

Critical Speed of Hyperbolic Disks and Eccentricity Effects on The Stability of The Rotor Bearing System

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Abstract

The critical speed of hyperbolic disks for the selected model is calculated in three different cases, anisotropic material and thermal case, isotropic material and thermal case finally isotropic material and thermal with generated heat case. The second case is found to be the best because the critical speed occurs above maximum speed values.

For recognition of the eccentricity (e) effects on the stability, the original bearings of the selected model were tilting pads bearing replaced by adjustable hydrodynamic pads bearing. The results proved that stability criteria (Sc), critical mass (\bar{M}_c), whirling frequency ratio (\bar{f}_n^2) of variable rotational speed of the adjustable hydrodynamic pads bearing are stable for the range of eccentricity (e) about 0.02 mm to 0.09 mm. Finally the thermohydrodynamic (THD) analysis of eccentricity ratio (e) has effects on the stability of the rotor-bearing system.

Keywords: hyperbolic disk, rotor-bearing system, critical speed, stability criteria

السرعة الحرجة للأقراص ذات القطع المكافئ وتأثيرات انحراف المراكز على اتزان منظومة محور و مسند

الخلاص

تم حساب السرعة الحرجة للأقراص نوع القطع المكافئ للنموذج المختار لهذا البحث لثلاث حالات اشتغال و هي: منظومة حرارية غير متجانسة الخواص, منظومة حرارية متجانسة الخواص, و أخيرا منظومة حرارية متجانسة الخواص مع تولد حراري. وأثبتت النتائج بأفضلية الحالة الثانية نظرا لوقوع السرعة الحرجة عند قيم السرعة الدورانية العالية. لمعرفة تأثيرات انحراف المراكز (e) على اتزان المنظومة تم تغيير مسندي النموذج وهما من نوع مسند ذو وسائد قابلة للإمالة بنوع آخر ذات قابلية على تعيير وسائد المسند. أثبتت النتائج إن معيار الاتزان, الكتلة الحرجة و نسبة سرعة التدويم عند سرع دوران مختلفة تكون متزنة عند النوع المستبدل من المساند لمدى من القيم انحراف المراكز (e) تتراوح من 0.02 mm - 0.09 mm. وأخيرا إن التحليل بطريقة (THD) لنسبة انحراف المراكز سوف تؤثر على اتزان المنظومة.

Notation

a, b= outer and inner radius of the disk respectively mm
 B_c = bearing centre -
 B_c^* = new bearing centre after tilting angle -
 C_r = radial clearance of the bearing (bearing radius-journal radius) mm

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e = eccentricity	mm
$h(r)$ = thickness of a disk as function of radius	mm
L = length of shaft	mm
J_c = journal centre	-
m = total mass of rotor	kg
r = radius of disk	m
r_j = radius of journal (shaft)	m
F_r, F_θ = tensile force in radial and tangential directions per unit length respectively	N/m
σ_r, σ_θ = stress in radial and tangential directions	N/m ²
$u(r)$ = displacement of disk in radial direction	mm
w = load	N
ϕ = altitude angle	degree
ψ = temperature rise parameter	-
ω = angular speed of rotor	rad/sec
ω_c = critical angular speed	rad/sec
μ = viscosity of lubricant	N.sec/m ²
e = eccentricity ratio (with effect THD)	-
e_o = steady state eccentricity ratio	-
λ = constant value which depends on the pad material	-

Introduction

The rotordynamics of turbomachinery encompasses the structural analysis of rotors (shafts and disks) and the design of fluid film bearings that determine the best dynamic performance given the required operating conditions. This best performance is denoted by well-characterized natural frequencies and critical speeds with amplitudes of synchronous dynamic response within required standards and demonstrated absence of subsynchronous vibration instabilities, [1]. Lewis et al,(1987), [2] have developed a spilt bearing with a new configuration, this bearing surfaces consists of a five pad tilting pad fluid bearings with one pad having a mean for being

moved radially. This radial motion causes changes in the proportionality constants, the stiffness and the damping capabilities of the bearing which in turn changes the critical speeds of the rotor system Wang and Shin, (1990), [3] improved the stability of rotor bearing system by optimization technique used to find the optimum diameters of shaft elements so that the optimized rotor can sustain maximum fluid leakage excitation. Chivens and Nelson (2000),[4] presented an analytical investigation into the influence of disk flexibility on the transverse bending natural frequencies and critical speeds of a rotating shaft-disk system. The partial differential equations governing the system motion and the associated exact

solution form had been developed. Numerical solutions are presented covering a wide range of non-dimensional parameters and general conclusions are drawn. Krodkiwski and Sun (2002),[5] presented the modeling and analysis of a rotor - bearing system with a new type of active oil bearing, the active bearing is supplied with a flexible sleeve whose deformation can be changed during operation of the rotor. The flexible sleeve is also a part of a hydraulic damper whose parameters can be controlled during operation as well, the self exciting vibration can be eliminated or greatly reduced during operation by properly controlled deformation of the flexible sleeve and optimal choice of the hydraulic damper parameters. In this paper the topic has been divided into two parts, the first is confined the evaluation of the critical speeds of hyperbolic disk by thermoelastic relations in three operating cases, while the second contains the replacement of the two tilting bearings type used in the selected model with two hydrodynamic adjustable pads bearings and also study the effects of the eccentricity on the stability criteria at high speeds, and the effect of thermohydrodynamic (THD) analysis on the stability of rotor-bearing system.

Theoretical Analysis of The Model

It takes similar characteristics of centrifugal compressor, it's consisting of three disks (hyperbolic profile) clamped on a shaft which is

supported by two tilting pads bearings driven by steam turbine using in "southern refineries company", all dimensions in origin specifications as shown in figure (1).

Mechanical Specification

(i)-Type of compressors RB43B;
Serial No. 33F3047

(ii)-shaft properties :

-total length: 1411 mm

-shaft material; all properties as shown in table (1).

-maximum diameter:142 mm

-minimum diameter:108 mm at bearing and 63 mm (at shaft end).

(iii)- Two tilting pads bearing (right and left).

(iv)-Numbers and type of disks; three hyperbolic profile variable thickness disk, some whose dimensions:

-inner and outer diameters respectively :139mm - 420mm

-disk thickness: 100 mm (at inner diameter)

-disk material; all properties as shown in table (1)

Operating Data

(i)-Suction pressure: 11 kg/cm²

(ii)-Discharge pressure : 12.5 kg/cm²

(iii)-Process gas: recycle gas

(iv)-Suction temperature: 45 °C

(v)-Discharge temperature: 70 °C

(vi)-Normal speed without suction (8000 r.p.m -10670 r.p.m)

(vii)-Rated speed: 11670 r.p.m

(viii)-Maximum speed: 12000 r.p.m

Rotating Disks with Variable Thickness and Inhomogeneity

Disks of hyperbolic axial thickness with profile shown in figure (2) and described by [6].

$$h(r) = h_2 (r/a)^{-b} \quad (1)$$

where:

h_2 =outer thickness at radius (a)

β = constant in variable thickness expression

The governing equations for thermo-elastic analysis of an inhomogeneous, rotating disk with arbitrary varying thickness, small deformations and rotational symmetry are assumed. If the thickness of the disk is small compared to its radius, the axial stress is negligible and the disk is then in a state of plane stress. The material is assumed to be isotropic but inhomogeneous along the radial direction. That is the Young's modulus $E=E(r)$, the density of the disk material $\rho = \rho(r)$, the coefficient of linear expansion $\alpha = \alpha(r)$, and the Poisson's ratio $\nu = \nu(r)$. The temperature field is steady, axisymmetric but inhomogeneous, that is, $T = T(r)$ where T is the temperature. From equilibrium consideration of an element as shown in figure (2),[7].

$$d(F_r \cdot r \cdot dq) + \frac{r}{2} \cdot w^2 \cdot h \cdot dr \cdot dq - F_q \cdot \frac{dr}{2} - F_r \cdot dr \cdot dq = 0 \quad (2)$$

The exact elastic displacement in radial direction $u(r)$ can be written as:

$$u(r) = (81.26 \cdot r + \frac{221711}{r} - 0.028 \cdot r^3) \times 10^{-3} \text{ mm} \quad (3)$$

Now the exact value of radial and tangential forces per unit length

components are then obtained as follows:

$$F_r(r) = (1.784 + 23736 \frac{1}{r^2} - 20118 \times 10^{-3} r^2) \times 10^{10} \text{ N/m} \quad (4)$$

$$F_q(r) = (1.784 + 23736 \frac{1}{r^2} - 11583 \times 10^{-4} r^2) \times 10^{10} \text{ N/m} \quad (5)$$

Thermal Effect on The Critical Speed of Height-Speed Disk

Anisotropic elastic disk of radius an assumed to be spinning uniformly about its polar axis with an angular velocity " ω " in a steady -state heat flow, for such a disk, following stress -strain relations, [8].

$$\left. \begin{aligned} s_r &= C_{11} \cdot e_r + C_{12} \cdot e_q - (C_{11} a_r + C_{12} a_q) \Delta T \\ s_q &= C_{12} \cdot e_r + C_{22} \cdot e_q - (C_{12} a_r + C_{22} a_q) \Delta T \\ t_{rq} &= 0 \end{aligned} \right\} \quad (6)$$

$$w^2 = \frac{T \cdot (C_{11} C_{22} - C_{12}^2) (3a_q - a_r)}{(3C_{11} + C_{12}) r \cdot a^2} \quad (7)$$

where : C_{11}, C_{12}, C_{22} = elastic constants
 ΔT = differential temperatures
(°C)

T = operating temperature (°C)

$$\left. \begin{aligned} \frac{C_{22}}{C_{11} C_{22} - C_{12}^2} &= \frac{1}{E_1} \\ \frac{C_{11}}{C_{11} C_{22} - C_{12}^2} &= \frac{1}{E_2} \\ u_r &= \frac{C_{12}}{C_{11}} \\ u_q &= \frac{C_{12}}{C_{22}} \end{aligned} \right\} \quad (8)$$

$$W_c = \left[\frac{T \cdot E_2 (3a_r - a_r)}{(3+u_r) \cdot r \cdot a^2} \right]^{1/2} \quad (9)$$

The critical speed (ω_c) depends on thermo-mechanical anisotropy of the disk material, size of the disk and temperature at center of the disk, so that the selection of material and size of the disk become important for design consideration.

Equation (9) can be written for disk material which is isotropic material:

$$\left. \begin{aligned} E_1 &= E_2 \\ u_r &= u_q \\ a_r &= a_q \\ w_c &= \left[\frac{2T \cdot E_1 \cdot a_r}{(3+u_r) \cdot r \cdot a^2} \right]^{1/2} \end{aligned} \right\} \quad (10)$$

where:

α_r, α_θ = thermal coefficient of expansion in radial and tangential direction respectively.

E_1, E_2 = Young's modulus in radial and tangential directions respectively.

u_r, u_q = radial and tangential

Poisson's ratio respectively.

if we consider such a thermal situation that rate of generation of heat is fixed amount "Q" per unit volume, the critical speed:

$$w_c = \left[\frac{E_1 \cdot a_r \cdot Q}{2K^* \cdot r \cdot (3+u_r)} \right]^{1/2} \quad (11)$$

where: K^* = thermal conductivity
(W/m°C.)

Q = generated heat in disk
(W/m³)

Adjustable Hydrodynamic Pads Bearing

This bearing has four pads arranged as shown in Fig.(3), [9] these pads were designed to be adjustable, i.e each pad could be tilted about its leading edge in a clockwise (cw) or counter clockwise (ccw) direction. The running surface of each pad, at zero tilt, fits into a circle which corresponds to the bearing surface circle in a conventional bearing.

The stability margins of the rotor bearing system, in terms of critical mass (M_c), threshold speed (Ω), and whirl frequency (f_n), can be established by applying **Routh's criteria** to the dynamic coefficients as stiffness and damping coefficients, [10].

$$K_{xx} = \frac{W}{e} \cos f \quad (12)$$

$$K_{xz} = W \frac{\partial f}{\partial e} \cos f + \frac{\partial W}{\partial e} \sin f \quad (13)$$

$$K_{zx} = -\frac{W}{e} \sin f \quad (14)$$

$$K_{zz} = \frac{\partial W}{\partial e} \cos f - W \sin f \frac{\partial f}{\partial e} \quad (15)$$

$$C_{xx} = -\frac{2}{w} \left[-\frac{W}{e} \sin f \right] \quad (16)$$

$$C_{xz} = C_{zx} = \frac{2}{w} \left[\frac{W}{e} \cos f \right] \quad (17)$$

$$C_{zz} = -\frac{2}{w} \left[\frac{W}{e} \cos f + \frac{\partial W}{\partial e} \sin f \right] \quad (18)$$

$$\frac{K_{xx}}{w}, \frac{K_{xz}}{w}, \frac{K_{zx}}{w}, \frac{K_{zz}}{w} = \frac{(K_{xx}K_{xz}, K_{zx}K_{zz})C_r^3}{(w.m r^4)} \quad (19)$$

$$\overline{C_{xx}}, \overline{C_{xz}}, \overline{C_{zx}}, \overline{C_{zz}} = \frac{(\overline{C_{xx}}, \overline{C_{xz}}, \overline{C_{zx}}, \overline{C_{zz}}) \overline{C_r}^3}{m r^4} \quad (20)$$

$$K_1 = \frac{\overline{K_{xx}} \overline{C_{zz}} + \overline{K_{zz}} \overline{C_{xx}} - \overline{K_{xz}} \overline{C_{zx}} - \overline{K_{zx}} \overline{C_{xz}}}{\overline{C_{xx}} + \overline{C_{zz}}} \quad (21)$$

$$\overline{f_n}^2 = \frac{(\overline{K_{xx}} - K_1)(\overline{K_{zz}} - K_1) - \overline{K_{xz}} \overline{K_{zx}}}{\overline{C_{xx}} \overline{C_{zz}} - \overline{C_{xz}} \overline{C_{zx}}} \quad (22)$$

where:

$\overline{K_{xx}}, \overline{K_{xz}}, \overline{K_{zx}}, \overline{K_{zz}}$ = Stiffness

coefficients in xx, xz, zx, zz

directions

$\overline{C_{xx}}, \overline{C_{xz}}, \overline{C_{zx}}, \overline{C_{zz}}$ = Dynamic damping

coefficients in xx, xz, zx, zz

directions

$\overline{K_{xx}}, \overline{K_{xz}}, \overline{K_{zx}}, \overline{K_{zz}}$ = Dimensionless

stiffness coefficients in xx, xz, zx, zz

directions

$\overline{C_{xx}}, \overline{C_{xz}}, \overline{C_{zx}}, \overline{C_{zz}}$ = Dimensionless

damping coefficients in xx, xz, zx, zz

directions

A rotor-bearing system is asymptotically stable when the rotor mass (\overline{M}_r) is less than critical mass

(\overline{M}_c) for operating condition

determined by bearing geometry and rotor speed, also stable when the operating speed of the rotor is less than the speed defined by threshold speed parameter ($\overline{\Omega}$), a negative value of critical mass (\overline{M}_c) means the

rotor bearing system will be stable at all the values of rotor mass (\overline{M}_r).

Similarly, a negative value of the

whirling frequency ratio ($\overline{f_n}^2$) implies the absence of whirl, [10].

(i). Critical mass of rotor (\overline{M}_c), (Kg)

$$\overline{M}_c = \frac{K_1}{\overline{f_n}^2}$$

$$M_c = \overline{M}_c \frac{m r^4}{w \cdot C_r^3} \quad \text{and}$$

$$\overline{M}_r = \overline{M}_r \frac{m r^4}{w \cdot C_r^3} \quad (23)$$

(ii). The computation of the threshold speed ($\overline{\Omega}$) including the thermal effects is as follow [11].

Step 1- Compute the sommerfeld number (S) with viscosity of oil evaluated at the supply temperature

$$S = \frac{m \cdot w \cdot L \cdot r^3}{p \cdot w \cdot C_r^2}$$

Step 2- Compute the eccentricity ratio (ϵ), as function of sommerfeld number, [12].

$$\epsilon = -0.36667 + \frac{0.49737}{\sqrt{I}} - 0.20986 \ln(S)$$

Step 3- Compute the temperature rise parameter (Ψ)

$$\Psi = \frac{b \cdot m \cdot w}{r \cdot C_n} \cdot \left(\frac{r}{C_r} \right)^2$$

Step 4- Compute the threshold speed ($\overline{\Omega}$) corresponding to the temperature rise parameter computed in

step 3 and eccentricity
ratio computed in step 2.

$$\bar{W} = \frac{W \cdot C_r^2}{m \cdot W \cdot r^4}$$

$$\bar{\Omega} = \sqrt{\frac{\bar{M}_c}{\bar{W}}} \quad \text{or}$$

$$\bar{\Omega} = \bar{w} \sqrt{\frac{g}{C_r}} \quad (24)$$

(iii). whirl frequency (f_n)

$$f_n^{-2} = f_n^{-2} \cdot \bar{w} \quad (25)$$

where:

\bar{M}_c = dimensionless critical mass

f_n^{-2} = whirl frequency ratio

$\bar{\Omega}$ = dimensionless threshold speed;

g=gravitational acceleration

$$9.81 \text{ m/sec}^2$$

\bar{w} = dimensionless load-carrying
capacity,

Study The Rotor Bearing System Parametric Stability Analysis

Method of analysis based on Liapunov's direct method is used for studying the behavior of the nonlinear system of differential equations governing the motion of a rotor-bearing system in the neighborhood of its equilibrium point. Among the results reached in Liapunov's direct method is the demonstration of the roles played by, [13].

$$U_{ik} = \begin{bmatrix} \left[\frac{w_{xx}^2}{w^2} - 1 \right] & \bar{a} & 0 & 0 \\ \bar{a} & \left[\frac{w_{zz}^2}{w^2} - 1 \right] & 0 & 0 \\ 0 & 0 & l^2 \left[\frac{w_{xx}^2}{w^2} - \bar{H} \right] & l^2 \bar{a} \\ 0 & 0 & l^2 \bar{a} & l^2 \left[\frac{w_{zz}^2}{w^2} - \bar{H} \right] \end{bmatrix}$$

$$\bar{a} = \frac{(w_{xz}^2 + w_{zx}^2)}{2w^2} \quad (26)$$

$$w_{ik}^2 = \frac{K_{ik}}{m}$$

Applying *Sylvester's Theorem* to test the positive definiteness of (U_{ik}), its successive principal minor determinants must be positive definite, thus yielding the following sufficient conditions for system whirl stability.

$$\frac{w_{xx}^2}{w^2} > 1 \quad (27)$$

$$\left[\frac{w_{xx}^2}{w^2} - 1 \right] \left[\frac{w_{zz}^2}{w^2} - 1 \right] > \bar{a}^2 \quad (28)$$

$$\frac{w_{xx}^2}{w^2} > \bar{H} \quad (29)$$

and

$$Sc = \left[\frac{w_{xx}^2}{w^2} - \bar{H} \right] \left[\frac{w_{zz}^2}{w^2} - \bar{H} \right] > \bar{H}^2 \quad (30)$$

where:

Sc=Stability criteria

$$\frac{w_{xx}^2}{w^2}, \frac{w_{zz}^2}{w^2}, \frac{\bar{a}, \bar{H}}{H} =$$

nondimensional parameters on the whirl stability

Discussion of Results

The results of this paper are illustrated below. Fig. (4) show the relationship between temperature rise (suction and discharge temperatures) and increasing value of critical speed (ω_c) for three cases, also shows the critical speed (ω_c) as a variable quantity. The second case is found to be the best case because the critical speed occurs at over the range of the high rotational speed disk $N=12000$ r.p.m. (*centrifugal compressor driven by steam turbine*).

It is quite easy to observe the dependence of critical speed (ω_c) on the thermo-mechanical anisotropy of the disk material (size of the disk and temperature at center of the disk). From the values of the critical speed (ω_c) given in equations (9,10 and 11) it is noted that the critical speed (ω_c) is independent of the variation of disk thickness that is the critical speed (ω_c) will remain the same for all hyperbolic profiles. From Fig.(5), the rotor critical mass (\bar{M}_c) increases as the eccentricity (e) increases to reach system stability. The rotor critical mass (\bar{M}_c) must be negative this means the curve no.3 is more stable especially at high values of eccentricity (e). Fig.(6) shows the relationship between whirling

frequency ratio (\bar{f}_n^2) with change of eccentricity ratio (ϵ). The system becomes more stable at the negative value of this ratio as shown in curve (3) at rotational speed equal 12000 r. p .m.. Whirl begins with the rotor operating relatively close to center of the bearing. When the journal is close to the center of the bearing (the eccentricity ratio is small) the bearing stiffness is much lower than the shaft stiffness. When the journal is located relatively close to the bearing wall (the eccentricity ratio is near 0.9) the bearing stiffness is typically much higher than rotor shaft stiffness. One can improved the stability of rotor bearing system by selecting softer surface material of bearings.

It can be seen from Fig.(7) the thermohydrodynamic (THD) analysis effects on the threshold speed at all eccentricity ratios (ϵ). The best value of threshold speed (Ω) at $\epsilon=0.4$ is in the stable region. This figure shows the increase of temperature rise parameter (ψ) with increase of eccentricity ratio (when the maximum rotational speed $N=12000$ r.p.m the value of $\epsilon=0.66$ at $\psi=0.2$ but when $\epsilon=0.696$ at $\psi=0.26$). Table (2) shows the relationship between the stability criteria with change of eccentricity (e). It also shows the stability criteria increases with the increase of eccentricity, but the values of stability criteria must be larger than (\bar{a}^2).

Conclusions

From the results of this work the following conclusions can be obtained:

- 1-The critical speed is increasing as the operation temperature increase.
- 2- When eccentricity increases the stability criteria increases as shown in Table (2). This means the stiffness and damping coefficients are improved .
- 3- The best value of threshold speed is at eccentricity ratio equal 0.421 for all values of temperature rise parameter when using thermohydrodynamic analysis.
- 4- When the adjustable hydrodynamic bearing is used the rotor-system becomes more stable. The critical speed occurs at over rotational speed range.
- 5- Elastic deformation of bearing surface is useful to reduce the instability whirl. This is controlled by selecting softer bearing material.

References

- [1]. Luis Andrés," Bearings and Seals", Eurasia Publishing House (P) Ltd, 2000
- [2]. Lewis, et al, "Vibration Limiting of Rotating Machinery Through Control Means", United States Patent 4,643,592. htm, February 17, 1987.
- [3].Wang F. H., and Shin F. M., " Improve the Stability of Rotor Subjected to Fluid Leakage by Optimum Diameters Design ", Journal of Vibration and Acoustic, Vol. 112, 1990, pp.59-64.
- [4]. Chivens, D.R. and Nelson, H.D "The Natural of A rotating, Flexible Shaft-Disk System". Journal of Engineering for Industry,pp.334-347 August, 2000.
- [5].Krodkiwski, J. M., Cen, Y.,and Sun, L.," Improvement of Stability of Rotor System By Introducing A Hydraulic Damper Into An Active Journal Bearing". Mechanical Vibration and Noise Inc., Victoria 3052, Australia,2002.
- [6]. Shahab, Alaa Aldeen, "Couple Vibration of Multi-shaft Disk System",Ph.D. Thesis, University of Surrey, Mech.Eng.Dept., 1985.
- [7]. Kai-Yuan Yeh, and Han, R. P. S. "Analysis of High-Speed Rotating Disks with Variable Thickness and Inhomogeneity ", Transactions of the ASME, Vol.61,March 1994,pp.186-191.
- [8]. Ghosh, N. C., "Thermal Effect on The Transverse Vibration of High-Speed Rotating an isotropic Disk", Journal of Applied Mechanics, Vol.52, 1985, pp.543-547.
- [9]. Muhanad Z.K., " Design of Adjustable Hydrodynamic Bearing", Ph.D. Thesis, Military

- College of Engineering, Mech. Eng. Dept., February 22, 2003.
- [10] Chandrawat H.N. and Sinhasan R., "A Study of Steady State and Transient Performance Characteristics of a Flexible Shell Journal Bearing", Tribology Internationals, Vol.21, No.3, 1988, pp.37-49.
- [11]. Jang J. Y. and Khonsari M. M., "Design of bearings on the basis of Thermohydrodynamic analysis", Proc. Inst. Mech. Engrs., Part J, 218., 2004
- [12].Sumit Singhal," A Simplified Thermohydrodynamic Stability Analysis of The Plain Cylindrical Hydrodynamic Journal Bearings ",M. S., Thesis, The Graduate Faculty of the Louisiana State University , Agricultural and Mechanical College, Mech. Eng. Dep., August 2004.
- [13].El-Marhomy, Abd-Alla, " Parametric Stability Analysis of Rotor-Bearing System", Proc. Natl. Sci. Coun. Roc (A), Vol. 23, No.1, 1999, pp. 42–49.

Table (1) Mechanical Properties of the Shaft and Hyperbolic Disk, [1]-[5].

	Shaft Material Properties (High Strength Steel cgs)		Disk Material Properties (High Strength Steel mk)	
	Properties		Properties	
I	Stiffness	values	Stiffness	values
	<i>Young modulus (E)</i>	199.95 GPa	<i>Young modulus E(r)</i>	$1.5 \times 10^{12} \cdot r^{0.1}$ (N/m ²)
	<i>Shear modulus (G)</i>	75.842 GPa	<i>Shear modulus (G)</i>	74.376GPa
	<i>Poissons ratio (n)</i>	0.32	<i>Poisson's ratio (n)</i>	0.3
II	Limit stress	values	Limit stress	values
	<i>Tension</i>	148.24 MPa.	<i>Tension</i>	145.37 MPa.
	<i>compression</i>	165.47 MPa.	<i>Compression</i>	162.27 MPa.
	<i>Shear</i>	107.56 MPa.	<i>Shear</i>	105.48 MPa.
III	Thermal	values	Thermal	values
	<i>Expansion coefficient (a)</i>	11.88×10^{-6} 1/°C	<i>Expansion coefficient (a)</i>	12.34×10^{-6} 1/°C
	<i>Conductivity (k*)</i>	0.0868095 W/m. °C	<i>Conductivity (k*)</i>	0.0943211 W/m. °C

Table (2) Effects of The Nondimensional Cross- Coupling Parameter on
The Stability Condition in the ($Sc > \bar{a}^2$)

<i>Eccentricity, e (mm)</i>	\bar{a}^2	$Sc = \left[\frac{W_{xx}^2}{W^2} - 1 \right] \left[\frac{W_{zz}^2}{W^2} - 1 \right]$
0.02	0.8913	0.9989961
0.03	0.8451	0.9990153
0.04	0.7327	0.9990404
0.05	0.6911	0.9990703
0.06	0.6237	0.9991037
0.07	0.5541	0.99901392
0.08	0.4113	0.9991758
0.09	0.4026	0.9992124

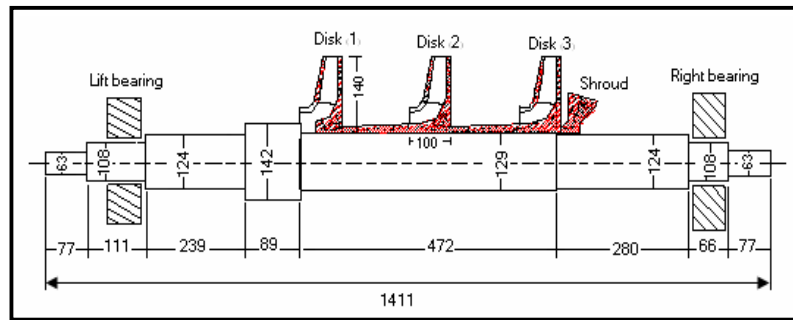
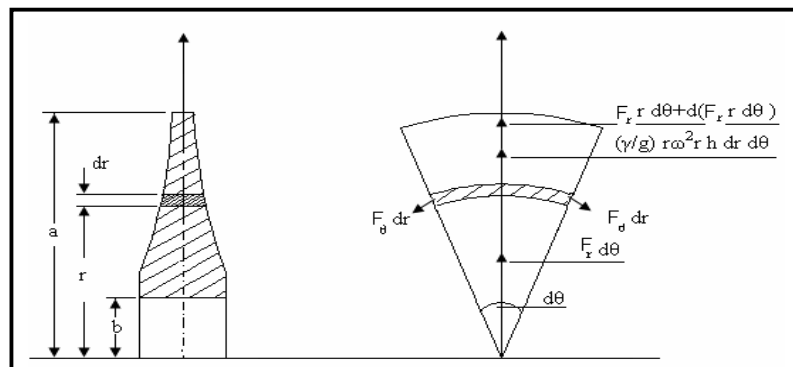


Figure (1) Rotor –Bearing of Centrifugal type RB4-3B

Figure (2) Profile of The Hyperbolic Disk and The Force
Acting on An Element,[8]

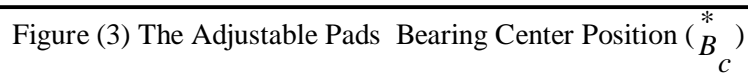


Figure 10 is a line graph showing the relationship between Critical Speeds, ω_c (rad/sec) on the Y-axis and Temperature, T (C) on the X-axis. The Y-axis ranges from 1150 to 1400 in increments of 25. The X-axis ranges from 45 to 70 in increments of 5. Three curves are plotted, labeled Case (1), Case (2), and Case (3). Case (1) is the lowest curve, Case (2) is the middle curve, and Case (3) is the highest curve. All three curves show an increasing trend with temperature. A horizontal line is drawn at $\omega = 1256$ rad/sec, which intersects Case (1) at approximately 60°C, Case (2) at approximately 55°C, and Case (3) at approximately 48°C.

Temperature, T (C)	Case (1) ω_c (rad/sec)	Case (2) ω_c (rad/sec)	Case (3) ω_c (rad/sec)
45	1185	1240	1295
50	1205	1275	1325
55	1230	1315	1360
60	1260	1360	1405
65	1295	1410	-
70	1335	1465	-

Legend:

- case (1)=anisotropic material and thermal
- case (2)=isotropic material and thermal
- case (3)=isotropic material, thermal and heat generated

Figure (4) Variation Critical Speed (ω_c) with Temperature (T) for Three Operating Cases

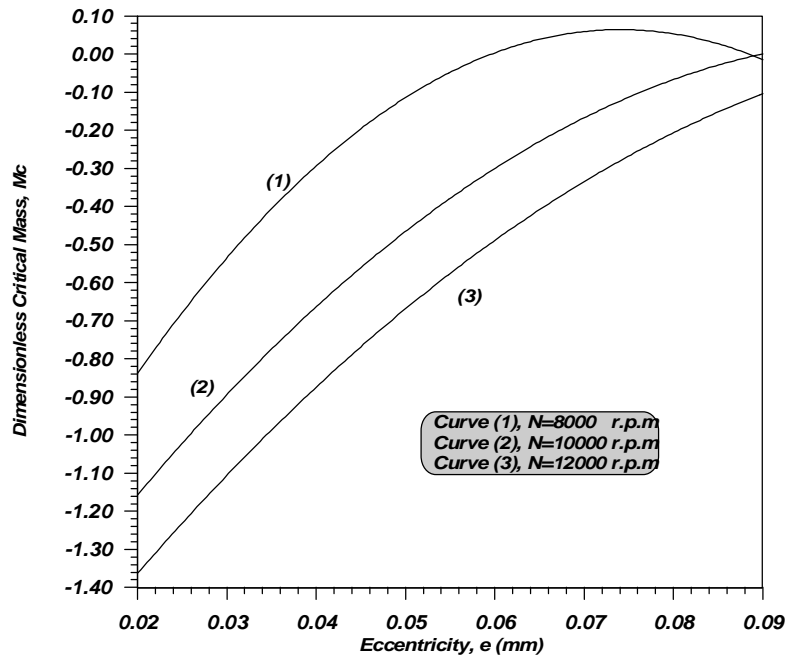


Figure (5) The Eccentricity Effect on The Stability Criteria
(dimensionless critical mass)

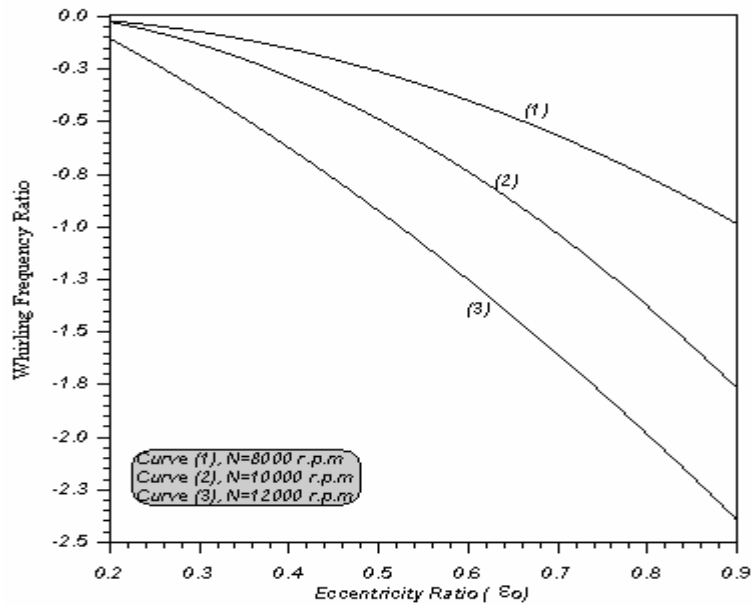


Figure (6) The Eccentricity ratio (ϵ_0) Effect on
The Whirling Frequency Ratio ($\frac{-2}{f_n}$)

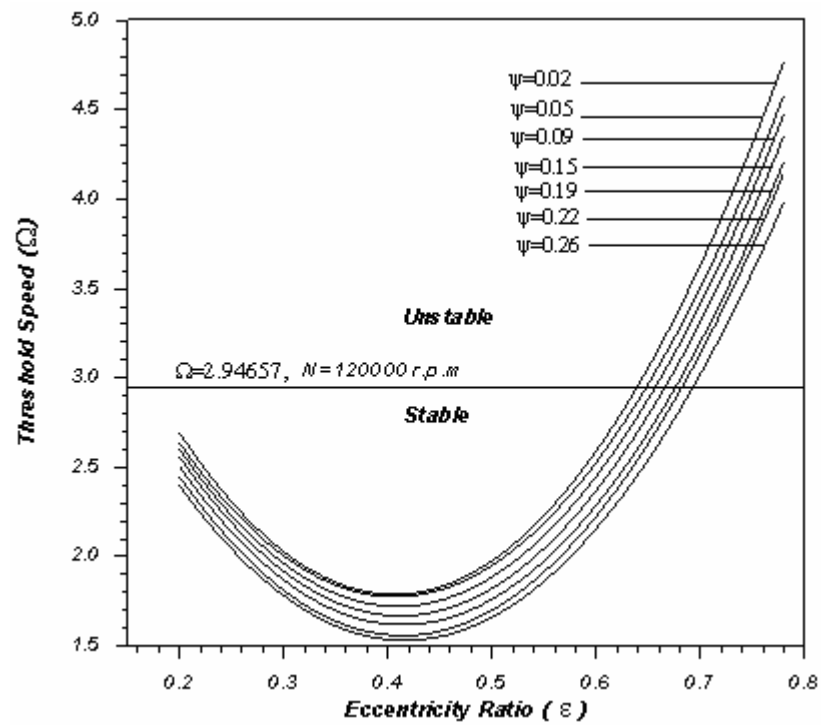


Figure (7) The Eccentricity Ratio (ϵ) With Temperature Rise Parameter (ψ) Effect on The Stability