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Effect of Airplane Tail Aspect Ratio on Lateral- Directional Stability

Dr. Mohammed A.R. Yass*
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Abstract

For lateral-directional stability analysis, the horizontal tail geometric parameters effect has not been considered. In the present work, the effect of horizontal tail aspect ratio on the lateral-directional stability derivatives, rolling static, lateral dynamic stability and Routh discriminate of the airplane is investigated.

The increasing of the horizontal tail aspect ratio improves both the rolling static stability and damping ability in the rolling convergence mode and decreases the damping ability in spiral mode. The results could be used as real design requirements for further configuration improvements of the airplane.

الخلاصة

عند تحليل الاستقرارية العرضية-الاتجاهية فان تأثير معاملات الشكل للذيل الافقي على الاغلب لاتؤخذ بنظر الاعتبار. في هذا البحث تم دراسة تأثير النسبة الباعية للذيل الافقي على مشتقات الاستقرارية العرضية-الاتجاهية، الاستقرارية الستاتيكية التدحرجية، الاستقرارية الديناميكية العرضية ومحددة راوث للطائره. عند زيادة النسبة الباعية للذيل الافقي فان الاستقرارية الستاتيكية التدحرجية وقابلية الاخماد في الطور التدحرجي التقاربي ستتحسن بينما تقل قابلية الاخماد في الطور الحلزوني.

يمكن اعتماد نتائج هذا البحث كمتطلبات تصميمية حقيقية للتطويرات اللاحقة لهيكل الطائرة.

List of symbols

Symbol	Description	unit
A	aerodynamic Matrix, Aerodynamic stability constant.	-----
AR	aspect ratio.	-----
A	lift curve slope.	rad
B	aerodynamic Matrix, Aerodynamic stability constant.	-----
b	span.	m
C	aerodynamic stability constant.	-----
CL	lift Coefficient.	-----
Cl_p	rolling moment coefficient due to rate of roil.	1/rad
Cl_r	rolling moment coefficient due to rate of yaw.	1/rad
$Cl\beta$	rolling moment coefficient due to rate of sideslip.	1/rad
Cnp	yawing moment coefficient due to rate of roil.	1/rad
Cnr	yawing moment coefficient due to rate of yaw.	1/rad
$Cn\beta$	directional static stability derivatives.	1/rad
Cyp	side force coefficient due to rate of roil.	1/rad
Cyr	side force coefficient due to rate of yaw.	1/rad
$Cy\beta$	side force coefficient due to slideslip.	1/rad
D.E.F	aerodynamic stability constants.	1/rad

* Dept. of Technical Education / Mech. Branch, UOT., Baghdad-IRAQ.

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2412-0758/University of Technology-Iraq, Baghdad, Iraq

g	gravitational acceleration.	1/rad
I	identity matrix.	1/rad
I	moment inertia.	1/rad
K	aerodynamic Coefficient, Aerodynamic stability constant	-----
Kn	static margin.	m
L	lift force	N
ℓ	distance from c.g position to aerodynamic center of the part considered.	m
N	yawing moment.	N/m
P	Rate of roll.	1/sec
Q	Dynamic pressure ratio.	-----
q	Dynamic pressure.	N/m^2
S	Surface area.	m^2
V	Volume ratio.	----
Z	Vertical distance of the considered part axis to the center line of the fuselage.	m
α	Angle of attack.	deg
φ	Flight path angle.	deg
β	Side slip angle.	deg
θ	Pitch angle.	deg
ψ	Azimuth angle.	deg
$A_{1/4}$	Sweep back on c/4, the quarter chord.	deg
η	Efficiency factor.	----
σ	Side wash angle.	deg
$\partial\sigma$	Side wash gradient	1/deg
$\partial\beta$	Roll wash gradient	1/deg
M_0	Undisturbed velocity along x-axis.	m/sec
v	Side velocity.	m/sec
δa	Aileron deflection.	deg
δr	Radder defection.	deg

Subscript :

f	-	Fin
H	-	Horizontal tail.
V	-	Vertical tail.
w	-	Wing.
wf	-	Wing-fuselage.

1. Introduction:

The horizontal tail in the neutral or streamlined position forms an aerofoil which produces a lift at varying angle of attack and the lift in turn producing either upward or downward and restoring moment to balance the wing pitching moments about the airplane's center of gravity [1].

The aerodynamic design of the horizontal tail is based on many specific requirements. Usually the design of horizontal tail is an interactive process based on choosing an initial certain shape

parameters such as aspect ratio [2]. The aspect ratio is the measured of how long is the span with respect to the mean chord. Usually at low speed increasing in aspect ratio of the wing results in a decreasing of the tail induced drag [3].During flight with angular velocity in roll, this motion is instantaneously like that of screw and it affects the airflow at each section of the wing and tail. The angle of attack due to rate of roll varies linearly across the span.
This antisymmetric angle of attack distribution produces an antisymmetric increment in the lift distribution. In the linear range, this is superimposed on the symmetric lift distribution associated with the wing angle of attack in undisturbed flight [4].

1.1 Stability Coupling Effect

The coupling effect between longitudinal and lateral- directional stability is one of the most interest analyses that must be done for complete rolling and yawing characteristics of the specific aircraft. This analysis requires an estimation of static and dynamic lateral-directional stability derivatives for the solution of equation of motion to estimate the lateral modes and their characteristics.
The process of lateral- directional stability analysis is usually done in conjunction with the mass, inertia, center of gravity, the aerodynamic parameters, and the dynamic derivatives .Stalford, [5] estimates the dynamic derivatives and stability coupling using the Estimation – Before – Modeling (EBM) system and comparisons between the wind tunnel and identified state and control derivatives were done with a good agreement.

The tail and its support structure must first of all be designed for symmetrical tail loads that are required for balance the tail – off moments and to maneuver the airplane in pitch.
Also the design must consider gust loads and unsymmetrical loads due to sideslip [6].

The stability of an aircraft with an angle of attack and sideslip equilibrium conditions serves as an indication of the aircraft motion in the neighborhood of these equilibrium. This requires a boundary of alpha (α) – beta (β) parameter plane of stable equilibrium points yields an indication of the aircraft's allowable alpha-beta combinations flight [7].
This stability boundary was estimated by method of Ref. [8], which requires determining the equilibrium points and solving the resulting eigenvalue problem. Moul and Paulson [9] determined the stability bounds for both high alpha with zero beta, and for high angles of attack and with sideslip [10].Due to the success in the longitudinal case, a preliminary attempt to develop a simplified model for unsteady effects in lateral aircraft dynamics was made by Wells, Banda, and Quam [11].
A model to account for unsteadiness in the side slipping flight of an aircraft is utilized in Ref. [12] to estimate the rolling and yawing moments derivatives due to the sideslip, rate of roll, rate of yaw, rudder, and aileron deflections based on the cross coupling effects between longitudinal and lateral stabilities.

1.2 Objective

The tail aspect ratio on Lateral-Directional stability helps the designer to put in mind what the best overall value of aspect ratio shall be considered especially for high performance airplane.

2- Method of Analysis

The contribution of the aircraft components to directional stability often concentrates on the wing, fuselage and vertical tail contributions, so that the wing contributions are usually quite small in comparison to the fuselage provided the angle of attack is not large. The fuselage and engine nacelles, in general, create a destabilizing contribution to directional stability. The wing fuselage contribution can be calculated from the following empirical expression [13]:

$$C_{nb_{wf}} = -k_n k_{RI} \frac{s_{fs/f}}{s_w b} (per\deg) \quad (1)$$

Since the wing-fuselage contribution to directional stability is destabilizing, the other aircraft components must be properly sized to ensure that the aircraft has directional stability.
The side force acting on the vertical tail can be expressed as:

$$Y_v = -C_{Lav} a_v Q_v S_v \quad (2)$$

where

$$\alpha_V = \beta + \sigma \quad (3)$$

The moment produced by the vertical tail can be written as a function of side acting on it:

$$N_V = I_V Y_V = I_V C_{L_{aV}} (b+s) Q_V S_V \quad (4)$$

in coefficient from:

$$C_N = V_V h_V C_{L_{aV}} (b+s) \quad (5)$$

Then the contribution of the vertical tail to directional stability is:

$$\frac{\partial C_n}{\partial b}\Big)_v = C_{nbv} = V_V h_V c_{lav} (1+\frac{\partial s}{\partial b}) \quad (6)$$

The following simple algebraic equation is used for estimating the combined side wash and tail efficiency factor η_V in [1]:

$$h_v (1+\frac{\partial s}{\partial b}) = 0.724 + 306 \frac{S_V / S}{1+COS\Lambda_{C/4W}} + \frac{Z_w}{d} + 0.009 AR_w \quad (7)$$

The effect of horizontal tail is included in the effect of the other derivative. The total configuration derivatives are given by the following expression [1]:

$$C_{nb} = -\left[K(C_{La})_V (1+\frac{\partial s}{\partial b}) \frac{q_V}{q_\infty} \frac{S_v}{S_W} \right] \frac{(I_p \cos a + Z_p \sin a)}{B_W} \quad (8)$$

The linearized small disturbance lateral-directional equation of motion of aircraft is given by [14]

$$\left(\frac{d}{d_t} - Y_v \right) \Delta_v + (m_o - Y_r)\Delta_r - (g \cos q_o)\Delta_f = Y_{dr}\Delta_{dr} \quad (9)$$

$$-L_v \Delta_v + \left(\frac{d}{d_t} - L_p \right) \Delta_p - \left(\frac{I_{XZ}}{I_X} \frac{d}{d_t} + L_r \right) \Delta_r = L_{dA}\Delta_{dA} + L_{dr}\Delta_{dr} \quad (10)$$

$$-L_v\Delta_v+\left(\frac{I_{XZ}}{I_Z}\frac{d}{d_t}-L_p\right)\Delta_p-\left(\frac{d}{d_t}+N_r\right)\Delta_r=L_{dA}\Delta_{dA}+L_{dr}\Delta_{dr} \qquad (11)$$

The equation of motion for pure rolling motion of the aircraft is:

$$\frac{\partial L}{\partial d_a}d_a+\frac{\partial L}{\partial p}p=I_X\ddot{\Phi} \qquad (12)$$

and the pure yawing motion is:

$$\Delta N=I_z\Delta\ddot{y} \qquad (13)$$

Equations (12 & 13) are rearranged and collecting terms the above equation can be written in state variable form:

$$x=Ax+Bx \qquad (14)$$

and the matrices A and B are defined as follows:

$$A=\begin{bmatrix} Y_V & Y_P & -(m-Y_r) & g\cos q \\ L_V^*+\frac{I_{xz}}{Ix}N_V^* & L_P^*+\frac{I_{XZ}}{I_X}N_P^* & L_r^*+\frac{I_{XZ}}{I_X}N_r^* & 0 \\ N_V^*+\frac{I_{XZ}}{I_Z}I_V^* & N_P^*+\frac{I_{XZ}}{I_Z}I_P^* & N_r^*+\frac{I_{XZ}}{I_Z}L_r^* & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad (15)$$

$$B=\begin{bmatrix} 0 & Y_{dr} \\ L_{da}^*+\frac{I_{XZ}}{I_X}N_{da}^* & L_{dr}^*+\frac{I_{XZ}}{I_X}N_{dr}^* \\ N_{da}^*+\frac{I_{XZ}}{I_Z}L_{da}^* & N_{dr}^*+\frac{I_{XZ}}{I_Z}L_{dr}^* \\ 0 & 0 \end{bmatrix} \qquad (16)$$

$$=\begin{bmatrix}\Delta v\\ d_P\\ d_r\\ \end{bmatrix} \quad and \quad h=\begin{bmatrix}\Delta d_a\\ \Delta d_r\end{bmatrix} \qquad (17)$$

when the starred derivatives are defined as follows:

$$L_V^*=\frac{L_V}{\left(1-\left(I_{XZ}^2/I_XI_Z\right)\right)} ,$$

$$N_V^*=\frac{N_V}{\left(1-\left(I_{XZ}^2/I_XI_Z\right)\right)} \quad ect \qquad (18)$$

The product of inertia Iα = 0, the equations of motions reduce to the following form with using the side slip Δβ instead of the side velocity Δv
when

$$\Delta b\approx\tan^{-1}\frac{\Delta v}{m_o}=\frac{\Delta v}{m_o} \text{ [12]}$$

$$\begin{bmatrix} \dot{\Delta v} \\ \dot{\Delta p} \\ \dot{\Delta r} \\ \dot{\Delta \Phi} \end{bmatrix} = \begin{bmatrix} Y_V & Y_P & -(m_O - Y_r) & g\cos q_o \\ L_V & L_P & L_r & 0 \\ N_V & N_P & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta \Phi \end{bmatrix} + \begin{bmatrix} o & Y_{dr} \\ l_{da} & Y_{dr} \\ N_{da} & N_{dr} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta_{da} \\ \Delta_{dr} \end{bmatrix} \quad (19)$$

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The solution of equation (19) gives the characteristic equation by expanding the following determinant:

$$|lI - A| = 0 \qquad (20)$$

where I and A are the identity and lateral stability matrices, respectively. The characteristic equation determined from the stability matrix A yields a quadric equation.

$$A l^4 + B l^3 + C l^2 + D l + E = 0 \qquad (21)$$

where A, B, C, D and E are aerodynamic stability coefficients which are functions of mass lateral directional stability derivatives, and inertia characteristics of the aircraft.

The root of the characteristic equation in composed of two real root, and a pair of complex roots. The roots will be such that the aircraft response can be characterized by the following motions:

(a) A slow convergent or divergent motion, called spiral mode.
(b) A highly convergent motion, called rolling mode.
(c) A lightly damped oscillatory motion having a low frequency, called the Dutch roll mode.

3- <u>Results and Discussion:</u>

The analysis was done on the advanced jet trainer L-39. All the required data were taken from Ref. [14].

The following results were obtained:

a. Figure (1) shows that there is no effect of horizontal tail aspect ratio on static stability derivatives due to sideslip in lateral-directional motion. Rolling stability derivatives due to rate of roll dose not changes with horizontal tail aspect ratio as shown in Fig (2).
b. The damping in roll derivative decreases with the increasing of horizontal tail aspect ratio as shown in Figure (3), while there is no effect of this parameter on the yawing stability derivatives as represented in Figure (4).
c. The spiral mode real root increases in its absolute value with the increasing of horizontal tail aspect ratio, while the rolling convergence mode real root decreases as shown in Figures (5), and (6).
d. Figures (8), (9), and (10) show that there is no effect of horizontal tail aspect ratio on lateral numerical

parameters, natural frequency, damping ratio, and characteristic equation coefficient, E. The Routh discriminant (R) of lateral-directional stability increases significantly with the increasing of this parameter as shown in Figure (11).

4- Conclusions:

The design of the horizontal tail for optimum performance, stability and control is concentrated on it efficiency in producing the required lift and pitching moment. In the process of lateral-directional stability analysis, the horizontal tail design must consider the boundaries of both angle of attack and sideslip.
This requires a detailed investigation of the design parameters of horizontal tail, especially, the aspect ratio on the stability of the aircraft in lateral-directional motion. For the jet trainer L-39, the above analysis shows the following important conclusions:

1- The horizontal tail aspect ratio has a significant effect on damping in roll. For the range of the aspect ratio considered in the analysis, the aircraft still stable, is an indication for heavy damped total design.
2- For spiral mode, there is an effect of the horizontal tail aspect ratio with a good convergent, stable level for the range considered.
3- The rolling convergence mode and Routh discrimenant (R) are improved with the increase in horizontal tail aspect ratio.
4- There are no effects of the horizontal tail aspect ratio as a design parameter on the lateral-directional stability derivatives, damping, frequency of Dutch roll mode, lateral numerical parameters of all modes and the characteristic equation coefficient, E.
5- These results are sufficient for the designer for further developments of the lateral-directional stability characteristics of this aircraft according to the requirements.

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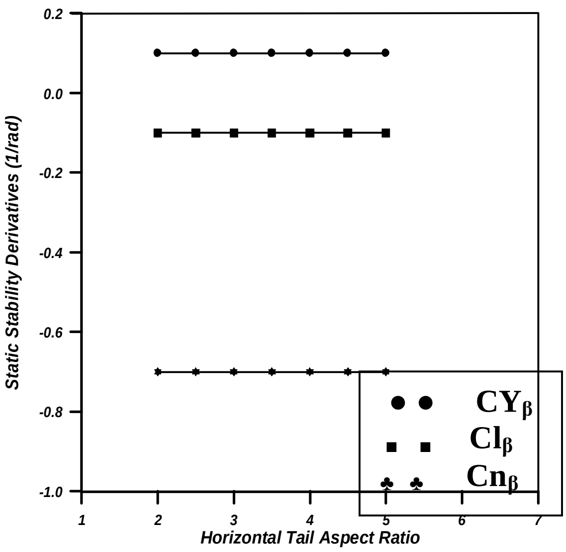

Fig (1) Lateral Static Stability Derivatives with Horizontal Tail Aspect Ratio

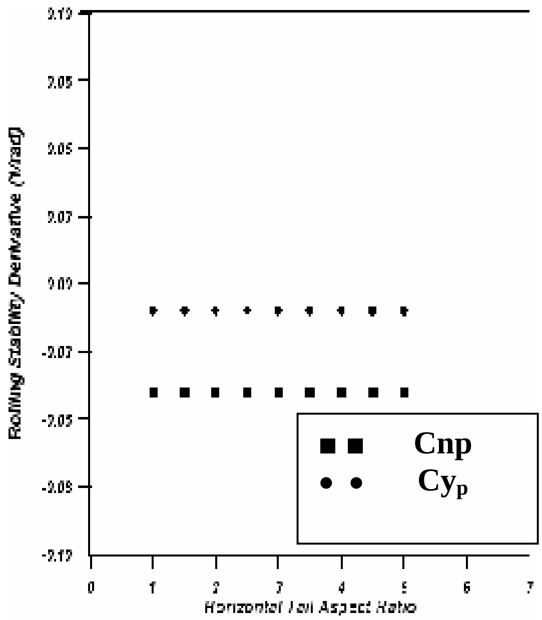

Fig (2) Rolling Stability Derivatives with Horizontal Tail Aspect Ratio

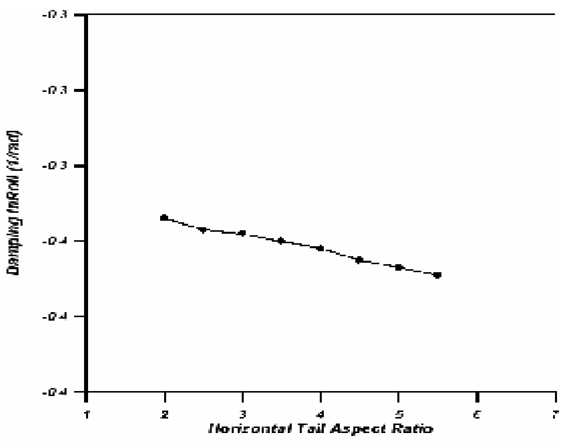

Fig (3) Damping in Roll Derivatives with Horizontal Tail Aspect Ratio

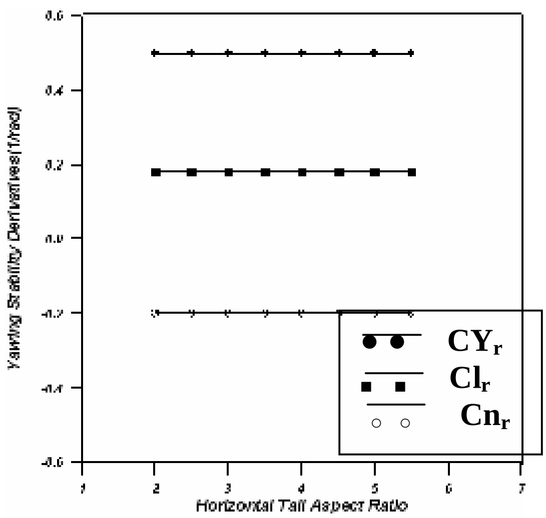

Fig (4) Yawing Stability Derivatives with Horizontal Tail Aspect Ratio

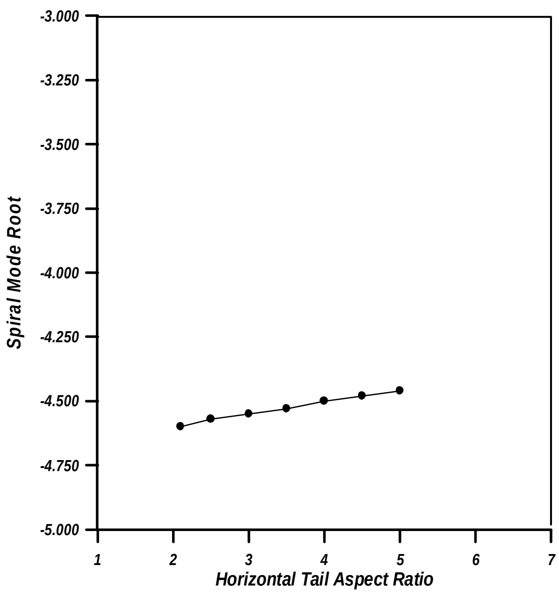

Fig (5) Spiral Mode Root (n) Multiplied by 10^4 with Horizontal Tail Aspect Ratio

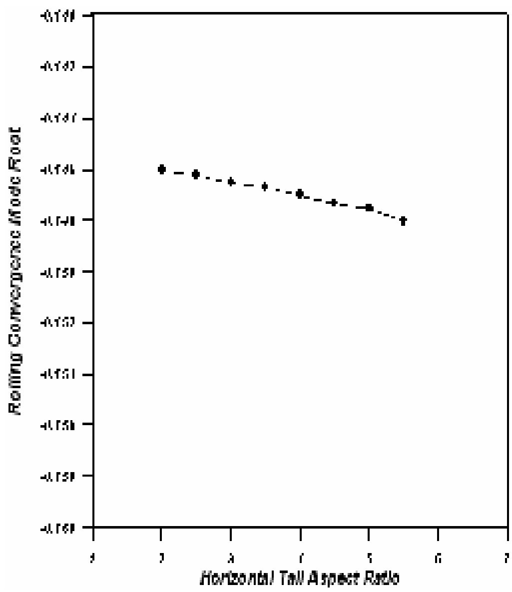

Fig (6) Rolling Convergence Mode Root with Horizontal Tail Aspect Ratio

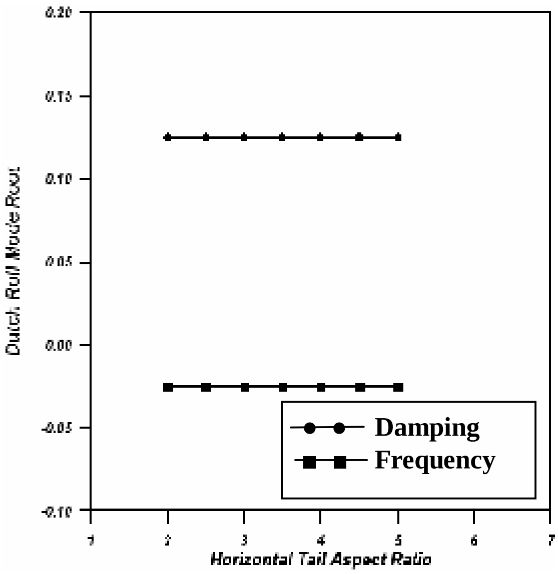

Fig (7) Damping and Frequency of Dutch Roll Mode Root with Horizontal Tail Aspect Ratio

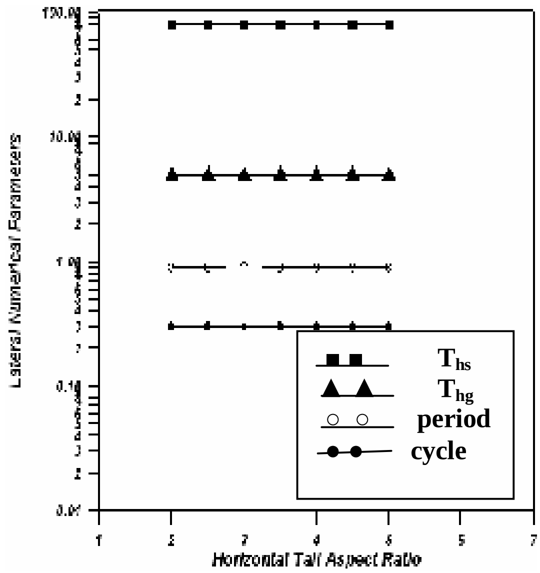

Fig (8) Lateral Numerical Parameters of the three Motion Modes with Horizontal Tail Aspect Ratio

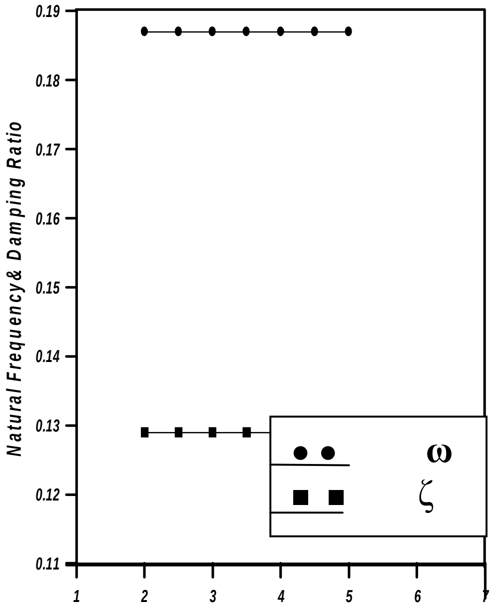

Fig (9) Natural Frequency & Damping Ratio with Horizontal Tail Aspect Ratio

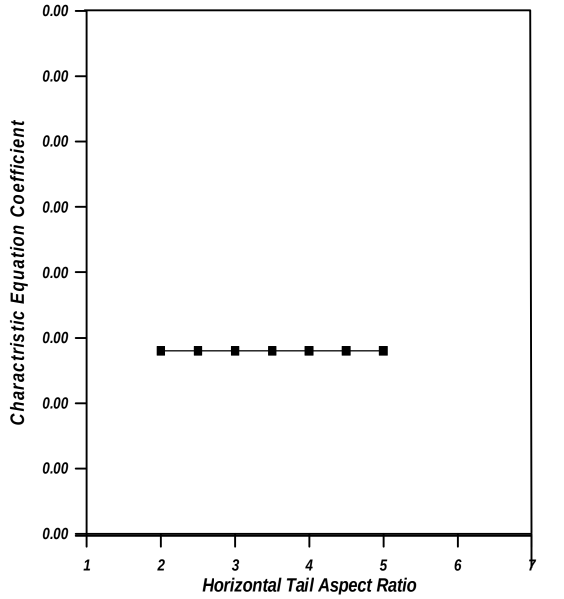

Fig (10) Characteristic Equation Coefficient (E) with Horizontal Tail Aspect Ratio

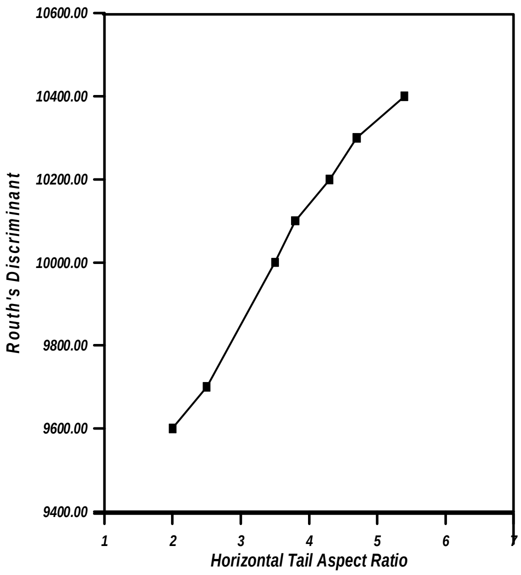

Fig (11) Routh's Disriminant (R) with Horizontal Tail Aspect Ratio

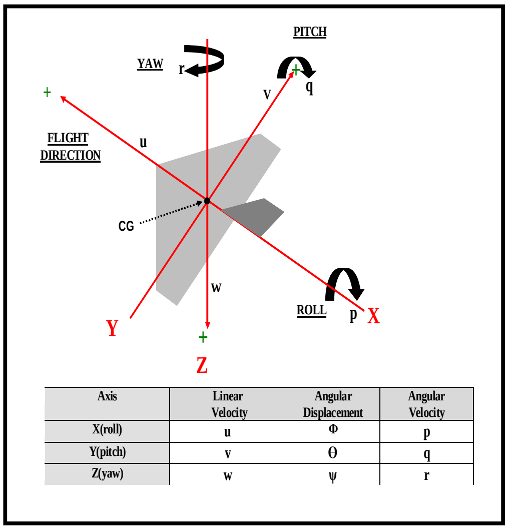

Fig (12) Airplane Axis