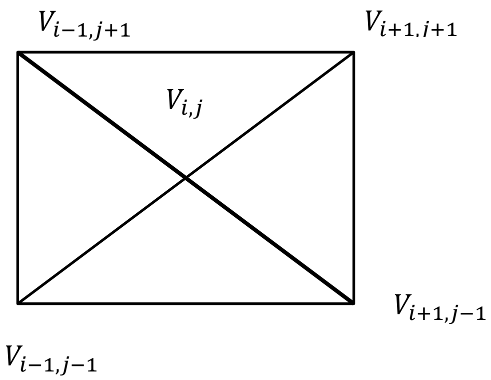


Fig.2

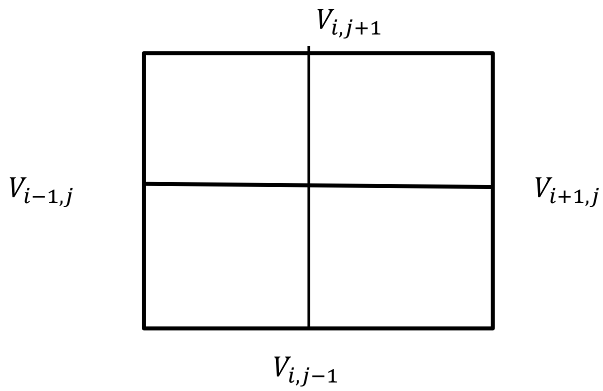
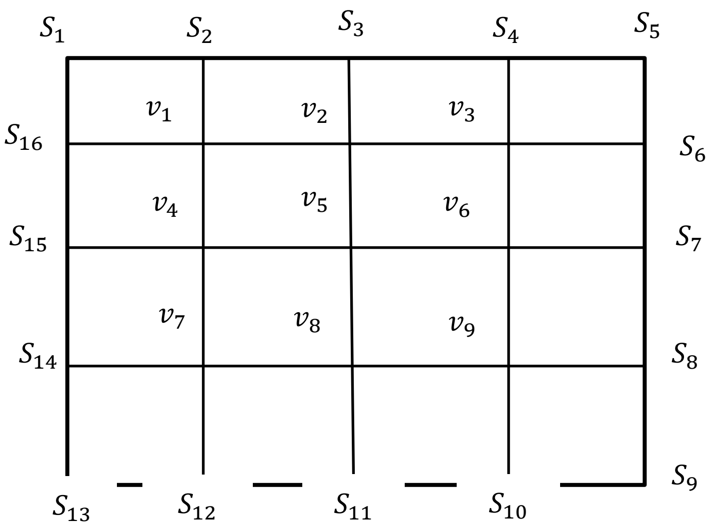


Fig.3



From $v_1\ to\ v_9$ are represents the interior points because to the square mesh as show in Fig.3 , Now, we using the Standard Five Point Equation(SFPE) to evaluate initial value of $v_5^{(0)}$

from equation (4)as.

$v_5^{(0)}(=)\frac{1}{4-h^2}[s_3+s_7+s_{11}+s_{15}-h^2(g_5)]$…………………….. (5)

We can write equation (4) in some specifics

$$\left[v_5^{(0)}\right]^{(\alpha)}\ (=)\left[\frac{J_{3,1}+J_{7,1}+J_{11,1}+J_{15,1}-h^2g_{5,1}}{4-h^2},\frac{J_{3,2}+J_{7,2}+J_{11,2}+J_{15,2}-h^2g_{5,2}}{4-h^2},\frac{J_{3,3}+J_{7,3}+J_{11,3}+J_{15,3}-h^2g_{5,3}}{4-h^2}\right]\dots\dots\dots\dots\dots\dots..(6)$$

Fuzzy membership function (f.m.f) are competent α -cuts of $s_1, s_3, s_5, s_7, s_9, s, s_{13}$ and s_{15} are respectively as

$$\mu s_i\,(x)=\left\{\frac{x-J_{i,1}}{J_{i,2}-J_{i,1}}\ ;where\ J_{i,1}\le x\le J_{i,2}\ \ \frac{x-J_{i,3}}{J_{i,2}-J_{i,3}}\ ;where\ J_{i,2}\le x\le J_{i,3}\ 0\qquad ;\quad otherwise\right\}$$

Hence the α-cuts of s_i is given by

$[s_i]^{(\alpha)}(=)\left[J_{i,1}+\alpha\left(J_{i,2}-J_{i,1}\right), J_{i,3}+\alpha(J_{i,2}-J_{i,3})\right]$ where i=1,2,3,…,16. Then from equation (6)we have

$$v_5^{(0)}=\left\{\frac{(J_{3,2}-J_{3,1})+(J_{7,2}-J_{7,1})+(J_{11,2}-J_{11,1})+(J_{15,2}-J_{15,1})-h^2(g_{5,2}-g_{5,1})}{4-h^2}\alpha+\frac{(J_{3,1}+J_{7,1}+J_{11,1}+J_{15,1}+h^2g_{5,1})}{4-h^2}\ \frac{(J_{3,1}-J_{3,3})+(J_{7,2}-J_{7,3})+(J_{11,2}-J_{11,3})+(J_{15,2}-J_{15,3})-h^2(g_{5,2}-g_{5,3})}{4-h^2}\alpha+\frac{(J_{3,3}+J_{7,3}+J_{11,3}+J_{15,3}-h^2g_{5,3})}{4-h^2}\right\}$$

Or

$\mu_{v_5^{(0)}}(x)=\left\{\frac{T_{5,2}-T_{5,1}}{4-h^2}\alpha+\frac{T_{5,1}}{4-h^2}\ \frac{T_{5,2}-T_{5,3}}{4-h^2}\alpha+\frac{T_{5,3}}{4-h^2}\right\}$ where $\{T_{5,1}=J_{3,1}+J_{7,1}+J_{11,1}+J_{15,1}-h^2g_{5,1}\ T_{5,2}=J_{3,2}+J_{7,2}+J_{11,2}+J_{15,2}-h^2g_{5,2}\ T_{5,3}=J_{3,3}+J_{7,3}+J_{11,3}+J_{15,3}-h^2g_{5,3}\}$

Let

$x_1=\frac{T_{5,2}-T_{5,1}}{4-h^2}\alpha+\frac{T_{5,1}}{4-h^2}\ \ and\ \ x_2=\frac{T_{5,2}-T_{5,3}}{4-h^2}\alpha+\frac{T_{5,3}}{4-h^2}$; solution for α .

So, We have $\alpha=\frac{4x_1-T_{5,1}}{T_{5,2}-T_{5,1}}$.Hence (f.m.f) for $v_5^{(0)} is$

$$\mu_{v_5^{(0)}}(x)=\left\{\frac{(4-h^2)x-T_{5,1}}{T_{5,2}-T_{5,1}}\ ;\qquad \frac{1}{4-h^2}T_{5,1}\le x\le\frac{1}{4-h^2}T_{5,2}\ \ \frac{(4-h^2)x-T_{5,3}}{T_{5,2}-T_{5,3}}\ ;\qquad \frac{1}{4-h^2}T_{5,2}\le x\le\frac{1}{4-h^2}T_{5,3}\ 0\qquad ;otherwise\right\}\dots\dots\dots\dots..(7)$$

Where;α between the interval [0,1]

We want to find the initial values of v_3, v_7, v_9 and v_1 using 5- points diagonally (DFPE) i.e. equation (4), and so we want to find The initial values of v_4, v_6, v_8 and v_2 by (SFPE) i.e. equation (3).The initial value of $v_1^{(0)}$ we use the equation (4) interval of confidence.

$$v_1^{(0)} = \left[\frac{J_{1,1}+J_{3,1}+v_{5,1}^{(0)}+J_{15,1}-2h^2g_{1,1}}{4-h^2}, \frac{J_{1,2}+J_{3,2}+v_{5,2}^{(0)}+J_{15,2}-2h^2g_{1,2}}{4-h^2}, \frac{J_{1,3}+J_{3,3}+v_{5,3}^{(0)}+J_{15,3}-2h^2g_{1,3}}{4-h^2}\right] \text{…………..(8)}$$

then(f.m.f) for s_1 is

$$\mu_{s_1}(x) = \left[\frac{x-J_{1,1}}{J_{1,2}-J_{1,1}} \ ; where\ J_{1,1} \le x \le J_{1,2}\ \frac{x-J_{1,3}}{J_{1,2}-J_{1,3}}\ ;\ where\, J_{1,2} \le x \le J_{1,3}\ 0 \qquad ;otherwise\right]$$

,so then α-cuts of s_1

$$[s_1]^\alpha = \left[J_{1,1} + \alpha(J_{1,2} - J_{1,1});\quad J_{1,3} + \alpha(J_{1,2} - J_{1,3})\right].$$

Also, the interval confidence of $s_{15}, s_3\ and\ v_5^{(0)}$ are

$$[s_{15}]^{(\alpha)} = \left[J_{15,1} + \alpha(J_{15,2} - J_{15,1}),\quad J_{15,3} + \alpha(J_{15,2} - J_{15,3})\right],$$

$[s_3]^\alpha = \left[J_{3,1} + \alpha(J_{3,2} - J_{3,1}),\quad J_{3,3} + \alpha(J_{3,2} - J_{3,3})\right], and\ \left[v_5^{(0)}\right]^\alpha = \left[v_{5,1}^{(0)} + \alpha\left(v_{5,2}^{(0)} - v_{5,1}^{(0)}\right),\quad v_{5,3}^{(0)} + \alpha(v_{5,2}^{(0)} - v_{5,3}^{(0)})\right].$

So it is, the α-cuts of $v_1^{(0)}$ is

Let $x_1 = \frac{L_{1,2}-L_{1,1}}{4-h^2}\alpha + \frac{L_{1,1}}{4-h^2}$ and $x_2 = \frac{L_{1,3}-L_{1,2}}{4-h^2}\alpha + \frac{L_{1,3}}{4-h^2}$, solution for α

$$\alpha = \frac{4x_1-L_{11}}{L_{1,2}-L_{1,1}}\ and\ \alpha = \frac{4x_2-L_{1,3}}{L_{1,2}-L_{1,3}}$$

$$v_1^{(0)} = \left[\frac{L_{1,2}-L_{1,1}}{4-h^2}\alpha + \frac{L_{1,1}}{4-h^2}\ \frac{L_{1,2}-L_{1,3}}{4-h^2}\alpha + \frac{L_{1,3}}{4-h^2}\right] \quad \text{where} \quad \left\{L_{1,1} = J_{1,1} + J_{3,1} + J_{15,1} + v_{5,1}^{(0)} - 2h^2g_{1,1}\ L_{1,2} = J_{1,2} + J_{3,2} + J_{15,2} + v_{5,2}^{(0)} - 2h^2g_{1,1}\ L_{1,3} = J_{1,3} + J_{3,3} + J_{15,3} + v_{5,3}^{(0)} - 2h^2g_{1,3}\right\}$$

Then (f.mf) for $v_1^{(0)}$ is

$$\mu_{v_1^{(0)}}(x) = \left\{\frac{(4-h^2)x-L_{1,1}}{L_{1,2}-L_{1,1}}\ where\ \frac{1}{4-h^2}L_{1,1} \le x \le \frac{1}{4-h^2}L_{1,2}\ \frac{(4-h^2)x-L_{1,3}}{L_{1,2}-L_{1,3}}\ where\ \frac{1}{4-h^2}L_{1,2} \le x \le \frac{1}{4-h^2}L_{1,3}\ 0 \quad ; \quad otherwise\right\} \text{……………………..(9)}$$

This process can be repeated with $v_3, v_9\ and\ v_7$ .In the same way we can find $v_2, v_4\ and\ v_6, for\ v_2^{(0)}$

We , now find the interval of confidence of $s_3, v_3^{(0)}, v_5^{(0)}\ and\ v_1^{(0)}\ we\ get$

$v_2^{(0)} = \left[\frac{J_{3,1}+v_{3,1}^{(0)}+v_{5,1}^{(0)}+v_{1,1}^{(0)}-h^2 g_{2,1}}{4-h^2}, \frac{J_{3,2}+v_{3,2}^{(0)}+v_{5,2}^{(0)}+v_{1,2}^{(0)}-h^2 g_{2,2}}{4-h^2}, \frac{J_{3,3}+v_{3,3}^{(0)}+v_{5,3}^{(0)}+v_{1,3}^{(0)}-h^2 g_{2,3}}{4-h^2}\right]$ …………(10)

Let $x_1 = \frac{R_{2,2}-R_{2,1}}{4-h^2}\alpha + \frac{R_{2,1}}{4-h^2}$ and $x_2 = \frac{R_{2,2}-R_{2,3}}{4-h^2}\alpha + \frac{R_{2,3}}{4-h^2}$, solution for α

$\alpha = \frac{4x_1-R_{2,1}}{R_{2,2}-R_{2,1}}$ and $\alpha = \frac{4x_2-R_{2,3}}{R_{2,2}-R_{2,3}}$

$v_2^{(0)} = \left[\frac{R_{2,2}-R_{2,1}}{4-h^2}\alpha + \frac{R_{2,1}}{4-h^2} \; \frac{R_{2,2}-L_{2,3}}{4-h^2}\alpha + \frac{R_{2,3}}{4-h^2}\right]$ where $\{R_{2,1} = J_{3,1} + v_{5,1}^{(0)} + v_{1,1}^{(0)} + v_{3,1}^{(0)} - 2h^2 g_{2,1}\; R_{2,2} = J_{3,2} + v_{5,2}^{(0)} + v_{1,2}^{(0)} + v_{3,2}^{(0)} - 2h^2 g_{2,2}\; R_{2,3} = J_{3,3} + v_{5,3}^{(0)} + v_{1,3}^{(0)} + v_{3,3}^{(0)} - 2h^2 g_{2,3}\}$;

The (f.m.f) for $v_2^{(0)}$ is:

$\mu_{v_2^{(0)}} \left\{ \frac{(4-h^2)x-R_{2,1}}{R_{2,2}-R_{2,1}} \; where \; \frac{1}{4-h^2}R_{2,1} \le x \le \frac{1}{4-h^2}R_{2,2} \;\; \frac{(4-h^2)x-R_{2,3}}{R_{2,2}-R_{2,3}} \; where \; \frac{1}{4-h^2}R_{2,2} \le x \le \frac{1}{4-h^2}R_{2,3} \;\; 0 \;\; otherwise \right\}$……………………..….(11)

Next successive approximations with their f.m.f. as it is required be obtain from previous approximations and specified boundary conditions.

4. Numerical example for the Helmholtz equation

Let us consider the Helmholtz example deals with , g(x,y).

$\frac{d^2 v}{dx^2} + \frac{d^2 v}{dy^2} + v_{i,j} = g_{i,j}$ …………………..(12)

We have : $g_{i,j} = xe^y$ and = 0,

In the region R=$[(x,y): 0 \le x \le 4 \;,\; 0 \le y \le 4]$. The boundary condition:-

$v(x,0) = x \;;\; v(0,y) = 0$

$v(4,y) = 4e^{-y} \;;\; v(x,4) = xe^{-4}$ .the exact solve is given by $v(x,y) = xe^{-y}$.

<u>Solution:</u> In the region $0 \le x \le 4 \,,\; 0 \le x \le 4$ with the boundary interview for the points as shown in table-2-

We calculated the boundary conditions from equations in table-2-

s_1=0	s_2=0.01831563	s_3=0.0366312	s_4=0.0549469	s_5=0.0732625
s_{16}=0	v_7	v_8	v_9	s_6=0.1991482
s_{15}=0	v_4	v_5	v_6	s_7=0.5413411
s_{14}=0	v_1	v_2	v_3	s_8=1.4715177
s_{13}=0	s_{12}=1	s_{11}=2	s_{10}=3	s_9=4

Table-2-

We will be applied Leibmann's to solve equation (12)

So ,we can calculated the initial values v_i=1,2,3,…,9 from the standard five point and the diagonal five points equation, to get the approximate solution .from equation (5)and(7) we have

$v_5^{(0)}(=)\frac{1}{4-h^2}[s_3+s_7+s_{11}+s_{15}-h^2(g_5)]$…………………….. (5)

$\mu_{v_5^{(0)}}(x)(=)\begin{cases}\frac{(4-h^2)x-T_{5,1}}{T_{5,2}-T_{5,1}} ; & \frac{1}{4-h^2}T_{5,1}\le x\le \frac{1}{4-h^2}T_{5,2}\\ \frac{(4-h^2)x-T_{5,3}}{T_{5,2}-T_{5,3}} ; & \frac{1}{4-h^2}T_{5,2}\le x\le \frac{1}{4-h^2}T_{5,3}\\ 0 & ; otherwise\end{cases}$

where $\{T_{5,1}=J_{3,1}+J_{7,1}+J_{11,1}+J_{15,1}-h^2g_{5,1}\ T_{5,2}=J_{3,2}+J_{7,2}+J_{11,2}+J_{15,2}-h^2g_{5,2}\ T_{5,3}=J_{3,3}+J_{7,3}+J_{11,3}+J_{15,3}-h^2g_{5,3}\}$

$v_5^{(0)}$ (=)[31.492603060, 31.493603060, 31.494603060]………….(13)

Now, we find (f.m.f) its own interval confidence these five s_i for i=1,16,15,14and13 as such follows

$\mu_{s_i^{(0)}}(x)(=)\begin{cases}\frac{x+0.001}{0.001+0.001} ; & where\ 0.001\le x\le 0\\ \frac{x-0.001}{0.001-0.001} ; & where\ 0.001\le x\le 0.001\\ 0 ; & otherwise\end{cases}$,

$[s_i]^{(\alpha)}(=)[0.001\alpha+0.001,-0.001\alpha+0.001]$, where i=1,16,15,14 and13 we are find(f.m.f)by same way.

$\mu_{s_{12}^{(0)}}(x)(=)\begin{cases}\frac{x-0.999}{1-0.999} ; & where\ 0.999\le x\le 1\\ \frac{x-1.001}{1-1.001} ; & where\ 1\le x\le 1.001\\ 0 ; & otherwise\end{cases}$,

$[s_{12}]^{(\alpha)}(=)[0.001\alpha+0.999,-0.001\alpha+1.001]$,

$\mu_{s_{11}^{(0)}}(x)(=)\begin{cases}\frac{x-1.999}{2-1.999} ; & where\ 1.999\le x\le 2\\ \frac{x-2.001}{2-2.001} ; & where\ 2\le x\le 2.001\\ 0 ; & otherwise\end{cases}$,

$[s_{11}]^{(\alpha)}(=)[0.001\alpha+1.999,-0.001\alpha+2.001]$,

$\mu_{s_{10}^{(0)}}(x)(=)\begin{cases}\frac{x-2.999}{3-2.999} ; & where\ 2.999\le x\le 3\\ \frac{x-3.001}{3-3.001} ; & where\ 3\le x\le 3.001\\ 0 ; & otherwise\end{cases}$,

${s_{10}}^{(\alpha)}(=)[0.001\alpha+2.999,-0.001\alpha+3.001]$,

$\mu_{s_9^{(0)}}(x)(=)\left\{\frac{x-3.999}{4-3.999}\right.$; $where\ 3.999 \leq x \leq 4$ $\frac{x-4.001}{4-4.001}$; $where\ 4 \leq x \leq$ 4.001 0 ; $otherwise \}$,

$[s_9]^{(\alpha)}(=)[0.001\alpha + 3.999, -0.001\alpha + 4.001]$,

$[s_8]^{(\alpha)}(=)[0.001\alpha + 10.8721273138, -0.001\alpha + 10.8741273138]$,

$[s_7]^{(\alpha)}(=)[0.001\alpha + 29.5552243957, -0.001\alpha + 29.5572243957]$,

$[s_6]^{(\alpha)}(=)[0.001\alpha + 80.3411476928, -0.001\alpha + 80.3431476928]$,

$[s_5]^{(\alpha)}(=)[0.001\alpha + 218.3916001326, -0.001\alpha + 218.3936001326]$,

$[s_4]^{(\alpha)}(=)[0.001\alpha + 163.79345000994, -0.001\alpha + 163.79545000994]$,

$[s_3]^{(\alpha)}(=)[0.001\alpha + 109.19533000663, -0.001\alpha + 109.19733000663]$,

$[s_2]^{(\alpha)}(=)[0.001\alpha + 54.5971500331, -0.001\alpha + 54.5991500331]$, with $\alpha \in [0,1]$

Now ,to find $v_5^{(0)}$ from equation (7).

Let x_1=0.001 α +31.492603060 and x_2=-0.001 α +31.494603060 to solve for α we get

$\alpha = \frac{x_1-31.492603060}{0.001}$ and $\alpha = \frac{x_1-31.494603060}{0.001}$ the(f.m.f)for $v_5^{(0)}$

$v_5^{(0)} =$
$\left[\frac{x-31.492603060}{31.493603060-31.492603060}\right.$; $where\ 31.492603060 \leq x \leq$
31.493603060 $\frac{x-31.494603060}{31.493603060-31.494603060}$; $where\ 31.493603060 \leq x \leq$
31.494603060 0 ; $otherwise$ $]$

In the same way we can find (f.m.f) to $v_1^{(0)}, v_4^{(0)}, v_7^{(0)}, v_5^{(0)}, v_8^{(0)}, v_9^{(0)}, v_6^{(0)}, v_3^{(0)}andv_2^{(0)}$ are respectively of.

$\mu_{v_1^{(0)}}x)(=$
$)\left[\frac{x-0.3906}{0.3906-0.3906}\right.$, $where\ 25.08453629232 \leq x \leq$
25.08553629232 $\frac{x-25.08653629232}{25.08453629232-25.08653629232}$, $where\ 25.08453629232 \leq x \leq$
25.08653629232 0 , $otherwise$ $]$

$\mu_{v_3^{(0)}}x)(=$
$)\left[\frac{x-67.0303840154}{67.0313840154-67.0303840154}\right.$, $where\ 67.0303840154 \leq x \leq$
67.0313840154 $\frac{x-67.0323840154}{67.0313840154-67.0323840154}$, $where\ 67.0313840154 \leq x \leq$
67.0323840154 0 , $otherwise$ $]$

$\mu_{v_7^{(0)}}x)(=$
$) \Big[\frac{x-7.6928303079}{7.6938303079-7.6928303079}$ $, where\ 7.6928303079 \leq x \leq$
$7.6938303079\ \frac{x-7.6948303079}{7.6938303079-7.6948303079}$ $, where\ 7.6938303079 \leq x \leq$
$7.6948303079\ 0$, $otherwise$ $\Big]$

$\mu_{v_9^{(0)}}x)(=$
$) \Big[\frac{x-12.6840341212}{12.6850341212-12.6840341212}$ $, where\ 12.6840341212 \leq x \leq$
$12.6850341212\ \frac{x-12.6860341212}{12.6850341212-12.6860341212}$ $, where\ 12.6850341212 \leq x \leq$
$12.6860341212\ 0$, $otherwise$ $\Big]$

$\mu_{v_2^{(0)}}x)(=$
$) \Big[\frac{x-48.1579450397}{48.1589450397-48.1579450397}$ $, where\ 48.1579450397 \leq x \leq$
$48.1589450397\ \frac{x-48.1599450397}{48.1589450397-48.1599450397}$ $, where\ 48.1589450397 \leq x \leq$
$48.1599450397\ 0$, $otherwise$ $\Big]$

$\mu_{v_4^{(0)}}x)(=$
$) \Big[\frac{x-14.2299785481}{14.2309785481-14.2299785481}$ $, where\ 14.2299785481 \leq x \leq$
$14.2309785481\ \frac{x-14.2319785481}{14.2299785481-14.2309785481}$ $, where 14.2299785481 \leq x \leq$
$14.2319785481\ 0$, $otherwise$ $\Big]$

$\mu_{v_6^{(0)}}x)(=$
$) \Big[\frac{x-29.50474783}{29.50574783-29.50474783}$ $, where\ 29.50474783 \leq x \leq$
$29.50574783\ \frac{x-29.50674783}{29.50574783-29.50674783}$ $, where\ 29.50574783 \leq x \leq$
$29.50674783\ 0$, $otherwise$ $\Big]$

$\mu_{v_8^{(0)}}x)(=$
$) \Big[\frac{x-12.10797579575}{12.10897579575-12.10797579575}$ $, where\ 12.10877579575 \leq x \leq$
$12.10897579575\ \frac{x-12.10997579575}{12.10897579575-12.10997579575}$ $, where\ 12.10897579575 \leq x \leq$
$12.10997579575\ 0$, $otherwise$ $\Big]$

Now ,In the following there are f.m.f of the 5^{th} approximations using 9-points by the way of the Lebmann's iteration process applied for equation (4) have been found as

$\mu_{v_1^{(5)}}x)(=$
$) \Big[\frac{x-22.054999397}{22.055999397-22.054999397}$ $, where\ 22.054999397 \leq x \leq$

$22.055999397\ \frac{x-22.056999397}{22.055999397-22.056999397}\ , where\ 22.055999397 \leq x \leq$

$22.056999397\ 0\qquad ,\qquad otherwise\ \Big]$

$\mu_{v_2^{(5)}}x)(=$

$)\Big[\frac{x-43.0911950194}{43.0921950194-43.0911950194}\ , where\ 43.0911950194 \leq x \leq$

$43.0921950194\ \frac{x-43.0931950194}{43.0921950194-43.0931950194}\ , where\ 43.0921950194 \leq x \leq$

$43.0931950194\ 0\qquad ,\qquad otherwise\ \Big]$

$\mu_{v_3^{(5)}}x)(=$

$)\Big[\frac{x-62.8920276785}{62.8930276785-62.8920276785}\ , where\ 62.8920276785 \leq x \leq$

$62.8930276785\ \frac{x-62.8940276785}{62.8930276785-62.8940276785}\ , where\ 62.8930276785 \leq x \leq$

$62.8940276785\ 0\qquad ,\qquad otherwise\ \Big]$

$\mu_{v_4^{(5)}}x)(=$

$)\Big[\frac{x-9.34261884292}{9.34361884292-9.34261884292}\ , where\ 9.34361884292 \leq x \leq$

$9.34361884292\ \frac{x-9.34461884292}{9.34361884292-9.34461884292}\ , where\ 9.34361884292 \leq x \leq$

$9.34461884292\ 0\qquad ,\qquad otherwise\ \Big]$

$\mu_{v_5^{(5)}}x)(=$

$)\Big[\frac{x-17.3755369899}{17.3765369899-17.3755369899}\ , where\ 17.3755369899 \leq x \leq$

$17.3765369899\ \frac{x-17.3775369899}{17.3765369899-17.3775369899}\ , where 17.3765369899 \leq x \leq$

$17.3775369899\ 0\qquad ,\qquad otherwise\ \Big]$

$\mu_{v_6^{(5)}}x)(=$

$)\Big[\frac{x-24.2563972427}{24.2573972427-24.2573972427}\ , where\ 24.2563972427 \leq x \leq$

$24.2573972427\ \frac{x-24.2583972427}{24.2573972427-24.2583972427}\ , where\ 24.2573972427 \leq x \leq$

$24.2583972427\ 0\qquad ,\qquad otherwise\ \Big]$

$\mu_{v_7^{(5)}}x)(=$

$)\Big[\frac{x-3.7174637405}{3.7184637405-3.7174637405}\ , where\ 3.7174637405 \leq x \leq$

$3.7184637405\ \frac{x-3.7194637405}{3.7184637405-3.7194637405}\ , where\ 3.7184637405 \leq x \leq$

$3.7194637405\ 0\qquad ,\qquad otherwise\ \Big]$

$\mu_{v_8^{(5)}}x)(=$

$)\Big[\frac{x-6.7563513193}{6.7573513193-6.7563513193}\ , where\ 6.7563513193 \leq x \leq$

$6.7573513193 \ \frac{x-6.7583513193}{6.7573513193-6.7583513193}$ $, where \ 6.7573513193 \leq x \leq$
$6.7583513193 \ 0 \qquad , \qquad otherwise \ ]$

$\mu_{v_9^{(5)}} x)(=$
$) \left[\frac{x-9.1822575976}{9.1832575976-9.1822575976} \right.$ $, where \ 9.1822575976 \leq x \leq$
$9.1832575976 \ \frac{x-9.1832575976}{9.1832575976-9.1842575976}$ $, where \ 9.1832575976 \leq x \leq$
$9.1842575976 \ 0 \qquad , \qquad otherwise \ ]$

5. CONCLUSION

To the given initial values the 4^{th} approximations to dissolve the above example numerically is very important results in comparison with example dissolved in [8] using 5-points only. However may increased the accuracy as desired if we take more iterations. Also, using 9-points is more accurate than 5- points.

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