On Soft Group Spaces

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Published online 29-11-2022

Abstract. The major of this paper is to study the concept of soft action of soft topological groups, and introduce some types of soft group space. Finally, some examples and properties about them are given.

Keywords: Soft topological group, Soft group space, invariant set, transitive soft group space, homogenous soft group space.

1. INTRODUCTION

Oguz et al explored the actions of soft groups in [4] & discovered several interesting features. Other algebraic characteristic of soft sets was also investigated in [1,2,5]. Oguz defined the action of soft topological groups and examined some of its features in 2020[3], as well as presenting some research on soft orbit spaces and soft transitive spaces. In this paper, by using these concepts we are going to introduce some types of soft group space, and give some examples and propositions about them.

2. SOFT ACTION

We recall some notions and properties about the soft topological group used throughout the paper.

Definition (2.1)[3]

Let $\mathbb{G}$ be a group with topology Γ, and $(F, \hat{E})$ be a non-null soft set (S-set) over $\mathbb{G}$, then $(\mathbb{G}, \Gamma, F, \hat{E})$ is called a soft topological group (Sʈg) over $\mathbb{G}$ if for all $\omega \in \hat{E}$:

i) $F(\omega) < \mathbb{G}$ ($F(\omega)$ is a subgroup of $\mathbb{G}$),

ii) the mapping $\mu_{\omega}(\chi, y) = \chi y$ of the topological space $F(\omega) \times F(\omega)$ onto $F(\omega)$ and the inversion $\psi_{\omega} : F(\omega) \to F(\omega)$, $\psi_{\omega}(\chi) = \chi^{-1}$ are continuous.

Definition (2.2)[3]

Let $(F, \hat{E}, \Gamma)$ be a Sʈg over $\mathbb{G}$, and let $(K, \hat{E}, \Gamma')$ be a Sʈs over X . A soft action of $(F, \hat{E}, \Gamma)$ on $(K, \hat{E}, \Gamma')$ is a continuous map $\varphi_{\omega} : F(\omega) \times K(\omega) \to K(\omega)$ $\forall \omega \in \hat{E}$ such that:

(i) $\varphi_{\omega}(e_G, \chi) = \chi$, $\forall \chi \in K(\omega)$

(ii) $\varphi_{\omega}(g, \varphi_{\omega}(g*, \chi)) = \varphi_{\omega}(gg*, \chi)$, $\forall\ g, g* \in F(\omega)$ & $\chi \in K(\omega)$.

The Sʈs $(K, \hat{E}, \Gamma')$ is called "Soft Group space" which is denoted by ($S\mathbb{G}$-space).

Example (2.3)

Let $(\mathbb{G}, \Gamma, F, \hat{E})$ be a Sʈg, then every Sʈs $(K, \hat{E}, \Gamma')$is $S\mathbb{G}$-space, where $\varphi_{\omega} : F(\omega) \times F(\omega) \to F(\omega)$ defined by $\varphi_{\omega}(g, g*) = gg*$, $\forall \omega \in \hat{E}$.

Example (2.4)

Let $\mathbb{G} = (Z_2, +_2)$ with discrete topology Γ, $\hat{E} = \{\omega_1, \omega_2\}$ & $(F, \hat{E})$ be a S-set over $\mathbb{G}$ where $F(\omega_1) = \{0\}, F(\omega_2) = Z_2$. Then $(K, \hat{E}, \Gamma')$ is a $S\mathbb{G}$ -space where $\varphi_\omega : F(\omega) \times F(\omega) \to F(\omega)$ defined by $\varphi_\omega(g, g*) = g +_2 g* +_2 g^{-1}$, $\forall \omega \in \hat{E}$.

Proposition (2.5)

Let $(K, \hat{E}, \Gamma')$ be a $S\mathbb{G}$ -space, then $\forall \omega \in \hat{E}$ the map $\phi_{\omega g}: K(\omega) \to K(\omega)$, which is defined by $\phi_{\omega g}(\chi) = \varphi_\omega(g, \chi)$, $\forall \chi \in K(\omega)$ is a homeomorphism.

Proof:

Let $\{\eta_\beta\}_{\beta \in \Lambda}$ be a net in $K(\omega)$ such that $\eta_\beta \to \chi$ in $K(\omega)$.

Then $\varphi_\omega(g, \eta_\beta) \to \varphi_\omega(g, \chi)$. So $\phi_{\omega g}(\eta_\beta) \to \phi_{\omega g}(\chi)$. Hence $\phi_{\omega g}$ is continuous.

Also, its inverse $\phi_{\omega g^{-1}}$ is continuous.

Then $\phi_{\omega g}$ is homeomorphism.

Definition (2.6)[3]

Let $(K, \hat{E}, \Gamma')$ be a $S\mathbb{G}$ -space, then $\forall \omega \in \hat{E}$:

i) The set $Orb_\omega(\chi) = \{\varphi_\omega(g, \chi): g \in F(\omega)\}$, $\forall$ $x \in K(\omega)$ is called the soft Orbit of χ.

ii) The set $Stab_\omega(\chi) = \{g \in F(\omega): \varphi_\omega(g, \chi) = \chi\}$ is called the soft stabilizer of χ.

iii) The set $Ker\phi_\omega = \{g \in F(\omega): \varphi_\omega(g, \chi) = \chi, \ \forall \chi \in K(\omega)\}$ is called the soft kernel of the soft action φ_ω

Proposition (2.7)

Let $(K, \hat{E}, \Gamma')$ be a $S\mathbb{G}$ -space, then $\forall \omega \in \hat{E}$:

(i) $Stab_\omega(\chi) < F(\omega)$.

ii) $Ker\varphi_\omega = \bigcap_{x \in K(\omega)} Stab_\omega(\chi)$.

iii) $Ker\varphi_\omega \lhd F(\omega)$ (normal subgroup).

Proof:

(i) It is clear that $e_G \in Stab_\omega(\chi)$

Then $\varphi_\omega(g_1 g_2, \chi) = \chi$

Then $g_1 g_2 \in Stab_\omega(\chi)$ & $\varphi_\omega(g^{-1}, \chi) = \chi$, $\forall$ $g_1, g_2 \in Stab_\omega(\chi)$

Therefore $Stab_\omega(\chi) < F(\omega)$.

ii) let $g \in Ker\varphi_\omega \leftrightarrow \varphi_\omega(g, \chi) = \chi, \forall \chi \in K(\omega) \leftrightarrow g \in Stab_\omega(\chi), \forall \chi \in K(\omega) \leftrightarrow g \in \bigcap_{\chi \in K(\omega)} Stab_\omega(\chi)$

Then $Ker\varphi_\omega = \bigcap_{x \in K(\omega)} Stab_\omega(\chi)$.

iii) $Ker\varphi_\omega < F(\omega)$ from (i & ii).

Let $g \epsilon F(\omega)$ and $h \in Ker\varphi_\omega$, then $\varphi_\omega(ghg^{-1}, \chi) = \chi$, $\forall \chi \in K(\omega)$

Then $g^{-1}(Ker\varphi_\omega)g \subset Ker\varphi_\omega$, & therefore $Ker\varphi_\omega \subseteq g(Ker\varphi_\omega)g^{-1}$.

Thus $g(Ker\varphi_\omega)g^{-1} = Ker\varphi_\omega$, $\forall g \in F(\omega)$. Hence $Ker\varphi_\omega \lhd F(\omega)$.

Proposition (2.8)

Let $(K, \hat{E}, \Gamma')$ be a $S\mathbb{G}$ -space, if X be a Hausdorff space, then $\forall \omega \in \hat{E}$:

i) $Ker\varphi_\omega$ is a closed normal subgroup of $F(\omega)$,

ii) $Stab_\omega(\chi)$ is a closed subgroup of $F(\omega)$, $\forall \chi \in K(\omega)$.

proof:

i) $Ker\varphi_{\omega} \lhd F(\omega)$ from proposition (2.7(iii))

Now let $g \in \overline{Ker\varphi_{\omega}}$, then $\exists$ a net $\{g_{\beta}\}_{\beta\in\Omega}$ in $Ker\varphi_{\omega}$ such that $g_{\beta} \to g$

Then $\varphi_{\omega}(g_{\beta},\chi) \to \varphi_{\omega}(g,\chi)$, $\forall\chi \in K(\omega)$

But $g_{\beta} \in Ker_{\omega}\varphi$. Then $\varphi_{\omega}(g_{\beta},\chi) = \chi$, $\forall\chi \in K(\omega)$

Since $K(\omega)$ is a Hausdorff Space, then $\varphi_{\omega}(g,\chi) = \chi$, $\forall\chi \in K(\omega)$

Then $g \in Ker\varphi_{\omega}$.

Hence $Ker\varphi_{\omega}$ is closed.

ii) $Stab_{\omega}(\chi) < F(\omega)$, $\forall\chi \in K(\omega)$ from proposition (2.7(i))

Let $g \in \overline{Stab_{\omega}(\chi)}$

Then there is a net $\{g_{\beta}\}_{\beta\in\Omega}$ in $Stab_{\omega}(\chi)$ such that $g_{\beta} \to g$.

But $\varphi_{\omega}(g_{\beta},\chi) = \chi$ & $\varphi_{\omega}(g_{\beta},\chi) \to \varphi_{\omega}(g,\chi)$

Since $K(\omega)$ is a Hausdorff Space, then $\varphi_{\omega}(g,\chi) = \chi$

So $g \in Stab_{\omega}(\chi)$

Thus $Stab_{\omega}(\chi)$ is closed set.

3. SOME TYPES OF SOFT GROUP SPACES

In this section, we introduce some types of soft group space, and investigate some propositions and examples about them.

Definition (3.1) [3]

A soft action of the Sʈg $(F, \hat{E}, \Gamma)$ on the Sʈs $(K, \hat{E}, \Gamma')$ is called:

i) Transitive if $Orb_{\omega}(\chi) = K(\omega)$, $\forall\chi \in K(\omega)$ & $\forall\omega \in \hat{E}$.

ii) Effective (faithful) if $Ker\varphi_{\omega} = \{e_G\}$, $\forall\omega \in \hat{E}$.

iii) Free if $Stab_{\omega}(\chi) = \{e_G\}$, $\forall\chi \in K(\omega)$ & $\forall\omega \in \hat{E}$.

iv) Trivial if $Ker\varphi_{\omega} = F(\omega)$, $\forall\omega \in \hat{E}$.

v) Regular if it is both transitive and free.

vi) Semi-free if $Stab_{\omega}(\chi) = \{e_G\}$ or $Stab_{\omega}(\chi) = F(\omega)$, $\forall\chi \in K(\omega)$ & $\forall\omega \in \hat{E}$.

Definition (3.2)

A $S\mathbb{G}$ -Space $(K, \hat{E}, \Gamma')$ is called:

i) Transitive $S\mathbb{G}$ -Space if its soft action is transitive.

ii) Free $S\mathbb{G}$ -Space if its soft action is free.

iii) Effective $S\mathbb{G}$ -Space if its soft action is effective.

(iv) Regular $S\mathbb{G}$ -Space if its soft action is regular.

(v) Semi-free $S\mathbb{G}$ -Space if its soft action is semi-free.

Example (3.3)

In example (2.3), the Sʈs $(K, \hat{E}, \Gamma')$ is regular $S\mathbb{G}$ -space.

Proposition (3.4)

Every free $S\mathbb{G}$ -space is an effective $S\mathbb{G}$ -space.

Proof:

Let $(K, \hat{E}, \Gamma')$ be free $S\mathbb{G}$ -space, then the soft action φ_{ω} is free $\forall\omega \in \hat{E}$.

Then $Ker\varphi_{\omega} = \bigcap_{\chi\in K(\omega)} Stab_{\omega}(\chi) = \{e_G\}$

So $(K, \hat{E}, \Gamma')$ is an effective SG-space.

As the following example demonstrates the opposite of (3.4) isn't true.

Example (3.5)

Let $(\mathbb{G},\cdot) = (\{-1,1\},.)$ with discrete topology, $\hat{E} = \{\omega_1, \omega_2\}$, let $(F, \hat{E})$ be a S-set over $\mathbb{G}$ defined by $F(\omega_1) = \{1\}$, $F(\omega_2) = \{-1,1\}$. Let $(K, \hat{E}, \Gamma')$ be a Sts over $\mathfrak{R}$ such that $K(\omega_1) = K(\omega_2) = \mathfrak{R}$. Define $\varphi_{\omega_1} : F(\omega_1) \times K(\omega_1) \to K(\omega_1)$ by $\varphi_{\omega_1}(1, \chi) = \chi$, $\forall \chi \in K(\omega_1)$. & $\varphi_{\omega_2} : F(\omega_2) \times K(\omega_2) \to K(\omega_2)$ by $\varphi_{\omega_2}(1, \chi) = \chi$, $\varphi_{\omega_2}(-1, \chi) = -\chi$, $\forall \chi \in K(\omega_2)$. Then $(K, \hat{E}, \Gamma')$ is an effective $S\mathbb{G}$ -space but not free $S\mathbb{G}$ -space.

Proposition (3.6)

Every free $S\mathbb{G}$ -space is semi-free $S\mathbb{G}$ -space.

Proof:

Let $(K, \hat{E}, \Gamma')$ be a free $S\mathbb{G}$ -space, then the soft action φ_ω is free, $\forall \omega \in \hat{E}$.

Then $Stab_\omega(\chi) = \{e_G\}$.

Hence $(K, \hat{E}, \Gamma')$ is semi-free $S\mathbb{G}$ -space.

As the following example demonstrates the opposite of (3.6) isn't true.

Example (3.7)

Let $(\mathbb{G}, \Gamma, F, \hat{E})$ be commutative Stg. If $\varphi_\omega : F(\omega) \times F(\omega) \to F(\omega)$ defined by, $\varphi_\omega(g,h) = ghg^{-1}$ $\forall \omega \in \hat{E}$ Then $(K, \hat{E}, \Gamma')$is semi-free $S\mathbb{G}$ -space, but not free $S\mathbb{G}$ -space.

Remark (3.8)

Let $(K, \hat{E}, \Gamma')$ be a $S\mathbb{G}$ -Space, we define a relation * in $K(\omega)$ as follows:

$\chi * b \leftrightarrow \exists g \in F(\omega)$ such that $\varphi_\omega(g,\chi) = b$

we claim that * is an equivalence relation in $K(\omega)$

i) * is reflexive: We have $\chi * \chi$, $\forall \chi \in K(\omega)$ Since $\varphi_\omega(e_G, \chi) = \chi$

ii) * is symmetric: Suppose that $\chi * b$, then $\exists g \in F(\omega)$ such that $\varphi_\omega(g,\chi) = b$.

Then $\chi = \varphi_\omega(e_G,\chi) = \varphi_\omega(g^{-1}g,\chi) = \varphi_\omega(g^{-1},\varphi_\omega(g,\chi))$

$= \varphi_\omega(g^{-1},b)$. Thus $\varphi_\omega(g^{-1},b) = \chi$ which shows that $b * \chi$.

iii) * is transitive: suppose $\chi * b$ & $b * c$.

Then there are $g_1, g_2 \in F(\omega)$ such that $\varphi_\omega(g_1,\chi) = b$ & $\varphi_\omega(g_2,b) = c$

Now $\varphi_\omega(g_2 g_1,\chi) = \varphi_\omega(g_2,\varphi_\omega(g_1,\chi)) = \varphi_\omega(g_2,b) = c$ which shows that a*c.

Thus * is an equivalence relation in $K(\omega)$, and we have that the equivalence class $[\chi]$ of a point $\chi \in K(\omega)$ equals

$[\chi] = \{b \in K(\omega) : \chi * b\}$ $= \{b \in K(\omega) : b = \varphi_\omega(g,a), g \in F(\omega)$ $\} = Orb_\omega(\chi)$

Thus, the equivalence class of a under * is exactly the $Orb_\omega(\chi)$.

By $K(\omega)/F(\omega)$, We denote the set of equivalence classes under *, that is $K(\omega)/F(\omega)$ denotes the set of orbits in $K(\omega)$.

By $\Pi_\omega : K(\omega) \to K(\omega)/F(\omega)$, we denote the natural projection.

We give $K(\omega)/F(\omega)$ the quotient topology induced by $\Pi_\omega : K(\omega) \to K(\omega)/F(\omega)$. We call $K(\omega)/F(\omega)$ the orbit space of the $S\mathbb{G}$ -space$(K, \hat{E}, \Gamma')$.

Proposition (3.9)

The map $\Pi_{\omega} : K(\omega) \rightarrow K(\omega)/F(\omega)$ is an open map $\forall \omega \in \hat{E}$.

Proof:

Let A be an open set in $K(\omega)$ $\forall \omega \in \hat{E}$.

Then $\Pi_{\omega}^{-1}(\Pi_{\omega}(A)) = \bigcup_{g \in F(\omega)} \phi_{\omega g}(A)$ is an open in $K(\omega)$.

So $\Pi_{\omega}(A)$ is an open set in $K(\omega)/F(\omega)$

Hence Π_{ω} is an open map $\forall \omega \in \hat{E}$.

Proposition (3.10)

Let $(K, \hat{E}, \Gamma')$ be Hausdorff $S\mathbb{G}$ -space, and $(\mathbb{G}, \Gamma, F, \hat{E})$ be a compact Stg, then φ_{ω} is a closed map $\forall \omega \in \hat{E}$.

Proof:

Let V be a closed set in $F(\omega) \times K(\omega)$ and let $\chi \in \overline{\varphi_{\omega}(V)}$, then there is a net $\{(g_{\beta}, x_{\beta})\}_{\beta \in \Omega}$ in V such that $\varphi_{\omega}(g_{\beta}, x_{\beta}) \rightarrow x,\ \ \forall \omega \in \hat{E}$.

Then $\{g_{\beta}\}$ has a convergent subnet, say $\{g_{\beta}\}$ such that $g_{\beta} \rightarrow g$in $F(\omega)$(because $F(\omega)$ is a compact).

Since $\chi_{\beta} = \varphi_{\omega}(g_{\beta}^{-1}, \varphi_{\omega}(g_{\beta}, \chi_{\beta})) \rightarrow \varphi_{\omega}(g^{-1}, \chi)$

Then $(g_{\beta}, \chi_{\beta}) \rightarrow (g, \varphi_{\omega}(g^{-1}, \chi))$, but $\{(g_{\beta}, \chi_{\beta})\}_{\beta \in \Omega}$is a net in V

Then $(g, \varphi_{\omega}(g^{-1}, \chi)) \in V$, So $\varphi_{\omega}(g, \varphi_{\omega}(g^{-1}, \chi)) \in \varphi_{\omega}(V)$

Hence $\varphi_{\omega}(V) = \overline{\varphi_{\omega}(V)}$. Thus φ_{ω} is a closed map, $\forall \omega \in \hat{E}$.

Definition (3.11)

Let $(K, \hat{E}, \Gamma')$ be a $S\mathbb{G}$ -space. A subset V of $K(\omega)$ is called an invariant of $F(\omega)$ if $F_{\omega}(V) = V$, where $F_{\omega}(V) = \{\varphi_{\omega}(g, \chi) : g \in F(\omega), \chi \in V\}$.

Example (3.12)

Let $\mathbb{G} = (Z, +)$ with discrete space, $\hat{E} = \{\omega_1, \omega_2\}$ and $(F, \hat{E})$ be a s-set over $\mathbb{G}$defined by $F(\omega_1) = F(\omega_2) = Z$. If $\varphi_{\omega} : F(\omega) \times F(\omega) \rightarrow F(\omega)$ defined by $\varphi_{\omega}(g_1, g_2) = g_1 + g_2,\ \forall \omega \in \hat{E}$.

(i) If $V = Z$, then V is an invariant set.

(ii) If $V = Z_0$ (the set of odd integer numbers), then V isn't invariant.

Proposition (3.13)

Let $(K, \hat{E}, \Gamma')$ be a Hausdorff $S\mathbb{G}$ -space, where $(\mathbb{G}, \Gamma, F, \hat{E})$ a compact Stg, and let V be a closed subset of $K(\omega)$.

i) If V is closed set, then $F_{\omega}(V)$ is closed in $K(\omega)$.

ii) If V is compact, then $F_{\omega}(V)$ is compact.

Proof:

i) If V is closed subset of $K(\omega)$, then $F(\omega) \times V$ is a closed subset of $F(\omega) \times K(\omega)$.

Hence by proposition (3.10), we have $\varphi_{\omega}(F(\omega) \times V) = F_{\omega}(V)$ is closed set in $K(\omega)$.

2) If V is compact, then $F(\omega) \times V$ is compact, and hence $\varphi_{\omega}(F(\omega) \times V) = F_{\omega}(V)$ is compact.

Theorem (3.14)

Let $(K, \hat{E}, \Gamma')$ is a Hausdorff $S\mathbb{G}$ -Space, where $(\mathbb{G}, \Gamma, F, \hat{E})$is a compact Stg, then $\forall \omega \in \hat{E}$:

i) The natural projection $\Pi_{\omega} : K(\omega) \to K(\omega)/F(\omega)$ is a closed map.

ii) The Orbit space $K(\omega)/F(\omega)$ is Hausdorff.

iii) The map $\Pi_{\omega} : K(\omega) \to K(\omega)/F(\omega)$ is proper.

iv) $K(\omega)$ is compact iff $K(\omega)/F(\omega)$ is compact.

Proof:

i) Suppose U is a closed subset of $K(\omega)$.

Then $\Pi_{\omega}^{-1}(\Pi_{\omega}(U)) = F_{\omega}(U)$

By proposition (3.13), we have that $F_{\omega}(U)$ is closed in $K(\omega)$ and hence $\Pi_{\omega}(U)$ is closed in $K(\omega)/F(\omega)$.

Then Π_{ω} is closed map.

ii) Let $a', b' \in K(\omega)/F(\omega)$ such that $a' \neq b'$

Then there are $a, b \in K(\omega)$ such that $\Pi_{\omega}(a) = a'$, $\Pi_{\omega}(b) = b'$

Then $\Pi_{\omega}^{-1}(a') = Orb_{\omega}(a)$ and $\Pi_{\omega}^{-1}(b') = Orb_{\omega}(b)$

The $Orb_{\omega}(a)$ and $Orb_{\omega}(b)$ are compacts (by proposition 3.13) and disjoint.

Hence there are disjoint open sets A and B such that $Orb_{\omega}(a) \subset A$, $Orb_{\omega}(b) \subset B$.

So $\overline{A} \cap Orb_{\omega}(b) = \phi$

Now: $b' = \Pi_{\omega}(b) \notin \Pi_{\omega}(\overline{A})$. Furthermore $\Pi_{\omega}(\overline{A})$ is closed in $K(\omega)/F(\omega)$ by (i).

Thus $K(\omega)/F(\omega) - \Pi_{\omega}(\overline{A})$ is an open neighborhood of b in $K(\omega)/F(\omega)$.

Since $\Pi_{\omega} : K(\omega) \to K(\omega)/F(\omega)$ is an open map, then $\Pi_{\omega}(A)$ is an open neighborhood of $a' = \Pi_{\omega}(a)$ in $K(\omega)/F(\omega)$.

Since $\Pi_{\omega}(A) \cap (K(\omega)/F(\omega) - \Pi_{\omega}(\overline{A})) = \phi$. This completes the proof.

iii) Let A be a compact set in $K(\omega)/F(\omega)$, we claim that $\Pi_{\omega}^{-1}(A)$ is compact.

Let $\{A_{\beta} : \beta \in \Omega\}$ be an open cover of $\Pi_{\omega}^{-1}(A)$. For all $b \in K(\omega)/F(\omega)$ we have that $\Pi_{\omega}^{-1}(b)$ is compact.

Since $\Pi_{\omega}^{-1}(b) = Orb_{\omega}(a)$ where $\Pi_{\omega}(a) = b$.

Thus, for all $b \in H(\omega)$ there is a finite subset Ω_b of Ω such that

$\Pi_{\omega}^{-1}(b) = \bigcup_{\beta \in \Omega_b} A_{\beta} = A_b$, then $B_b = K(\omega)/F(\omega) - \Pi_{\omega}(A_b^c)$

Since A_b^c is closed in $K(\omega)$, we have by (i) that $\Pi_{\omega}(A_b^c)$ is closed in $K(\omega)/F(\omega)$

Thus B_b is an open in $K(\omega)/F(\omega)$.

We claim that $\Pi_{\omega}^{-1}(B_b) \subset A_b$ and $b \in B_b$

Let $a \in \Pi_{\omega}^{-1}(B_b)$. Then $\Pi_{\omega}(a) \in B_b = K(\omega)/F(\omega) - \Pi_{\omega}(A_b^c)$

Thus $\Pi_{\omega}(a) \notin \Pi_{\omega}(A_b^c)$. Hence $a \notin A_b^c$, and thus $a \in A_b$

Since $\Pi_{\omega}^{-1}(b) \subset A_b$, then $b \notin \Pi_{\omega}(A_b^c)$, that is $b \in (K(\omega)/F(\omega) - \Pi_{\omega}(A_b^c)) = B_b$

The sets $B_b, b \in A$ form a covering of A by open subsets of $K(\omega)/F(\omega)$

Since A is compact, then there are $b_1, ..., b_n$ such that $A \subset \bigcup_{i=1}^{n} B_{b_i}$

Then $\Pi_{\omega}^{-1}(A) \subset \bigcup_{i=1}^{n} \Pi_{\omega}^{-1}(B_{b_i}) \subset \bigcup_{i=1}^{n} A_{b_i} = \bigcup_{i=1}^{n} \bigcup_{\beta \in \Omega_{b_i}} A_{\beta}$.

Thus, the finite sub cover $A_{\beta} : \beta \in \Omega_{b_i} : 1 \le i \le n\}$ of $\{A_{\beta}\}_{\beta \in \Omega}$ covers $\Pi_{\omega}^{-1}(A)$

Then $\Pi_{\omega}^{-1}(A)$ is compact, hence Π_{ω} is proper map.

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