Fibrewise Multi-Separation Axioms

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ABSTRACT

The aim of the research is to apply fibrewise multi-emisssions of the paramount separation axioms of normally topology namely fibrewise multi-T0. spaces, fibrewise multi-T1 spaces, fibrewise multi-R0 spaces, fibrewise multi-Hausdorff spaces, fibrewise multi-functionally Hausdorff spaces, fibrewise multi-regular spaces, fibrewise multi-completely regular spaces, fibrewise multi-normal spaces and fibrewise multi-functionally normal spaces. Also we give many score regarding it.

1.Introduction

We beginning our work by the concept of category of fibrewise (briefly, 𝔽.𝕎.) sets on a known set, named the base set. If the base set is stated with D then a 𝔽.𝕎. set on D apply of a set E with a function X is X: E → D, named the projection (briefly, project.). For every point d of D the fibre on d is the subset $E_d = X^{-1}(d)$ of E; fibres will be empty let we do not require X to be surjection, also for every subset D^* of D we regard $E_{D^*} = X^{-1}(D^*)$ as a 𝔽.𝕎. set on D^* with the project. determined by X. A multi- function [2] Ω of a set E in to F is a correspondence such that Ω (e) is a nonempty subset of F for every e ∈ E. We will denote such a multi- function by Ω : E → F . For a multi- function Ω, the upper and lower inverse set of a set K of F, will be denoted by $\Omega^+(K)$ and $\Omega^-(K)$ respectively that is $\Omega^+(K) = \{e \in E : \Omega(e) \subseteq K\}$ and $\Omega^-(K) = \{e \in E: \Omega(e) \cap V \neq \emptyset \}$.

Definition 1.1. [5] Suppose that E and F are 𝔽.𝕎. sets on D, with project. X_E: E → D and Y_F: F → D, respectively, a function Ω: E → F is named to be 𝔽.𝕎. if $Y_F O\Omega = X_E$, that is to say if $\Omega(X_d) \subset F_d$ for every point d of D.

It should be noted that a 𝔽.𝕎. function Ω: E → F on D determines, by restriction, 𝔽.𝕎. function Ω_{D^*}: $E_{D^*} \to F_{D^*}$ on D^* for every D^* of D.

Let $\{E_r\}$ be an indexed family of 𝔽.𝕎. sets on D the 𝔽.𝕎. product $\prod_D E_r$ is stated, as a 𝔽.𝕎. set on D, and comes included with the family of 𝔽.𝕎. projection $\pi_r : \prod_D E_r \to E_r$. Specifically, the 𝔽.𝕎. product is stated as the subset of the normally product $\prod E_r$ where in the fibres are the products of the relevant fibers of the strain E_r. The 𝔽.𝕎. product is recognized by the following Cartesian property: for every 𝔽.𝕎. set E on D the 𝔽.𝕎. functions $\Omega: E \to \prod_r E_r$ correspond exactly to the families of 𝔽.𝕎. functions $\{\Omega_r\}$, with $\Omega_r = \pi_r o\Omega: E \to E_r$. For example if $E_r = E$ for every index r the diagonal Δ : $E \to \prod_D E$, is stated so that $\pi_r o\Delta = idE$ for every r. If $\{E_r\}$ is as before, the 𝔽.𝕎. coproduct $\coprod_D E_r$ is with stated, as 𝔽.𝕎. set on D, and comes included with the family of F.𝕎. insertions $\sigma: E_r \to \coprod_D E_r$, specifically the 𝔽.𝕎. coproduct synchronize, as a set, with the normally coproduct (saparated union), the fibres being the coproducts of the relevant fipers of the summands E_r. The 𝔽.𝕎. coproduct is recognized by the following Cartesian property, for every 𝔽.𝕎. set E on D the 𝔽.𝕎. functions φ: $\coprod_D E_r \to E$ correspond exactly to the families of 𝔽.𝕎. functions $\{\varphi_r\}$, where in $\varphi_r = \varphi o\sigma_r: E_r \to E$. For example, if $E_r = E$ for every index r the codiagonal ∇: $\coprod_D E \to E$ is stated so that, $\nabla o\sigma_r = idE$ for

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every r. The notation $E \times_D F$ is used for the 𝔽.𝕎. product in the case of the family {E, F}, of two F.W. sets and similarity for finite families generally. As well as, we builte on some of the result in [1,6,7-15]. For other concepts or informataon that are undefined here we follow nearly I.M.James [5], R.Engelking [4] and N. Bourbaki [3].

Recall that [5] Let D be topological space, the 𝔽.𝕎. topology space (briefly, 𝔽.𝕎.T.S.) on a 𝔽.𝕎. set E on D, mean any topology on E for that the project. X is continuous.

Remark 1.1. [5]

(a) The smaller topology is the topology trace with X, where in the open sets of E are exactly the pre image of the open sets of D, this is named the 𝔽.𝕎. indiscrete topology.

(b) The 𝔽.𝕎.T.S. on D is stated to be a 𝔽.𝕎. set on D with a 𝔽.𝕎.T.S.

We regard the topology product $D \times T$, for any topological space T, as a 𝔽.𝕎.T.S. on D using the category of first projection. The equivalences in the category of 𝔽.𝕎.T.S. are named 𝔽.𝕎.T. equivalences. If E is 𝔽.𝕎.T. equivalent to $D \times T$, for some topological space T, we say that E is trivial, as a 𝔽.𝕎.T.S. on D. In 𝔽.𝕎.T. the form neighborhood(briefly, ηℙ𝕕) is used in exactly in the same sense as it is in normally topology, but the forms 𝔽.𝕎. basic may need some illustration, so let E be 𝔽.𝕎.T.S. on D, if e is a point of E_d where in $d \in D$, appear a family N(e) of nbd of e in E as 𝔽.𝕎. basic if as every ηℙ𝕕 H of e we have $E_w \cap K \subset H$, for some element K of N(e) and ηℙ𝕕 W of d in D. As example, in the case of the topological product $D \times T$, where in T is a topological spaces, the family of Cartesian products $D \times N(t)$, where in N(t) runs through the ηℙ𝕕s of t, is 𝔽.𝕎. basic for (d,t).

Definition 1.2. [5] The 𝔽.𝕎. functions Ω: E→ F; E and F are 𝔽.𝕎. spaces on D is named:

(a) continuous (briefly, cont.) if every $e \in E_d$; $d \in D$, the $\Omega^{-1}(e)$ is open set of e.

(b) open if for every $e \in E_d$, $d \in D$, the direct image of every open set of e is an open set of $\Omega(e)$.

Definition 1.3. [5] The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. closed (resp., open) if the project. X is closed (resp., open) functions.

Definition 1.4. [2] Let Ω: E → F be a multi-function. Then Ω is upper cont. (briefly, U. cont.) iff $\Omega^{+}(K)$ open in E for all V open in F. That is, $\Omega^{+}(K) = \{x \in E: \Omega(x) \subseteq K\}$. $K \subseteq F$.

Definition 1.5. [2] Let Ω: E → F be a multi-function. Then Ω is lower cont. (briefly, L. cont.) iff $\Omega^{-}(K)$ open in E for all K open in F. That is, $\Omega^{-}(K) = \{e \in E: \Omega(e) \cap K \neq \emptyset \}$. $K \subseteq F$

Let Ω: E → F be a multi-function. Then Ω is multi cont. (briefly, M. cont.) iff it is U. cont. and L. cont.

2 . Fibrewise Multi-$T_\circ$, Multi-T_1 and Multi-Hausdorff spaces.

Before that offer the stated of fibrewise multi- separation axioms we offer the following stated:

Definition 2.1. Suppose that E be 𝔽.𝕎.T.S. on D. Then E is named 𝔽.𝕎. upper $T_\circ$space (briefly, 𝔽.𝕎.U.$T_\circ$.S.) if whenever $e_1, e_2 \in E_d^{+}$, where in $d \in D$ and $e_1 \neq e_2$, either there exists an open set of e_1 that does not including e_2 in E, or rice versa.

Definition 2.2. Suppose that E be 𝔽.𝕎.T.S. on D. Then E is named 𝔽.𝕎. lower $T_\circ$ space(briefly, 𝔽.𝕎.L.$T_\circ$.S.) if whenever $e_1, e_2 \in E_d^{-}$, where in $d \in D$ and $e_1 \neq e_2$, either there exists an open set of e_1 that does not including e_2 in E, or rice versa.

The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. multi-T_0space(briefly, 𝔽.𝕎.M.$T_\circ$.S.) if E is 𝔽.𝕎.U.T_0 .S. and 𝔽.𝕎.L.T_0 .S.

Remark 2.1.

(a) E is $\mathbb{F}.\mathbb{W}.U.T_0$ space (resp., $\mathbb{F}.\mathbb{W}.L.T_0$ and $\mathbb{F}.\mathbb{W}.M.T_0$) if and only if each fiber E_d is upper T_0 (resp., lower T_0 and multi-T_0).

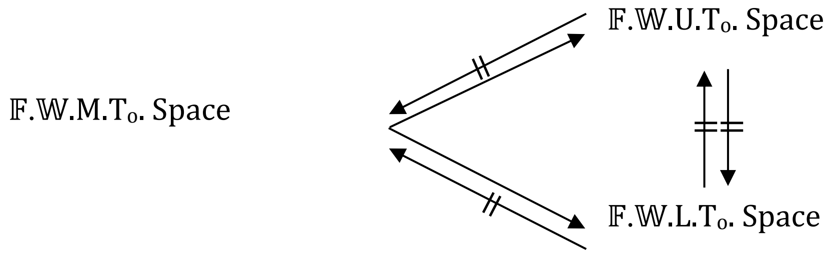
(b) Subspace of $\mathbb{F}.\mathbb{W}.U.T_0$ space (resp., $\mathbb{F}.\mathbb{W}.L.T_0$ and $\mathbb{F}.\mathbb{W}.M.T_0$) are $\mathbb{F}.\mathbb{W}.U.T_0$ space (resp., $\mathbb{F}.\mathbb{W}.L.T_0$ and $\mathbb{F}.\mathbb{W}.M.T_0$).

(c) Every $\mathbb{F}.\mathbb{W}.M.T_0$ space is $\mathbb{F}.\mathbb{W}.T_0$ space, but the convers is not true.

(d) Every $\mathbb{F}.\mathbb{W}.M.T_0$ space is $\mathbb{F}.\mathbb{W}.U.T_0$ space, but the convers is not true.

(e) Every $\mathbb{F}.\mathbb{W}.M.T_0$ space is $F.W.L.T_0$ space, but the convers is not true.

(f) The $\mathbb{F}.\mathbb{W}.U.T_0$ space and $\mathbb{F}.\mathbb{W}.U.T_0$ space are independence.



Planned 2.1.

Example 2.1.

(a) Let $E = \{a, b, c\}$, $\tau_{(E)}$ = discrete topology. $D = \{1,2\}$, $\rho = \{\emptyset, D, \{1\}\}$. Define the project. $X: (E, \tau_{(E)}) \to (D, \rho)$ by $X(a) = X(b) = X(c) = \{1\}$, E is $\mathbb{F}.\mathbb{W}.U.T_0.S.$, $\mathbb{F}.\mathbb{W}.L.T_0.S.$, and $\mathbb{F}.\mathbb{W}.M.T_0.S.$

(b) Let $E = \mathbb{R}$ with the usual topology τ and let $D = \{a, b, c\}$ with the topology $\rho = \{\emptyset, D, \{a\}, \{b\}, \{a, b\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\} ; & e \leq 0 \\ \{a, c\} ; & e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}.$ $L.T_0.S.$ But not $\mathbb{F}.\mathbb{W}.U.T_0.S.$ And not $\mathbb{F}.\mathbb{W}.M.T_0.S.$

(c) Let $E = \mathbb{R}$ with the usual topology τ and $D = \{a, b\}$ with the topology $\rho = \{\emptyset, D, \{a\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\} ; & e \leq 0 \\ \emptyset ; & e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}.U.T_0.S.$, but not $\mathbb{F}.\mathbb{W}.L.T_0.S.$ and not $\mathbb{F}.\mathbb{W}.M.T_0.S.$

(d) Let $E = \mathbb{R}$ with the usual topology τ and $D = \{a, b, c\}$ with the topology $\rho = \{\emptyset, D, \{a\}\}$. Define multi-function

$$X : (\mathbb{R},\tau) \rightarrow (D,\rho) \text{ by } X(e) = \begin{cases} \{a\}\,;\ e < 0 \\ \{a,b\}\,;\ e = 0 \\ \{c\}\,;\ e > 0 \end{cases}$$

E is not $\mathbb{F}.\mathbb{W}.U.T_0.S.$, not $\mathbb{F}.\mathbb{W}.L.T_0.S.$ and not $\mathbb{F}.\mathbb{W}.M.T_0.S.$

Of course, we can formulate a $\mathbb{F}.\mathbb{W}.$ emission of the $U.T_1.S.$ (resp., $L.T_1.S.$ and $M.T_1.S.$) in a similarly shape as follows "Let E be $\mathbb{F}.\mathbb{W}.T.S.$ on D. Then E is named $\mathbb{F}.\mathbb{W}.U.T_1$(resp., $\mathbb{F}.\mathbb{W}.L.T_1$ and $\mathbb{F}.\mathbb{W}.M.T_1$) if whenever $e_1,e_2 \in E_d^+$ (resp., E_d^-), where in $d \in D$ and $e_1 \neq e_2$, there exist an open set H_1,H_2 in E such that $e_1 \in H_1$, $e_2 \notin H_1$ and $e_1 \notin H_2$, $e_2 \in H_2$", but it work that there is no fact use for it is in what we are going to do. Alternatively we do some use of different axiom "The axiom is that every multi-open set including the closure of every of its points", and use the from multi-R_0 space. This is right for multi-T_1 spaces and for multi-regular spaces. It is clear that is a weak form of multi-regularity.

Definition 2.3. The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is named $\mathbb{F}.\mathbb{W}.$ upper $R_\circ$ space (briefly, $\mathbb{F}.\mathbb{W}.U.R_\circ.S.$), if for every point $e \in E_d^+$, where in $d \in D$, and every open set V of e in E, there exist a $\eta\mathbb{P}\mathbb{d}$ W of d in D such that the closure of $\{e\}$ in E_W is including in V (i.e. $E_w^+ \cap Cl\{e\} \subset V$).

For example, $D\times T$ is $\mathbb{F}.\mathbb{W}.U.R_0.S.$ for all upper R_0 spaces T.

Definition 2.4. The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is named $\mathbb{F}.\mathbb{W}.$ lower $R_\circ$space (briefly, $\mathbb{F}.\mathbb{W}.L.R_\circ.S.$), if for every point $e \in E_d^-$, where in $d \in D$, and each open set V of e in E, there exist a$\eta\mathbb{P}\mathbb{d}$ W of d in D such that the closure of $\{e\}$ in E_W is including in V (i.e. $E_w^- \cap Cl\{e\} \subset V$).

For example, $D\times T$ is $F.W.L.R_0$ .S. for all lower R_0 spaces T.

The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is named $\mathbb{F}.\mathbb{W}.$multi-$R_\circ$*space* (briefly, $\mathbb{F}.\mathbb{W}.M.R_\circ.S.$), if E is $\mathbb{F}.\mathbb{W}.U.R_\circ.S.$ and $\mathbb{F}.\mathbb{W}.L.R_0.S.$

Remark 2.2.

(a) E is $\mathbb{F}.\mathbb{W}.U.R_0.S.$ (resp., $F.W.L.R_0.S.$ and $F.W.M.R_0.S.$) if and only if each fiber E_d is upper R_0 (resp., lower R_0 and multi-R_0).

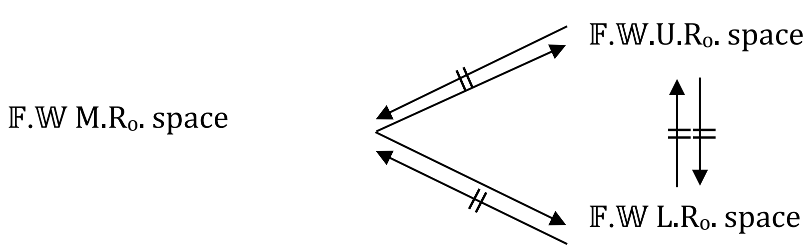
(b) Subspace of $\mathbb{F}.\mathbb{W}.U.R_0\,.S.$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0.S.$ and $\mathbb{F}.\mathbb{W}M.R_0.S.$) are $\mathbb{F}.\mathbb{W}.U.R_0\,.S.$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$ and $\mathbb{F}.\mathbb{W}.M.R_0$).

(c) Every $\mathbb{F}.\mathbb{W}.M.R_0$ space is $\mathbb{F}.\mathbb{W}.R_0$ space, but the convers is not true.

(d) Every $\mathbb{F}.\mathbb{W}.M.R_0$ space is $\mathbb{F}.\mathbb{W}.U.R_0$ space, but the convers is not true.

(e) Every $\mathbb{F}.\mathbb{W}.M.R_0$ space is $\mathbb{F}.\mathbb{W}.L.R_0$ space, but the convers is not true.

(f) The $\mathbb{F}.\mathbb{W}.U.R_0$ space and $\mathbb{F}.\mathbb{W}.U.R_0$ space are independence.



Example 2.2.

(a) Let E = {a, b, c} , $\tau_{(E)}$ = discrete topology. D = {1,2}, $\rho = \{\emptyset, D, \{2\}\}$. Define the project. $X: (E, \tau_{(E)}) \to (D, \rho)$ by X(a) = X(b) = X(c) = {2} E is $\mathbb{F}.\mathbb{W}.U.R_0$.S., $\mathbb{F}.\mathbb{W}.L.R_0$.S., and $\mathbb{F}.\mathbb{W}.M.R_0$.S.

(b) Let E = ℝ with the usual topology τ and let D = N with the cofinite topology ρ. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} N \setminus \{1,2\} ; & e \le 0 \\ N ; & e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}$. $L.R_0$.S. But not $\mathbb{F}.\mathbb{W}.U.R_0$.S. And not $\mathbb{F}.\mathbb{W}.M.R_0$.S.

(c) Let E =ℝ with the usual topology τ and D = {a, b} with the discrete topology ρ. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\} ; & e \le 0 \\ \emptyset ; & e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}.U.R_0$.S., but not $\mathbb{F}.\mathbb{W}.L.R_0$.S. and not $\mathbb{F}.\mathbb{W}.M.R_0$S.

(d) Let E = ℝ with the usual topology τ and D = {a, b, c} with the topology $\rho = \{\emptyset, D, \{a\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\} ; & e < 0 \\ \{a, b\} ; & e = 0 \\ \{c\} ; & e > 0 \end{cases}$$

E is not $\mathbb{F}.\mathbb{W}$ $U.R_0$.S., not $\mathbb{F}.\mathbb{W}$ $L.R_0$S. and not $\mathbb{F}.\mathbb{W}$ $M.R_0$.S.

Proposition 2.1. let $\Omega : E \to E^*$ be a $\mathbb{F}.\mathbb{W}$. embedding, where in E and E^* are $\mathbb{F}.\mathbb{W}$ T.S. on D. If E^* is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}$ $L.R_0$) then so is E.

Proof. Suppose that $e \in E_d^+$ (resp., E_d^-), where in d ∈ D, and let V be an open set of e in E. Then K = Ω^{-1}(K*), where in K* is an open set of e* = Ω(e) in E*. Since E* is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $F.W.L.R_0$) there exists a $\eta\mathbb{P}𝕕$ W of d such that $(E^*)_w^+ \cap Cl\{e^*\} \subset K^*$ (resp., $(E^*)_w^- \cap Cl\{e^*\} \subset K^*$). Then $E_w^+ \cap Cl\{e\} \subset \Omega^{-1}$ (resp., $E_w^- \cap Cl\{e\} \subset \Omega^{-1}$) $((E^*)_w^+ \cap Cl\{e^*\}) \subset \Omega^{-1}(K^*)$ (resp., $(E^*)_w^- \cap Cl\{e^*\}) \subset \Omega^{-1}(V^*)$) = K, and so E is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$).

The class of $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$) space*s* is finitely multiplicative.

Corollary 2.1. let $\Omega : E \to E^*$ be a $\mathbb{F}.\mathbb{W}$. embedding, where in E and E^* are $\mathbb{F}.\mathbb{W}$.T.S. on D. If E^* is $\mathbb{F}.\mathbb{W}.M.R_0$ then so is E.

Proposition 2.2. let $\{E_r\}$ be a finite family of $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$) spaces on D. Then the $\mathbb{F}.\mathbb{W}$. topological product $E = \prod_D E_r$ is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$).

Proof. Let $e \in E_d^+$ (resp., E_d^-), where in d ∈ D. Consider an open set $V = \prod_D K_r$ of e in E, where in K_r is an open set of $\pi_r(e) = e_r$ in E_r for every index r. Since E_r is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$) there exists a $\eta\mathbb{P}𝕕$ W_r of d ∈ D such that $(E_r|W_r)^+ \cap Cl(e_r) \subset K_r$ (*resp.*, $(E_r|W_r)^- \cap Cl(e_r) \subset K_r$). Then the intersection W of the W_r is a nbd of d such that $E_w^+ \cap Cl\{e\} \subset K$ (resp., $E_w^- \cap Cl\{e\} \subset K$) and so $E = \prod_D$

E_r is $\mathbb{F}.\mathbb{W}.U.R._0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$). The same conclusion holds for infinite $\mathbb{F}.\mathbb{W}.$ products provided every of the strain is $\mathbb{F}.\mathbb{W}.$ nonempty.

Corollary 2.2. let $\{E_r\}$ be a finite family of $\mathbb{F}.\mathbb{W}.M.R_0$ spaces on D. Then the $\mathbb{F}.\mathbb{W}.$ topological product $E = \prod_D E_r$ is $\mathbb{F}.\mathbb{W}.M.R_0$.

Proposition 2.3. let $\Omega : E \to F$ be a closed F.W. surjection, where in E and F are $\mathbb{F}.\mathbb{W}.T.S.$ on D. If E is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$), then so is F.

Proof. Let $f \in F_d^+$ (resp., F_d^-), where in $d \in D$, and let V be an open set of f in F. Pick $e \in \Omega^{-1}(f)$. Then $H = \Omega^{-1}(K)$ is an open set of e. Since E is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.L.R_0$) there exists a $\eta\mathbb{P}\mathbb{d}$ W of d such that $E_w^+ \cap Cl\{e\} \subset H$ (resp., $E_w^- \cap Cl\{e\} \subset H$). Then $F_W^+ \cap \Omega(Cl\{e\}) \subset \Omega(H)$ (resp., $F_W^- \cap \Omega(Cl\{e\}) \subset \Omega(H)) = K$. Since Ω is closed, then $\Omega(Cl\{e\}) = Cl(\Omega\{e\})$. Therefore $F_W^+ \cap \Omega(Cl\{e\}) \subset K$ (resp., $F_W^- \cap \Omega(Cl\{e\}) \subset K$) and so F is $\mathbb{F}.\mathbb{W}.U.R_0$ (resp., $\mathbb{F}.\mathbb{W}.U.R_0$), as asserted.

Corollary 2.3. let $\Omega : E \to F$ be a closed $\mathbb{F}.\mathbb{W}.$ surjection, where in E and F are $\mathbb{F}.\mathbb{W}.T.S.$ on D. If E is $\mathbb{F}.\mathbb{W}.M.R_0$, then so is F.

Now we offer the emission of $\mathbb{F}.\mathbb{W}.$ upper (resp., lower and multi) Hausdorff spaces as follows:

Definition 2.5. The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is named $\mathbb{F}.\mathbb{W}.$ upper Hausdorff (briefly, $\mathbb{F}.\mathbb{W}.U.$ Hausd.) if whenever $e_1, e_2 \in E_d^+$, where in $d \in D$ and $e_1 \neq e_2$, there exist saparated open sets H_1, H_2 of e_1, e_2 in E.

Definition 2.6. The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is named $\mathbb{F}.\mathbb{W}.$ lower Hausdorff (briefly, $\mathbb{F}.\mathbb{W}.L.$ Hausd.) if whenever $e_1, e_2 \in E_d^-$, where in $d \in D$ and $e_1 \neq e_2$, there exist saparated open sets H_1, H_2 of e_1, e_2 in E.

The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is named $\mathbb{F}.\mathbb{W}.$ multi-Hausdorff (briefly, $\mathbb{F}.\mathbb{W}.M.$ Hausd.) if E is $\mathbb{F}.\mathbb{W}.U.$ Hausd. and $\mathbb{F}.\mathbb{W}.L.$ Hausd..

Remark 2.3. If E is $\mathbb{F}.\mathbb{W}.U.$ Hausd. (resp., $\mathbb{F}.\mathbb{W}.L.$ Hausd. and $\mathbb{F}.\mathbb{W}.M.$ Hausd.) space on D, then E_{D^*} is $\mathbb{F}.\mathbb{W}.U.$ Hausd. (resp., $\mathbb{F}.\mathbb{W}.L.$ Hausd. and $\mathbb{F}.\mathbb{W}.M.$ Hausd.) space on D* for every subspace D* of D. In particular the fibres of E are U. Hausd. (resp., L. Hausd. and M. Hausd.) spaces. However a $\mathbb{F}.\mathbb{W}.T.S.$ with U. Hausd. (resp., L. Hausd. and M. Hausd.) fibres is not necessarily U. Hausd. (resp., L. Hausd. and M. Hausd.) : for example take E = D with D indiscrete.

Proposition 2.4. The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is $\mathbb{F}.\mathbb{W}.U.$ Hausd. (resp., $\mathbb{F}.\mathbb{W}.L.$ Hausd.) iff the diagonal embedding $\Delta : E \longrightarrow E\times_D E$ is closed.

Proof. ($\Leftarrow$) suppose that $e_1, e_2 \in E_d^+$ (resp., E_d^-), where in $d \in D$ and $e_1 \neq e_2$. Let $\Delta(E)$ is closed in $E\times_D E$, then (e_1, e_2), being a point of the completeness, admits a $\mathbb{F}.\mathbb{W}.$ product open set $V_1 \times_D V_2$ that does not meet $\Delta(E)$, and then K_1, K_2 are separated open set of e_1, e_2.

($\Rightarrow$) The revise direction is similarity.

Corollary 2.4. The $\mathbb{F}.\mathbb{W}.T.S.$ E on D is $\mathbb{F}.\mathbb{W}.M.$ Hausd. iff the diagonal embedding $\Delta : E \longrightarrow E\times_D E$ is closed.

Subspaces of $\mathbb{F}.\mathbb{W}.U.$ Hausd. S. (resp., $\mathbb{F}.\mathbb{W}.L.$ Hausd. S.) are $\mathbb{F}.\mathbb{W}.U.$ Hausd. (resp., $\mathbb{F}.\mathbb{W}.L.$ Hausd.).

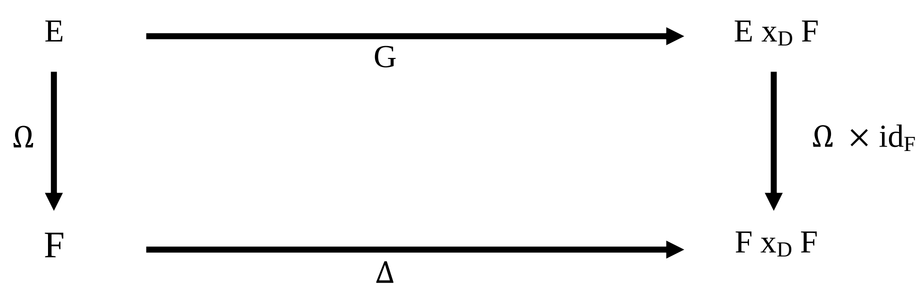
Proposition 2.5. Let Ω : E$\rightarrow$ E* be an embedding$\mathbb{F}.\mathbb{W}$. function, where in E and E*are $\mathbb{F}.\mathbb{W}$.T.S. on D. If E* is $\mathbb{F}.\mathbb{W}$.U. Hausd. (resp., $\mathbb{F}.\mathbb{W}$.L. Hausd.) then so is E.

Proof. Let $e_1, e_2 \in E_d^+(\text{resp.}, E_d^-)$, where in d $\in$ D and $e_1 \neq e_2$. Then $\Omega(e_1)$, $\Omega(e_2) \in E^*_d$ are distinct, since E* is $\mathbb{F}.\mathbb{W}$.U. Hausd. (resp., $\mathbb{F}.\mathbb{W}$.L. Hausd), there exist an open sets K_1, K_2 of $\Omega(e_1)$, $\Omega(e_2)$ in E* that are separated. Their inverse images $\Omega^{-1}(V_1)$, $\Omega^{-1}(V_2)$ are open sets of e_1, e_2 in E that are separated and so E is $\mathbb{F}.\mathbb{W}$.U. Hausd. (resp., $\mathbb{F}.\mathbb{W}$.L. Hausd.). Alternatively (2.4) can de used.

Corollary 2.5. Let Ω : E$\rightarrow$ E* be an embedding $\mathbb{F}.\mathbb{W}$. function, where in E and E*are $\mathbb{F}.\mathbb{W}$.T.S. on D. If E* is $\mathbb{F}.\mathbb{W}$.M. Hausd. then so is E.

Proposition 2.6. Let Ω : E$\rightarrow$ F be a $\mathbb{F}.\mathbb{W}$. functions, where in E and F are $\mathbb{F}.\mathbb{W}$.U.T.S. (resp., $\mathbb{F}.\mathbb{W}$.L.T.S.) on D. If F is $\mathbb{F}.\mathbb{W}$.U. Hausd. (resp., $\mathbb{F}.\mathbb{W}$.L. Hausd.), then the $\mathbb{F}.\mathbb{W}$. graph G : E $\rightarrow$ E$\times_D$F of Ω is closed embedding.

Proof. The $\mathbb{F}.\mathbb{W}$.graph is defined in the same way as the normally graph, but with values in the $\mathbb{F}.\mathbb{W}$. product, so that the diagram show below is commutative.



Planned 2.3.

Since Δ(F) is closed in F$\times_D$ F, by (2.4), so G(E) = $(\Omega \times \text{id}_F)^{-1}(\Delta(F))$ is closed in E$\times_D$ F, as asserted.

Corollary 2.6. Let Ω : E$\rightarrow$ F be a $\mathbb{F}.\mathbb{W}$. functions, where in E and F are $\mathbb{F}.\mathbb{W}$.M.T.S. on D. If F is $\mathbb{F}.\mathbb{W}$.M. Hausd., then the $\mathbb{F}.\mathbb{W}$. graph G : E $\rightarrow$ E$\times_D$F of Ω is closed embedding.

The class of $\mathbb{F}.\mathbb{W}$.U. Hausd. S. (resp., $\mathbb{F}.\mathbb{W}$.L. Hausd. S.) is multiplicative, in the following sense.

Proposition 2.7. Let $\{E_t\}$ be a family of $\mathbb{F}.\mathbb{W}$.U. Hausd. S. (resp., $\mathbb{F}.\mathbb{W}$.L. Hausd. S.) on D. Then the $\mathbb{F}.\mathbb{W}$. topological product E = $\prod_B E_r$ with the family of $\mathbb{F}.\mathbb{W}$. projection π_r: E = $\prod_B E_r \longrightarrow E_r$ is $\mathbb{F}.\mathbb{W}$.U. Hausd. (resp., $\mathbb{F}.\mathbb{W}$.L. Hausd.).

Proof. Let $e_1, e_2 \in E_d^+(\text{resp.}, E_d^-)$, where in d $\in$ D and $e_1 \neq e_2$. Then $\pi_r(e_1) \neq \pi_r(e_2)$ for some index r. Since E_r^+ (resp., E_r^-) is $\mathbb{F}.\mathbb{W}$.U. Hausd. (resp., F.W.L. Hausd.), there exist an open sets K_1, K_2 of $\pi_r(e_1)$, $\pi_r(e_2)$ in E_r^+ (resp., E_r^-) that are separated. Since π_r is projection, then the inverse images π_r^{-1} (K_1), π_r^{-1} (K_2) are separated an open sets of e_1, e_2 in E, as required.

Corollary 2.7. Let $\{E_t\}$ be a family of $\mathbb{F}.\mathbb{W}$.M. Hausd. S. on D. Then the $\mathbb{F}.\mathbb{W}$. topological product E = $\prod_B E_r$ with the family of $\mathbb{F}.\mathbb{W}$. projection π_r: E = $\prod_B E_r \longrightarrow E_r$ is $\mathbb{F}.\mathbb{W}$.M. Hausd.

The upper functionally (resp., lower functionally and multi- functionally) emission of the 𝔽.𝕎.U. Hausd. (resp., 𝔽.𝕎.L. Hausd. and 𝔽.𝕎.M. Hausd.) axiom is finer than the non multi-functional emission, but its properties are fairly similarly. Here and elsewhere we use I to denote the closed unit interval $[0, 1]$ in the real line IR.

Definition 2.7. The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. upper functionally (briefly, F.W.U. funct.) Hausd. if whenever $e_1, e_2 \in E^+{}_d$, where in $d \in D$ and $e_1 \neq e_2$, there exist a ηℙ𝕕 W of d and separated open sets H, K of e_1, e_2 in E and a cont. function λ: $E^+_W \longrightarrow I$ such that $E^+{}_d \cap H \subset \lambda^{-1}(I)$.

Definition 2.8. The 𝔽.𝕎.T.S. E on D is named F. W. lower functionally (briefly, 𝔽.𝕎.L. funct.) Hausd. if whenever $e_1, e_2 \in E^-{}_d$, where in $d \in D$ and $e_1 \neq e_2$, there exist a ηℙ𝕕 W of d and saparated open sets H, K of e_1, e_2 in E and a continuous function λ: $E^-_W \longrightarrow I$ such that $E^-{}_d \cap H \subset \lambda^{-1}(I)$.

The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. multi- functionally (briefly, 𝔽.𝕎.M. funct.) Hausd. if whenever $e_1, e_2 \in E_d$, where in $d \in D$ and $e_1 \neq e_2$, there exist a a ηℙ𝕕 W of d and separated open sets H, K of e_1, e_2 in E and a cont. function λ: EW $\longrightarrow$ I such that $E_d \cap H \subset \lambda^{-1}(I)$.

For example, D×T is 𝔽.𝕎.U. funct. Hausd.S. for all upper funct. Hausd. spaces T.

For example, D×T is 𝔽.𝕎.L. funct. Hausd.S. for all lower funct. Hausd. spaces T.

For example, D×T is 𝔽.𝕎.M. funct. Hausd.S. for all multi-funct. Hausd. spaces T.

Remark 2.4. If E is 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L. funct. and 𝔽.𝕎.M. funct.) Hausd. S. on D, then E* = E_{D^*} is 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L. funct. and 𝔽.𝕎.M. funct.) Hausd. on D* for every subspace D* of D. In particular the fibers of E are upper funct. (resp., lower funct. and multi- funct.) Hausd.S.

Subspaces of 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L. funct. and 𝔽.𝕎.M. funct.) Hausd. S. are 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L. funct. and 𝔽.𝕎.M. funct.) Hausd.S. In fact, we have

Proposition 2.8. Let $\Omega : E \longrightarrow E^*$ be an embedding 𝔽.𝕎. function, where in E and E* are 𝔽.𝕎.T.S. on D. If E* is 𝔽.𝕎.U. funct. Hausd. (resp., 𝔽.𝕎.L. funct. Hausd.) then so is E.

Corollary 2.8. Let $\Omega : E \longrightarrow E^*$ be an embedding 𝔽.𝕎. function, where in E and E* are 𝔽.𝕎.T.S. on D. If E* is 𝔽.𝕎.M. funct. Hausd., then so is E.

Moreover, the class of 𝔽.𝕎.U. funct. Hausd. S. (resp., 𝔽.𝕎.L. funct. Hausd. S.) is multiplicative, as stated in.

Proposition 2.9. Let $\{E_t\}$ be a family of 𝔽.𝕎.U. funct. Hausd.S. (resp., 𝔽.𝕎.L. funct. Hausd.S.) on D. Then the 𝔽.𝕎. topological product $E = \prod_D E_r$ with the family of 𝔽.𝕎. projection $\pi_r : E = \prod_D E_r \rightarrow E_r$ is 𝔽.𝕎.U. funct. Hausd. (resp., 𝔽.𝕎.L. funct. Hausd.)..

Corollary 2.9. Let $\{E_t\}$ be a family of 𝔽.𝕎.M. funct. Hausd.S. on D. Then the 𝔽.𝕎. topological product $E = \prod_D E_r$ with the family of 𝔽.𝕎. projection $\pi_r : E = \prod_D E_r \rightarrow E_r$ is 𝔽.𝕎.M. funct. Hausd.

The proofs of Proposition (2.8.) and (2.9.) (resp., Corollary (2.8) and (2.9)) are similarly to those for the corresponding results in the non-functional case and will therefore be omitted

3. Fibrewise Multi-regular and Multi-normal spaces

We now progress to look the $\mathbb{F}.\mathbb{W}.$ emissions of the higher multi- separation axioms, starting by multi-regularity and multi-completely regularity.

Definition 3.1. The $\mathbb{F}.\mathbb{W}.$T.S. E on D is named $\mathbb{F}.\mathbb{W}.$ upper regular (briefly, $\mathbb{F}.\mathbb{W}.$U. re.) if for every point $e \in E_d^+$, where in $d \in D$, and for every open set K of e in E, there exists a $\eta\mathbb{P}\mathbb{d}$ W of d in D and an open set H of e in E_W^+ such that closure of H in is E_W^+ancluding in K (i.e. $E_W^+ \cap Cl(H) \subset K$).

Definition 3.2. The $\mathbb{F}.\mathbb{W}.$T.S. E on D is named $\mathbb{F}.\mathbb{W}.$ lower regular (briefly, $\mathbb{F}.\mathbb{W}.$L. re.) if for every point $e \in E_d^-$, where in $d \in D$, and for every open set K of e in E, there exists a $\eta\mathbb{P}\mathbb{d}$ W of d in D and an open set H of e in E_W^- such that closure of H in is E_W^-ancluding in K (i.e. $E_W^- \cap Cl(H) \subset K$).

The $\mathbb{F}.\mathbb{W}.$T.S. on D is named $\mathbb{F}.\mathbb{W}.$ multi- regular (briefly, $\mathbb{F}.\mathbb{W}.$M. re.), if E is $\mathbb{F}.\mathbb{W}.$U. re and $\mathbb{F}.\mathbb{W}$ L. re.

Remark 3.1.

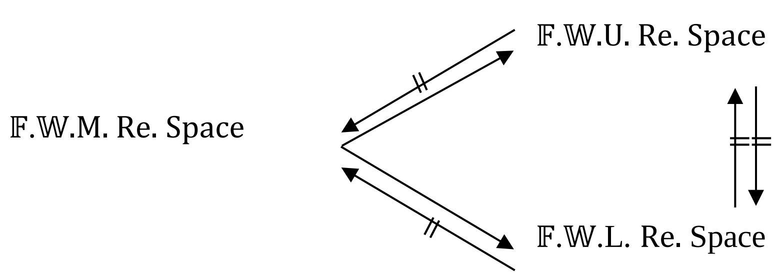
(a) The $\eta\mathbb{P}\mathbb{d}$ s of e are given by $\mathbb{F}.\mathbb{W}.$ basis it is sufficient if the condition in definition (3.1) is satisfied for all $\mathbb{F}.\mathbb{W}.$ basic $\eta\mathbb{P}\mathbb{d}\ s$.

(b) Every $\mathbb{F}.\mathbb{W}.$M. re. space is $\mathbb{F}.\mathbb{W}.$ re. space, but the convers is not true.

(c) Every $\mathbb{F}.\mathbb{W}.$M. re. space is $\mathbb{F}.\mathbb{W}.$U. re. space, but the convers is not true.

(d) Every $\mathbb{F}.\mathbb{W}.$M. re. space is $\mathbb{F}.\mathbb{W}.$L. re. space, but the convers is not true.

(e) The $\mathbb{F}.\mathbb{W}.$U. re. space and $\mathbb{F}.\mathbb{W}.$U. re. space are independence.



Planned 3.1.

Example 3.1.

(a) Let $E = \{a, b, c\}$, $\tau_{(E)} = \{E, \emptyset, \{a\}, \{b, c\}\}$. $D = \{1,2\}$, $\rho = \{\emptyset, D, \{1\}\}$. Define the project. $X: (E, \tau_{(E)}) \rightarrow (D, \rho)$ by , $X(a) = X(b) = X(c) = \{1\}$

E is $\mathbb{F}.\mathbb{W}.$U. re. space $\mathbb{F}.\mathbb{W}.$L. re. space and $\mathbb{F}.\mathbb{W}.$M. re. space.

(b) Let E = $\mathbb{R}$ with the usual topology τ and let $D = \{a, b, c\}$ with the topology $\rho = \{\emptyset, D, \{a\}, \{b\}, \{a, b\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\}\,;\ e \le 0 \\ \{a, c\}\,;\ e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}$. L. re. space, but not $\mathbb{F}.\mathbb{W}$.U. re. space and not $\mathbb{F}.\mathbb{W}$.M. re. space.

(c) Let E =$\mathbb{R}$ with the usual topology τ and $D = \{a, b\}$ with the topology $\rho = \{\emptyset, D, \{a\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\}\,;\ e \le 0 \\ \emptyset\,;\ e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}$.U. re. space, but not $\mathbb{F}.\mathbb{W}$.L. re. space and not $\mathbb{F}.\mathbb{W}$.M. re. space.

(d) Let E = $\mathbb{R}$ with the usual topology τ and $D = \{a, b, c\}$ with the topology$\rho = \{\emptyset, D, \{a\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\}\,;\ e < 0 \\ \{a, b\}\,;\ e = 0 \\ \{c\}\,;\ e > 0 \end{cases}$$

E is not $\mathbb{F}.\mathbb{W}$.U. re. space, not $\mathbb{F}.\mathbb{W}$.L. re. space and not $\mathbb{F}.\mathbb{W}$.M. re. space.

If E is $\mathbb{F}.\mathbb{W}$.U. re. (resp., $\mathbb{F}.\mathbb{W}$.L. re. and $\mathbb{F}.\mathbb{W}$.M. re.) space on D* for every subspace D* of D. Subspace of $\mathbb{F}.\mathbb{W}$.U. re. (resp., F.W $\mathbb{F}.\mathbb{W}$.L. re. and $\mathbb{F}.\mathbb{W}$.M. re.) spaces $\mathbb{F}.\mathbb{W}$.U. re. (resp., $\mathbb{F}.\mathbb{W}$.L. re. and $\mathbb{F}.\mathbb{W}$.M. re.) space.

Proposition 3.1. Let Ω: E → E* be a $\mathbb{F}.\mathbb{W}$. embedding function, where in E and E* are $\mathbb{F}.\mathbb{W}$.T.S. on D. If E* is $\mathbb{F}.\mathbb{W}$.U. re. (resp., $\mathbb{F}.\mathbb{W}$.L. re.) then so is E.

Proof. Let $e \in E_d^+$ (resp., E_d^-), where in $d \in D$, and let K be an open set of e in E. Then $K = \Omega^1(K^*)$, where in K* is an open set of e* =Ω(e) in E*. Since E* is $\mathbb{F}.\mathbb{W}$.U. re. (resp., $\mathbb{F}.\mathbb{W}$.L. re.), there exists a $\eta\mathbb{P}\mathbb{d}$ W of d and an open set H* of e* in E^*_W such that $(E^*)_w^+ \cap CI(H^*) \subset K^*$ (resp., $(E^*)_w^- \cap CI(H^*) \subset K^*$). Then $H = \Omega^{-1}(H^*)$ is an open set of e in E_W such that $E_w^+ \cap CI(H) \subset K$(resp., $E_w^- \cap CI(H) \subset K$), and so E is $\mathbb{F}.\mathbb{W}$.U. re. (resp., $\mathbb{F}.\mathbb{W}$.L. re.).

Corollary 3.1. Let Ω: E → E* be a $\mathbb{F}.\mathbb{W}$. embedding function, where in E and E* are $\mathbb{F}.\mathbb{W}$.T.S. on D. If E* is $\mathbb{F}.\mathbb{W}$.M. re. then so is E.

The class of $\mathbb{F}.\mathbb{W}$.U. re. (resp., $\mathbb{F}.\mathbb{W}$.L. re. and $\mathbb{F}.\mathbb{W}$.M. re.) spaces is $\mathbb{F}.\mathbb{W}$. multiplicative, in the following sense.

Proposition 3.2. Let $\{E_r\}$ be a finite family of 𝔽.𝕎.U. re. S. (resp., 𝔽.𝕎.L. re. S.) on D. Then the 𝔽.𝕎.T. product $E = \prod_D E_r$ is 𝔽.𝕎.U. re. (resp., 𝔽.𝕎.L. re.).

Proof. Let $e \in E_d^+$ (resp., E_d^-), where in $d \in D$. Consider an open set $K = \prod_D E_r$ of e in E, where in K_r is an open set of $\pi_r(e) = e_r$ in E_r for every index r. Since E_r is 𝔽.𝕎.U. re. (𝔽.𝕎.U. re.)

there exists a ηℙ𝕕W_r of d in D and an open set H_r of e_r in $(E_r|W_r)^+$ $(resp., (E_r|W_r)^-)$ such that the closure $(E_r|W_r)^+ \cap CI(H_r)$ (resp., $(E_r|W_r)^- \cap CI(H_r)$) of U_r in $(E_r|W_r)^+$ $(resp., (E_r|W_r)^-)$is including in K_r. Then the intersection W of the W_r is a ηℙ𝕕 of b and $H = \prod_D E_r$ is an open set of e in E_w^+(resp., E_w^-), such that the closure $E_w^+ \cap CI(H)$(resp., $E_w^- \cap CI(H)$) of H in E_w^+(resp., E_w^-), is including in K, and so $E = \prod_D E_r$ is 𝔽.𝕎.U. re. (resp., 𝔽.𝕎.L. re.). The same conclusion holds for infinite F. W. products provided every of the strain is 𝔽.𝕎. non-empty.

Corollary 3.2. Let $\{E_r\}$ be a finite family of 𝔽.𝕎.M.re.S. on D. Then the 𝔽.𝕎. T. product $E = \prod_D E_r$ is 𝔽.𝕎.M. re..

Proposition 3.3. Let $\Omega: E \to F$ be an open and closed 𝔽.𝕎. surjection, where in E and F are 𝔽.𝕎.T.S. on D. If E is 𝔽.𝕎.U. re. (resp., 𝔽.𝕎.L. re.), then so is F.

Proof. Let $f \in F_d^+$ (resp., F_d^-) , where in $d \in D$, and let K be an open set of f in F. Pick $e \in \Omega^{-1}(f)$. Then $H = \Omega^{-1}(K)$, is an open set of e. Since E 𝔽.𝕎.U. re. (resp., 𝔽.𝕎.L. re.) there exists a ηℙ𝕕 W of d and an open set H* of e such that $E_w^+ \cap CI(H^*) \subset H$(resp., $E_w^- \cap CI(H^*) \subset H$). Then $F_w^+ \cap \Omega(CI(H^*)) \subset \varphi(H)$(resp., $F_w^- \cap \Omega(CI(H^*)) \subset \Omega(H)$) = K. Let φ is closed, then $\Omega(CI(H^*)) = CI(\Omega(H^*))$ and since Ω is open, then $\Omega(H^*)$ is an open set of y. Thus, F is 𝔽.𝕎.U. re. (resp., 𝔽.𝕎.L. re.), as asserted.

Corollary 3.3. Let $\Omega: E \to F$ be an open and closed 𝔽.𝕎. surjection, where in E and F are 𝔽.𝕎.T.S. on D. If E is 𝔽.𝕎.M. re., then so is F.

The upper (resp., lower and multi) funct. emission of the 𝔽.𝕎.U. re. (resp., 𝔽.𝕎.L. re. and resp., F.W 𝔽.𝕎.M. re.) axiom is finer than the non- upper (resp., lower and multi) funct. emission, but its properties are fairly similarly. In the normally theory the term completely re. is always used instead of funct. re. and we extend this usage to the 𝔽.𝕎 theory.

Definition 3.3. The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. upper completely (briefly, 𝔽.𝕎.U. comp.) re. if for every point $e \in E_d^+$, where in $d \in D$ and for every open set K of e, there exist a ηℙ𝕕 W of d and an open set H of e in E_w^+ and a continuous function $\lambda : E_w^+ \longrightarrow I$ such that $E_d^+ \cap H \subset \lambda^{-1}(0)$ and $E_w^+ \cap (E_w^+ - K) \subset \lambda^{-1}(1)$.

Definition 3.4. The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. lower completely (briefly, 𝔽.𝕎.L. comp.) re. if for every point $e \in E_d^-$, where in $d \in D$ and for each open set K of e, there exist a ηℙ𝕕 W of d and an open set H of e in E_w^- and a continuous function $\lambda : E_w^- \longrightarrow I$ such that $E_d^- \cap H \subset \lambda^{-1}(0)$ and $E_w^- \cap (E_w^- - K) \subset \lambda^{-1}(1)$.

The 𝔽.𝕎.T.S. on D is named 𝔽.𝕎. multi-completely (briefly, 𝔽.𝕎.M. comp.) re., if E is 𝔽.𝕎.U. comp. re. and 𝔽.𝕎.L. comp. re..

Remark 3.2.

(a) The ηℙ𝕕s of e are given by a 𝔽.𝕎. basis it is sufficient if the condition in Definition (3.2.) is satisfied for every 𝔽.𝕎. basic ηℙ𝕕s.

(b) If E is 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp. and 𝔽.𝕎.M. comp.) re.S. on D, then E_{D^*} is 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp. and 𝔽.𝕎.M. comp.) re.S. on D* for every subspace D* of D.

Subspaces of 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp. and 𝔽.𝕎.M. comp.) re.S. are 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp. and 𝔽.𝕎.M. comp.) re. S. In fact, we have.

Proposition 3.4. Let Ω: E → E* be a 𝔽.𝕎. embedding function, where in E and E* are 𝔽.𝕎.T.S. on D. If E* is 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp.) re. then so is E.

Proof. The proof is similarly to the proof of (Proposition 3.3.), so it is omitted.

Corollary 3.4. Let Ω: E → E* be a 𝔽.𝕎. embedding function, where in E and E* are 𝔽.𝕎.T.S. on D. If E* is 𝔽.𝕎 M. comp. re. then so is E.

The class of is 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp. and 𝔽.𝕎.M. comp.) re.S. is finitely multiplicative, in the following sense.

Proposition 3.5. Let $\{E_r\}$ be a finitely family of 𝔽.𝕎.U. comp. (resp., F.W.L. comp.) re.S. on D. Then the 𝔽.𝕎.T. product E = $\prod_D E_r$ is 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp.) re.

Proof. Let $e \in E_d^+(\text{resp.}, E_d^-)$, where in d ∈ D. Consider a 𝔽.𝕎. open set $\prod_D K_r$ of e in E, where in K_r is an open set of $\pi_r(e) = e_r$ in E_r for every index r. Let E_r is 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp.) re. there exists a ηℙ𝕕 W_r of d and an open set H of e_r in E_r and a continuous function $\lambda_r : X_{w_r}^+ \longrightarrow I$(resp., $\lambda_r : X_{w_r}^- \longrightarrow I$) such that $(E_r)_d^+ \cap H \subset \lambda_r^{-1}(0)$(resp., $(E_r)_d^- \cap U \subset \lambda_r^{-1}(0)$) and $E_{w_r}^+ \cap (E_{w_r}^+ - K_r) \subset \lambda_r^{-1}(1)$(resp., $E_{w_r}^- \cap (E_{w_r}^- - K_r) \subset \lambda_r^{-1}(1)$). Then the intersection W of the W_r is a ηℙ𝕕 of b and $\lambda: E_w^+ \longrightarrow I$(resp., $\lambda: E_w^- \longrightarrow I$) is a continuous function where

$$\lambda(\varepsilon) = \inf_{r=1,2,\ldots,n}\{\lambda_r(\varepsilon_r)\} \text{ for } \varepsilon = (\varepsilon_r) \in E_w^+(\text{resp.}, E_w^-)$$

Since $E_d^+ \cap \pi_r^{-1}(H) \subset \pi_r^{-1}$ (resp., $E_d^- \cap \pi_r^{-1}(H) \subset \pi_r^{-1}$) $((Er)_{d}^+{}_b \cap H) \subset \pi_r^{-1}(\lambda_r^{-1}(0)$ (resp., $(Er)_{d}^-{}_b \cap H) \subset \pi_r^{-1}(\lambda_r^{-1}(0))$) $= (\lambda_r o\, \pi_r)^{-1}(0)$ and E_{wr}^+ (resp., E_{wr}^-) $\cap\ \pi_r^{-1}(E_{wr}^+$(resp., E_{wr}^-) $- K_r) \subset \pi_r^{-1}(E_{wr}^+$(resp., E_{wr}^-) $\cap (E_{wr}^+$ (resp., E_{wr}^-) $- K_2) \subset \pi_r^{-1}(\lambda_r^{-1}(1))) = (\lambda_r o \pi_r)^{-1}(1)$, this proves the result. The same conclusion holds for infinitely ηℙ𝕕. products provided that every of the strain is ηℙ𝕕. non-empty.

Corollary 3.5. Let $\{E_r\}$ be a finite family of 𝔽.𝕎.M. comp. re.S. on D. Then the 𝔽.𝕎.T. product E = $\prod_D E_r$ is 𝔽.𝕎.M. comp. re..

Proposition 3.6. Let Ω: E → F be an open and closed 𝔽.𝕎. surjection, where in E and F are 𝔽.𝕎.T.S. on D. If E is 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp.) re., then so is F.

Proof. Let $f \in F_d^+(\text{resp.}, F_d^-)$, where in d ∈ D, and let K be an open set of f. Pick $e \in E_d^+(\text{resp.}, E_d^-)$, so that $K_e = \Omega^{-1}(K_f)$ is an open set of e. Since E, 𝔽.𝕎.U. comp. (resp., 𝔽.𝕎.L. comp.) re., there exists a ηℙ𝕕 W of d and an open set H_e of e in X_W and a continuous function $\lambda: X_W \longrightarrow I$ such that $X_b \cap H_e \subset \lambda^{-1}(0)$ and $E_w^+ \cap (E_w^+ - K_e) \subset \lambda^{-1}(1)$ (resp., $E_w^- \cap (E_w^- - K_e) \subset \lambda^{-1}(1)$). Using (Proposition 1.3.) in [5]

we obtain a continuous function $\Omega : F_d^+ \longrightarrow I$(resp., $\Omega : F_d^- \longrightarrow I$) such that $F_d^+ \cap H_f \subset \varphi^{-1}(0)$(resp., $F_d^- \cap H_f \subset \varphi^{-1}(0)$) and $F_w^+ \cap (F_w^+ - K_y) \subset \Omega^{-1}(1)$ (resp., $F_w^- \cap (F_w^- - K_y) \subset \Omega^{-1}(1)$).

Now we offer the emission of $\mathbb{F}.\mathbb{W}.$ upper (resp., lower and multi) normal spaces as follows:

Definition 3.5. The $\mathbb{F}.\mathbb{W}.$T.S. E on D is named $\mathbb{F}.\mathbb{W}.$ upper normal (briefly, F.W.U. no.) if for every point d of D and every pair U, V of separated closed sets of E, there exist a $\eta\mathbb{P}\mathbb{d}$ W of d and a pair of separated open sets H, K of $E_w^+ \cap U$, $E_w^+ \cap V$ in E_w^+.

Definition 3.5. The $\mathbb{F}.\mathbb{W}.$T.S. E on D is named $\mathbb{F}.\mathbb{W}.$ lower normal (briefly, F.W.L. no.) if for every point d of D and every pair U, V of separated closed sets of E, there exist a $\eta\mathbb{P}\mathbb{d}$ W of d and a pair of separated open sets H, K of $E_w^- \cap U$, $E_w^- \cap V$ in E_w^-.

The $\mathbb{F}.\mathbb{W}.$T.S. on D is named $\mathbb{F}.\mathbb{W}.$ multi-normal (briefly, F.W.M. no.), if E is $\mathbb{F}.\mathbb{W}.$U. no. and F.W.L. no.

Remark 3.3.

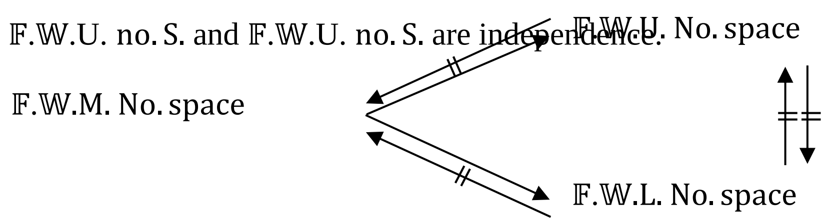
(a) If E is $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no. and $\mathbb{F}.\mathbb{W}.$M. no.) space on D, then E_{D^*} is $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no. and $\mathbb{F}.\mathbb{W}.$M. no.) on D* for each subspace D* of D

(b) Every $\mathbb{F}.\mathbb{W}.$M. no. S. is $\mathbb{F}.\mathbb{W}.$ no. S., but the convers is not true.

(c) Every $\mathbb{F}.\mathbb{W}.$M. no. S. is $\mathbb{F}.\mathbb{W}.$U. no. S., but the convers is not true.

(d) Every $\mathbb{F}.\mathbb{W}.$M. no. S. is $\mathbb{F}.\mathbb{W}.$L. no. S., but the convers is not true.

(e) The $\mathbb{F}.\mathbb{W}.$U. no. S. and $\mathbb{F}.\mathbb{W}.$U. no. S. are independence.



Planned 3.2.

Example 3.2.

(a) Let E = {a, b, c} , $\tau_{(E)}$ = {E, ∅, {a}}. D = {1,2}, ρ = {∅, D, {1}}. Define the project. X: $(E, \tau_{(E)}) \rightarrow (D, \rho)$ by X(a) = X(b) = X(c) = {1}

E is $\mathbb{F}.\mathbb{W}.$U. no. S. $\mathbb{F}.\mathbb{W}.$L. no. S. and $\mathbb{F}.\mathbb{W}.$M. no. S.

(b) Let E = {a, b, c} , $\tau_{(E)}$= discrete topology and let D = {a, b, c} with the topology ρ = {∅, D, {a}, {b}, {a, b}}. Define multi-function

$$X : (\mathbb{R}, \tau) \rightarrow (D, \rho) \text{ by } X(e) = \begin{cases} \{a\}\,;\ e \leq 0 \\ \{a, c\}\,;\ e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}.$L. no. S., but not $\mathbb{F}.\mathbb{W}.$U. no. S. and not $\mathbb{F}.\mathbb{W}.$M. no. S.

(c) Let E = $\{a, b, c\}$, $\tau_{(E)}$= discrete topology and D = $\{a, b\}$ with the topology $\rho = \{\emptyset, D, \{a\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\} ; & e \leq 0 \\ \emptyset ; & e > 0 \end{cases}$$

E is $\mathbb{F}.\mathbb{W}.$U. no. S., but not $\mathbb{F}.\mathbb{W}.$L. no. S.and not $\mathbb{F}.\mathbb{W}.$M. no. S.

(d) Let E = $\mathbb{R}$ with the usual topology τ and D = $\{a, b, c\}$ with the topology$\rho = \{\emptyset, D, \{a\}\}$. Define multi-function

$$X : (\mathbb{R}, \tau) \to (D, \rho) \text{ by } X(e) = \begin{cases} \{a\} ; & e < 0 \\ \{a, b\} ; & e = 0 \\ \{c\} ; & e > 0 \end{cases}$$

E is not $\mathbb{F}.\mathbb{W}.$U. re. S, not $\mathbb{F}.\mathbb{W}.$L. re. S. and not $\mathbb{F}.\mathbb{W}.$M. re. S.

Closed subspace*s* of $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no. and $\mathbb{F}.\mathbb{W}.$M. no.) space*s* are $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no. and $\mathbb{F}.\mathbb{W}.$M. no.). In fact, we have:

.

Proposition 3.7. Let Ω: E → E* be a closed $\mathbb{F}.\mathbb{W}.$ embedding, where in E and E* are $\mathbb{F}.\mathbb{W}.$T.S. on D. If E* is $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no.) then so is E.

Proof. Let d be a point of D and let U, V be separated closed sets of E. Then Ω(U), Ω(V) are separated closed sets of E*. since E* is $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no.)there exists a $\eta\mathbb{P}\mathbb{d}$ W of d and a pair of separated open sets H, K of $(E^*)^+_w \cap \Omega(U)$(resp., $(E^*)^-_w \cap \Omega(U)$) , $(E^*)^+_w \cap \Omega(V)$(resp., $(E^*)^-_w \cap \Omega(V)$) in $(E^*)^+_w$(resp., $(E^*)^-_w$) . Then Ω^{-1}(H) and Ω^{-1}(K) are separated open sets of $E^+_w \cap U$(resp., $E^-_w \cap U$) **,** $E^+_w \cap V$(resp., E^-_w) in E^+_w(resp., E^-_w)**.**

Corollary 3.7. Let Ω: E → E* be a closed $\mathbb{F}.\mathbb{W}.$ embedding, where in E and E* are $\mathbb{F}.\mathbb{W}.$T.S. on D. If E* is $\mathbb{F}.\mathbb{W}.$M. no. then so is E.

Proposition 3.8. Let Ω: E → F be an closed continuous$\mathbb{F}.\mathbb{W}.$ surjection, where in E and F are $\mathbb{F}.\mathbb{W}.$T.S. on D. If E is $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no.), then so is F.

Proof. Let d be a point of D and let U, V be separated closed sets of F. Then Ω^{-1}(U), Ω^{-1}(V) are separated closed sets of E. Since E is $\mathbb{F}.\mathbb{W}.$U. no. (resp., $\mathbb{F}.\mathbb{W}.$L. no.) there exists a $\eta\mathbb{P}\mathbb{d}$ W of d and a pair of separated open sets H, K of $E^+_w \cap \Omega^{-1}$(U)(resp., $E^-_w \cap \Omega^{-1}$(U)) and $E^+_w \cap \Omega^{-1}$(V)(resp., $E^-_w \cap \Omega^{-1}$(V)). Since Ω is closed, the sets F^+_w - Ω (E^+_w- H)(resp., F^+_w - Ω (E^+_w- H)), F^+_w - Ω (E^+_w- K)(resp., F^+_w - Ω (E^+_w- K)) are open in F^+_w, and from a separated pair of an open sets of $F^+_w \cap$ U(resp., $F^-_w \cap$ U) , $F^+_w \cap$ V(resp., $F^-_w \cap$ V) in F^+_w, as required.

Corollary 3.8. Let Ω: E → F be an closed continuous$\mathbb{F}.\mathbb{W}.$ surjection, where in E and F are $\mathbb{F}.\mathbb{W}.$T.S. on D. If E is $\mathbb{F}.\mathbb{W}.$M. no. (then so is F.

Finally, we offer the emission of 𝔽.𝕎. upper functionally (resp., F.W. lower functionally and 𝔽.𝕎. multi- functionally) normal space as follows:

Definition 3.7. The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. upper functionally (briefly, 𝔽.𝕎.U. funct.) no. if for every point d of D and every pair U, V of separated closed sets of E, there exist a ηℙ𝕕 W of d and a pair of separated open sets K, H and a continuous function $\lambda : X_w^+ \to I$ such that $X_w^+ \cap U \cap H \subset \lambda^{-1}(0)$ and $X_w^+ \cap V \cap K \subset \lambda^{-1}(1)$ in X_w^+.

Definition 3.8. The 𝔽.𝕎.T.S. E on D is named 𝔽.𝕎. lower functionally (briefly, F.W.L. funct.) no. if for every point d of D and every pair U, V of separated closed sets of E, there exist a ηℙ𝕕 W of d and a pair of separated open sets H, K and a continuous function $\lambda : X_w^- \to I$ such that $X_w^- \cap U \cap H \subset \lambda^{-1}(0)$ and $X_w^- \cap V \cap K \subset \lambda^{-1}(1)$ in X_w^-.

The 𝔽.𝕎.T.S. on D is named 𝔽.𝕎. multi- functionally (briefly, F.W.M. funct.) no., if E is 𝔽.𝕎.U. funct. no. and 𝔽.𝕎.L. funct. no.

Remark 3.3. If E is 𝔽.𝕎.U. funct. (resp., F.W.L funct. and F.W.M. funct.) no. S. on D, then E_{D^*} is 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L . funct. and 𝔽.𝕎.M. funct.) no. S. on D* for every subspace D* of D.

Closed subspaces of 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L funct. and 𝔽.𝕎.M. funct.) no. S. are 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L funct. and 𝔽.𝕎 M. funct.) no. In fact, we have:

Proposition 3.9. Let Ω: E → E* be a closed 𝔽.𝕎. embedding, where in E and E* are 𝔽.𝕎.T.S. on D. If E* is 𝔽.𝕎.U. funct. (resp., 𝔽.𝕎.L funct.) no. then so is E.

Proof. Let d be a point of D and let U, V be separated closed sets of E. Then Ω(U), Ω(V) are separated closed sets of E*. since E* is 𝔽.𝕎.U. funct. (resp., F.W.L funct.) no. there exists a ηℙ𝕕 W of d and a pair of separated open sets H, K and a continuous function $\lambda : (E^*)_w^+ \to I$(resp., $\lambda : (E^*)_w^- \to I$) such that $(E^*)_w^+ \cap \Omega(U) \cap H \subset \lambda^{-1}(0)$(resp., $(E^*)_w^+ \cap \Omega(U) \cap H \subset \lambda^{-1}(0)$) and $(E^*)_w^+ \cap \Omega(V) \cap K \subset \lambda^{-1}(1)$(resp., $(E^*)_w^+ \cap \Omega(V) \cap K \subset \lambda^{-1}(1)$) in $(E^*)_w^+$(resp., $(E^*)_w^-$), then **φ** = αoΩ is a continuous function $\lambda : E_w^+ \to I(\lambda : E_w^- \to I)$ such that $E_w^+ \cap U \cap \Omega^{-1}(H) \subset \varphi^{-1}(0)$(resp., $E_w^- \cap U \cap \Omega^{-1}(H) \subset \varphi^{-1}(0)$) and $E_w^+ \cap V \cap \Omega^{-1}(K) \subset \varphi^{-1}(1)$(resp., $E_w^- \cap V \cap \Omega^{-1}(K) \subset \varphi^{-1}(1)$) in$E_w^+$(resp., E_w^+),. As required.

Corollary 3.9. Let Ω: E → E* be a closed 𝔽.𝕎. embedding, where in E and E* are 𝔽.𝕎.T.S. on D. If E* is 𝔽.𝕎.M. funct. no. then so is E.

Proposition 3.10. Let Ω: E → F be an open, closed and continuous𝔽.𝕎. surjection, where in E and F are 𝔽.𝕎.T.S. on D. If E is 𝔽.𝕎.U. funct. (resp., F.W.L funct.) no., then so is Y.

Proof. Let d be a point of D and let U, V be separated closed sets of F. Then $\Omega^{-1}(U)$, $\Omega^{-1}(V)$ are separated closed sets of E. Since E is 𝔽.𝕎.U. funct. (resp., F.W.L funct.) no. there exists a ηℙ𝕕 W of d and a pair of separated open sets H, K and a continuous function $\lambda : X_w^+ \to I$ (resp., $\lambda : X_w^- \to I$) such that $X_w^+ \cap \Omega^{-1}(U) \cap H \subset \lambda^{-1}(0)$ (resp., $X_w^- \cap \Omega^{-1}(U) \cap H \subset \lambda^{-1}(0)$) and $X_w^+ \cap \Omega^{-1}(V) \cap K \subset \lambda^{-1}(1)$ (resp., $X_w^- \cap \Omega^{-1}(V) \cap K \subset \lambda^{-1}(1)$) in X_W. Now a function $\varphi : F_w^+ \to I$ (resp., $F_w^+ \to I$) is given by:

$(f) = sup_{e \in \Omega^{-1}(f)} \lambda(e); f \in F_w^+$ (resp., F_w^-)

Since Ω is open and closed, as well as continuous, it follows that φ is continuous. Since $F_W^+ \cap U \cap \Omega$ (H) ⊂ Ω^{-1} (0) (resp., $F_W^+ \cap U \cap \Omega$ (H) ⊂ Ω^{-1} (0)) and $F_W^+ \cap V \cap \Omega$ (H) ⊂ Ω^{-1} (1) (resp., $F_W^+ \cap V \cap \Omega$ (H) ⊂ Ω^{-1} (1)) in E_W^+ (resp., F_W^-). This proves the proposition.

Corollary 3.9.. Let Ω: E → F be an open, closed and continuous𝔽.𝕎. surjection, where in E and F are 𝔽.𝕎.T.S. on D. If E is 𝔽.𝕎.M. funct. no., then so is Y.

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