

## Particle Swarm Optimization for Optimal Reliability Allocation with Discrete Cost-reliability Data for Components

Authors Names	ABSTRACT
<p><i>Hayder Kareem Hammood<sup>a</sup></i>  <i>Bushra Kadhum Awaad<sup>b</sup></i></p> <p><b>Publication data:</b> 18 /12 /2023</p> <p><b>Keywords:</b> Optimization for reliability, reliability allocation, and reliability</p>	<p>In this work, the reliability function of a complex system is calculated, together with the ideal reliability distribution among the system's components that will be constructed. The majority of research to date has adopted the exponentially growing closed functions to associate cost and reliability, despite the fact that it is commonly known that component cost is an increasing function of their reliability. In actuality, these functions are sometimes ill-defined or challenging to construct, hence it is frequently logical to discuss cost-reliability connections between discrete data sets. We take into account scenarios where each component is offered in a range of reliability levels at varying pricing. A non-linear integer program is the consequence of a design optimization challenge. We introduce particle swarm optimization (PSO) to calculate the system reliability assignment for each system component as well as to compute the overall system reliability since each system configuration has an equivalent representation as either a serial connection to parallel subsystems or a parallel connection to serial subsystems. Each system component's cost was determined using three cost functions.</p>

### 1. Introduction

In this research, we looked at the complicated system's installed reliability [9, 10]. By leveraging short pathways across connection matrices, this system's dependability was discovered. All paths are obtained using Boolean algebra, and nodes are then eliminated to produce minimal paths [3, 6, 11, 12]. To find out more about how safe it is to utilize the installed sophisticated system, a reliability function is sought after. Despite the historical basis of the networks, we also examine the mathematical issue of distributing optimal reliability in this study. Based on location importance, the dependability standards for each component of a complex system are optimized. The objective is to increase the system's lifespan and dependability while lowering overall costs [5, 7, 8]. Depending on where they are located in the system, some components may require a high allocation in order to increase overall reliability. Engineers encounter a number of challenges when trying to improve mechanical and electrical systems [4, 6, 14]. This study focuses on the reliability of complex systems as well as the distributing and enhancing the system cost, which can be expressed in terms of size, weight, or other metrics. There are two main elements that affect this component's dependability: The model must be cost-based before the input element may be validated. The suggested cost parameter's specifications can be altered. This facilitates the engineers' analysis of the financial allocations for each system and planning for the attainment of the bare minimum dependability required for each machine component. The analytical dependability of the input system must also be considered by the model. When applied to bigger systems, simple systems can occasionally pose a substantial challenge. The outcomes were obtained using the Particle Swarm, a tool for complicated systems to tackle optimization issues. A logarithmic model, an exponential behavior model, and an exponential behavior feasibility factor were used to calculate the cost.

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## 2. Complex system optimization and reliability allocation

Take into account a complicated system with components linked to reliability [5, 17]. We make use of the notes below:

$C_i(R_i)$  = element  $i$  cost;

$0 \leq R_i \leq 1$  = reliability  $i$  component;

$R_s$  = reliability of the system;

$C(R_1, \dots, R_n) = \sum_{i=1}^n a_i C_i(R_i)$  is the total system cost, in which  $a_i$  is greater than 0;

RG = objective of systems reliability.

There are many possible outcomes due to the system's modular design and the unique functions of each component. The same capacity is provided to us via a variety of system components, each with various degrees of dependability. The system's ability to correctly allocate resources to all components or selected ones is the ultimate goal. Problems are necessary for nonlinear programming [9, 16, 17]. Despite not being linear, the constraint serves a purpose and incurs costs that can be researched:

$$\text{Minimized } C(R_1, \dots, R_n) = \sum_{i=1}^n a_i C_i(R_i), \quad a_i > 0,$$

Subject to:

$$\begin{aligned} R_s &\geq R_G \\ 0 \leq R_i &< 1, \text{ in which } i = 1, \dots, n \end{aligned} \quad (1)$$

Let the partial cost function be reasonable and  $C_i(R_i)$  satisfies some conditions [12], Positive, differentiated functions, increasing from  $\left[ \Rightarrow \frac{dC_i}{dR_i} \geq 0 \right]$ .

The part costs function of the Euclidean convexity  $C_i(R_i)$  analogous to the reality that its derivatives  $\frac{dC_i}{dR_i}$  are monotonically increased, i. e.  $\frac{d^2 C_i}{dR_i^2} \geq 0$ .

The system reliability restriction is lowered under  $R_G$ , and the prior plan's objective is to achieve an all-out framework cost base [12].

## 3. Implementation in a complex system

In order to estimate the complex system, we need to transform it into a more approachable network, similar to how we would transform a series of objects into a parallel network. In parallel and series networks with  $n$  components, the dependability is, respectively:

$$R_s = \prod_{i=1}^n R_i \quad (2)$$

$$R_s = 1 - \prod_{i=1}^n (1 - R_i) \quad (3)$$

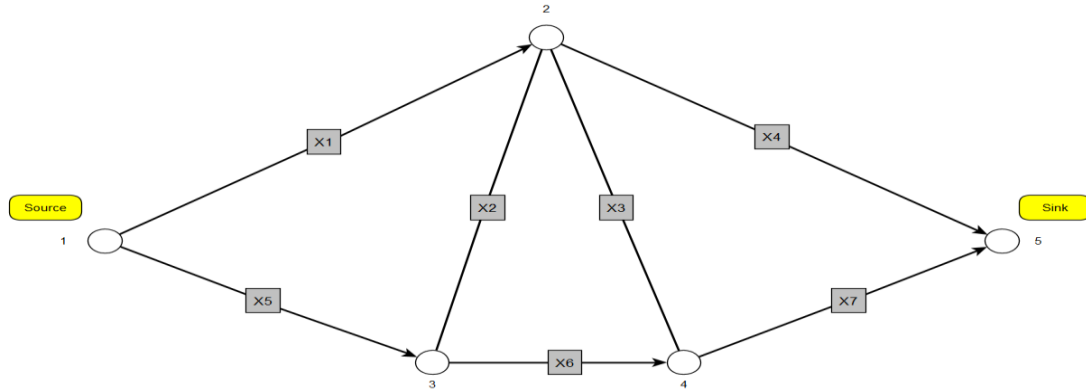
Here  $R_N$  represents the reliability network and  $R_i$  is the reliability of the component  $i$  [6,8].

From equations (1) and (2) we will compare the reliability of each complex network with  $p$  minimum paths that are given via

$$R_s = 1 - \prod_{z=1}^p (1 - \prod_{j=\alpha}^{\omega} R_j) \quad (4)$$

Here  $\alpha$  is the index of the first component, and  $\omega$  is the index of the last component of a minimal path  $z$ .

Equation can be used to determine the dependability of the complicated network in Fig. 1 below (3).



**Figure 1.** complicated network

The sets:

$$S = \{\{x_1x_4\}, \{x_2x_4x_5\}, \{x_1x_3x_7\}, \{x_5x_6x_7\}, \{x_2x_3x_5x_7\}, \{x_1x_2x_6x_7\}, \{x_3x_4x_5x_6\}, \{x_1x_2x_3x_4x_6\}\}.$$

$$R_s = 1 - [1 - p_r(x_1x_4)] \times [1 - p_r(x_2x_4x_5)] \times [1 - p_r(x_1x_3x_7)] \times [1 - p_r(x_5x_6x_7)] \times [1 - p_r(x_2x_3x_5x_7)] \times [1 - p_r(x_1x_2x_6x_7)] \quad (5)$$

**Note:** When the  $i$  – th component succeeds, then  $R_i = 1$ , and when it fails, then  $R_i = 0 \forall i = 1, \dots, 7$ , these lead to  $R_i^n = R_i$  [7,9].

By using the note above, equation (5) becomes the following polynomial

$$R_s = R_1R_4 + R_1R_3R_7 + R_2R_4R_5 + R_5R_6R_7 - R_1R_2R_4R_5 - R_1R_3R_4R_7 + R_1R_2R_6R_7 + R_2R_3R_5R_7 + R_3R_4R_5R_6 - R_1R_2R_3R_5R_7 - R_1R_2R_3R_6R_7 - R_1R_3R_4R_5R_6 - R_1R_2R_4R_6R_7 - R_2R_3R_4R_5R_6 - R_1R_2R_5R_6R_7 - R_2R_3R_4R_5R_7 - R_1R_3R_5R_6R_7 - R_1R_4R_5R_6R_7 - R_2R_3R_5R_6R_7 - R_2R_4R_5R_6R_7 - R_3R_4R_5R_6R_7 + R_1R_2R_3R_4R_5R_6 + R_1R_2R_3R_4R_5R_7 + R_1R_2R_3R_4R_6R_7 + 2R_1R_2R_3R_5R_6R_7 + 2R_1R_2R_4R_5R_6R_7 + 2R_2R_3R_4R_5R_6R_7 + 2R_2R_3R_4R_5R_6R_7 - 3R_1R_2R_3R_4R_5R_6R_7.$$

#### 4. PSO method

A swarm of particles is a collection of entities with the ideal number of attributes or values to include in a swarm problem space [17, 14]. Communities of people form so that information can be shared. Using the bit string "01110" as an example, a neighborhood is defined in mathematics as "the set of points around a certain position, each within a specified distance from the stated point." The third bit is the one that leaves the location that is given (middle bit). The full bit string, two on the left and two on the right, will fit in the neighborhood of size 3. These neighborhoods can have a range of topologies, despite the fact that their structures are substantially different from the topologies of the ANN. In particle swarm settings, a spherical or star-shaped topology is frequent.

## Implementation of PSO

The evolutionary algorithm PSO requires the creation of random numbers. The PSO algorithm's output is influenced by the caliber and volume of the statistics that are generated. The initial iteration is dispersed across the entire search area. Fig. 2 displays the fundamental implementation of the PSO.

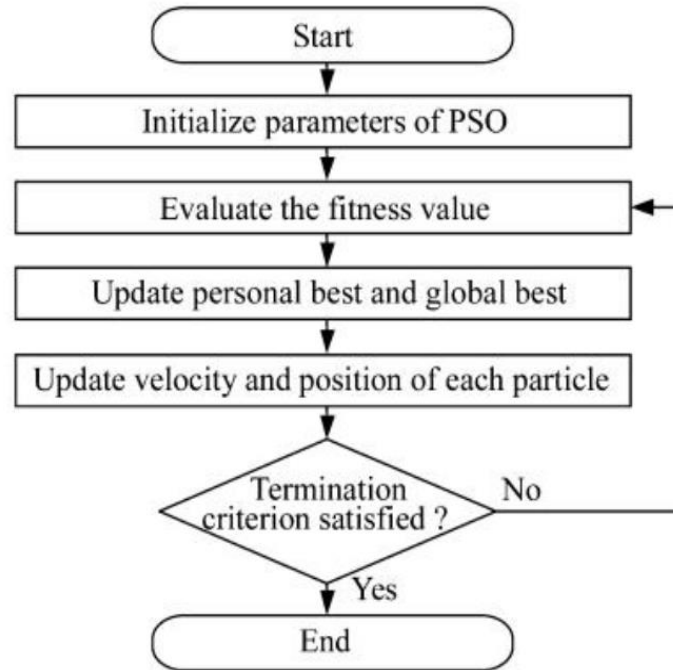


Figure 2. Flow chart of Particle Optimization

Three important reliability cost models

Calculus-based exponential feasibility model

Assume  $0 < f_i < 1$  is a feasibility factor [12],  $R_{i,max}$  is maximum reliability, and  $R_{i,min}$  is minimum reliability.

$$C_i(R_i) = \exp[(1 - f_i) \frac{R_i - R_{i,min}}{R_{i,max} - R_i}],$$

$$R_{i,min} \leq R_i \leq R_{i,max}, i = 1, 2, \dots, n.$$

The issue with optimization arises

$$\text{Minimize } C(R_1, \dots, R_n) = \sum_{i=1}^n a_i \exp[(1 - f_i) \frac{R_i - R_{i,min}}{R_{i,max} - R_i}],$$

$$\text{in which } i = 1, 2, \dots, n.$$

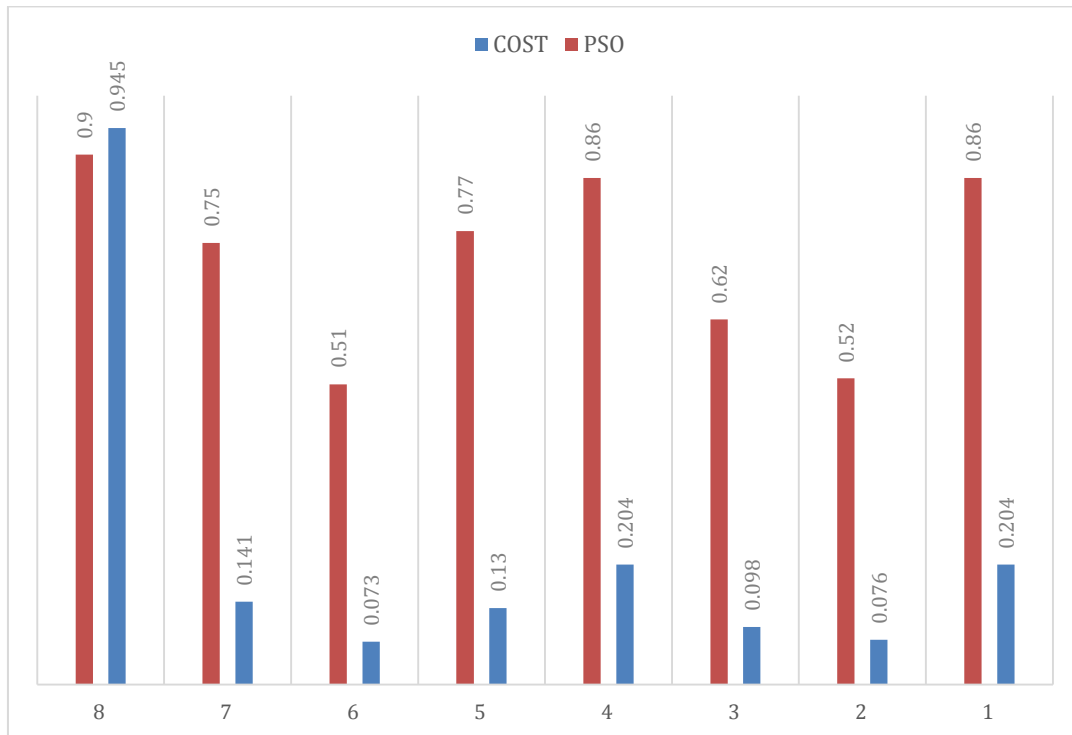
Subjected to :

$$R_s \geq R_G$$

$$R_{i,min} \leq R_i \leq R_{i,max}, i = 1, \dots, n.$$

**Table 1:** Optimum reliability allocation utilizing PSO and GA with an applied cost function.

Components	PSO	COST
$R_1$	0.82	3.44
$R_2$	0.72	1.63
$R_3$	0.6	1.18
$R_4$	0.81	3.09
$R_5$	0.8	2.8
$R_6$	0.68	1.46
$R_7$	0.81	3.09
$R_{system}$	0.9	16.71

**Figure 2:** Utilizing PSO to allocate reliability using the given feasibility factor model

## 5. The model of exponential behavior

Assume the constants  $a_i$  and  $b_i$ , in which  $i=1,2,\dots,n$  and  $0 \leq R_i < 1$ , in which  $i = 1, \dots, n$ . [12-16] proposed it in the following format.

$$(R_i) = a_i e^{\left(\frac{b_i}{1-R_i}\right)}, \quad b_i > 0, a_i > 0,$$

in which  $i = 1, 2, 3, \dots, n$

The optimizing problem emerges as:

$$\text{Minimizing } C(R_1, \dots, R_i) = \sum_{i=1}^n a_i e^{\left(\frac{b_i}{1-R_i}\right)}, i = 1, 2, 3, \dots, n.$$

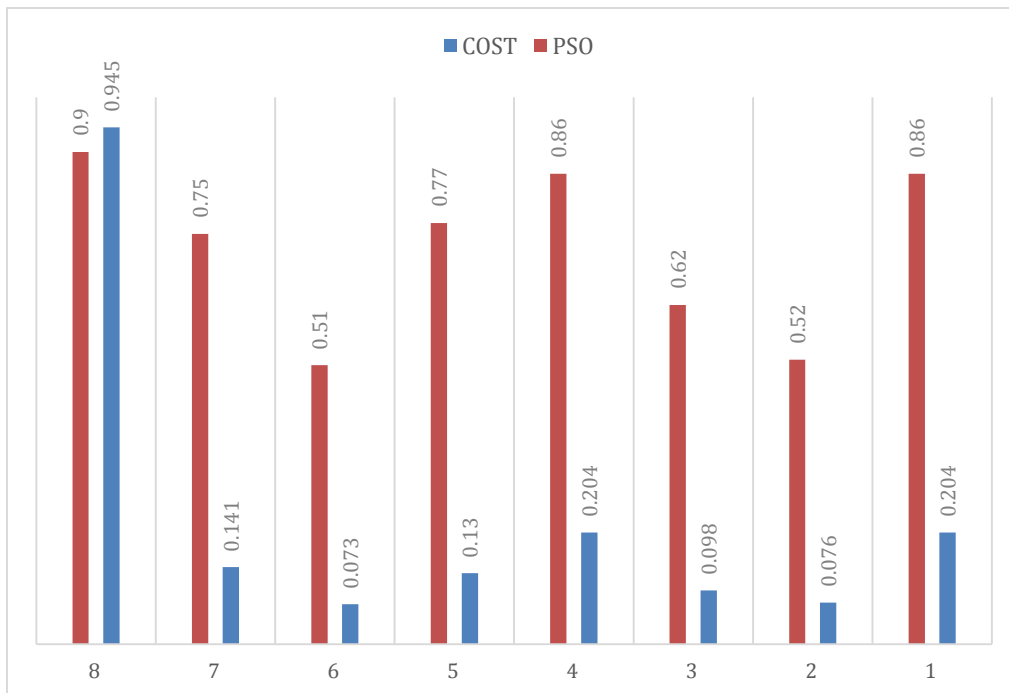
Subjected to :

$$R_s \geq R_G$$

$$0 \leq R_i < 1, \text{ in which } i = 1, 2, 3, \dots, n$$

**Table 2** Optimum reliability allocation utilizing PSO and GA with an applied cost function.

Components	PSO	COST
$R_1$	0.82	19.91
$R_2$	0.55	0.77
$R_3$	0.56	0.81
$R_4$	0.81	14.83
$R_5$	0.8	9.001
$R_6$	0.78	5.97
$R_7$	0.8	9.001
$R_{system}$	0.9	60.31



**Figure 3:** Utilizing PSO to allocate reliability using the exponential behavior model.

## 6. The Logarithmic model

Assuming that  $0 < R_i < 1$ , and  $a_i$  are constants, in which  $i = 1, \dots, n$ . [12, 16] proposed it in the following format.

$$C_i(R_i) = a_i \ln \left( \frac{1}{1-R_i} \right), a_i > 0,$$

in which  $i = 1, \dots, n$

The optimizing problem would become:

$$\text{Minimizing } C(R_1, \dots, R_n) = \sum_{i=1}^n a_i \ln \left( \frac{1}{1-R_i} \right),$$

in which  $i = 1, 2, 3, \dots, n$ .

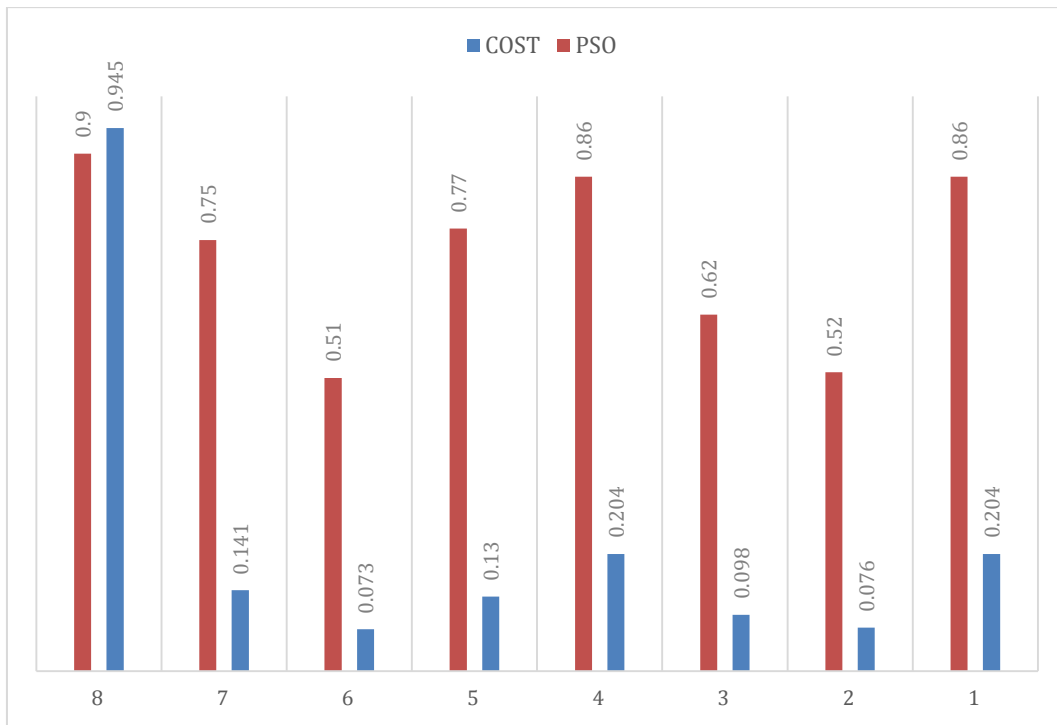
Subjected to :

$$R_s \geq R_G$$

$$0 \leq R_i < 1, \text{ in which } i = 1, 2, 3, \dots, n$$

**Table 3:** Optimum reliability allocation utilizing PSO and GA with an applied cost function.

Components	PSO	COST
$R_1$	0.86	0.204
$R_2$	0.52	0.076
$R_3$	0.62	0.098
$R_4$	0.86	0.204
$R_5$	0.77	0.13
$R_6$	0.51	0.073
$R_7$	0.75	0.141
$R_{system}$	0.9	0.945



**Figure 4:** Utilizing PSO to allocate reliability using the provided Logarithmic mod

## 7. Conclusion

In this study, the topic of increasing reliability for a certain complicated network is covered. a system optimization issue where each system component's reliability is tailored using engineering principles. A nonlinear programming problem with three cost functions and business constraints has also been used to address this topic (reliability of complex systems). The reliability assignment problem was tackled using particle swarm optimization, and after comparing the results, we came to the conclusion that the feasibility factor model, which is described above and has Rs values of 0.9, is the best model for the three cost functions we chose. Similar to the dependability assignment problem, component (1, 4) received the greatest assignment and cost, whereas component (2, 3), as shown in the above tables, received the lowest assignment. because of where these parts are located inside the complex system. The benefit of this paradigm is that each software will be able to do the highly difficult math techniques used.

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