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Numerical Stabilization for Prey-Predator Model

Authors Names	**ABSTRACT**
Shaymaa Hussain Salih ***Publication data***: 18 /12 /2023 ***Keywords***: Numerical Stabilization, Prey-Predator Model .	The dynamical system is a notion that is used to explain how many events behave in our everyday lives. Both linear and nonlinear varieties are available. Stability and chaos, two fundamental characteristics that are further divided into discrete and continuous categories for models showing chaotic behavior and occasionally requiring stabilization and synchronization, define the latter. For this, there are numerous strategies. In this study, the prey-predator model's chaotic behavior is stabilized without the addition of any control factors. This method is thought to be effective for models with difficult-to-find analytical solutions. Also, the Jacobian matrix eigenvalues' modulus exceeds one. The viability and efficiency of this stabilizing approach are finally shown

1- Introduction

In many areas, the mathematical model is used to explain how a system behaves and how its parts interact. Mathematical models can be either continuous or discontinuous. Differential equations describe the formal, whereas It is explained through difference equations. The complexity that these systems' dynamics draws scholarly interest. As instances of the continuous kind, we recommend that the reader look at [1-3].

Difference equations or discrete-time dynamical systems, on the other hand have come to be more and more popular for modeling ecosystems and biological systems where Each of them has a difference in the passage of time[4-5]. Iterative maps are utilized in this modeling approach. In nonlinear systems dynamics, iterative maps play a crucial role. They enable us to fit the previous state's output to the upcoming iteration of the system. To explicitly solve a system of difference equations is, however, generally not simple.

The stability of differential equation solutions and the orbits of dynamical systems are two topics covered utilizing stability theory to explore discrete-time dynamical systems. From the viewpoint of dynamical systems, The bifurcation theory focuses on the modifications within the qualitative or topological structure of a family of difference equation solutions. [6-7]. Dynamical systems have chaotic states that are frequently guided by deterministic laws, and one area of research in this field is called chaos theory. Its solutions behave erratically and are extremely susceptible to starting conditions [8]. Applications of the discrete-time chaotic system are numerous. in virtually all disciplines of applied sciences. This is because it may accurately explain a variety of real-world situations' behaviors. The majority of the animals and plants, for instance, that reproduce once a year. They included, but were not limited to, those in engineering [10], economics and finance [11], security and encoding [9], biology and medical [9], Despite the fact that in some instances chaotic behavior is required, it should nevertheless be avoided due to strong sensitivity to starting conditions, unpredictable influence, and difficulty of long-term prediction. It was necessary to Prey-Predator model in discrete time has a traditional work pieces with the following structure:

$$x_{i+1} = x_i e^{a(1-x_i-y_i)}$$
$$y_{i+1} = c\, x_i y_i - b y_i \qquad \text{... (1)}$$

The Ricker function, which depicts $y_i$the Predator growth is directly correlated with the amount of available prey in the current predator population, is used to explain the growth of the prey population where x_i represents the current prey population.

The prey's growth rate, which is also its consumption rate by the predator, is determined by the

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parameter a, and the predator's conversion rate is determined by the value c. The decay rate of the predator in this instance is represented by the parameter b.

The organization of this essay to be as follows: The variable consideration of the Prey-Predator model is presented inside Section 2. Section 3 discusses the Prey-Predator model's dynamic behavior. The main approach to controlling discrete chaotic systems was introduced in Section 4. The prey-predator model's numerical stabilization technique is presented in Section 5. In section 6, the analytical result is statistically supported. The paper was ended at section 7.

2-Analysis Evaluation of the Prey-Predator Model

The discrete-time system's fixed points and stability are identified requirements are looked. The potential fixed locations are discovered via resolving the equation system.

$$x_{i+1} = x_i e^{a(1-x_i-y_i)} = f_1(x_i, y_i)$$
$$y_{i+1} = c\, x_i y_i - b y_i = f_2(x_i, y_i) \qquad \ldots(2)$$

The system (2) has three fixed points, according to a crude calculation. Jacobian matrix for (2) is as follows:

$$J(x,y) = \begin{pmatrix} e^{a(1-x-y)}(1-ax) & -axe^{a(1-x-y)} \\ cy & cx-b \end{pmatrix}$$

The following equilibrium points are obtained by solving this system (2).$E_0 = (0,0)$ & $E_1 = (1,0), E^* = (x^*, y^*) = (\frac{b+1}{c}, 1 - \frac{b+1}{c})$ Each fixed point's Jacobin matrix is determined as follows:

1-At point $E_0 = (0,0)$, there are:

$$J_{E_0} = \left[\begin{pmatrix} e^a & 0 \\ 0 & -b \end{pmatrix}\right]$$

The eigenvalues of E_0 are $\lambda_1 = e^a$ and $\lambda_2 = -b$, because of the e^a is always greater than 1 .Hence, A quick calculation demonstrates the local dynamics of fixed point E_0:

E_0 is a source point if and only if b>1 and is saddle point if b<1 and is non-hyperbolic if b=1

E_0 is a saddle point if $b < 1$

E_0 is a non-hyperbolic point if $b = 1$

2-At point $E_1 = (1,0)$ we have,

$$J_{E1} = \left[\begin{pmatrix} 1-a & -a \\ 0 & c-b \end{pmatrix}\right]$$

The eigenvalues of E_1 are; $\lambda_1 = (1-a)$ and $\lambda_2 = (c-b)$. Therefore, Proposition 2 gives an example of the local dynamics of E_1:

Proposition 1: About the fixed point E_1, the next station holds.

E_1 is a sink point if and only $|\lambda_1| < 1$ & $|\lambda_2| < 1$ if $0 < a < 2$ & $1 - b < c < 1 + b$

E_1 is a non-hyperbolic point if and only $|\lambda_1| = 1$ if $a = 2$&$c = b - 1$

About the fixed point $E^* = (x^*, y^*) = (\frac{b+1}{c}, 1 - \frac{b+1}{c})$ the Jacobian matrix of 2 is

$$J_{E^*} = \begin{bmatrix} 1 - a(\frac{1+b}{c}) & -a(\frac{1+b}{c}) \\ c-1-b & 1 \end{bmatrix}$$

E_* is a source point if and only if $|\lambda_1| > 1$ and $|\lambda_2| > 1$ if $a = -1$&$b = c + 1$.

iii. E_* is a saddle point if and only for $|\lambda_1| < 1$ and $|\lambda_2| > 1$ if $-1 < b < c - 1$&$a < \frac{2c}{b+1}$

E_* is a non-hyperbolic point if and only if $|\lambda_2| = 1$, for $a = -1$ &$b = -1$.

3- Dynamic Analysis of Prey-Predator

Bifurcation Analysis:

The bifurcation occurs when a parameter's value has an impact on how the system behaves. A diagram that depicts the changing dynamic is used to describe this phenomenon. Figure 2 depicts this paragraph's bifurcation diagram (1).

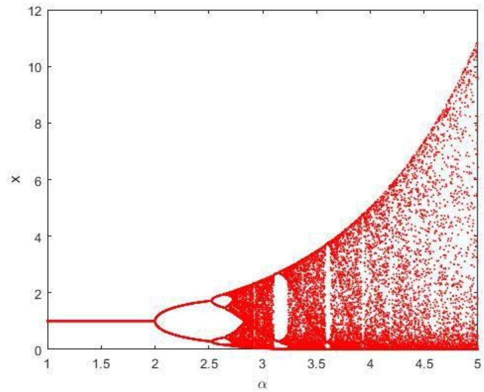

Figure 2: Bifurcation diagram of the Prey-Predator model

Lyapunov Exponent (LE):

The nonlinear dynamical systems' high sensitivity to the initial condition is fundamentally what defines their chaos. Consider a dynamical system with two adjacent trajectories that diverge exponentially. In such instance, the Lyapunov exponent, a chaotic system, is described by this arbitrary invariant. The LE examines (1)'s chaotic nature, as seen in Fig 3. It demonstrates t that the proposed system shows chaotic behavior depending on various factors. To determine the local instability of a given system, one uses the average of LE.keep in mind that the chaotic behavior of system (1) with the range a∈[0,5]

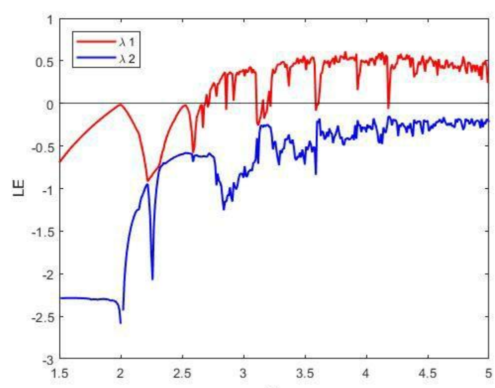

Figure 3: Lyapunov Exponent of the prey-predator

Complexity-Based Sample Entropy:

Using a fundamental method called Sample Entropy, this part investigates the 2D-complexity. ICSM's (SamEn). SamEn was developed by the authors of [12] to determine how much additional data is needed to anticipate the (t + 1)th output of a trajectory based on the (t) outputs from the trajectory's previous (t) iterations.Lower levels of regularity on a chaotic map are indicated by SamEn with greater values. In other words, the chaotic map demonstrates a great degree of complexity and unpredictable behavior.

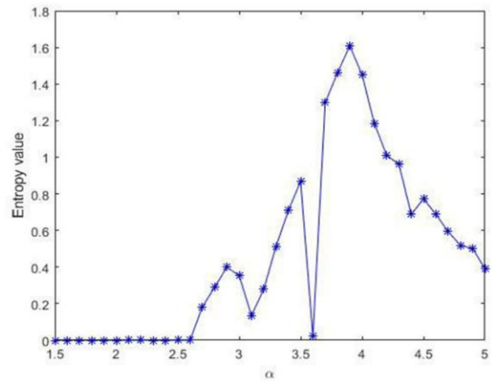

Figure 3. SamEn values of the 2D-ICSM as its parameters change.

4- Controlling Chaotic Discrete System

Let's the an n-dimensional dynamic system :

$x_{k+1}=f(x_k)$ …(3)

where f is a nonlinear vector-valued function and $x \in R^n$ is an n-dimensional vector. Let x_f represent the system's fixed point (1). We select variable feedback control, which is described by: to stabilize the chaotic orbit of

$x_{k+1} = f(x_k) + u(x_k)$ .... (4)

Substitute in (4) feedback control $u(x_k) = \mathcal{M}(f(x_k) - x_k)$ we get

$x_{k+1} = f(x_k) + \mathcal{M}(f(x_k) - x_k)$ ... (5)

Describe a infinitesimal deviation of x_k from x_f as

$\delta x_k = x_k - x_f$. Then,from 5 using the Taylor series, from about x_f , we have

$$x_{k+1} \cong f(x_f) + \frac{\partial f}{\partial x}(x_k - x_f) + \frac{1}{2}\frac{\partial^2 F}{\partial^2 x}(x_k - x_f)^2 + \cdots + \mathcal{M}\left(\frac{\partial F}{\partial x}(x_k - x_f) - 0\right)$$

$$x_{k+1} \cong x_f + J\delta x_k + \mathcal{M}J\,\delta x_k$$

$$x_{k+1} - x_f \cong J\delta x_k + \mathcal{M}(J - I)\,\delta x_k$$

$$\delta_{x_{k+1}} = J\delta x_k + \mathcal{M}(J - I)\,\delta x_k$$

…(6)

where $J=\frac{\partial f}{\partial x_k}\Big|_{x_k=x_f}$ the Jacobin matrix of the initial-state f system that was assessed in the fixed point x_f and I represents the nxn identity matrix..

Making is the aim of control here.$[\delta x_k] = 0$

It requires in order reaching this aim

$$\delta x_{k+1} = Q\delta x_k \quad ...(7)$$

where Q is a n×n matrix and takes the form

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}$$

…(8)

Eq. (7) and Eq. (8) being substituted into Eq (6)

selection of a particular matrix form Q=qI, q∈(-1,1)

We get

$$\delta x_{k+1} = J\delta x_k + \mathcal{M}(J - I)\,\delta x_k$$

$$Q\delta x_k = J\delta x_k + \mathcal{M}(J - I)\,\delta x_k$$

where $(J + \mathcal{M}(J - I) - Q)\,\delta x_k = 0$.

We observe $J + \mathcal{M}(J - I) - Q = 0$.

Note that $\mathcal{M}(J-I) = Q - J$.
At last, we have

$$\mathcal{M} = (qI - J)(J - I).^{-1} \quad \dots (9)$$

5-Stabilizing of the Prey-Predator
The following that algorithm used to stabilize the Prey-Predator system:
<u>Input Data :</u> The chaotic system

$$x_1(k+1) = x_1(k)\, e^{a(1-x_1(k)-x_2(k))}$$
$$x_2(k+1) = cx_1(k)x_2(k) - bx_2(k)$$
$$k = 0,1,\dots,n \quad \dots(10)$$

<u>Output</u> stability of the system.
<u>Algorithm Steps:</u>
Step 1. Calculate fixed point of the Prey-Predator system
Obtaining the fixed point (0.9045, 0.3455) by fixed point iteration method.
Step 2. Calculate the corresponding Jacobain matrix to the fixed point (x_{1f}, x_{2f}) such that:

$$J = \begin{pmatrix} (1 - ax_{1f})e^{a(1-x_{1f}-x_{2f})} & -ax_1 e^{a(1-x_{1f}-x_{2f})} \\ cx_{2f} & cx_1 - b \end{pmatrix} \quad \dots(11)$$

Step 3.
Calculate the matrix $\mathcal{M}$ from Eq. (9) after calculation matrix J from (11) and Calculate the inverse $\mathcal{M}(J-I)^{-1}$

$$\mathcal{M} = \begin{pmatrix} q + (ax_{1f} - 1)e^{-a(x_{1f}+x_{2f}-1)} & ax_{1f}e^{-a(x_{1f}+x_{2f}-1)} \\ -cx_{2f} & b + q - cx_{1f} \end{pmatrix} \quad \dots (12)$$

Step4. Choose the parameter (x_{1f}, x_{2f})=(0, 0),(q_1, q_2)=(0.3,0.5) in (10) we get,
Step 5. Calculation the matrix $\mathcal{M}$ in Eq. (8) at $q_1 = 0.3$& $q_2 = 0.5$
Specifically, we obtain

$$\mathcal{M}_1 = \begin{pmatrix} -1.0131 & 0 \\ 0 & -0.6111 \end{pmatrix}$$

$$\mathcal{M}_2 = \begin{pmatrix} -1.0093 & 0 \\ 0 & -0.7222 \end{pmatrix} \quad \dots(13)$$

In this part, Figures 4 and 5 display the numerical outcomes. Figure 4(a) shows the Prey-Predator map in chaos before stabilization for x_i is added with various values of a∈[0,5], whereas Figure 4(b) shows the Prey-Predator in stability after stabilization for $x_i$is added for a∈[0,5]. The Prey-Predator in Figure 5(a) is chaotic before to adding stabilization for y_i with various settings of a∈[0,5], but the Prey-Predator model in Figure 5(b) is stable following the addition of stabilization for $y_i$with a∈[0,5].

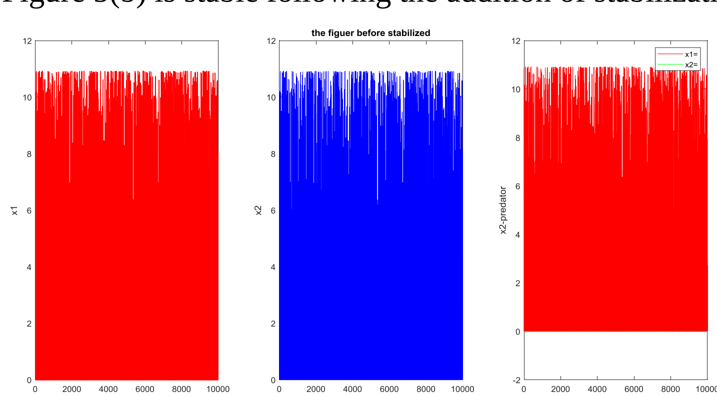

FIGURE (4) the phase portrait of Prey- Predator before being stabilized

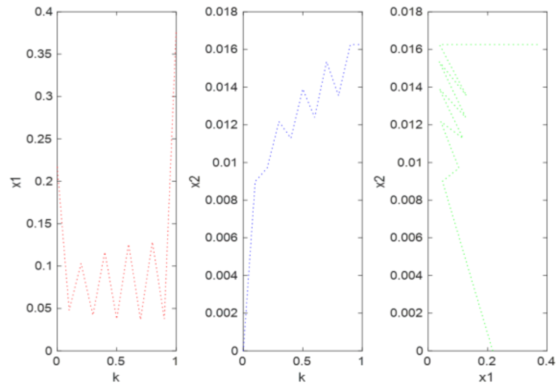

Figure (5) the phase portrait after being stabilized

5- Conclusion

The prey-predator is taken into account in this essay. It is fastened at three locations. To demonstrate the chaotic behavior of these points for particular eigenvalues, their behavior is discussed. The chaotic behavior of this map is stabilized by a simple approach. The chaotic behavior might be controlled using this strategy without the need for changing parameters via simulations for both before and after stabilization various ranges of value a that severely influenced a stable of the prey-predator system; the results illustrate the effectiveness of this strategy.

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