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Numerical Simulation Of Two Dimensional Transient Natural Convection Heat Transfer From Isothermal Horizontal Cylindrical Annuli

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Abstract

Numerical solutions are presented for the transient natural convection heat transfer problem in horizontal isothermal cylindrical annuli, enclosed in heated inner and cooled outer cylinders. Solutions for laminar case were obtained within Grashof number based on the inner diameter which varied from $1x10^2$ to $1x10^5$ in air. Both vorticity and energy equations were solved using alternating direction implicit (ADI) method and stream function equation by successive over relaxation (SOR) method. The structure of fluid flow such as a velocity vector and temperature distribution as well as Nusselt number were obtained and the effect of diameter ratio on them was examined. In addition, the Grashof number was changed with the influence of variation in Prandtle number and diameter ratio. Our numerical calculation are summarized by Nussult number vs. Grashof number curves with diameter ratios and Prandtl as a parameter, which serves as a guide to natural convection heat transfer calculated from annulus. Good agreement with previous data is obtained.

Keyword: Natural Convection, Numerical Simulation, Isothermal, Cylindrical Annui.

الخلاصة

قدَّم حَلّ عددي لمسالةِ انتقال الحرارةِ العابرةِ بالحمل ِ الطبيعي في العمود الحلقي الإسطواني ثابت درجة الحرارةِ بوضع الأفقي، حيث تكونَ الاسطوانة الداخليةِ مسخنةِ وَ الإسطوانة َ الخارجية مبردة. تم حل المسالة لحالةِ الجريان الطباقي ولرقم كراشوف مستندة على القطر الداخلي يتراوح بين $1x10^2$ الى $1x10^5$ للهواء ,معادلة الدوامية ومعادلة الطاقةِ حلتا بإستعمال بطريقة الإتّجاهَ الضمنيَ المتناوبُ ومعادلة الانسياب بطريقة فوق التراخي المتعاقبة.تم الحصول علىّ تركيبَ جريان المائع كمتجه السرعةِ وتوزيع درجةِ حرارة بالإضافة إلى رقم نسلت، تم اختبار تأثير نسبةِ القطرِ عليها .بالإضافة الى دراسة تأثير إختلافِ رقم كراشوف مع إختلافِ رقم برانتل و نسبة القطرَ عليها .تم اجمال الحسابات العددية بواسطة منحنيات رقم نسلت مع رقم كراشوف و نسبة القطرَ ورقم برانتل .التي يمكن الاستفادة منها كدليل لحسابات انتقال الحرارة بالحمل الطبيعي من الشكل الحلقي.تم الحصول على توافق جيد بين النتائج الحالية وبياناتِ البحوث السابقةِ.

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Nomenclature

Dr mesh interval in r-direction
$D\theta$ mesh interval in θ-direction
a diameter ratio, radius ratio, r_o/r_i
c_p specific heat at constant pressure
g gravitational acceleration
Gr Grashof number, $g\,\beta\,(T_h\text{-}T_c)\,r_i^3\,/\upsilon^2$
k thermal conductivity
Nu local Nusselt number
Pr Prandtl number, υ/α
r radial distance
R dimensionless radial coordinate, r/r_i
Ra Rayleigh number, $Ra= Gr.Pr$
T temperature
U dimensionless radial velocity, ur_i/υ
u radial velocity
V dimensionless tangential velocity, vr_i/υ
v tangential velocity

Greek symbols

ψ dimensionless stream function
t dimensionless time, $t\upsilon/r_i^2$
ρ fluid density
α thermal diffusivity, $(k/r\ c_p)$
β thermal expansion coefficient of fluid
Θ dimensionless temperature, $(T\text{-}T_c)/(T_h\text{-}T_c)$
υ kinematics viscosity
Ω dimensionless vorticity

Subscripts

h,c hot and cold, respectively
i,o inner and outer, respectively

Superscripts

— Mean

Introduction

In recent years, natural convection heat transfer in a cylindrical annulus has attracted much attention with relation to thermal storage systems, solar collectors, spent nuclear air fuel cooling, nuclear reactors, aircraft fuselages insulation, cooling of electrical equipments. The horizontal convection isothermal cylinders were used pressurized gas underground electric transsimition cables Pederson et.al. (1971). In this paper, we present a brief review of selected experimental papers and relatedtheoretical studies. Liu et.al. (1961) measured the overall heat transfer and radial temperature profiles of air, water and silicone fluid. Qualitative flow descriptions were given for each fluid. Photographs of flow patterns in air using smoke were presented by Bishop and Carley (1966) and Bishop et.al (1968). Different flow regimes depending on the Grashof number and diameter ratio were delineated by Powe et.al. (1969). The first determination of local heat transfer coefficients in annular geometry with air was made by Eckert and Soehngen (1970) using Mach-Zender interferometer The first numerical solution of natural convection between horizontal convection cylinders was obtained by Crawford and Lemlich (1962) using Gauss-Seidel iteration approach for Prandtl number of 0.7 and for diameter ratio of 2, 8 and 57. Abbot (1964) obtained a solution for diameter ratio close unity using matrix inversion techniques. Mack and Bishop (1968) employed a power series expansion valid in the range diameter ratios from 1.15 to 4.15. However, as pointed out by Hodnett (1973), if the diameter ratio becomes too large, there is a region in the annulus where convection effects are as important as conduction effects. Such a problem has been attacked by Hodnett (1973) using a perturbation method. Powe et. al. (1971) examined the transition from steady to unsteady flow for air with Prandtl around 0.7 by determining the critical Rayleigh number at which an eddy forms and turns in the opposite direction of the main cells. Kuehn and Goldstein (1978) performed experimental and theoretical–numerical studies for air and water at Rayleigh numbers from $2.1x10^4$ to $9.8x10^5$ at diameter ratio of 2.6. Charrier-Mojtabi et al (1979) presented numerical solutions at a Prandtl number of 0.7 and 0.02 with various diameter ratios and Rayleigh numbers. Tsui and Tremblay (1983) carried out theoretical-numerical study at Grashof number from $7x10^2$ to $9x10^4$ and Prandtl number of 0.7 with diameter ratio of 1.2, 1.5 and 2. A numerical investigation has been performed by Hand and Back (1999), to examine the interaction between radiation and steady laminar natural convection in cylindrical annuli filled with a dry gas. Radiation was found to play an important role in determining thermo-fluid dynamics behavior in natural convection induced by hot inner cylinder under large temperature difference. All references cited except references Charrier-Mojtabi et.al.(1979) and Tsui and Trembaly (1983) are confined to the steady-state analysis. Even Charrier-Mojtabi et al (1979) gives the steady- state results only and Tsui and Tremblay (1983) presents the transition-state results with Prandtl around 0.7 only.

The purpose of this paper is to present the transient-state results with the effect of variation of Prandtl number and diameter ratio, which are new to the author's knowledge.

Mathematical Formulation

The physical model and the coordinate system in the present analysis are shown in Figure 1. A fluid layer is enclosed between two concentric cylinders with radii r_i and r_o. Temperatures at the heated inner cylinder surface and the cooled outer one, designated by T_h and T_c, respectively, are to be constant. Flow and temperature fields are assumed to have a symmetric nature with respect to vertical plane (*θ=0^o* and *180^o*) and the region of computation is limited between *θ=0^o* and *180^o*.

The physical system consists of a Newtonian fluid air, in an annulus bounded by two isothermal surfaces. To formulate the problem it is assumed that: (a) the fluid motion and temperature distribution are two-dimensional (2-D), (b) the fluid is viscous and incompressible, (c) frictional heating is negligible, (d) the difference in temperature between the two isothermal boundaries is small compared with *1/β*, (e) the fluid properties are constant except for the density variation with temperature. Thus, within the Boussinesq approximation, four governing equations (two momentum, one energy and continuity) in polar coordinate are as follows (Torrance 1985) and (Chun-Yen 1979):

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{1}{r}\frac{\partial v}{\partial q}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial q}=gb(T-T_c)\cos q-\frac{1}{r}\frac{dp}{dr}$$
$$+u(\nabla^2 u-\frac{u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial q}) \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial q})=gb(T-T_c)\sin q-\frac{1}{r}\frac{1}{r}\frac{dp}{dr}$$
$$+u(\nabla^2 v-\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial q}) \qquad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+\frac{v}{r}\frac{\partial T}{\partial q}=a(\nabla^2 T) \quad (4)$$

where all constants, variables and operators are dimensional.

The coordinates *r*, are measured from the center of the system, and *θ*, measured counterclockwise from the downward vertical line. The radial velocity *u* is positive radially outwards, and the tangential (angular) velocity *v* positive in the counterclockwise direction for *0^o£ θ £ p* .

The vector potential Ψ and vorticity vector Ω are introduced (Torrance 1985)

$$V=\nabla x\Psi \qquad (5)$$

$$\Omega=\nabla xV \qquad (6)$$

where Ψ and Ω satisfy the following solenoid condition

$$\nabla.\Psi=0 \qquad (7)$$

$$\nabla.\Omega=0 \qquad (8)$$

The vector potential satisfies the equation of continuity, equation (1), automatically. Then, the relation between Ψ and Ω is presented in the following dimensionless form

$$\Omega=-\nabla^2\Psi \qquad (9)$$

Taking the curl of Eqs.(2)and(3) to eliminate the pressure term, the vorticity transport equation is obtained in the dimensionless form

$$\frac{\partial\Omega}{\partial t}+U\frac{\partial\Omega}{\partial R}+\frac{V}{R}\frac{\partial\Omega}{\partial q}=$$
$$Gr\left(\cos q\frac{1}{R}\frac{\partial(R\Theta)}{\partial R}-\sin q\frac{1}{R}\frac{\partial\Theta}{\partial q}\right)$$
$$+\nabla^2\Omega \qquad (10)$$

In the same manner, the dimensionless form of the energy equation is written as

$$\frac{\partial\Theta}{\partial t}+U\frac{\partial\Theta}{\partial R}+\frac{V}{R}\frac{\partial\Theta}{\partial q}=\frac{1}{\Pr}\nabla^2\Theta \qquad (11)$$

$$\text{also,} U=\frac{1}{R}\frac{\partial\Psi}{\partial q}, V=-\frac{\partial\Psi}{\partial R} \qquad (12)$$

where all constants, variables and operators are dimensionless.

In the eqs. (9-12), the following dimensionless variables and parameters are used

$$R=\frac{r}{r_i},\ q=q\ \ U=\frac{ur_i}{u},\ V=\frac{vr_i}{u},$$
$$t=\frac{tv}{r_i^2},\qquad \Theta=\frac{T-T_c}{T_h-T_c},$$
$$Gr=\frac{gb(T_h-T_c)r_i^3}{u^2},$$
$$\nabla^2=r_i^2\nabla^2 \qquad (13)$$

Eqs. (10) and (11) are coupled through the buoyancy force. Furthermore, both the vorticity Eq (10) and energy eq. (11) are non-linear due to the convective terms. It is to be noted that both the vorticity Eq. (10) and energy Eq. (11) are of the parabolic type and the stream function Eq. (9) is of the elliptic type.

Eq. (9) is coupled with Eqs. (10) and (11) through Eq. (12) which relates the stream function to the velocities. Our problem is to seek $\Theta(r,\theta,t)$, $\Omega(r,\theta,t)$ and $\psi(r,\theta,t)$ which satisfy three partial differential Eqs. (9), (10) and (11) as well as the following initial and boundary conditions. To begin with, the fluid in the annulus is stationary with a uniform temperature:

$$\Omega=\Psi=T=0$$
every where at $t=0$ (14)

The boundary conditions are

$$\Psi=\frac{1}{R}\frac{\partial\Psi}{\partial q}=\frac{\partial\Psi}{\partial R}=0$$
On both walls ,i.e. $R=R_i$ and $R=R_{o,}$ (15)

$\theta=1$ at $R=1_,$ (16-a)
$\theta=0$ at $R=$ (16-b)

Eqs (5) and (9-11) are the final form of governing equations, which were transformed into the finite difference equations and solved numerically (Chun-Yen 1979). The relaxation factors chosen (1.7) for the stream function, and the number of nodal points in the grid was 41, 21 for the *R-θ* respectively the choice of 41x21 uniform grid points was found to provide grid independence for the results reported in this paper.

Numerical Solution

In the field of heat transfer, several numerical methods have been developed to deal with complicated physical problems. The finite difference method is one of the most widely used numerical methods for decades. The present work is concerned with numerical simulation of two dimensional transient natural convection flow, by means of alternating direction implicit (ADI) method for vorticity and energy equation, and by successive over relaxation (SOR) method for stream function equations. The time increment (Tsui and Trembaly1983) is

$$\Delta t = \frac{2}{2\left(\frac{1}{(\Delta r)^2} + \frac{1}{(\Delta q)^2}\right) + \frac{U}{\Delta r} + \frac{V}{\Delta q}} \quad (17)$$

The convergence criteria for Nusselt number is

$$\frac{Nu^{n+1} - Nu^{n}}{Nu^{n+1}} \le e \quad (18)$$

The way for checking the convergence is to compare the mean Nusselt numbers at the inner and outer radius. These are usually within $e = 10^{-4}$ at convergence. This convergence criterion is employed in this paper.

In order to gain confidence in our numerical results, we tried to compare ours with previously published results (Charrier-Mojtabi et.al.1979) and (Tsui and Trembaly1983). Figure 2-a, which depicts streamlines and isotherms for Gr=10 000, Pr=0.7 and a=2, resemble results presented by (Tsui and Tremblay 1983) at Gr =10000, Pr=0.7, and a=2.0. Figure 2-b which shows streamlines and isotherms for Gr =38800, Pr=0.71 and a=2, is similar to one given by (Charrier-Mojtabi et al. 1979) at Ra = 3x10^4, Pr = 0.7 and a = 2.0. We see good agreement in results at diameter ratio of 2.0.

After obtaining confidence in our results see Table1, we process to compute the mean transient Nusselt numbers at inner and outer radius for our calculations, which cover the Grashof number 10^2 to 10^5 including physical realistic cases.

Local Nuselt numbers at the inner and outer radius Nu_i and Nu_o are defined as follows:

$$Nu_i = -\ln a\left[R\frac{\partial\Theta}{\partial R}\right]_{r=R_i} \quad (19a)$$

$$Nu_o = -\ln a\left[R\frac{\partial\Theta}{\partial R}\right]_{r=Ro} \quad (19b)$$

The mean Nusselt numbers $\overline{Nu_i}$ and $\overline{Nu_o}$ are the angular average of their local values over the cylinder inner and outer surface and can be carried out using numerical integration by Trapezoidal rule (Gerald 1970), through eq. (19c).

$$\overline{Nu} = \frac{1}{A}\int_A Nu.dA \quad (19c)$$

Both mean transient $\overline{Nu_i}$ and $\overline{Nu_o}$, vs. dimensionless time, τ, are plotted in Figure 3, which included physical realistic cases (Tsui and Trembaly 1983). As τ increases, both $\overline{Nu_i}$ and $\overline{Nu_o}$ approach their steady- state values and should be equal based on a simple energy balance. In fact, due to the numerical techniques involved, the values actually obtained differ somewhat. Generally, we can note that the dimensionless time increases with increases in diameter ratio and decreases or is constant with increased Grashof number.

The effect of Prandtl number on results was determined by varying Prandtl number at values 0.7, 5.0, and 10.0 respectively, corresponding to the same values of Grashof numbers and diameter ratios. Looking at the effect of variation in Prandtl number that when the diameter ratio changes from 1.2 to 2.0, It seen that, at a diameter ratio a=1.2, there is no significantly change in the convection heat transfer, i.e., $\overline{Nu}$ even Gr=1x10^5
The maximum non-dimensional transition time increases with increases Prandtl number and high convection occur, see Figure 4 The values of mean Nusselt number of higher Prandtl number are higher than those for air.

The convection heat transfer increases very rapidly when the diameter ratio increases further from 1.5 to 2.0 and especially at high Grashof number. A review of Figure 3 and **4** shows both $\overline{Nu_i}$ and $\overline{Nu_o}$ approach unity as time increase.
For high values of Prandtl number such as Pr=10 show that the effect of Grashof number is clear where the maximum dimensionless time decreases for Grashof number increases for different values of diameter ratio (a) as shown in Figure 4a and Figure 4c and this effect decreased for decreased prenatal number as shown in Figure 4b

Local Nusselt number is generally smaller and more uniform at lower Prandtl number as, including an approach to conduction. The numerical data obtained in the present study are correlated to one-fourth power law see Figure 5.

Results and Discussion

Our range of interest covers Grashof numbers from approximately 1x10^2 to 1x10^5 and diameter ratio from 1.2, 1.5 and 2.0. Three steady state mean Nusselt number, $\overline{Nu}$ vs. Grashof number, Gr, curves are shown in Figure 5 with diameter ratio, a, as a parameter. It is seen that, at a diameter ratio a=1.2, there is no or little convective heat transfer even Gr=1x10^5 because the variation of $\overline{Nu}$ vs. Grashof number is small that from the values of constant(c) of the equation (cGr^n) ,for diameter ratio (a=1.2) and Prandtl number Pr=10 and Pr=5 have the same value of constant (c) because no convective heat transfer and only conductive heat transfer which has been substantiated by Kuehn and Goldstein's calculation (Kuehn and Goldstein 1978).

It is clear that the maximum increment in the amount of convection heat transfer with larger Prandtl number (Gr=10^5) by 31% and 25% is a=1.5, 2.0 respectively compared with corresponding values at low Prandtle number.

Looking at the variation of $\overline{Nu}$ vs. a at fixed Gr it follows that in when the diameter ratio, a, changes from 1.2 to 1.5, the mean Nusselt number increases very rapidly. When the diameter ratio increases further from 1.5 to 2.0, there is a substantial enhancement in the convective heat transfer .However; the rate of increase of $\overline{Nu}$ vs. a slows down (with increase in a than 2.0). After (a) it reaches 2, the rate of increase of convective heat transfer flattens out that appear from increase the value of constant (c) in correlation equation of the diameter ratio of (a=2), This is demonstrated by Kuehn and Goldstein (1976), despite a collection of experimental data from previous authors. From an engineering viewpoint, there is no advantage in increasing the diameter ratio beyond two as far as natural convection is concerned.

The flow and heat transfer results can be divided in to several regimes (Kuehn and Goldstein 1978).Near Grashof number of 10^2 the maximum stream function or center of rotation is near 90^o . The flow in the top and bottom portions of the annulus is symmetric about the 90^o position. The velocity profiles at any one position are similar, with the magnitudes directly proportional to the Grashof number. The velocities are too small to affect the temperature distribution, which remains essentially as in pure conduction, see Figure 6.

This makes the convection terms in Eqs. (10) and (11) vanish. Therefore, Eqs. (10) and (11) can be approximated by

$$\nabla^2\Omega = Gr\left(\sin q\,\frac{1}{R}\frac{\partial\Theta}{\partial q} - \cos q\,\frac{\partial\Theta}{\partial R}\right) \qquad (19)$$

$$\nabla^2\Theta = 0 \qquad (20)$$

A transition region exists for Grashof numbers between 10^2 and 10^4. The flow remains in essentially the same pattern but becomes strong enough to influence the temperature field. As the Grashof number increases, the center of rotation moves upwards.

The isotherms begin to resemble eccentric circles near a Grashof of 10^3, as can be seen in Figure 6 at different diameter ratio. This has been called the 'pseudo-conductive regime' (Grigull and Hauf 1966), since the overall heat transfer remains essentially that of conduction.

With further increase in Grashof number and increasing diameter ratio, the temperature distribution becomes distorted, resulting in an increase in mean Nusselt number. From a plot of streamlines and isotherms at Grashof number of 10^4, the radial temperature inversion appears indicating the separation of the inner and outer cylinder thermal boundary layers which become obvious at the top portion of a=2.0. The cross indicates the location of maximum value of the stream function, which would be the center of rotation. This maximum is located near the 70^o position. Local heat transfer flux values are becoming further distorted from those of conduction. Essentially heat is being convected from the lower portion of inner cylinder to the outer cylinder. The vorticity in the central core is almost constant, including a region approaching solid-body rotation; see Figure 6c at high Grashof number.

In addition, steady laminar boundary layer regime exists between Grashof number of 10^4 and 10^5. Streamlines and isotherms in this region are shown in Figure 6. Boundary layer exists on both cylinders although the lower portion of the annulus is practically stagnating.

As the Grashof number increases further, the flow above the inner cylinder will become turbulent. This will create a turbulent boundary layer on the outer cylinder while the inner boundary layer remains laminar (Lis(1966). Eventually, the inner boundary layer will also become turbulent.

The effect of diameter ratio on the results is determined by varying R_o/R_{in} from 1.2 to 2.0 for the whole range of our numerical calculations. The flow pattern does change significantly at lower Grashof numbers although the center of rotation moves towards the top with increasing diameter ratio (a), but separation is clear at high Grashof numbers as shown in Figure 6. The maximum Nusselt number occurs near a=2.0 at $Gr=10^5$ but occurs at smallest value at larger a. However, the mean Nusselt number increased as the outer cylinder is made large at constant Grashof number as shown in Figure 5. As the outer cylinder becomes large relative to the inner diameter, the mean temperature in the annulus decreased. This indicates that the thermal resistance around the inner cylinder is becoming the dominant factor in the mean Nusselt number. As the outer cylinder becomes infinitely large, the only thermal resistance is around the inner cylinder with the temperature in the gap equal to that of the outer cylinder. At large diameter ratio, the total heat flow will be essentially that from a single horizontal cylinder in an infinite medium.

Fluids with larger Prandtle number will remain steady until larger Grashof number is attained. This is observed (Liu et.al. 1980) and (Charrier-Mojtabi 1979) and confirmed by the present numerical results. Figure 6 shows streamlines and isotherms at different Grashof number and diameter ratio with Prandtl around 0.7.

The maximum stream function is about 15^o from the top with lower portion of annulus particularly stagnant. The vorticity approaching to zero in the central portion of the annulus, indicate the beginning of stationary core region. The center of rotation moved near the top as the Prandtl number increases.

Also, with further increase in Grashof number and increasing diameter ratio, the temperature distribution becomes distorted, resulting in an increase in mean Nusselt number. From a plot of streamlines and isotherms at Grashof number of 10^4, the radial temperature inversion appears indicating the separation of the inner and outer cylinder thermal boundary layers which is obvious at the top portion of a=2.0. As the Grashof number increases further, the flow above the inner cylinder will become turbulent. This will create a turbulent boundary layer on the outer cylinder while the inner boundary layer remains laminar (Lis 1966).

Eventually, the inner boundary layer will also become turbulent. An oscillating laminar flow regime begins near Grashof number of 10^5.

At low Prandtl number, the velocity profile at any one position is similar with the magnitude directly proportional to the Grashof number. The velocities are too small at low

Grashof number and increases with increased Garshof number and Prandtle number causing the separation of inner and outer cylinder thermal boundary layer.

The velocity profile in the outer cylinder boundary layer in the top half of the annulus ($30^0 \leq \theta \leq 90^o$) is independent of angular position. As the fluid moves down past the 90^o position the outer boundary layer weakens, and disappears entirely near the bottom, see Figure 7, at high Grashof number. The velocities at the bottom of the annulus are very low compared to the velocities at the middle and top regions (Kuehn and Goldstein 1978).

On the basis of the good agreement between numerical results of the present study and experimental and numerical results of previous work, it seems possible to determine heat transfer parameters free convection in enclosures using either method. The experiments have the advantage of being applicable to unsteady flow and turbulence, where the numerical computation becomes unstable. However, the numerical analysis gives more information, including the velocity vector, which is difficult to obtain experimentally. The errors in numerical results arise from the constant property assumption, the finite number of grid of nodes and the convergence level of the solution though not perfect owing to the consideration mentioned above is quite good lending validity to results compared with previous work.

Summary

The numerical study of natural convection heat transfer and fluid flow between horizontal isothermal concentric cylinders has been presented. Quantities obtained numerically include temperature distribution, local and average Nusselt numbers. The numerical solutions confirm velocity distribution and extend results to lower Grashof numbers. Numerical solutions cover the range of Grashof numbers from pure conduction to steady laminar boundary layer flow for R=1.2, 1.5, and 2.0. The flow was steady for all Grashof numbers investigated. The influence of diameter ratio and Prandtl number was determined. Good agreements with available previous published results have been obtained.

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Table 1. Mean Nusselt number results for a=2.0, Pr=0.7

a	Gr_{Ri}	$\overline{Nu}$ (Tsui and Trembaly 1983).	$\overline{Nu}$ (Present study)
2.0	10 000	1.64	1.658
	38 800	2.4	2.42
	88 000	3.08	2.99

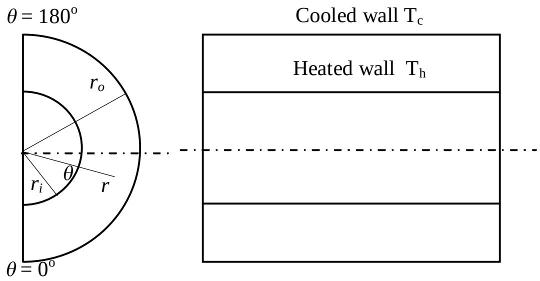
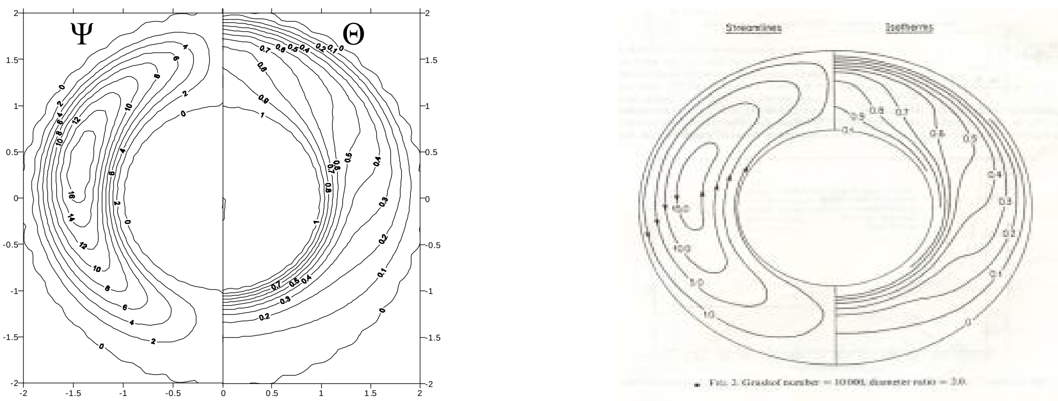
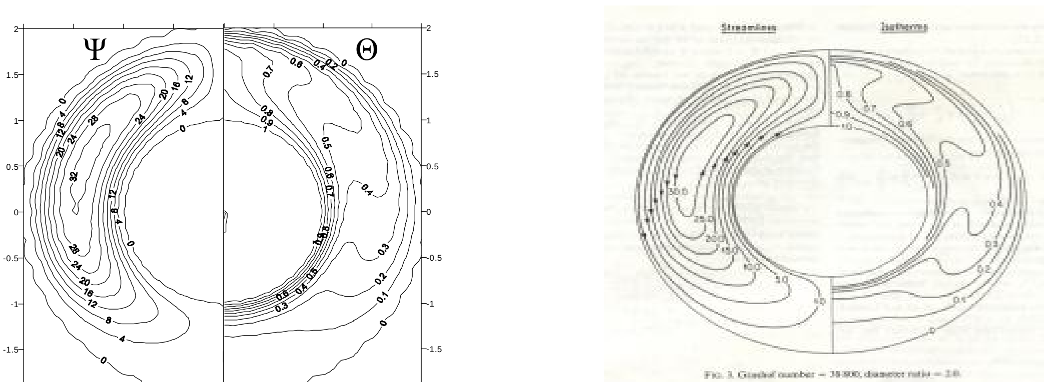


Fig.1. Natural convection in air filled annulus bounded by two isothermal walls



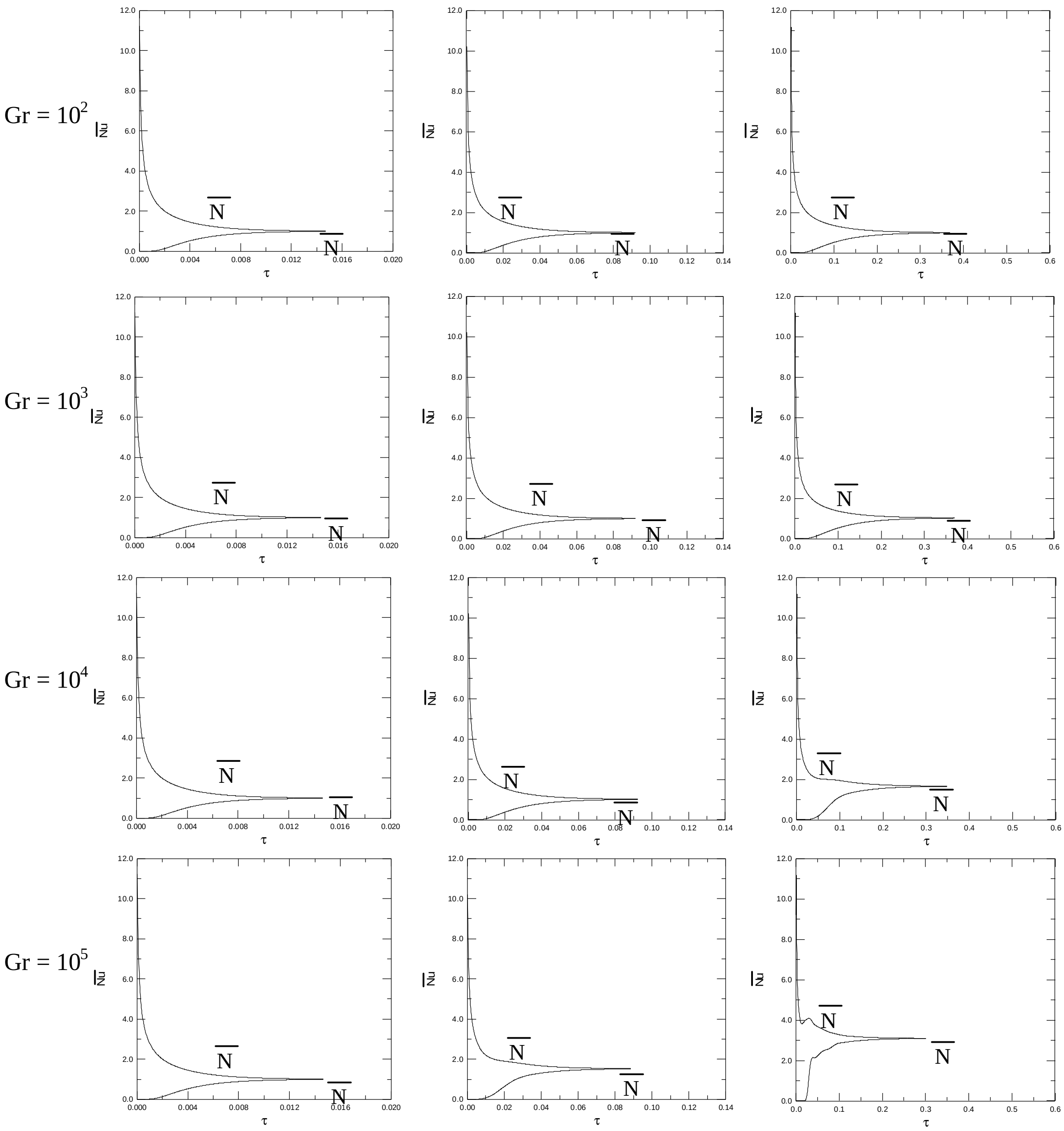
a) Gr=10 000



b) Gr=38 800

(Present study) (Tsui and Trembaly 1983).

Figure 2: Streamlines and isotherms, diameter ratio=2.0, Pr = 0.7



a)a=1.2 b)a=1.5 c)a=2.0

Figure 3: Mean Nusselt number vs. dimensionless time, Pr=0.7

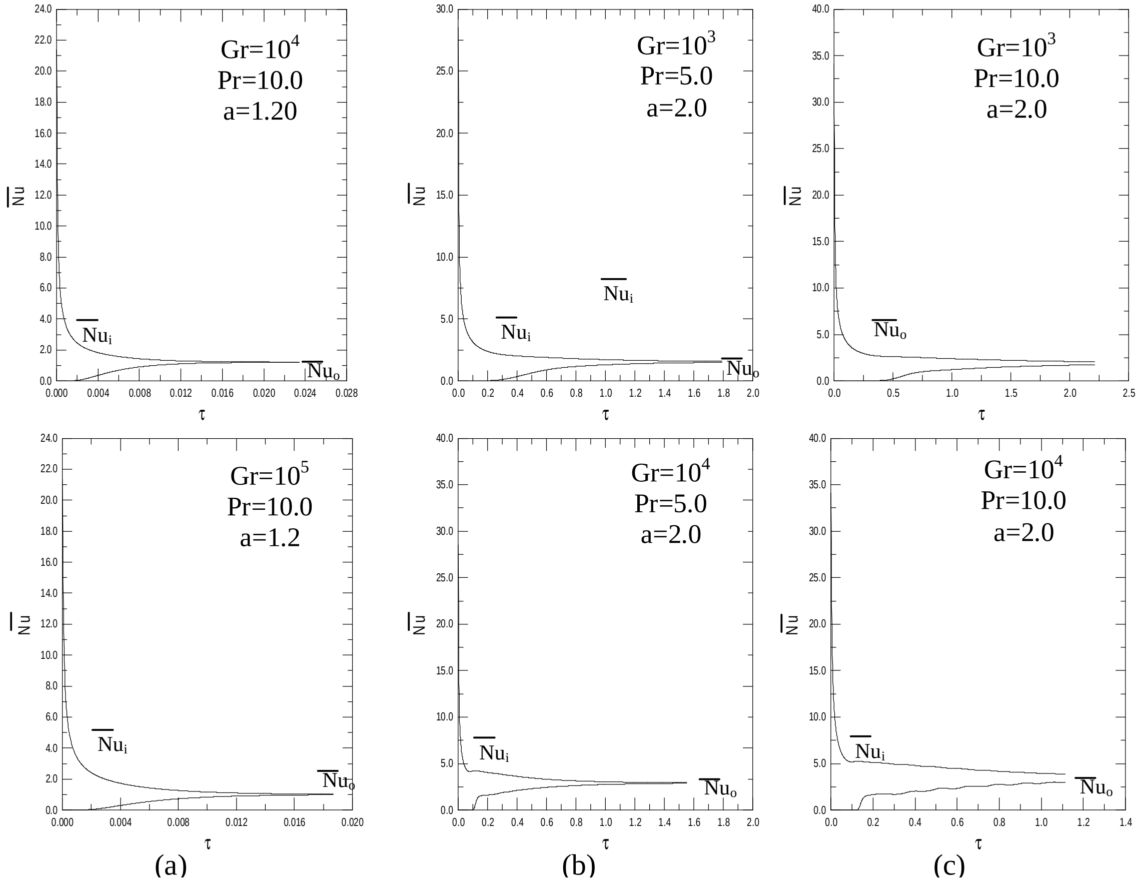


Figure 4: Variation in dimensionless time τ at different prandtl numbers

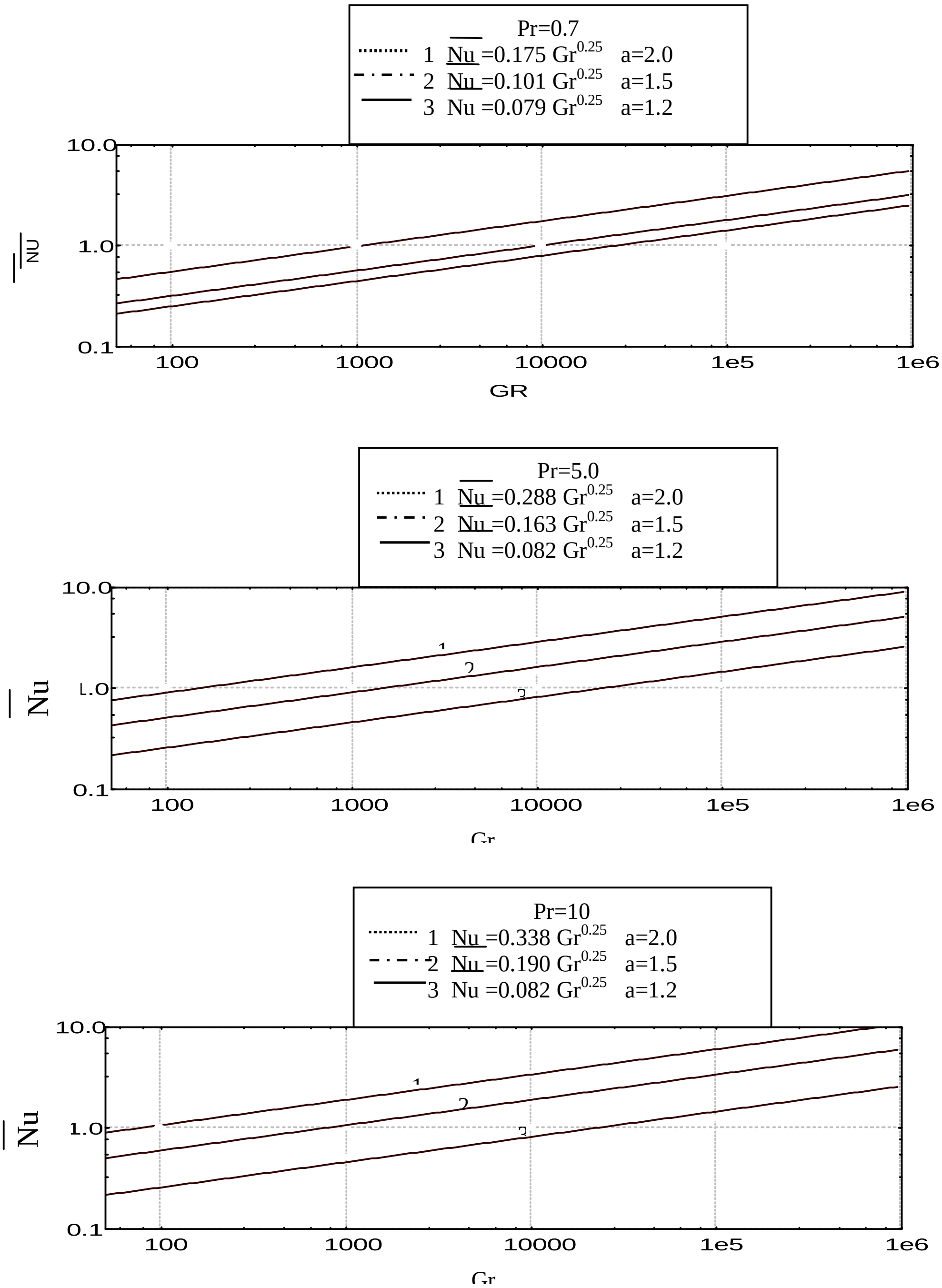


Figure 5: Correlation of mean Nusselt number as a function of Grashof number for different Prandtl number

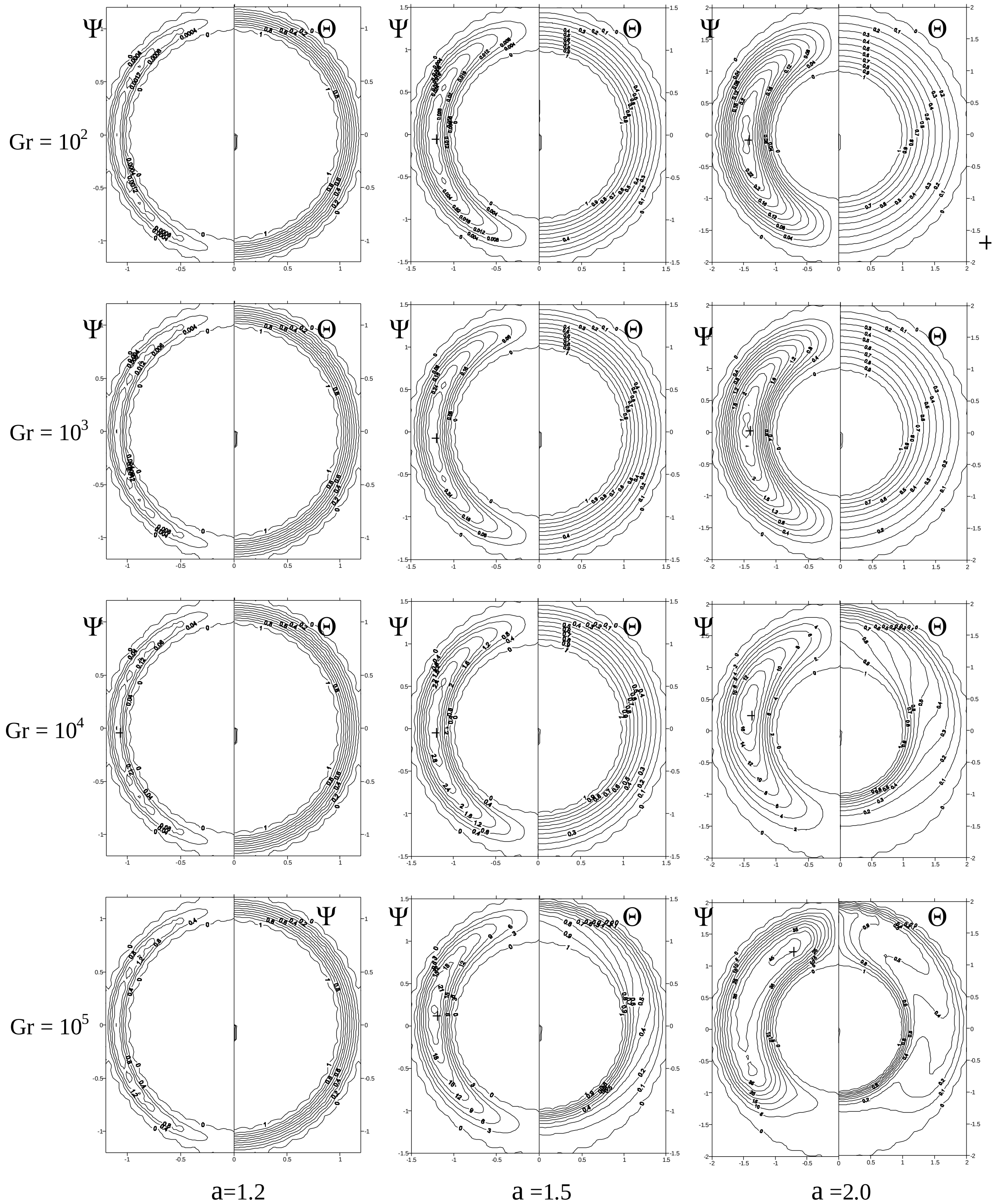


Figure 6: Streamlines and isotherms at different Grashof number, Pr=0.7

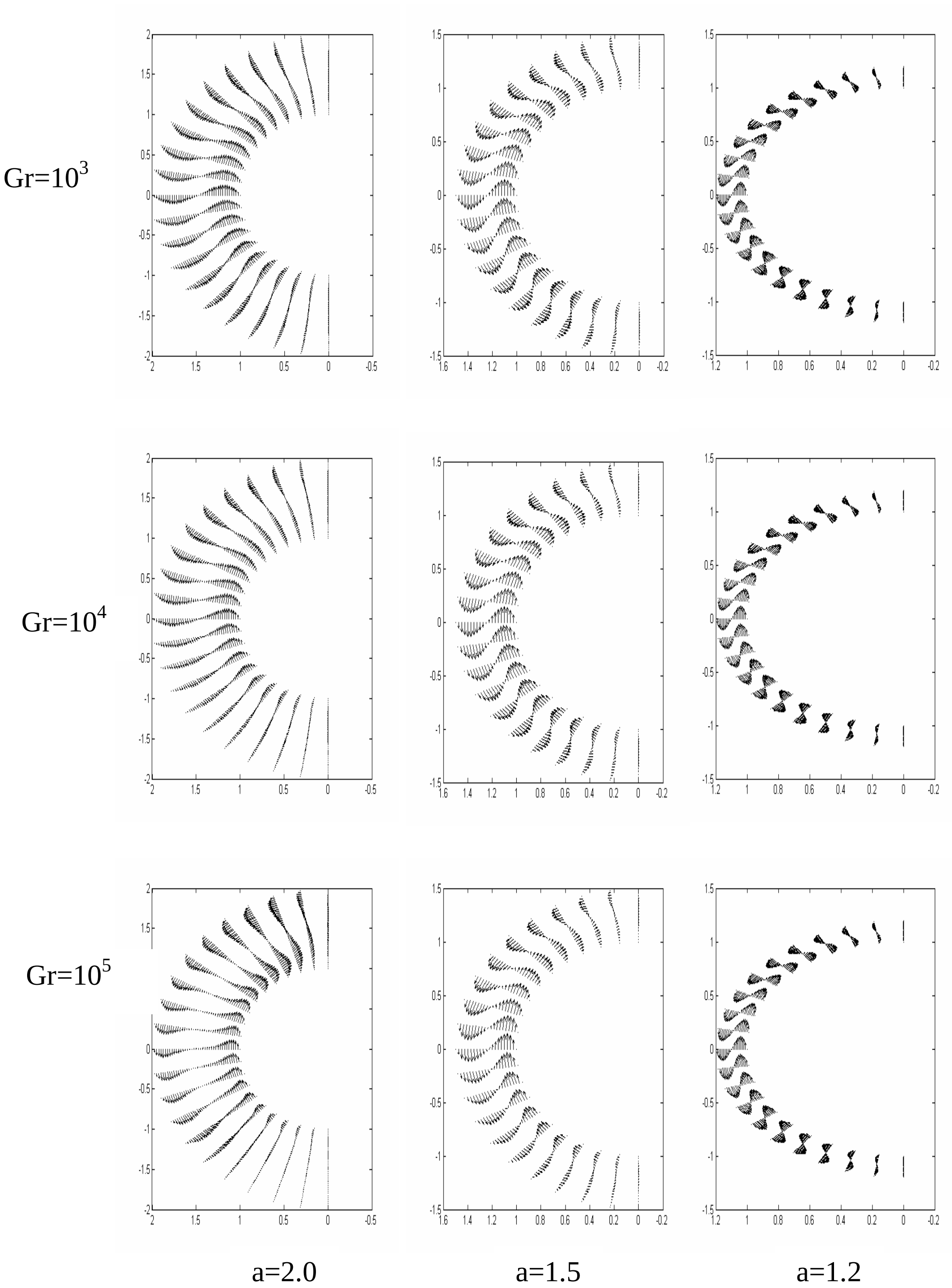


Figure 7: Velocity vector for natural convection in an annulus at Pr=0.7