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Fuzzy soft topological vector spaces

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ABSTRACT

In the present paper a notion of fuzzy soft topological vector spaces has been presented and some basic properties of such spaces studied. Fuzzy soft norm on a fuzzy soft linear space defined and the problem of fuzzy soft norm ability of fuzzy soft topological vector spaces has been addressed.

1.Introduction

Most of the practical problems in economics, engineering. environmental sci ence, social science, medical science and so forth cannot be dealt with classical methods because of various types of uncertainties present in these problems. Several theories, for example. probability theory. fuzzy set theory, rough set theory, interval analysis etc. are evolved to address the different kinds of uncertainty problems. But each of these theories has their own difficulties and limitations, perhaps, due to the lack of the parametrization tools of the theory as it was indicated by Molodtsov in [5]. Molodtsov initiated a new mathematical tool, namely, soft set as a generaliza tion of fuzzy set, for dealing with uncertainties and applied it in many different fields such as smoothness of functions, game theory, Ricmann integration, Perron integra tion, probability theory etc. Maji et al. [20,21] dealt with the algebraic operations over soft sets. Recently investigations are going on in developing different mathe matical structures such as algebraic, topological, algebraico-topological ete. over soft sets. To mention some of them, Aktas and Cagman [8] have introduced soft groups: Jun [35 , 36] applied soft sets to the theory of BCK/BCI algebras and intrduced the concept of soft BCK/BCI-algebras; Feng et al. [7] defined soft semi-rings: Shabir and Ali [14] studied soft semi-groups and soft ideals; Babitha and Sunil [10] studied soft set relations and functions; Kharal and Ahmed [2]as well as Majumdar and Samanta [21] studied soft mappings. Shabir and Naz [15] came up with an idea of opological spaces. Afterwords Zorlutuna et al. [15], Cagman et al. [16], Hussain ed [|10], Hazra et al. [9]. Georgiou et al.[6]. Áygunoglu et al. [3], Babitha et al. [11], Mondal et al. [31] M Chinev et al. [17] and many other authors studied topological spaces. Recently metric space, linear space, topological group, Banach algebra, topological vector space structures in soft setting are also studied [16, 25, 26, 28, 29, 23].

In this paper, we introduce a notion of fuzzy soft topological vector space. For doing this we consider the vector space to be a fuzzy soft vector space and the underlying topology is taken to be a new type of fuzzy soft topology which is defined and developed by using the concepts of clementary union, intersection and complement of fuzzy soft sets although, interestingly, with respect to these operations the relevant distributive properties and the law of excluded middle do not hold. Also in this paper, we study some basic properties of this space and finally the problem of fuzzy soft normability of fuzzy soft topological vector space is addressed to follows: In Section 2, we briefly review some basic notions. This paper is organized as and facts on fuzzy soft sets which are used to prove or illustrate

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results in subsequence sections. Section 3 is devoted to study some properties of balanced, convex and Labsorbing fuzzy soft sets. The concept of a fuzzy soft topological vector space is introduced in section 4 along with some basic properties of such spaces. Finally, in Section 5, we present the conclusion.

2.Preliminaries

Let U , V are a universal set and E be a set of parameters . Let P(U) denote the power set of U and A be a subset of E .and I = [0,1] .

Definition 2.1: [6]: The $\mathcal{S}$. set is F^A over U, where F is a mapping given be F^A : A→ P(U).

Definition 2.2: [38]: A $\mathcal{F}$. set f over U is defined by the mapping $f : U \longrightarrow I$. The set of all $\mathcal{F}$. set in U is I^U.

Example 2.3: Every ordinary set is $\mathcal{F}$. set . In other words if B is a subset of U (ordinary set) , then the membership in B is often viewed as characteristic function $U_B$from U to {0,1} such that

$$U_B(a) = \begin{cases} 1 & , \ a \in U \\ 0 & , \ a \notin U \end{cases}$$

Definition 2.4: [38]: Let $f_1$and f_2 be $\mathcal{F}$. sets over U .

(i) $f_1 \,\breve{\subseteq}\, f_2$. If $f_1(u) \le f_2(u)$ for any $u \in U$.

(ii) $f_1 \,\breve{=}\, f_2$ iff $f_1 \,\breve{\subseteq}\, f_2$ and $f_2 \,\breve{\subseteq}\, f_1$.

(iii) $f_1 \,\breve{\cup}\, f_2$ is $(f_1 \cup f_2)(u) = \max\{f_1(u) , f_2(u)\}$ for all $u \in U$.

(iv) $f_1 \,\breve{\cap}\, f_2$ is $(f_1 \cap f_2)(u) = \min\{f_1(u), f_2(u)\}$for all $u \in U$.

Definition 2.5: [19]: (i) The $\mathcal{F}$. set f is called absolute $\mathcal{F}$. set , denoted by $\bar{1}$, if $f(u) = 1 \ \forall u \in U$.

(ii)A $\mathcal{F}$. point u_t in U is special $\mathcal{F}$. set with membership function defined by

$$u\,(u^0) = \begin{cases} t & , \ 0 < t \le 1. \ u = u^0 \\ 0 & , \qquad\qquad u \neq u^0 \end{cases}$$

And denoted u_t.

Definition 2.6: [1] Let $f_1$and f_2 be $\mathcal{F}$.set in U ,V respectively . Denote by $f_1 \times f_2$the $\mathcal{F}$.set in $U \times V$

For which $(f_1 \,\breve{\times}\, f_2)(u,v) = \min\{f_1(u) , f_2(v)\}$, $(u,v) \in U \times V$.

Definition 2.7: [1] Let $f_1$and f_2 be $\mathcal{F}$.set in a linear space U over $\mathbb{F}$, let $\lambda \in \mathbb{F}$ then

(i)$(f_1 \,\breve{+}\, f_2)\,(u_3)$ = sup {min{ $f_1(u_1)$, $f_2(u_2)$ }: $u_3 = u_1 + u_2$, $u_3 , u_1 , u_2 \in U$ } .

(ii) $(\lambda f_1)(u) = \begin{cases} f_1(\frac{1}{\lambda}u) & , \lambda \neq 0 \\ 0 & , \lambda = 0 , \ u \neq 0 \\ \sup\{f_1(u) : u \in U \} & , \lambda = 0 , \ u \neq 0 \end{cases}$

(iii) $f(u) < (\lambda f)(u)$ for all $u \in U$.

(iv) $f_1 \breve{+} f_2$ and λf_1 are $\mathcal{F}$. sets in U .

Definition 2.8: [1] let f is $\mathcal{F}$. set in a vector space U .

(i) f is $\mathcal{F}$. sub space if $f(b_1u_1 + b_2u_2) \geq \min\{ f(u_1) , f(u_2) \}$ for all $b_1 , b_2 \in \mathbb{F}$ and $u_1 , u_2 \in U$.

(ii) f is $\mathcal{F}$. convex if $f(\lambda u_1 + (1-\lambda)u_2) \geq \min\{ f(u_1) , f(u_2) \}$ for all $u_1 , u_2 \in U$ and $0 \leq \lambda \leq 1$.

(iii) f is $\mathcal{F}$. balaced if $f(\lambda u_1) \geq f(u_1)$ all $u_1 , u_2 \in U$ and $|\lambda| \leq 1$.

(iv) f is $\mathcal{F}$. bsorbing if $\text{sub}_{\lambda>0} f(\lambda u) = 1$.

Definition 2.9: [1] Let f_1 and f_2 be $\mathcal{F}$. set in a linear space U over $\mathbb{F}$, let $\lambda \in \mathbb{F}$ then

(i) If f_1 , f_2 are $\mathcal{F}$. sub space in U , then $f_1 \breve{\cap} f_2$, βf_1 are $\mathcal{F}$. sub space in U .

(ii) If f_1 , f_2 are $\mathcal{F}$. convex in U , then $f_1 \breve{\cap} f_2$, βf_1 are $\mathcal{F}$. convex in U .

(iii) If f_1 , f_2 are $\mathcal{F}$. balaced in U , then $f_1 \breve{\cap} f$, βf_1 are $\mathcal{F}$. balaced in U .

(iv) If f_1 , f_2 are $\mathcal{F}$. bsorbing in U , then $f_1 \breve{\cap} f_2$, βf_1 are $\mathcal{F}$. bsorbing in U.

Definition 2.10:[14]: Let U be a universal set , E be a set of parameter and $A \subseteq E$. A $\mathcal{F}.\mathcal{s}$. set Γ_A over U, is a mapping $\Gamma_A : A \longrightarrow I^U$, which is defined as $\Gamma_A(e) \cong f^e_{\Gamma_A} \breve{\in} I^U$.

Example 2.11 : Let $U = \{u^1, u^2, u^3\}$ and $A = \{\varepsilon , \alpha , \gamma\}$ and $B = \{u^1, u^2\} \subseteq U$ .clear U_B is $\mathcal{F}$. set , define U_B by $U_B : U \longrightarrow I$, $U_B(u) = \{^{1\ ,\ u \in B}_{0\ ,\ u \notin B}$ then $\Gamma_A \hat{=} \{ (\varepsilon , U_B) , (\alpha , U_B) , (\gamma, U_B)\}$, is $\mathcal{F}.\mathcal{s}$. set .

Definition 2.12: [14]

(1) The $\mathcal{F}.\mathcal{s}$. set Γ_A is called null $\mathcal{F}.\mathcal{s}$. set and is denoted by Γ_ϕ. Here $\Gamma_A(e) \cong \bar{0}$ for all $e \in A$ where $\bar{0}(u) = 0$ for each $u \in U$.
(2) The $\mathcal{F}.\mathcal{s}$. set Γ_A is called absolute $\mathcal{F}.\mathcal{s}$. set and is donated by Γ_U . Here $\Gamma_A(e) \cong \bar{1}$ for all $e \in A$, where $\bar{1}(u) = 1$ for each $u \in U$.
(3) For two $\mathcal{F}.\mathcal{s}$. sets $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ over a universe U. We say that $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$. subset of $\Gamma_{2_{A_2}}$ i.e $\Gamma_{1_{A_1}} \hat{\subseteq} \Gamma_{2_{A_2}}$ if $\Gamma_{1_{A_1}}(e) \breve{\subseteq} \Gamma_{2_{A_2}}(e)$ for all $e \in A_1$ and $A_1 \subseteq A_2$.
(4) For two $\mathcal{F}.\mathcal{s}$. sets $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ over a universe U . $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathcal{s}$. equal if $\Gamma_{1_{A_1}} \hat{\subseteq} \Gamma_{2_{A_2}}$ and $\Gamma_{2_{A_2}} \hat{\subseteq} \Gamma_{1_{A_1}}$.
(5) The union of two $\mathcal{F}.\mathcal{s}$. sets $\Gamma_{1_{A_1}}$ and $\Gamma_{2_{A_2}}$ over the universe U is the $\mathcal{F}.\mathcal{s}$. set

$\Gamma_{3_{A_3}} = \Gamma_{1_{A_1}} \hat{\cup} \Gamma_{2_{A_2}}$, where $A_3 = A_1 \cup A_2$ and for all $e \in A_3$

$$\Gamma_{3_{A_3}}(e) \cong \begin{cases} \Gamma_{1_{A_1}}(e) , & \text{if } e \in A_1 - A_2 \\ \Gamma_{2_{A_2}}(e) , & \text{if } e \in A_2 - A_1 \\ \Gamma_{1_{A_1}}(e) \cup \Gamma_{2_{A_2}}(e) , & \text{if } e \in A_1 \cap A_2 \end{cases}$$

(6) The intersection of $\Gamma_{1_{A_1}}$ and $\Gamma_{2_{A_2}}$ is a $\mathcal{F}.\mathcal{s}.$ set $\Gamma_{3_{A_3}} = \Gamma_{1_{A_1}} \hat{\cap} \Gamma_{2_{A_2}}$, where

$A_3 = A_1 \cap A_2$ and $\Gamma_{3_{A_3}}(e) \cong \Gamma_{1_{A_1}}(e) \breve{\cap} \Gamma_{2_{A_2}}(e)$, $\forall$ $e \in A_3$.

In particular if $A_1 \cap A_2 = \emptyset$ or $\Gamma_{1_{A_1}}(e) \breve{\cap} \Gamma_{2_{A_2}}(e) \cong \bar{0}$ for every $e \in A_1 \cap A_2$, Then $\Gamma_{3_{A_3}}(e)$ $\cong \bar{0}$.

Definition 2.13: [14] The complement of a $\mathcal{F}.\mathcal{s}.$ set Γ_A is defined by Γ_A^c , where Γ_A^c:A$\longrightarrow I^U$ is a mapping given by $\Gamma_A^c(e) \cong \bar{1} / \Gamma_A(e)$, for all e $\in$ A .

Definition 2.14:[39]: Let Γ_{1_A}, Γ_{2_A} are $\mathcal{F}.\mathcal{s}.$ set in U ,V respectively . then their product denoted is defiend by $(\Gamma_{1_A} \hat{\times} \Gamma_{2_A}) = (\Gamma_1 \hat{\times} \Gamma_2 , A)$, where $(\Gamma_{1_A} \hat{\times} \Gamma_{2_A})(e) = \Gamma_{1_A}(e) \breve{\times} \Gamma_{2_A}(e)$, $e \in A$. It is clear that $(\Gamma_1 \hat{\times} \Gamma_2 , A)$ is $\mathcal{F}.\mathcal{s}$ .set over U $\times$ V .

Definition 2.15: [35] : (i) The $\mathcal{F}.\mathcal{s}.$ set Γ_A is called a $\mathcal{F}.\mathcal{s}.$ point over U , denoted by $\hat{u}_{f^e_{\Gamma_A}}$

$$f^e_{\Gamma_A}(u) = \begin{cases} t & , \text{if } u = u_o \text{ and } e = e_o \in A \\ 0 & , \text{if } u \in U - \{u_0\} \text{ or } e \in A - \{e_o\} \end{cases}$$

Where t $\in$ (0,1] .

(ii)The $\mathcal{F}.\mathcal{s}.$ point $\hat{u}_{f^e_{\Gamma_A}}$ is said to be in the $\mathcal{F}.\mathcal{s}.$ set $\Gamma_{1_{A_1}}$ (belongs to it), donated by $\hat{u}_{f^e_{\Gamma_A}} \hat{\in} \Gamma_{1_{A_1}}$ if for the element e $\in$ A , we have $\Gamma_A(e) \breve{\subseteq} \Gamma_{1_{A_1}}(e)$.

Definition 2.16: [23] Let $\mathbb{R}$ be the set of all real numbers, E be a set of parameters, A$\subseteq$E and $\mathcal{B}$ be the collection of all non null $\mathcal{F}.$ bounded subsets of $\mathbb{R}$. A $\Re_A$ is called a $\mathcal{F}.\mathcal{s}.$ real set over $\mathbb{R}$ and is defined as a set of ordered pairs $\Re_A$={(e, $\Re_A(e)$) :e $\in$ A , $\Re_A(e) \in \mathcal{B}$ }, Where $\Re$ is mapping given by $\Re_A$:A $\longrightarrow$ $\mathcal{B}$, A is called the support of $\Re_A$.

Definition 2.17: [23]: The $\mathcal{F}.\mathcal{s}.$ real set $\Re_A$ is called a $\mathcal{F}.\mathcal{s}.$ real number in $\mathbb{R}$, denoted by $\hat{r}$, if it is a singleton $\mathcal{F}.\mathcal{s}.$ real set . $\mathbb{R}$(A) denotes the set of all $\mathcal{F}.\mathcal{s}.$ real numbers and $\mathbb{R}^+$(A) denotes the set of all non-negative $\mathcal{F}.\mathcal{s}.$ real numbers.

Definition 2.18:[25]: A $\mathcal{F}.\mathcal{s}.$ topology T_{Γ_A} on (U, E) is a family of $\mathcal{F}.\mathcal{s}.$ sets over (U, E) .Satisfying the following properties

(i) Γ_A , Γ_ϕ .

(ii) if $\Gamma_{i_{A_i}} \hat{\in} T_{\Gamma_A}$ then $\cap_{i=1}^{2} \Gamma_{i_{A_i}} \hat{\in} T_{\Gamma_A}$.

(iii) if $\Gamma_{i_{A_i}} \hat{\in} T_{\Gamma_A}$ for all i $\in$ I, an index set, then $\cup_{i\in I} \Gamma_{i_{A_i}} \hat{\in} T_{\Gamma_A}$.

Definition 2.19 :[25]: If T_{Γ_A} is a $\mathcal{F}.\mathcal{s}$. topology on (U, E) , the apire (Γ_A , T_{Γ_A}) is said to be $\mathcal{F}.\mathcal{s}$. topological space. Each member of T_{Γ_A} is called $\mathcal{F}.\mathcal{s}$. open set in (Γ_A, T_{Γ_A}) .

Definition 2.20:[35] Let (Γ_A, T_{Γ_A})be a $\mathcal{F}.\mathcal{s}$. topological space. A $\mathcal{F}.\mathcal{s}$. set is called $\mathcal{F}.\mathcal{s}$. closed if its complement isa number of T_Γ.

Proposition 2.21:[35]: Let (Γ_A, T_{Γ_A}) be a $\mathcal{F}.\mathcal{s}$. topological space and let $T^*{}_{\Gamma_A}$ be the collection of all $\mathcal{F}.\mathcal{s}$. closed sets. Then

(i) Γ_A , Γ_ϕ .

(ii) if $\Gamma_{i_{A_i}} \widehat{\in} T_\Gamma$ then $\widehat{\cup}_{i=1}^{2} \Gamma_{i_{A_i}} \widehat{\in} T^*{}_{\Gamma_A}$.

(iii) if $\Gamma_{i_{A_i}} \widehat{\in} T_\Gamma$ for all i ∈ I, an index set, then $\widehat{\cap}_{i\in I} \Gamma_{i_{A_i}} \widehat{\in} T^*{}_{\Gamma_A}$

Definition 2.22:[35]: Let (Γ_A, T_{Γ_A}) be a $\mathcal{F}.\mathcal{s}$. topological space and $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$.set over U . then the collection $\widehat{T}_{\Gamma_{1_{A_1}}}$ ={ $\Gamma_{1_{A_1}} \widehat{\cap} \Gamma_{2_{A_2}}$: $\Gamma_{2_{A_2}} \widehat{\in} T_\Gamma$ }is a $\mathcal{F}.\mathcal{s}$. topology space on the $\mathcal{F}.\mathcal{s}$. subset $\Gamma_{1_{A_1}}$ relative to parameter set A . And $\widehat{T}_{\Gamma_{1_{A_1}}}$ is called $\mathcal{F}.\mathcal{s}$. subspace topology and ($\Gamma_{1_{A_1}}$, $\widehat{T}_{\Gamma_{1_{A_1}}}$) is called a $\mathcal{F}.\mathcal{s}$. topological subspace of (Γ_A, T_{Γ_A}) .

Definition 2.23:[28]: Let $\left(\Gamma_A, T_{\Gamma_A}\right)$, be a $\mathcal{F}.\mathcal{s}$. topological space. Then the $\Gamma_{1_{A_1}}$ is said to be a $\mathcal{F}.\mathcal{s}$. neighborhood (for short $\mathcal{F}.\mathcal{s}$. nbhd) of $\mathcal{F}.\mathcal{s}$. point $\widehat{u^3}_{f_3{}^{e_3}_{\Gamma_{3_{A_3}}}}$ if there exists a $\mathcal{F}.\mathcal{s}$. open set $\Gamma_{2_{A_2}}$ such that $\widehat{u^3}_{f_3{}^{e_3}_{\Gamma_{3_{A_3}}}} \widehat{\in} \Gamma_{2_{A_2}} \widehat{\subseteq} \Gamma_{1_{A_1}}$. The set of all $\mathcal{F}.\mathcal{s}$. nbhd of $\widehat{u^3}_{f_3{}^{e_3}_{\Gamma_{3_{A_3}}}}$ is denoted by $\mathcal{N}_{\widehat{u^3}_{f_3{}^{e_3}_{\Gamma_{3_{A_3}}}}}$.

Definition 2.24:[35]: (i) Let ((Γ_A, T_{Γ_A}), be a $\mathcal{F}.\mathcal{s}$. topological space. Let $\Gamma_{1_{A_1}}$ be a $\mathcal{F}.\mathcal{s}$. set over (U, E). Then the $\mathcal{F}.\mathcal{s}$. closure of $\Gamma_{1_{A_1}}$,denoted by $\overline{\Gamma_{1_{A_1}}}$, is defined as the intersection of all $\mathcal{F}.\mathcal{s}$. closed sets which contain Γ_{A_1} .That is $\overline{\Gamma_{1_{A_1}}} = \widehat{\cap} \{\Gamma_{2_{A_2}} : \Gamma_{2_{A_2}}$ is a $\mathcal{F}.\mathcal{s}$. closed and $\Gamma_{1_{A_1}} \widehat{\subseteq} \Gamma_{2_{A_2}}\}$.

Clearly, $\Gamma_{1_{A_1}}$ is the smallest $\mathcal{F}.\mathcal{s}$. closed set over (U, E) which contain $\Gamma_{1_{A_1}}$ .is also clear that $\overline{\Gamma_{1_{A_1}}}$, is $\mathcal{F}.\mathcal{s}$. closed and $\Gamma_{1_{A_1}} \widehat{\subseteq} \overline{\Gamma_{1_{A_1}}}$.

(ii) Let $\left(\Gamma_A, T_{\Gamma_A}\right)$ be a $\mathcal{F}.\mathcal{s}$. topological space. Let $\Gamma_{1_{A_1}}$, be a $\mathcal{F}.\mathcal{s}$. set over (U, E). Then the $\mathcal{F}.\mathcal{s}$. interior of $\Gamma_{1_{A_1}}$,denoted by $(\Gamma_{1_{A_1}})^\circ$, is defined as the union of all $\mathcal{F}.\mathcal{s}$. open sets contained in $(\Gamma_{1_{A_1}})^\circ$.That is $(\Gamma_{1_{A_1}})^\circ = \widehat{\cup} \{\Gamma_{2_{A_2}} : \Gamma_{2_{A_2}}$ is a $\mathcal{F}.\mathcal{s}$. open and $\Gamma_{2_{A_2}} \widehat{\subseteq} \Gamma_{1_{A_1}}\}$

Clearly, $(\Gamma_{1_{A_1}})^\circ$ is the largest $\mathcal{F}.\mathcal{s}$. open set over (U, E) which contained in $\Gamma_{1_{A_1}}$. It is also clear that $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$. closed and $(\Gamma_{1_{A_1}})^\circ \widehat{\subseteq} \Gamma_{1_{A_1}}$.

Definition 2.25:[35]: Let $\left(\Gamma_A, T_{\Gamma_A}\right)$be a fuzzy soft topological space . and $\Gamma_{1_{A_1}} \widehat{\subseteq} \Gamma_A$, $\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \Gamma_A$, $\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \overline{\Gamma_{1_{A_1}}}$ iff $\Gamma_{2_{A_2}} \widehat{\cap} \Gamma_{1_{A_1}} \widehat{\neq} \Gamma_\emptyset$, $\forall\, \Gamma_{2_{A_2}} \widehat{\in} \mathcal{N}_{\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}}$, $\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \Gamma_A$.

Definition 2.26: Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathcal{s}$.sets over U , then $\mathcal{F}.\mathcal{s}$. arelation from $\Gamma_{1_{A_1}}$ to $\Gamma_{2_{A_2}}$ is a $\mathcal{F}.\mathcal{s}$.subset of $\Gamma_{1_{A_1}} \times \Gamma_{2_{A_2}}$. In other words , a $\mathcal{F}.\mathcal{s}$. relation from $\Gamma_{1_{A_1}}$ to $\Gamma_{2_{A_2}}$ is of the form $\Gamma_{3_{A_3}}$

where $A_3 \subseteq A_1 \times A_2$ and $\Gamma_{3_{A_3}}(e_3, e''_3) = \Gamma_{4_{A_4}}(e_3, e''_3), \forall (e_3, e''_3) \in A_3$ where $\Gamma_{4_{A_4}} = \Gamma_{1_{A_1}} \times \Gamma_{2_{A_2}}$ .Any subset $\Gamma_{1_{A_1}} \times \Gamma_{1_{A_1}}$
Is called a $\mathcal{F}.\mathcal{s}$. relation on $\Gamma_{1_{A_1}}$. In an equivalent way,can define the relation $\mathcal{R}$ on the fuzzy soft set Γ_A in the parameterized form as follows: If $\Gamma_A = \{F(e), F(\alpha), \ldots..\}$, then $F(e)\ \mathcal{R}\ F(\alpha)$ iff $F(e) \times F(\alpha) \in \mathcal{R}$.

Definition 2.27: Let Γ_A, $\Upsilon_B$are $\mathcal{F}.\mathcal{s}$.sets over U , then $\mathcal{F}.\mathcal{s}$. arelation $\mathfrak{J} \widehat{\subseteq} \Gamma_A \widehat{\times} \Upsilon_B$ is a $\mathcal{F}.\mathcal{s}$.subset of $\Gamma_A \widehat{\times} \Upsilon_B$ is called a $\mathcal{F}.\mathcal{s}$. mapping from $\Gamma_A$to Υ_B[denoted by $\mathfrak{J}: \Gamma_A \widehat{\rightarrow} \Upsilon_B$] if the following conditions as satisfied : For each $\mathcal{F}.\mathcal{s}$. point $\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \Gamma_A$, there exists only one $\mathcal{F}.\mathcal{s}$ .point $\widehat{w^1}_{\hat{g}_1{}^{\alpha_1}_{\Upsilon_{1_{B_1}}}} \widehat{\in} \Upsilon_B$such that $\widehat{u^1}_{f^{e_1}_{\Gamma_{1_{A_1}}}}\ \mathfrak{J}\ \widehat{w^1}_{\hat{g}_1{}^{\alpha_1}_{\Upsilon_{1_{B_1}}}}$ which will be noted as $\mathfrak{J}(\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) \widehat{=} \widehat{w^1}_{\hat{g}_1{}^{\alpha_1}_{\Upsilon_{1_{B_1}}}}$. for each null $\mathcal{F}.\mathcal{s}$. point $\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \Gamma_A$, $\mathfrak{J}(\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}})$ is null $\mathcal{F}.\mathcal{s}$. point of Υ_B.

Definition 2.28: Let Γ_A , Υ_B are $\mathcal{F}.\mathcal{s}$.sets over U. and let $\mathfrak{J}: \Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$. mapping .

The $\mathcal{F}.\mathcal{s}$.set imag of $\Gamma_{1_{A_1}} \widehat{\subseteq} \Gamma_A$ under $\mathcal{F}.\mathcal{s}$. mapping $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$. set denoted by $\mathfrak{J}(\Gamma_{1_{A_1}})$ of the form $\mathfrak{J}(\Gamma_{1_{A_1}}) = \widehat{\cup}\{\mathfrak{J}(\hat{u}_{f^e_{\Gamma_A}}), \forall\ \widehat{u^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \widehat{\in} \Gamma_{1_{A_1}}$, $\mathfrak{J}(\Gamma_\emptyset) \widehat{=} \Gamma_\emptyset$ for each fuzzy soft mapping $\mathfrak{J}\}$.

Definition 2.29: Let Γ_A , Υ_B are $\mathcal{F}.\mathcal{s}$.sets over U. and let $\mathfrak{J}: \Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$. mapping .

The $\mathcal{F}.\mathcal{s}$. inverse imag of $\Upsilon_{1_{B_1}} \widehat{\subseteq} \Upsilon_B$ under $\mathcal{F}.\mathcal{s}$. mapping $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$. set denoted by $\mathfrak{J}^{-1}(\Upsilon_{1_{B_1}})$ of the form
$\mathfrak{J}^{-1}(\Upsilon_{1_{B_1}}) = \widehat{\cup}\{\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} : \widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \Gamma_A\ , \mathfrak{J}(\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) \widehat{\in} \Upsilon_{1_{B_1}}\}$

Definition 2.30: Let Γ_A , Υ_B are $\mathcal{F}.\mathcal{s}$.sets over U. and let $\mathfrak{J}: \Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$. mapping .

Is said to be ($\mathcal{F}.\mathcal{s}$.injective) if each $\mathcal{F}.\mathcal{s}$. point in Γ_A is related to a different in Υ_B .More formally $\mathfrak{J}(\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) \widehat{=} \mathfrak{J}(\widehat{w^1}_{\hat{g}_1{}^{\alpha_1}_{\Upsilon_{1_{B_1}}}})$ implies $\widehat{w^1}_{\hat{g}_1{}^{\alpha_1}_{\Upsilon_{1_{B_1}}}} \widehat{=} \widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}$.

Definition 2.31: Let Γ_A , Υ_B are $\mathcal{F}.\mathcal{s}$.sets over U. and let $\mathfrak{J}: \Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$. mapping .

Is said to be ($\mathcal{F}.\mathcal{s}$.injective) if fory every $\mathcal{F}.\mathcal{s}$. point in Υ_B , there is a $\mathcal{F}.\mathcal{s}$. point in $\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}$ in $\Gamma_A$there is $\mathcal{F}.\mathcal{s}$. point inΓ_A such that $\mathfrak{J}(\widehat{u^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) \widehat{=} \widehat{w^1}_{g_1{}^{\alpha_1}_{\Upsilon_{1_{B_1}}}}$.

Definition 2.32: Let Γ_A , Υ_B are $\mathcal{F}.\mathcal{s}$.sets over U. and let $\mathfrak{J}: \Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$. mapping .

Is said to be ($\mathcal{F}.\mathcal{s}$.bijective) if it is $\mathcal{F}.\mathcal{s}$.injective and $\mathcal{F}.\mathcal{s}$.injective .

Proposition 2.33 : Let $\mathfrak{J}: \Gamma_A \widehat{\rightarrow} \Upsilon_B$ a $\mathcal{F}.\mathcal{s}$.mapping

(i) If $\Gamma_{1_{A_1}}$ $\mathcal{F}.\mathcal{s}$.set in U , then $\Gamma_{1_{A_1}} \widehat{\subseteq} \mathfrak{J}^{-1}(\mathfrak{J}(\Gamma_{1_{A_1}}))$. In particular if $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$. injective then $\Gamma_{1_{A_1}} \widehat{=} \mathfrak{J}^{-1}(\mathfrak{J}(\Gamma_{1_{A_1}}))$

(ii) If γ_B is $\mathcal{F}.\mathcal{s}$.set in V , then $\mathfrak{J}(\mathfrak{J}^{-1}(\gamma_{1_{B_1}})) \widehat{\subseteq} \gamma_{1_{B_1}}$. In particular if $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$.surjective then $\gamma_{1_{B_1}} \widehat{=} \mathfrak{J}(\mathfrak{J}^{-1}(\gamma_{1_{B_1}}))$

Proof : Obviously

Definition 2.34: Let (Γ_A, T_{Γ_A}), $(\Upsilon_B, \Sigma_{\Upsilon_B})$ be two $\mathcal{F}.\mathscr{s}$. topological spaces and $\mathfrak{J} :(\Gamma_A, T_{\Gamma_A}) \longrightarrow (\Upsilon_B, \Sigma_{\Upsilon_B})$

Be a $\mathcal{F}.\mathscr{s}$. mapping . then $\mathfrak{J}$ is said to be

(i) An $\mathcal{F}.\mathscr{s}$. open mapping iff $\mathfrak{J}(\Gamma_{1_{A_1}}) \hat{\in} \Sigma_\Upsilon$ for all $\Gamma_{1_{A_1}} \hat{\in} T_{\Gamma_A}$.

(ii) A $\mathcal{F}.\mathscr{s}$. closed mapping iff $\mathfrak{J}(\Gamma_{1_{A_1}}) \hat{\in} \Sigma_{\Upsilon_B}$is soft closed in Σ_{Υ_B} for every $\mathcal{F}.\mathscr{s}$. closed set $\Gamma_{1_{A_1}}$ in T_Γ

(iii)A $\mathcal{F}.\mathscr{s}$. continuous mapping iff $\mathfrak{J}^{-1}(\Upsilon_{1_{B_1}}) \hat{\in} T_{\Gamma_A}$for all $\Upsilon_{1_{B_1}} \hat{\in} \Sigma_{\Upsilon_B}$.

Definition 2.35: Let (Γ_A, T_{Γ_A}), $(\Upsilon_B, \Sigma_{\Upsilon_B})$ be two $\mathcal{F}.\mathscr{s}$. topological spaces and $\mathfrak{J} :(\Gamma_A, T_{\Gamma_A}) \rightrightarrows (\Upsilon_B, \Sigma_{\Upsilon_B})$

Be a $\mathcal{F}.\mathscr{s}$. mapping . then $\mathfrak{J}$ is said to be fuzzy soft homeomorphism if $\mathfrak{J}$ is $\mathcal{F}.\mathscr{s}$. bijective , $\mathcal{F}.\mathscr{s}$. open , $\mathcal{F}.\mathscr{s}$. continuous mapping .

Theorem 2.36: Let $\mathfrak{J} :\Gamma_A \rightrightarrows \Upsilon_B$ be a $\mathcal{F}.\mathscr{s}$.bijective and $\mathcal{F}.\mathscr{s}$.continuous mapping . Then the following statements are equivalent

(i)$\mathfrak{J}^{-1}$ is $\mathcal{F}.\mathscr{s}$.continuous

(ii) $\mathfrak{J}$ is $\mathcal{F}.\mathscr{s}$.open

(iii)$\mathfrak{J}$ is $\mathcal{F}.\mathscr{s}$.closed

Proof :

(i)⇒(ii) Let $\Upsilon_{1_{B_1}}$be $\mathcal{F}.\mathscr{s}$.open set in $(\Upsilon_B, \Sigma_{\Upsilon_B})$ since $\mathfrak{J}^{-1}$ is $\mathcal{F}.\mathscr{s}$.continuous mapping then$(\mathfrak{J}^{-1})^{-1}(\Upsilon_{1_{B_1}})$ is $\mathcal{F}.\mathscr{s}$. open set in :(Γ_A, T_{Γ_A}) since $\mathfrak{J}$ is $\mathcal{F}.\mathscr{s}$.bijective then $(\mathfrak{J}^{-1})^{-1}(\Upsilon_{1_{B_1}}) = \Upsilon_{1_{B_1}}$, is $\mathcal{F}.\mathscr{s}$.open mapping.
(ii)⟹(iii) Let $\Gamma_{1_{A_1}}$be $\mathcal{F}.\mathscr{s}$.open set in (Γ_A, T_{Γ_A})then $(\Gamma_{1_{A_1}})^c$is $\mathcal{F}.\mathscr{s}$.open set in(Γ_A, T_{Γ_A}),hence $\mathfrak{J}((\Gamma_{1_{A_1}})^c)$ is $\mathcal{F}.\mathscr{s}$.open set in $(\Upsilon_B, \Sigma_{\Upsilon_B})$
$\mathfrak{J}((\Gamma_{1_{A_1}})^c) = \mathfrak{J}(\Gamma_A \backslash \Gamma_{1_{A_1}}) = \mathfrak{J}(\Gamma_A) \backslash \mathfrak{J}(\Gamma_{1_{A_1}}) = \Upsilon_B \backslash \mathfrak{J}(\Gamma_{1_{A_1}})$ is $\mathcal{F}.\mathscr{s}$.open, $\mathfrak{J}(\Gamma_{1_{A_1}})$ is $\mathcal{F}.\mathscr{s}$.closed set, $\mathfrak{J}$ is $\mathcal{F}.\mathscr{s}$.closed mapping.
(iii)→(i) Let $\Gamma_{1_{A_1}}$be $\mathcal{F}.\mathscr{s}$.open set in (Γ_A, T_{Γ_A})then $(\Gamma_{1_{A_1}})^c$ is $\mathcal{F}.\mathscr{s}$.closed set in (Γ_A, T_{Γ_A}) since $\mathfrak{J}$ is $\mathcal{F}.\mathscr{s}$.closed then $\mathfrak{J}(\Gamma_{1_{A_1}})$ is $\mathcal{F}.\mathscr{s}$.closed set in $(\Upsilon_B, \Sigma_{\Upsilon_B})$ then $\mathfrak{J}((\Gamma_{1_{A_1}})^c) = \mathfrak{J}(\Gamma_A \backslash \Gamma_{1_{A_1}}) = \mathfrak{J}(\Gamma_A) \backslash \mathfrak{J}(\Gamma_{1_{A_1}}) = \Upsilon_B \backslash \mathfrak{J}(\Gamma_{1_{A_1}})$ is $\mathcal{F}.\mathscr{s}$.closed set in$(\Upsilon_B, \Sigma_{\Upsilon_B})$ hence $\mathfrak{J}(\Gamma_{1_{A_1}}) = (\mathfrak{J}^{-1})^{-1}(\Gamma_{1_{A_1}})$) is $\mathcal{F}.\mathscr{s}$.open set in $(\Upsilon_B, \Sigma_{\Upsilon_B})$.

Theorem 2.37: Let $\mathfrak{J} :(\Gamma_A, T_{\Gamma_A}) \rightrightarrows (\Upsilon_B, \Sigma_{\Upsilon_B})$be a $\mathcal{F}.\mathscr{s}$.continuous mapping . Then the following statements are equivalent

(i)$\mathfrak{J}$ is $\mathcal{F}.\mathscr{s}$.continuous

(ii)$\mathfrak{J}^{-1}(\Upsilon_{1_{B_1}}) \hat{\in} T^*{}_\Gamma$for all $\Upsilon_{1_{B_1}} \hat{\in} \Sigma_\Upsilon$.

Proof : Obviously

Theorem 2.38: Let $\mathfrak{J}:\Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$.mapping . Then $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$.continuous iff for each $\Gamma_{1_{A_1}}$ of Γ_A , $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$.continuous iff for each $\Gamma_{1_{A_1}}$ of Γ_A , $\mathfrak{J}(\overline{\Gamma_{1_{A_1}}}) \widehat{\subseteq} \overline{\mathfrak{J}(\Gamma_{1_{A_1}})}$.

Proof :

(⇒)Let $\mathfrak{J}:\Gamma_A \widehat{\rightarrow} \Upsilon_B$ be $\mathcal{F}.\mathcal{s}$.continuous mapping $\mathfrak{J}(\Gamma_{1_{A_1}}) \widehat{\subseteq} \overline{\mathfrak{J}(\Gamma_{1_{A_1}})}$,

$\Gamma_{1_{A_1}} \widehat{\subseteq} \mathfrak{J}^{-1}(\mathfrak{J}(\Gamma_{1_{A_1}})) \widehat{\subseteq} \mathfrak{J}^{-1}(\overline{\mathfrak{J}(\Gamma_{1_{A_1}})})$. since $\overline{\mathfrak{J}(\Gamma_{1_{A_1}})}$is $\mathcal{F}.\mathcal{s}$.closed, $\mathfrak{J}$ is fuzzy soft continuous then $\mathfrak{J}^{-1}(\overline{\mathfrak{J}(\Gamma_{1_{A_1}})})$is $\mathcal{F}.\mathcal{s}$.closed but $\overline{\Gamma_{1_{A_1}}}$ is the smallest $\mathcal{F}.\mathcal{s}$.closed set containing $\Gamma_{1_{A_1}}$ so $\Gamma_{1_{A_1}} \widehat{\subseteq} \overline{\Gamma_{1_{A_1}}} \widehat{\subseteq} \mathfrak{J}^{-1}(\overline{\mathfrak{J}(\Gamma_{1_{A_1}})})$ and therefore $\mathfrak{J}(\overline{\Gamma_{1_{A_1}}}) \widehat{\subseteq} \overline{\mathfrak{J}(\Gamma_{1_{A_1}})}$.

(⇐) Let $\mathfrak{J}(\Gamma_{1_{A_1}}) = \Upsilon_{1_{B_1}}$ to prove for each $\Upsilon_{1_{B_1}}$ $\mathcal{F}.\mathcal{s}$.closed then $\mathfrak{J}^{-1}(\Upsilon_{1_{B_1}})$ $\mathcal{F}.\mathcal{s}$.closed, to prove $\Gamma_{1_{A_1}}$ $\mathcal{F}.\mathcal{s}$.closed .we need to prove $\overline{\Gamma_{1_{A_1}}} \widehat{\subseteq} \Gamma_{1_{A_1}}$ since $\mathfrak{J}(\overline{\Gamma_{1_{A_1}}}) \widehat{\subseteq} \overline{\mathfrak{J}(\Gamma_{1_{A_1}})}$. then $\mathfrak{J}(\overline{\Gamma_{1_{A_1}}}) \widehat{\subseteq} \overline{\Upsilon_{1_{B_1}}} \widehat{=} \Upsilon_{1_{B_1}}$so $\overline{\Gamma_{1_{A_1}}} \widehat{\subseteq} \mathfrak{J}^{-1}(\Upsilon_{1_{B_1}}) \widehat{=} \Gamma_{1_{A_1}}$.

Theorem 2.39: Let $\mathfrak{J}:\Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$.mapping . then

$\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$.continuos if and if for each $\mathcal{F}.\mathcal{s}$.set $\Gamma_{1_{A_1}}$of Γ_A then $\mathfrak{J}(\Gamma_{1_{A_1}})^{\circ} \widehat{\subseteq} ((\mathfrak{J}(\Gamma_{1_{A_1}}))^{\circ}$

Proof : Obviously

Theorem 2.40: Let $\mathfrak{J}:\Gamma_A \widehat{\rightarrow} \Upsilon_B$ be a $\mathcal{F}.\mathcal{s}$.mapping . then

(i) $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$.open if and if for each $\mathcal{F}.\mathcal{s}$.set $\Gamma_{1_{A_1}}$of Γ_A then $\overline{\mathfrak{J}(\Gamma_{1_{A_1}})} \widehat{\subseteq} \mathfrak{J}(\overline{\Gamma_{1_{A_1}}})$

(ii) $\mathfrak{J}$ is $\mathcal{F}.\mathcal{s}$.closed if and if for each $\mathcal{F}.\mathcal{s}$.set $\Gamma_{1_{A_1}}$of Γ_A then $(\mathfrak{J}(\Gamma_{1_{A_1}}))^{\circ} \widehat{\subseteq} \mathfrak{J}(\Gamma_{1_{A_1}})^{\circ}$

Proof : Obviousl

Throughout this work , Let U ,V are a vector space over a field $\mathbb{F}$ ($\mathbb{F}=\mathbb{R}$) and the parameter set $A=\mathbb{R}$.

Definition 2.41:[5] Let $\Gamma_A$be a mapping given be $\Gamma_A : A \longrightarrow I^U$ then Γ_A is called $\mathcal{F}.\mathcal{s}$. vector space over U if $\Gamma_A(e) \cong f^{e}_{\Gamma_A}$ is a $\mathcal{F}$.vector sub space of U .

Definition 2.42. [34] : The $\mathcal{F}.\mathcal{s}$. set Γ_A is called $\mathcal{F}.\mathcal{s}$. vector over U , denoted by($\hat{v}_{f^{e}_{\Gamma_A}}$) , if there is exactly one e ∈ A such that $f^{e}_{\Gamma_A}(v)= t \in (0,1]$ for some v ∈ U and $f^{\alpha}_{\Gamma_A}(v) = 0$ for all $\alpha \in A/\{e\}$.

The set of all $\mathcal{F}.\mathcal{s}$ .vectors of a $\mathcal{F}.\mathcal{s}$. vector Γ_A denoted by $\mathcal{F}\mathcal{s}\mathcal{V}(\Gamma_A)$ Is said to be a $\mathcal{F}.\mathcal{s}$ .vector space according to The following two operation.

(i) $\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{+} \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} = (\widehat{v^1 + v^2})_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}} + f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}$ for all $\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}, f_2{}^{e_2}_{\Gamma_{2_{A_2}}} \in \mathcal{FS}\,\mathcal{V}(\Gamma_A)$.

(ii) $\hat{r}.\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} = (\widehat{rv^1})_{f_1{}^{re_1}_{\Gamma_{1_{A_1}}}}$ for all $\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \in \mathcal{FS}\,\mathcal{V}(\Gamma_A)$ and $\hat{r} \in \mathbb{R}(A)$.

Definition 2.43 : Let $U=\ell^p$ is a vector space over a field $\mathbb{F}$ ($\mathbb{F}=\mathbb{R}$) and $A=\mathbb{R}$. Then

$\mathcal{FS}\,\mathcal{V}(\Gamma_U)=\{\ \hat{v}_{\bar{1}}: v \in \ell^p\ ;\ v=(v^1, v^2, v^3, \ldots..)\ ;\ v^i \in \mathbb{R}\ ;\ \sum_{i=1}^{\infty} |v^i|^p < \infty, 1 \le p < \infty\ \}$

is fuzzy soft vector over $\mathbb{R}(A)$ is according to The following two operation

(i) $\widehat{v^1}_{\bar{1}} \hat{+} \widehat{v^2}_{\bar{1}} \cong (\widehat{v^1 + v^2})_{\bar{1}+\bar{1}}$ for all $\widehat{v^1}_{\bar{1}}, \widehat{v^2}_{\bar{1}} \hat{\in} \mathcal{FS}\,\mathcal{V}(\Gamma_U)$.

(ii) $\hat{r}.\hat{v}_{\bar{1}} \cong (\hat{rv})_{\bar{1}}$ for all $\hat{v}_{\bar{1}} \hat{\in} \mathcal{FS}\,\mathcal{V}(\Gamma_U)$ and $\hat{r} \hat{\in} \mathbb{R}(A)$.

for all $\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}, \widehat{v^2}_{f_{2_{\Gamma_{A_2}}}^{e_2}} \hat{\in} \mathcal{FS}\,\mathcal{V}(\Gamma_U)$.

Definition 2.44.[34]: Let $\mathcal{FSV}(\Gamma_A)$ be a fuzzy soft vector space. Then, a mapping

$\|.\|_{\mathcal{FS}}: \mathcal{FSV}(\Gamma_A) \to \mathbb{R}^+(A)$ is said to be a fuzzy soft norm on Γ_A if $\|.\|_{\mathcal{FS}}$ satisfies the following conditions:

(i) $\|\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}\|_{\mathcal{FS}} \geqq \hat{0}$,for all $\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}} \hat{\in} \mathcal{FSV}(\Gamma_A)$ and $\|\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}\|_{\mathcal{FS}} \cong \hat{0} \Leftrightarrow \widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}} \cong \hat{0}$

(ii) $\|\hat{r}.\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}\|_{\mathcal{FS}} = |\hat{r}| \hat{\cdot} \|\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}\|_{\mathcal{FS}}$, $\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}} \hat{\in} \mathcal{FSV}(\Gamma_A)$ and $\hat{r} \hat{\in} \mathbb{R}(A)$

(iii) $\|\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}} \hat{+} \widehat{v^2}_{f_{2_{\Gamma_{2_{A_2}}}}^{e_2}}\|_{\mathcal{FS}} \leqq \|\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}\|_{\mathcal{FS}} \hat{+} \|\widehat{v^2}_{f_{2_{\Gamma_{2_{A_2}}}}^{e_2}}\|_{\mathcal{FS}}$, for all $\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}, \widehat{v^2}_{f_{2_{\Gamma_{2_{A_2}}}}^{e_2}} \hat{\in}$ $\mathcal{FSV}(\Gamma_A)$

Apair $(\mathcal{FSV}(\Gamma_A)\ , \|.\|_{\mathcal{FS}})$ is called a fuzzy soft normed spac

Remark 2.45: Every $\mathcal{F}.\mathcal{s}$.subvector space($\mathcal{FSV}(\Gamma_{1_{A_1}}), \|.\|_{1\mathcal{FS}}$) of $\mathcal{F}.\mathcal{s}$ .normed space($\mathcal{FSV}(\Gamma_A), \|.\|_{2\mathcal{FS}}$) is $\mathcal{F}.\mathcal{s}$. normed space .

Theorem 2.46:[4]: Every $\mathcal{F}.\mathcal{s}$.normed linear space is a $\mathcal{F}.\mathcal{s}$ metric space with the soft metric $\hat{d}\,(\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}, \widehat{v^2}_{f_{2_{\Gamma_{2_{A_2}}}}^{e_2}}) \cong \|\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}} \widetilde{+\,v^2}_{f_{2_{\Gamma_{2_{A_2}}}}^{e_2}}\|_{\mathcal{FS}}$ for all $\widehat{v^1}_{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}}, \widehat{v^2}_{f_{2_{\Gamma_{2_{A_2}}}}^{e_2}} \hat{\in} \mathcal{FSV}(\Gamma_A)$.

Definition 2.47: [4]: $(\mathcal{FSV}(\Gamma_A), \|.\|_{\mathcal{FS}})$ a $\mathcal{F}.\mathcal{s}$.normed space . let $\{v^n\}$ be a sequence in U where n=1,2,3 ,…. For each n , let $(\widehat{v^n}_{f_{n_{\Gamma_{n_{A_n}}}}^{e_n}})$ $\mathcal{F}.\mathcal{s}$.point in U such that ∀ e∈ E and ∀ u∈ U

$$f_{\Gamma_A}^{e}(v^n) = \begin{cases} t \in (0,1] \ , \text{if } v = v_o \text{ and } e = e_o \in A \\ \\ 0 \ , \ \text{if } v \in U - \{v_0\} \text{ or } e \in A - \{e_o\} \end{cases}$$

Definition 2.48:[4] : A sequence of $\mathcal{F}.\mathcal{s}$.vectors $\{\widehat{v^n}_{f_{n_{\Gamma_{n_{A_n}}}}^{e_n}}\}$ in a $\mathcal{F}.\mathcal{s}$.normed space $(\mathcal{FSV}(\Gamma_A), \|.\|_{\mathcal{FS}})$ is said to be $\mathcal{F}.\mathcal{s}$.convergent to $\widehat{v^0}_{f_{0_{\Gamma_{0_{A_0}}}}^{e_0}}$, if $\lim_{n\to\infty} \|\widehat{v^n}_{f_{n_{\Gamma_{n_{A_n}}}}^{e_n}} \widetilde{-\,v^0}_{f_{0_{\Gamma_{0_{A_0}}}}^{e_0}}\|_{\mathcal{FS}} \cong \hat{0}$

I.e, $\forall \hat{\varepsilon} \hat{>} \hat{0}$, $\exists n_0 \in \mathbb{N}$ such that $\| \widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{-} \widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \| \hat{<} \hat{\varepsilon}, \forall n \geq n_0$. It is denoted by

$\lim_{n\to\infty} \widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \hat{=} \widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}$, or briefly $\widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \underset{n\to\infty}{\widehat{\rightarrow}} \widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}$.

That is to say that $\| \widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{-} \widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \|_{\mathcal{FS}} \underset{n\to\infty}{\widehat{\rightarrow}} \hat{0}$.

Definition 2.49:[4] : A sequence of $\mathcal{F}.\mathcal{s}$.vectors $\{\widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}\}$ in a $\mathcal{F}.\mathcal{s}$.normed space

$(\mathcal{FSV}(\Gamma_A), \|.\|_{\mathcal{FS}})$ is Said to be $\mathcal{F}.\mathcal{s}$.Cauchy sequence . I,e., $\forall \hat{\varepsilon} \hat{>} \hat{0}$, $\exists n_0 \in \mathbb{N}$ such that

$\| \widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{-} \widehat{v^m}_{f_m{}^{e_m}_{\Gamma_{m_{A_m}}}} \| \hat{<} \hat{\varepsilon}, \forall n, m \geq n_0$, $n \geq m$. That is

$\| \widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{-} \widehat{v^m}_{f_m{}^{e_m}_{\Gamma_{m_{A_m}}}} \| \underset{n,m\to\infty}{\widehat{\rightarrow}} \hat{0}$.

Definition 2.50:[4] Let U and V be universal sets and E_1, E_2 are the parameter sets . .Let $(\mathcal{FSV}(\Gamma_A), \|.\|_{1\mathcal{FS}})$ is $\mathcal{F}.\mathcal{s}$ normed spaces on (U, E_1) and $(\mathcal{FSV}(\Upsilon_B), \|.\|_{2\mathcal{FS}})$ $\mathcal{F}.\mathcal{s}$. normed spaces on (V, E_2) .Then the $\mathcal{F}.\mathcal{s}$ .mapping

$\tilde{T} : (\mathcal{FSV}(\Gamma_A), \|.\|_{1\mathcal{FS}}) \widehat{\rightarrow} (\mathcal{FSV}(\Upsilon_B), \|.\|_{2\mathcal{FS}})$ Is called a $\mathcal{F}.\mathcal{s}$ .operator .

Definition 2.51.[4]: Let $\tilde{T} : (\mathcal{FSV}(\Gamma_A), \|.\|_{1\mathcal{FS}}) \widehat{\rightarrow} (\mathcal{FSV}(\Upsilon_B), \|.\|_{2\mathcal{FS}})$ be a $\mathcal{F}.\mathcal{s}$ .linear operator

Then $\tilde{T}$ fuzzy soft continuous at a $\mathcal{F}.\mathcal{s}$ .vector $\widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \hat{\in} \Gamma_A$ .If for every $\widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widehat{\rightarrow} \widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}$ we have $\tilde{T}(\widehat{v^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}) \widehat{\rightarrow} \tilde{T}(\widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}})$.

3. Balanced Convex and Absorbing $\mathcal{F}.\mathcal{s}$.setes

Definition 3.1: let Γ_A is $\mathcal{F}.\mathcal{s}$. set in a vector space U

(i) Γ_A is $\mathcal{F}.\mathcal{s}$. sub space if $\Gamma(e)$ is $\mathcal{F}$.sub space in U .

(ii) Γ_A is $\mathcal{F}.\mathcal{s}$. convex if $\Gamma(e)$ is $\mathcal{F}$. convex in U.

(iii) Γ_A is $\mathcal{F}.\mathcal{s}$. balanced if $\Gamma(e)$ is $\mathcal{F}$. balanced in U.
(iv) Γ_A is $\mathcal{F}.\mathcal{s}$. absorbing if $\Gamma(e)$ is $\mathcal{F}$. bsorbing in U.

Theorem 3.2: Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathcal{s}$. sub space in U, let $\hat{r} \hat{\in} \mathbb{R}(A)$ then $\Gamma_{1_{A_1}} \hat{\cap} \Gamma_{2_{A_2}}$, $\hat{r}\,\Gamma_{1_{A_1}}$ are $\mathcal{F}.\mathcal{s}$. sub space in U

Proof :

(i) $*$ Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathcal{s}$. sub space in U . then $\Gamma_{1_{A_1}}(e) \cong f_1{}^{e_1}_{\Gamma_{1_{A_1}}}$, $\Gamma_{2_{A_2}}(e) \cong f_2{}^{e_2}_{\Gamma_{2_{A_2}}}$ are $\mathcal{F}$. sub space in U .

$f_1{}^{e_1}_{\Gamma_{1_{A_1}}} \bar{\cap} f_2{}^{e_2}_{\Gamma_{2_{A_2}}} (b_1 a_1 + b_2 a_2) = \min\{ f_1{}^{e_1}_{\Gamma_{1_{A_1}}} (b_1 a_1 + b_2 a_2), f_2{}^{e_2}_{\Gamma_{2_{A_2}}} (b_1 a_1 + b_2 a_2) \}$

$\geq \min \{ \min\{ f_1{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1), f_1{}^{e_1}_{\Gamma_{1_{A_1}}}(a_2) \}, \min\{ f_2{}^{e_2}_{\Gamma_{2_{A_2}}}(a_1), f_2{}^{e_2}_{\Gamma_{2_{A_2}}}(a_2) \} \}$

$\geq \min \{ \min\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1), f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_1)\}, \min\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_2)\}, f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_2)\}\}$

$\geq \min \{ f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}} \,\overline{\cap}\, f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_1), \min\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}} \,\overline{\cap}\, f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_2)\}$

then $\Gamma_{1_{A_1}} \,\widehat{\cap}\, \Gamma_{2_{A_2}}$ is $\mathcal{F}.\mathcal{s}$. sub space in U

** Let $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$. sub space in U . then $\Gamma_{1_{A_1}}(e) \cong f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}$ is $\mathcal{F}$. sub space in U .
$\hat{r}\,\Gamma_{1_{A_1}}(e) \cong r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}$ and

$$r\, f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2) = \begin{cases} f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\left(\frac{1}{r}\right)(b_1a_1 + b_2a_2)\right), & r \neq 0 \\ 0 & , r = 0,\ b_1a_1 + b_2a_2 \neq 0 \\ \sup\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2) : b_1a_1 + b_2a_2 \in U\}, & r = 0,\ b_1a_1 + b_2a_2 \neq 0 \end{cases}$$

If $r \neq 0$

$$r\, f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2) = f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\left(\frac{1}{r}\right)(b_1a_1 + b_2a_2)\right) \geq \min\left\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_1\right), f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_2\right)\right\}$$

$$\geq \min\left\{r\, f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1), r\, f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_2)\right\}$$

If $r = 0$

$$r\, f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2) = \sup\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2) : b_1a_1 + b_2a_2 \in U\}.$$

since $f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a) < (r\, f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}})(a)$ for all $a \in U$

$$< \sup\{r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2)\}$$

$$= \sup\left\{f^{e1}_{\Gamma_{1_{A_1}}}\left(b_1\left(\frac{1}{r}a_1\right) + b_2\left(\frac{1}{r}a_2\right)\right)\right\} \geq \min\left\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(b_1\left(\frac{1}{r}a_1\right) + b_2\left(\frac{1}{r}a_2\right)\right)\right\}$$

$$\geq \min\left\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_1\right), f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_2\right)\right\}$$

$$\geq \min\{r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1), r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_2)\}$$

then $\hat{r}\,\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$. sub space in U

Theorem 3.3: Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathcal{s}$. convex in U , let $\hat{r} \,\widehat{\in}\, \mathbb{R}(A)$. then $\Gamma_{1_{A_1}} \,\widehat{\cap}\, \Gamma_{2_{A_2}}$, $\hat{r}\,\Gamma_{1_{A_1}}$ are $\mathcal{F}.\mathcal{s}$. convex in U

Proof :

* Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathcal{s}$. sub space in U . then $\Gamma_{1_{A_1}}(e) \cong f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}$, $\Gamma_{2_{A_2}}(e) \cong f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}$ are $\mathcal{F}$. sub space in U .

$f^{e1}_{\Gamma_{1_{A_1}}} \,\overline{\cap}\, f^{e2}_{\Gamma_{2_{A_2}}}(\lambda a_1 + (1-\lambda)a_2) = \min\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a_1 + (1-\lambda)a_2), f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(\lambda a_1 + (1-\lambda)a_2)\}$

for all $a_1, a_2 \in$ U and $0 \le \lambda \le 1$

$\ge$ min { min{ $f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1)$, $f^{e1}_{\Gamma_{1_{A_1}}}(a_2)$} , min {$f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_1)$, $f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_2)$}}

$\ge$ min { min {$f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1)$, $f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_1)$ } , min {$f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_2)$} , $f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_2)$}

$\ge$ min { $f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}} \bar{\cap} f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_1)$, min {$f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}} \bar{\cap} f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}(a_2)$ }

then $\Gamma_{1_{A_1}} \widehat{\cap} \Gamma_{2_{A_2}}$ is $\mathcal{F}.\mathcal{s}$. convex in U

**Let $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$. convex in U. then $\Gamma_{1_{A_1}}(e) \cong f^{e1}_{\Gamma_{1_{A_1}}}$ is $\mathcal{F}$.convex in U.

$\hat{r}\Gamma_{1_{A_1}}(e) \cong r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}$ and

$$r f^{e1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2) = \begin{cases} f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\left(\frac{1}{r}\right)(b_1a_1 + b_2a_2)\right), & r \neq 0 \\ 0 & , r = 0,\ b_1a_1 + b_2a_2 \neq 0 \\ \sup\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(b_1a_1 + b_2a_2) : b_1a_1 + b_2a_2 \in U\}, & r = 0,\ b_1a_1 + b_2a_2 \neq 0 \end{cases}$$

for all $a_1, a_2 \in$ Uand $0 \le \lambda \le 1$

If $r \neq 0$

$$r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a_1 + (1-\lambda)a_2) = f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\left(\frac{1}{r}\right)(\lambda a_1 + (1-\lambda)a_2\right) \ge \min\left\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_1\right),\ f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_2\right)\right\}$$

$$\ge \min\left\{r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1),\ r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_2)\right\}$$

If $r = 0$

$r f^{e1}_{\Gamma_{1_{A_1}}}(\lambda a_1 + (1-\lambda)a_2) = \sup\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a_1 + (1-\lambda)a_2) : (\lambda a_1 + (1-\lambda)a_2) \in U\}$.

since $f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a) < (r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}})(a)$ for all $a \in U$

$$< \sup\{r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a_1 + (1-\lambda)a_2)\} = \sup\left\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(b_1\left(\frac{1}{r}a_1\right) + b_2\left(\frac{1}{r}a_2\right)\right)\right.$$

$$\ge \min\left\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(b_1\left(\frac{1}{r}a_1\right) + b_2\left(\frac{1}{r}a_2\right)\right)\right\}$$

$$\ge \min\left\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_1\right), f^{e1}_{\Gamma_{1_{A_1}}}\left(\frac{1}{r}a_2\right)\right.$$

$$\ge \min\{r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1),\ r f^{e1}_{\Gamma_{1_{A_1}}}(a_2)\}$$

$$\ge \min\{r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_1),\ r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(a_2)\}$$

then $\hat{r}\,\Gamma_{1_{A_1}}$ is is $\mathcal{F}.\mathcal{s}$. convex in U

Theorem 3.4: Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathcal{s}$. balanced in U , let $\hat{r} \widehat{\in} \mathbb{R}(A)$then $\Gamma_{1_{A_1}} \widehat{\cap} \Gamma_{2_{A_2}}$, $\hat{r}\,\Gamma_{1_{A_1}}$are $\mathcal{F}.\mathcal{s}$. balaced in U .

Proof : * Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathscr{s}$. balanced in U. then $\Gamma_{1_{A_1}}(e) \cong f_{1_{\Gamma_{1_{A_1}}}}^{e_1}$, $\Gamma_{2_{A_2}}(e) \cong f_{2_{\Gamma_{2_{A_2}}}}^{e_2}$ are $\mathcal{F}$. balanced in U

$f_{1_{\Gamma_{1_{A_1}}}}^{e_1} \,\overline{\cap}\, f_{2_{\Gamma_{2_{A_2}}}}^{e_2} (\lambda a) = \min\{ f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a), f_{2_{\Gamma_{2_{A_2}}}}^{e_2}(\lambda a)\}$. $a_1 \in U$ and $|\lambda| \le 1$.

$\ge \min\{ f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(a), f_{2_{\Gamma_{2_{A_2}}}}^{e_2}(a)\}$

$\ge f_{1_{\Gamma_{1_{A_1}}}}^{e_1} \,\overline{\cap}\, f_{2_{\Gamma_{2_{A_2}}}}^{e_2}(a)$

then $\Gamma_{1_{A_1}} \,\widehat{\cap}\, \Gamma_{2_{A_2}}$ is $\mathcal{F}.\mathscr{s}$. balanced in U

** Let $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathscr{s}$. balanced in U. then $\Gamma_{1_{A_1}}(e) = f_{1_{\Gamma_{1_{A_1}}}}^{e_1}$ is $\mathcal{F}$. balanced in U .

$\hat{r}\,\Gamma_{1_{A_1}}(e) \cong r f_{1_{\Gamma_{1_{A_1}}}}^{e_1}$ and

$$r\, f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a) = \begin{cases} f_{1_{\Gamma_{1_{A_1}}}}^{e_1}\left(\left(\frac{1}{r}\right)\lambda a\right), & r \neq 0 \\ 0, & r = 0,\ \lambda a \neq 0 \\ \sup\{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a) : \lambda a \in U\}, & r = 0,\ \lambda a,\ \lambda a \neq 0 \end{cases}$$

for all $a_1 \in U$ and $|\lambda| \le 1$.

If $r \neq 0$

$r\, f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a) = f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda)(\frac{1}{r} a)) \ge f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\frac{1}{r} a) \ge r\, f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(a)$

If $r = 0$

$r\, f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a_1)) = \sup\{f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a_1)\}$. since $f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(a) < (r\, f_{1_{\Gamma_{1_{A_1}}}}^{e_1})(a)$ for all $a \in U$, then

$$< \sup\{r f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a_1)\} = \sup\{ f_{1_{\Gamma_{1_{A_1}}}}^{e_1}\lambda\left(\frac{1}{r}a_1\right)\} \ge \{ f_{1_{\Gamma_{1_{A_1}}}}^{e_1}\lambda(\frac{1}{r}a_1)$$
$$\ge f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\frac{1}{r}a_1) = r\, f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(a_1)$$

then $\hat{r}\,\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathscr{s}$. balaced in U

Theorem 3.5: Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathscr{s}$. absorbing in U , let $\hat{r} \,\widehat{\in}\, \mathbb{R}(A)$ then, then $\Gamma_{1_{A_1}} \,\widehat{\cap}\, \Gamma_{2_{A_2}}$, $\tilde{r}\,\Gamma_{1_{A_1}}$ are $\mathcal{F}.\mathscr{s}$. bsorbing in U .

Proof :* Let $\Gamma_{1_{A_1}}$, $\Gamma_{2_{A_2}}$ are $\mathcal{F}.\mathscr{s}$.a bsorbing in U.then $\Gamma_{1_{A_1}}(e) \cong f_{1_{\Gamma_{1_{A_1}}}}^{e_1}$, $\Gamma_{2_{A_2}}(e) \cong f_{2_{\Gamma_{2_{A_2}}}}^{e_2}$ $\mathcal{F}$. a bsorbingin U.

$sub_{\lambda>0} f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a) = 1$ and $sub_{\lambda>0} f_{2_{\Gamma_{2_{A_2}}}}^{e_2}(\lambda a) = 1$

$Sub_{\lambda>0}(f_{1_{\Gamma_{1_{A_1}}}}^{e_1} \,\overline{\cap}\, f_{2_{\Gamma_{2_{A_2}}}}^{e_2})(\lambda a) = Sub_{\lambda>0}\{\min\{ f_{1_{\Gamma_{1_{A_1}}}}^{e_1}(\lambda a), f_{2_{\Gamma_{2_{A_2}}}}^{e_2}(\lambda a)\}\}$. $a \in U$

$= \min \{ Sub_{\lambda>0} f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}} (\lambda a), Sub_{\lambda>0} f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}} (\lambda a)\}=1$

then $\Gamma_{1_{A_1}} \widehat{\cap} \Gamma_{2_{A_2}}$ is $\mathcal{F}.\mathcal{s}$.a bsorbing in U.

$**$ Let $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$.a bsorbing in U. then $\Gamma_{1_{A_1}}(e) = f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}$ is $\mathcal{F}$. a bsorbing in U.

$\hat{r}\,\Gamma_{1_{A_1}}(e) \cong r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}$ and

$$r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a) = \begin{cases} f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}((\frac{1}{r})(\lambda a)) \ , \ r \neq 0 \\ 0 \qquad , r = 0, \ \lambda a \neq 0 \\ \sup\{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a) : (\lambda a) \in U\}, r = 0, \lambda a \ , \lambda a \neq 0 \end{cases}$$

If $r \neq 0$

$sub_{\lambda>0} r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a) = sub_{\lambda>0} f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda)(\frac{1}{r} a)) = 1$

If $r = 0$

$sub_{\lambda>0}\ r f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a)) = sub_{\lambda>0} \sup \{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a)\} = \sup sub_{\lambda>0} \{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}(\lambda a)\} = 1$

then $\hat{r}\,\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$.a bsorbing in U.

Remark 3.6: let Γ_A is $\mathcal{F}.\mathcal{s}$. set in a vector space U .

(i) Γ_A is $\mathcal{F}.\mathcal{s}$. sub space if $\hat{\alpha}\,\Gamma_A \hat{+} \hat{\beta}\Gamma_A$, $\hat{\alpha}$, $\hat{\beta}$ $\in \mathbb{R}(A)$.

(ii) Γ_A is $\mathcal{F}.\mathcal{s}$. convex if $\hat{\alpha}\,\Gamma_A \hat{+} (\hat{1} - \hat{\alpha})\Gamma_A$ and $\hat{0} \leq \hat{\alpha} \leq \hat{1}$.

(iii) Γ_A is $\mathcal{F}.\mathcal{s}$. balanced if $\hat{\alpha}\,\Gamma_A$ and $|\hat{\alpha}| \lessapprox \hat{1}$.

4 . $\mathcal{F}.\mathcal{s}$.topological vector space

Definition 4.1 : A $\mathcal{F}.\mathcal{s}$.topology on a $\mathcal{F}.\mathcal{s}$.vector space $\mathcal{FSV}(\Gamma_A)$over $\mathbb{R}(A)$ is called

A linear $\mathcal{F}.\mathcal{s}$.topology if

Addition $\hat{+}$: $\mathcal{FSV}(\Gamma_A) \hat{\times} \mathcal{FSV}(\Gamma_A) \longrightarrow \mathcal{FSV}(\Gamma_A)$defined

$\hat{+}(\widehat{v^1}_{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}}, \widehat{v^2}_{f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}}) \,\hat{=}\, (\widehat{v^1}_{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}} \hat{+} \widehat{v^2}_{f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}}) \,\hat{=}\, \widehat{(v^1 + v^2)}_{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}} \mp f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}}$

Multiplication $\hat{\because}$: $\mathbb{R}(A) \hat{\times} \mathcal{FSV}(\Gamma_A) \rightarrow \mathcal{FSV}(\Gamma_A)$defined

$\hat{\because}(\hat{r}, \widehat{v^1}_{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}}) = \hat{r} \hat{\because} \widehat{v^1}_{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}} = (\widehat{rv})_{f_{1}{}^{re_1}_{\Gamma_{1_{A_1}}}}$, where for all $\widehat{v^1}_{f_{1}{}^{e_1}_{\Gamma_{1_{A_1}}}}, \widehat{v^2}_{f_{2}{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\hat{\in}\, \mathcal{FSV}(\Gamma_A)$, $\hat{r} \,\hat{\in}\, \mathbb{R}(A)$ are $\mathcal{F}.\mathcal{s}$.continuous .A $\mathcal{F}.\mathcal{s}$.vector space $\mathcal{FSV}(\Gamma_A)$endowed with a $\mathcal{L}\,\mathcal{FS}\,\mathcal{T}$ is called $\mathcal{F}.\mathcal{s}$.topological vector space(for short $\mathcal{FSTVS}$)

Theorem 4.2:

$\mathcal{FSV}(\Gamma_A)$be a $\mathcal{FSTVS}$ over $\mathbb{R}(A)$. for $\widehat{v^0}_{f_{0}{}^{e_0}_{\Gamma_{0_{A_0}}}} \,\hat{\in}\, \mathcal{FSV}(\Gamma_A)$and $\hat{0} \hat{\neq} \hat{k} \,\hat{\in}\, \mathbb{R}(A)$. then

(1) the $\mathcal{F}.\mathscr{s}.$ translation , $\mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}} : \mathcal{FSV}(\Gamma_A) \to \mathcal{FSV}(\Gamma_A)$, $\mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}(\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}) \widehat{=} \widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}} \widehat{+} \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}$

(2) the $\mathcal{F}.\mathscr{s}.$ multiplication , $\mathcal{M}_{\hat{k}} : \mathcal{FSV}(\Gamma_A) \to \mathcal{FSV}(\Gamma_A)$, $\mathcal{M}_{\hat{k}}(\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}) \widehat{=} \hat{k} . \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}$

Are $\mathcal{F}.\mathscr{s}.$ homeomorphism for all $\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} \widehat{\in} \mathcal{FSV}(\Gamma_A)$.

Proof :

To proof $\mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{A_0}}^{e_0}}} : \mathcal{FSV}(\Gamma_A) \to \mathcal{FSV}(\Gamma_A$, $\mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}(\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}) \widehat{=} \widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}} \widehat{+} \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}$ for all $\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} \widehat{\in} \mathcal{FSV}(\Gamma_A)$. is

$\mathcal{F}.\mathscr{s}.$ homeomorphism

(i) let $\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}, \widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}} \widehat{\in} \mathcal{FSV}(\Gamma_A) \ni \mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}(\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}) \widehat{=} \mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}(\widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}})$

$\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}} \widehat{+} \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} \widehat{=} \widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}} \widehat{+} \widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}} \Rightarrow$

$\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} = \widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}} \Rightarrow \mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}$ is $\mathcal{F}.\mathscr{s}.$ injective

let $\widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}} \widehat{\in} \mathcal{F}\mathscr{s}\mathcal{V}(\Gamma_A) \Rightarrow (\widehat{v^0}_{f_{0\Gamma_{A_0}}^{e_0}})^{-1} \widehat{+} \widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}} \widehat{\in} \mathcal{FSV}(\Gamma_A)$ and $\mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{A_0}}^{e_0}}}((\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}})^{-1} \widehat{+} \widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}})$

$= \widehat{v^0}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}} \widehat{+} (\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}})^{-1} \widehat{+} \widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}} = \widehat{v^2}_{f_{2\Gamma_{2A_2}}^{e_2}} \Rightarrow \mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}$ is $\mathcal{F}.\mathscr{s}.$ surjective

$\Rightarrow \mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}$ is $\mathcal{F}.\mathscr{s}.$ bijective

(ii) let $\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} \widehat{\in} \mathcal{FSV}(\Gamma_A) \Longrightarrow \widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}} \widehat{+} \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} \widehat{\in} \mathcal{FSV}(\Gamma_A) \widehat{\times} \mathcal{FSV}(\Gamma_A)$

Since $\mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}(\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}) = \widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}} \widehat{+} \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} = \widehat{+}(\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}, \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}})$ and $\widehat{+}$ is $\mathcal{F}.\mathscr{s}.$continuous

$\Longrightarrow \mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}}$ is $\mathcal{F}.\mathscr{s}.$ continuous

(iii)let $\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} \widehat{\in} \mathcal{FSV}(\Gamma_A) \Rightarrow (\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}})^{-1} \widehat{+} \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}} \widehat{\in} \mathcal{FSV}(\Gamma_A) \widehat{\times} \mathcal{FSV}(\Gamma_A)$

$(\mathcal{T}_{\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}}})^{-1} = \mathcal{T}_{(\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}})^{-1}} \Rightarrow \mathcal{T}_{(\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}})^{-1}}(\widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}) = (\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}})^{-1} \widehat{+} \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}}$

$= \widehat{+}((\widehat{v^0}_{f_{0\Gamma_{0A_0}}^{e_0}})^{-1}, \widehat{v^1}_{f_{1\Gamma_{1A_1}}^{e_1}})$

and $\widehat{+}$ is $\mathcal{F}.\mathcal{s}$.continuous $\Longrightarrow (\mathcal{T}_{\widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}})^{-1}$ is $\mathcal{F}.\mathcal{s}$. continuous

$\mathcal{T}_{\widehat{v^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}}$ is $\mathcal{F}.\mathcal{s}$. homeomorphism

(2) To proof $\mathcal{M}_{\hat{k}} : \mathcal{FSV}(\Gamma_A)) \widehat{\rightarrow} \mathcal{FSV}(\Gamma_A)$, $\mathcal{M}_{\hat{k}}(\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) = \hat{k}$.

$\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}$ for all $\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \mathcal{FSV}(\Gamma_A), \hat{0} \widehat{\neq} \hat{k} \widehat{\in} \mathbb{R}(A)$. is $\mathcal{F}.\mathcal{s}$. homeomorphism

let $\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}, \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{A_2}}} \widehat{\in} \mathcal{FSV}(\Gamma_A) \ni \mathcal{M}_{\hat{k}}(\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) = \mathcal{M}_{\hat{k}}(\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{A_2}}}) \Longrightarrow \hat{k} . \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} = \hat{k} . \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}$

$\Longrightarrow \hat{k} . \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} = \hat{k} . \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{A_2}}} \Longrightarrow \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} = \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}$

$\mathcal{M}_{\hat{k}}$ is $\mathcal{F}.\mathcal{s}$. injective

$\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \mathcal{FsV}(\Gamma_A) \Rightarrow \frac{\hat{1}}{\hat{k}} . \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \mathcal{FSV}(\Gamma_A) \Longrightarrow \mathcal{M}_{\hat{k}}(\frac{\hat{1}}{\hat{k}} . \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) = \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}$

$\mathcal{M}_{\hat{k}}$ is $\mathcal{F}.\mathcal{s}$. surjective

$\mathcal{M}_{\hat{k}}$ is $\mathcal{F}.\mathcal{s}$. bijective

(ii) let $\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{\in} \mathcal{FsV}(\Gamma_A) \Rightarrow \hat{k} . \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \in \mathbb{R}(A) \widehat{\times} \mathcal{FSV}(\Gamma_A)$. $\mathcal{M}_{\hat{k}}(\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) = \hat{k} . \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}$

$= \widehat{:}(\hat{k}, \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}})$ and $\widehat{:}$ is $\mathcal{F}.\mathcal{s}$.continuous

$\Longrightarrow \mathcal{M}_{\hat{k}}$ is $\mathcal{F}.\mathcal{s}$. continuous

(iii) let $\hat{0} \widehat{\neq} \hat{k} \widehat{\in} \mathbb{R}(A) \Longrightarrow \frac{\hat{1}}{\hat{k}} \widehat{\in} \mathbb{R}(A)$

$(\mathcal{M}_{\hat{k}})^{-1} = \mathcal{M}_{\frac{\hat{1}}{\hat{k}}} \Rightarrow (\mathcal{M}_{\hat{k}})^{-1}(\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) = \mathcal{M}_{\frac{\hat{1}}{\hat{k}}}(\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) = \frac{\hat{1}}{\hat{k}} \widehat{:} \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} = \widehat{:}(\frac{\hat{1}}{\hat{k}}, \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}})$ and $\widehat{:}$ is $\mathcal{F}.\mathcal{s}$.continuous

$\Longrightarrow (\mathcal{M}_{\hat{k}})^{-1}$ is $\mathcal{F}.\mathcal{s}$. continuous

$\mathcal{M}_{\hat{k}}$ is $\mathcal{F}.\mathcal{s}$. homeomorphism

Corollary 4.3: Let $\Gamma_{1_{A_1}}$ be a $\mathcal{F}.\mathcal{s}$. set of a $\mathcal{FSTVS}$ $(\mathcal{FSV}(\Gamma_A))$over $\mathbb{R}(A)$ .For $\hat{0} \widehat{\neq} \hat{k} \widehat{\in} \mathbb{R}(A)$, we have

(i) $\hat{k}(\Gamma_{1_{A_1}})^{\circ} \widehat{=} (\widehat{k}(\Gamma_A))^{\circ}$

(ii) $\hat{k}\ \overline{(\Gamma_{1_{A_1}})} \widehat{=} \overline{\hat{k}\ (\Gamma_{1_{A_1}})}$

Proof :

(i) since $\mathcal{M}_{\hat{k}}$ is $\mathcal{F}.\mathcal{s}$. homeomorphism.then $\mathcal{M}_{\hat{k}}$ is $\mathcal{F}.\mathcal{s}$. continuous and $\mathcal{F}.\mathcal{s}$. closed by using 2.39 and 2.40 ii

$\mathcal{M}_{\widehat{k}}\ (\Gamma_{1_{A_1}})^\circ \,\widehat{\subseteq}\, ((\mathcal{M}_{\widehat{k}}\,(\Gamma_{1_{A_1}}))^\circ$and $((\mathcal{M}_{\widehat{k}}\,(\Gamma_{1_{A_1}}))^\circ \,\widehat{\subseteq}\, \mathcal{M}_{\widehat{k}}\ (\Gamma_{1_{A_1}})^\circ$. thus $\widehat{k}\,(\Gamma_{1_{A_1}})^\circ \,\widehat{\subseteq}\, \widehat{k}(\Gamma_A)^\circ$

$\widehat{k}(\Gamma_A)^\circ \ \widehat{\subseteq}\ \widehat{k}\,(\Gamma_{1_{A_1}})^\circ$. we have $\widehat{k}\,(\Gamma_{1_{A_1}})^\circ \,\widehat{=}\, (\widehat{k}(\Gamma_A))^\circ$

(ii) since $\mathcal{M}_{\widehat{k}}$ is $\mathcal{F}.\mathscr{s}.$ homeomorphism.then $\mathcal{M}_{\widehat{k}}$ is $\mathcal{F}.\mathscr{s}.$ continuous and $\mathcal{F}.\mathscr{s}.$ open

by using 2.40 (i) and 2.38

$\overline{\mathcal{M}_{\widehat{k}}\,(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \mathcal{M}_{\widehat{k}}\ \overline{(\Gamma_{1_{A_1}})}$and $\mathcal{M}_{\widehat{k}}\ \overline{(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \overline{\mathcal{M}_{\widehat{k}}\,(\Gamma_{1_{A_1}})}$. thus $\overline{\widehat{k}\,(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \widehat{k}\ \overline{(\Gamma_{1_{A_1}})}$ and

$\widehat{k}\ \overline{(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \overline{\widehat{k}\,(\Gamma_{1_{A_1}})}$. we have $\widehat{k}\ \overline{(\Gamma_{1_{A_1}})} \,\widehat{=}\, \overline{\widehat{k}\,(\Gamma_{1_{A_1}})}$.

Corollary 4.4: Let $\Gamma_{1_{A_1}}$ be $\mathcal{F}.\mathscr{s}.$ set of a $\mathcal{F}\mathscr{s}\mathcal{T}\mathcal{L}\mathscr{s}$ $\mathcal{FSV}(\Gamma_A)$ over $\mathbb{R}(A)$. we have

(i) $\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, (\Gamma_{1_{A_1}})^\circ = (\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \Gamma_A)^\circ$

(ii) $\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \overline{(\Gamma_{1_{A_1}})} = \overline{(\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \Gamma_{1_{A_1}})}$

Proof :

(i)since $\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}$ is $\mathcal{F}.\mathscr{s}.$ homeomorphism.then $\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}$ is $\mathcal{F}.\mathscr{s}.$ continuous and $\mathcal{F}.\mathscr{s}.$ closed

by using 2.39 and 2.40 ii

$\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\ (\Gamma_{1_{A_1}})^\circ \,\widehat{\subseteq}\, ((\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\,(\Gamma_{1_{A_1}}))^\circ$and $((\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\,(\Gamma_{1_{A_1}}))^\circ \,\widehat{\subseteq}\, \mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\ (\Gamma_{1_{A_1}})^\circ$. thus
$\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}(\Gamma_{1_{A_1}})^\circ \,\widehat{\subseteq}\, (\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \Gamma_A)^\circ$

$(\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \Gamma_A)^\circ \ \widehat{\subseteq}\ \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, (\Gamma_{1_{A_1}})^\circ$. we have $\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, (\Gamma_{1_{A_1}})^\circ \,\widehat{=}\, (\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \Gamma_A)^\circ)$since $\mathcal{T}_{\widehat{v}_{f^e_{\Gamma_A}}}$ is
(i)$\mathcal{F}.\mathscr{s}.$ homeomorphism.then $\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}$ is $\mathcal{F}.\mathscr{s}.$ continuous and $\mathcal{F}.\mathscr{s}.$ closed

by using 2.40 (i) and 2.38

$\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\ \overline{(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \overline{\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\ (\Gamma_{1_{A_1}})}$ and $\overline{\mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\ (\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \mathcal{T}_{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}}\ \overline{(\Gamma_{1_{A_1}})}$. thus
$\overline{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, (\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \overline{(\Gamma_{1_{A_1}})}$ and

(iii) $\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \overline{\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, (\Gamma_{1_{A_1}})}$. we have $\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{=}\, \overline{(\widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \,\widehat{\mp}\, \Gamma_{1_{A_1}})}$.

Proposition 4.5: Let $\Gamma_{1_{A_1}}, \Gamma_{2_{A_2}}$ are a $\mathcal{F}.\mathscr{s}.$ set of a $\mathcal{FSTVS}$ (Γ_A) over $\mathbb{R}(A)$. we have

$\overline{\Gamma_{1_{A_1}}} \widehat{\mp} \overline{\Gamma_{2_{A_2}}} \subset \overline{\Gamma_{1_{A_1}} \widehat{\mp}\, \Gamma_{2_{A_2}}}$.

Proof :

let $\widehat{u^3}_{f_3}{}^{e_3}_{\Gamma_{3_{A_3}}} \,\widehat{\in}\, \overline{\Gamma_{1_{A_1}}} \widehat{+} \overline{\Gamma_{2_{A_2}}} \Rightarrow \widehat{u^3}_{f_3}{}^{e_3}_{\Gamma_{3_{A_3}}} \,\widehat{=}\, \widehat{u^1}_{f_1}{}^{e_1}_{\Gamma_{1_{A_1}}} \widehat{+} \widehat{u^2}_{f_2}{}^{e_2}_{\Gamma_{2_{A_2}}}$ where $\widehat{u^1}_{f_1}{}^{e_1}_{\Gamma_{1_{A_1}}} \,\widehat{\in}\, \overline{\Gamma_{1_{A_1}}}$, $\widehat{u^2}_{f_2}{}^{e_2}_{\Gamma_{2_{A_2}}} \,\widehat{\in}\, \overline{\Gamma_{2_{A_2}}}$ and let $\Gamma_{3_{A_3}}$ be a fuzzy soft neighborhood of $\widehat{u^1}_{f_1}{}^{e_1}_{\Gamma_{1_{A_1}}} \widehat{+} \widehat{u^2}_{f_2}{}^{e_2}_{\Gamma_{2_{A_2}}}$.

since A fuzzy soft mapping $\widehat{+} : \mathcal{FSV}(\Gamma_A) \widehat{\times}\, \mathcal{FSV}(\Gamma_A)) \widehat{\rightarrow} \mathcal{FSV}(\Gamma_A)$ is fuzzy soft continuous , there are fuzzy soft neighborhood $\Gamma_{4_{A_4}}$ of $\widehat{u^1}_{f_1}{}^{e_1}_{\Gamma_{1_{A_1}}}$ and $\Gamma_{5_{A_5}}$ of $\widehat{u^2}_{f_2}{}^{e_2}_{\Gamma_{2_{A_2}}}$ such that $\Gamma_{4_{A_4}} \widehat{+} \Gamma_{5_{A_5}} \,\widehat{\subset}\, \Gamma_{6_{A_6}}$. Since

$\widehat{u^1}_{f_1}{}^{e_1}_{\Gamma_{1_{A_1}}} \,\widehat{\in}\, \overline{\Gamma_{1_{A_1}}} \Rightarrow \Gamma_{4_{A_4}} \,\widehat{\cap}\, \Gamma_{1_{A_1}} \,\widehat{\neq}\, \Gamma_\emptyset \Rightarrow \exists \widehat{u^5}_{f_5}{}^{e_5}_{\Gamma_{5_{A_5}}} \,\widehat{\in}\, \Gamma_{4_{A_4}}$, $\widehat{u^5}_{f_5}{}^{e_5}_{\Gamma_{5_{A_5}}} \,\widehat{\in}\, \Gamma_{1_{A_1}}$

Since $\widehat{u^2}_{f_2}{}^{e_2}_{\Gamma_{2_{A_2}}} \,\widehat{\in}\, \overline{\Gamma_{2_{A_2}}} \Rightarrow \Gamma_{5_{A_5}} \,\widehat{\cap}\, \Gamma_{2_{A_2}} \,\widehat{\neq}\, \Gamma_\emptyset \Rightarrow \exists \widehat{u^6}_{f_6}{}^{e_6}_{\Gamma_{6_{A_6}}} \,\widehat{\in}\, \Gamma_{5_{A_5}}$, $\widehat{u^5}_{f_5}{}^{e_5}_{\Gamma_{5_{A_5}}} \,\widehat{\in}\, \Gamma_{2_{A_2}}$

$\widehat{u^5}_{f_5}{}^{e_5}_{\Gamma_{5_{A_5}}} \widehat{+} \widehat{u^6}_{f_6}{}^{e_6}_{\Gamma_{6_{A_6}}} \,\widehat{\in}\, \Gamma_{4_{A_4}} \widehat{+} \Gamma_{5_{A_5}} \,\widehat{\subset}\, \Gamma_{6_{A_6}} \Rightarrow \widehat{u^5}_{f_5}{}^{e_5}_{\Gamma_{5_{A_5}}} \widehat{+} \widehat{u^6}_{f_6}{}^{e_6}_{\Gamma_{6_{A_6}}} \,\widehat{\in}\, \Gamma_{6_{A_6}}$

$\widehat{u^5}_{f_5}{}^{e_5}_{\Gamma_{5_{A_5}}} \widehat{+} \widehat{u^6}_{f_6}{}^{e_6}_{\Gamma_{6_{A_6}}} \,\widehat{\in}\, \Gamma_{3_{A_3}} \,\widehat{\cap}\, (\Gamma_{1_{A_1}} \widehat{+} \Gamma_{2_{A_2}}) \Rightarrow \Gamma_{3_{A_3}} \,\widehat{\cap}\, (\Gamma_{1_{A_1}} \widehat{+} \Gamma_{2_{A_2}}) \,\widehat{\neq}\, \Gamma_\emptyset$

For each a fuzzy soft neighborhood $\Gamma_{3_{A_3}}$ of $\widehat{u^1}_{f_1}{}^{e_1}_{\Gamma_{1_{A_1}}} \widehat{+} \widehat{u^2}_{f_2}{}^{e_2}_{\Gamma_{2_{A_2}}} \Rightarrow \overline{\Gamma_{1_{A_1}}} \widehat{+} \overline{\Gamma_{2_{A_2}}} \subset \overline{\Gamma_{1_{A_1}} \widehat{+} \Gamma_{2_{A_2}}}$

Proposition 4.6: let $\mathcal{FSV}(\Gamma_A)$ be a $\mathcal{FSTVS}$ over $\mathbb{R}(A)$. and $\Gamma_{1_{A_1}} \subseteq \Gamma_A$ then

(i)If $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ sub space so is $\overline{(\Gamma_{1_{A_1}})}$.

(ii) If $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ convex so is $\overline{(\Gamma_{1_{A_1}})}$.

(iii) If $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ balaced so is $\overline{(\Gamma_{1_{A_1}})}$.

proof :

(i)Let $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ sub space then $\hat{\alpha}\, \Gamma_{1_{A_1}} \widehat{+} \hat{\beta}\, \Gamma_{1_{A_1}} \,\widehat{\subseteq}\, \Gamma_{1_{A_1}}$, $\hat{\alpha}$, $\hat{\beta} \,\widehat{\in}\, \mathbb{R}(A)$. then $\overline{\hat{\alpha}\, \Gamma_{1_{A_1}}} \widehat{+} \overline{\hat{\beta}\, \Gamma_{1_{A_1}}} \,\widehat{\subseteq}\, \overline{\Gamma_{1_{A_1}}}$. since $\hat{\alpha}\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{=}\, \overline{\hat{\alpha}(\Gamma_{1_{A_1}})}$ and $\hat{\beta}\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{=}\, \overline{\hat{\beta}(\Gamma_{1_{A_1}})}$ thus $\hat{\alpha}\, \overline{(\Gamma_{1_{A_1}})} \widehat{+} \hat{\beta}\, \overline{(\Gamma_{1_{A_1}})}$. hence $\overline{(\Gamma_{1_{A_1}})}$ is $\mathcal{F}.\mathcal{s}.$ sub space .

(ii)Let $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ convex set then $\hat{\alpha}\, \Gamma_{1_{A_1}} \widehat{+} (\hat{1} - \hat{\alpha})\, \Gamma_{1_{A_1}} \,\widehat{\subseteq}\, \Gamma_{1_{A_1}}$, $\hat{0} \,\widehat{\leq}\, \hat{\alpha} \,\widehat{\leq}\, \hat{1}$. then $\overline{\hat{\alpha}\, \Gamma_{1_{A_1}}} \widehat{+} \overline{(\hat{1} - \hat{\alpha})\Gamma_{1_{A_1}}} \,\widehat{\subseteq}\, \overline{\Gamma_{1_{A_1}}}$ since $\hat{\alpha}\, \overline{(\Gamma_{1_{A_1}})} \widehat{+} \overline{\hat{\alpha}(\Gamma_{1_{A_1}})}$ and $(\hat{1} - \hat{\alpha})\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{=}\, \overline{(\hat{1} - \hat{\alpha})(\Gamma_{1_{A_1}})}$ thus $\hat{\alpha}\, \overline{(\Gamma_{1_{A_1}})} \widehat{+} (\hat{1} - \hat{\alpha})\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \overline{\Gamma_{1_{A_1}}}$. hence $\overline{(\Gamma_{1_{A_1}})}$ is $\mathcal{F}.\mathcal{s}.$ convex set .

(iii) Let $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ balanced set then $\hat{\alpha}\, \Gamma_{1_{A_1}} \,\widehat{\subseteq}\, \Gamma_{1_{A_1}}$. then $\overline{\hat{\alpha}\, \Gamma_{1_{A_1}}} \,\widehat{\subseteq}\, \overline{\Gamma_{1_{A_1}}}$ since $\hat{\alpha}\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{=}\, \overline{\hat{\alpha}(\Gamma_{1_{A_1}})}$ thus $\hat{\alpha}\, \overline{(\Gamma_{1_{A_1}})} \,\widehat{\subseteq}\, \overline{\Gamma_{1_{A_1}}}$. hence $\overline{(\Gamma_{1_{A_1}})}$ is $\mathcal{F}.\mathcal{s}.$ balanced set .

Proposition 4.7: Let $\Gamma_{1_{A_1}}$ be a $\mathcal{F}.\mathcal{s}.$ set of a $\mathcal{FSTVS}$ ($\mathcal{FSV}(\Gamma_A)$)over $\mathbb{R}(A)$. we have

(i)If $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ sub space so is$(\Gamma_{1_{A_1}})^\circ$.

(ii) If $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}.$ convex set so is $(\Gamma_{1_{A_1}})^\circ$

Proof:

(i)Let $\widehat{\alpha}$, $\widehat{\beta}$ $\in \mathbb{R}(A)$. since $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$. sub space and $(\Gamma_{1_{A_1}})^{\circ} \subseteq \Gamma_{1_{A_1}}$. then $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{+}\ \widehat{\beta}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{\subseteq}\ \widehat{\alpha}\,\Gamma_{1_{A_1}} \widehat{+}\ \widehat{\beta}\,\Gamma_{1_{A_1}} \widehat{\subseteq}\ \Gamma_{1_{A_1}}$. Since $(\Gamma_{1_{A_1}})^{\circ}$ is $\mathcal{F}.\mathcal{s}$. Open , and $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{+}\ \widehat{\beta}\,(\Gamma_{1_{A_1}})^{\circ}$ is $\mathcal{F}.\mathcal{s}$ .open subset of $\Gamma_{1_{A_1}}$. Since $(\Gamma_{1_{A_1}})^{\circ}$ is the largest $\mathcal{F}.\mathcal{s}$ .open subset of $\Gamma_{1_{A_1}}$ thus $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{+}\ \widehat{\beta}\,(\Gamma_{1_{A_1}})^{\circ} \subseteq (\Gamma_{1_{A_1}})^{\circ}$. hence $(\Gamma_{1_{A_1}})^{\circ}$is $\mathcal{F}.\mathcal{s}$. sub space .

(ii)Let $\Gamma_{1_{A_1}}$ is is $\mathcal{F}.\mathcal{s}$. convex set then $\widehat{\alpha}\,\Gamma_{1_{A_1}} \widehat{+}\ (\widehat{1} - \widehat{\alpha})\,\Gamma_{1_{A_1}} \widehat{\subseteq}\ \Gamma_{1_{A_1}}$, $\widehat{0} \le \widehat{\alpha} \le \widehat{1}$. then $(\widehat{\alpha}\,\Gamma_{1_{A_1}})^{\circ} \widehat{+}\ ((\widehat{1} - \widehat{\alpha})\,\Gamma_{1_{A_1}})^{\circ} \widehat{\subseteq} (\Gamma_{1_{A_1}})^{\circ}$. since $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{=} (\widehat{\alpha}\,\Gamma_{1_{A_1}})^{\circ}$ and $(\widehat{1} - \widehat{\alpha})\,(\Gamma_{1_{A_1}})^{\circ} \widehat{=} ((\widehat{1} - \widehat{\alpha})\,\Gamma_{1_{A_1}})^{\circ}$ thus $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{+}\ (\widehat{1} - \widehat{\alpha})\,(\Gamma_{1_{A_1}})^{\circ} \widehat{\subseteq} (\Gamma_{1_{A_1}})^{\circ}$. hence $(\Gamma_{1_{A_1}})^{\circ}$is is $\mathcal{F}.\mathcal{s}$. convex set

(iii)Let $\widehat{\alpha} \in \mathbb{R}(A)$. since $\Gamma_{1_{A_1}}$ is $\mathcal{F}.\mathcal{s}$. convex set and $(\Gamma_{1_{A_1}})^{\circ} \widehat{\subseteq}\ \Gamma_{1_{A_1}}$. then $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{+} (\widehat{1} - \widehat{\alpha})(\Gamma_{1_{A_1}})^{\circ} \widehat{\subseteq}\ \widehat{\alpha}\,\Gamma_{1_{A_1}} \widehat{+}\ (\widehat{1} - \widehat{\alpha})\,\Gamma_{1_{A_1}} \widehat{\subseteq}\ \Gamma_{1_{A_1}}$. Since $(\Gamma_{1_{A_1}})^{\circ}$ is $\mathcal{F}.\mathcal{s}$. open , and $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{+}\ \widehat{\beta}\,(\Gamma_{1_{A_1}})^{\circ}$ is $\mathcal{F}.\mathcal{s}$ .open subset of $\Gamma_{1_{A_1}}$. Since $(\Gamma_{1_{A_1}})^{\circ}$is the largest $\mathcal{F}.\mathcal{s}$ .open subset of $\Gamma_{1_{A_1}}$ thus $\widehat{\alpha}\,(\Gamma_{1_{A_1}})^{\circ} \widehat{+}\ (\widehat{1} - \widehat{\alpha})\,(\Gamma_{1_{A_1}})^{\circ} \widehat{\subseteq} (\Gamma_{1_{A_1}})^{\circ}$. hence $(\Gamma_{1_{A_1}})^{\circ}$is $\mathcal{F}.\mathcal{s}$. sub space

Theorem 4.8: Every $\mathcal{F}.\mathcal{s}$. normed space($\mathcal{FSV}(\Gamma_A)$,$\|\tilde{.}\|$) is called $\mathcal{F}.\mathcal{s}$. topological vector space

Proof: To proof is

$$(\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}, \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}}) \;\widehat{\Rightarrow}\; \widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}} \widehat{+} \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \qquad \text{(ii)}\ (\widehat{r}\,\widehat{\cdot}\,\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}) \longrightarrow \widehat{r}\,\widehat{\cdot}\,\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}$$

where for all $\widehat{v^1}_{f_1{}^{e_1}_{\Gamma_{1_{A_1}}}}, \widehat{v^2}_{f_2{}^{e_2}_{\Gamma_{2_{A_2}}}} \widehat{\in}\ \Gamma_A\ \mathcal{FSV}(\Gamma_A)$, $\tilde{r} \in \mathbb{R}(A)$ are $\mathcal{F}.\mathcal{s}$. continuous .

Proof :

Let $\widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}$, $\widehat{(v^2)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \widehat{\in} \Gamma_A$and asequence of $\mathcal{F}.\mathcal{s}$ .vectors $\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}$, $\widehat{(v^2)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}$ in Γ_A. such that $\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}\ {}_{n\to\infty} \widehat{\Rightarrow} \widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}$ and $\widehat{(v^2)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}\ {}_{n\to\infty} \widehat{\Rightarrow} \widehat{(v^2)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}$

now $\| \widetilde{(\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widehat{+} \widehat{(v^2)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}})} \widehat{-} (\widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \widehat{+} \widehat{(v^2)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}) \|$

$= \| \widetilde{(\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widehat{-} \widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}})} \widehat{+} (\widehat{(v^2)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widehat{-} \widehat{(v^2)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}) \|$

$\le \| (\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{\widehat{-}} \widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}) \| \widehat{+} \| (\widehat{(v^2)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{\widehat{-}} \widehat{(v^2)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}) \|$

since $\| (\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{\widehat{-}} \widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \|\ {}_{n\to\infty} \widehat{\Rightarrow}\ \widehat{0}$

We have

$\| (\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widehat{+} \widehat{(v^2)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}) \widetilde{\widehat{-}} (\widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \widehat{+} \widehat{(v^2)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}) \|\ {}_{n\to\infty} \widehat{\Rightarrow}\ \widehat{0}$

$\| (\widehat{(v^1)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}} \widetilde{\widehat{+}} \widehat{(v^2)^n}_{f_n{}^{e_n}_{\Gamma_{n_{A_n}}}}) \|\ {}_{n\to\infty} \widehat{\Rightarrow}\ \| (\widehat{(v^1)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}} \widetilde{\widehat{+}} \widehat{(v^2)^0}_{f_0{}^{e_0}_{\Gamma_{0_{A_0}}}}) \|$

Hence $(\widehat{v^1}_{f_1{}_{\Gamma_1}{}_{A_1}}^{e_1}, \widehat{v^2}_{f_2{}_{\Gamma_2}{}_{A_2}}^{e_2}) \longrightarrow \widehat{v^1}_{f_1{}_{\Gamma_1}{}_{A_1}}^{e_1} \widehat{+} \widehat{v^2}_{f_2{}_{\Gamma_2}{}_{A_2}}^{e_2}$ is $\mathcal{F}.\mathcal{S}$ .continuous .

Let $\widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0} \widehat{\in} \Gamma_A$, $\widehat{\alpha} \widehat{\in} \mathbb{R}(A)$ and asequence of $\mathcal{F}.\mathcal{S}$. vectors $\widehat{v^n}_{f_n{}_{\Gamma_n}{}_{A_n}}^{e_n}$ in $\Gamma_A$such that

$\widehat{v^n}_{f_n{}_{\Gamma_{A_n}}}^{e_n}{}_{n\to\infty} \widehat{\Rightarrow} \widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0}$ and $\widehat{\alpha_n}{}_{n\to\infty} \widehat{\Rightarrow} \widehat{\alpha}$.

now

$$\| \widehat{\alpha_n} . \widehat{v^n}_{f_n{}_{\Gamma_n}{}_{A_n}}^{e_n} \widetilde{\widehat{-}} \widehat{\alpha} . \widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0} \|$$

$$= \| (\widehat{\alpha_n} . \widehat{v^n}_{f_n{}_{\Gamma_n}{}_{A_n}}^{e_n} \widehat{-} \widehat{\alpha_n} . \widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0}) \widehat{+} (\widehat{\alpha_n} . \widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0} \widetilde{\widehat{-}} \widehat{\alpha} . \widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0}) \|$$

$$= \| \widehat{\alpha_n} (\widehat{v^n}_{f_n{}_{\Gamma_n}{}_{A_n}}^{e_n} \widehat{-} \widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0}) \widehat{+} (\widetilde{\widehat{\alpha_n} \widehat{-} \widehat{\alpha}) \widehat{v^0}}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0} \|$$

$$\leq |\widehat{\alpha_n}| \| (\widehat{v^n}_{f_n{}_{\Gamma_{A_n}}}^{e_n} \widehat{-} \widehat{v^0}_{f_0{}_{\Gamma_{A_0}}}^{e_0}) \widehat{+} |\widetilde{\widehat{\alpha_n} \widehat{-} \widehat{\alpha}|} \| \widehat{v^0}_{f_0{}_{\Gamma_{A_0}}}^{e_0} \|$$

Since $\| \widehat{v^n}_{f_n{}_{\Gamma_n}{}_{A_n}}^{e_n} \widetilde{\widehat{-}} \widehat{v^0}_{f_0{}_{\Gamma_{A_0}}}^{e_0} \|_{n\to\infty} \widehat{\Rightarrow} \widehat{0}$ and $|\widehat{\alpha_n} \widehat{-} \widehat{\alpha}|_{n\to\infty} \widehat{\Rightarrow} \widehat{0}$

We have

$$\| \widehat{\alpha_n} . \widehat{v^n}_{f_n{}_{\Gamma_n}{}_{A_n}}^{e_n} \widetilde{\widehat{-}} \widehat{\alpha} . \widehat{v^0}_{f_0{}_{\Gamma_0}{}_{A_0}}^{e_0} \|_{n\to\infty} \widehat{\Rightarrow} \widehat{0}$$

$(\widehat{\alpha} \widehat{:} \widehat{v^1}_{f_1{}_{\Gamma_1}{}_{A_1}}^{e_1}) \longrightarrow \widehat{\alpha} \widehat{:} \widehat{v^1}_{f_1{}_{\Gamma_1}{}_{A_1}}^{e_1}$ is $\mathcal{F}.\mathcal{S}$. continuous.

5.Conclusion

in this paper we introduce fuzzy soft topological vector space, fuzzy soft norm, and study some basic properties of those concepts. Also, we study the problem of fuzzy soft normability of fuzzy soft topological vector spaces. There is an ample scope for further research on many problems such as the problems of finite dimensionality, metrizability, open mapping theorem, closed graph theorem etc. in fuzzy soft topological vector spaces.

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