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دراسة عملية التحليل الهرمي ومساحة الوزن النسبية في تحليل مغلف البيانات

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الملخص:
باختصار، يمكن لطريقة *GDM* الجديدة المقترحة تجنب تعقيدات التعديل، ولا تتطلب تجميع أرقام الفواصل. كما أنها تُنتج عددًا من النتائج، بما في ذلك متجه الأولوية المتوقع، ومؤشر القبول، وعدم قبول الترتيب بشكل عام، وهي معلومات واضحة جدًا لاتخاذ القرار الأمثل. بالإضافة إلى ذلك، من عيوب طريقة *GDM* الجديدة أن افتراض التوزيع الموحد للأوزان المهمة لصانعي القرار قد لا يكون الخيار الأمثل في بعض الأحيان، لذلك نرغب في دراسة إمكانية استخدام توزيعات عشوائية أخرى في المستقبل. يتم ذلك باستخدام محاكاة مونت كارلو. لذلك، وبالنظر إلى مخرجات محاكاة مونت كارلو، ستكون النتائج التي تم الحصول عليها لأداء الطريقة الجديدة مختلفة في كل مرة. تُنشئ الطريقة المقترحة إطارًا جديدًا، حيث يُستخدم تحليل قبول مساحة الوزن العشوائية *DEA* لاتخاذ القرارات الجماعية باستخدام *IMPRs*. بالمقارنة مع المناهج الأخرى المعروفة، فإن الطريقة المقترحة خالية من العيوب والتشوهات المحتملة الناتجة عن تعديل الاستقرار. بالإضافة إلى ذلك، لا تتطلب هذه الطريقة خطوة تحديد وزن متخذي القرار الآخرين. إن متجه الأولوية المتوقعة، ومؤشر القبول، ومجموع التقييمات غير المقبولة الناتجة عن الطريقة المقترحة واضحة جدًا لدعم عملية اتخاذ القرار الأمثل بشكل أفضل
الكلمات الرئيسية: قرار المجموعة - مساحة الوزن العشوائية - تحليل مغلف البيانات

Investigating the process of hierarchical analysis and relative weight space in data envelopment analysis

Qays Atshan Khalaf Almusawi

Abstract:

In summary, the proposed new GDM method can avoid the complication of adjustment adjustment and does not require the aggregation of interval numbers. It produces a number of outcomes, including the expected priority vector, the acceptability index, and the overall unacceptability of the ranking, which is very clear information for the best decision. In addition, a limitation of the new GDM method is that the assumption of uniform distribution of the important weights of decision makers may sometimes not be an optimal choice, so we want to consider the possibility of using other random distributions in the future. This is done using Monte Carlo simulation. Therefore, considering the Monte Carlo simulation output, the answers obtained for the performance of the new method will be different each time. The proposed method creates a new framework, in which DEA random weight space acceptance analysis is used for group decision making with IMPRs. Compared to other known approaches, the proposed method is free from possible shortcomings and distortions due to stability adjustment. In addition, the step required to determine the weight of other decision makers is not required. The expected priority vector, acceptance index and total unacceptable ratings generated by the proposed method are too obvious to better support optimal decision making.

Keywords: Group Decision - Random Weight Space - Data Envelopment Analysis

1- مقدمة

تحليل مغلف البيانات (*DEA*) هو طريقة تعتمد على البرمجة الرياضية لتقييم كفاءة مجموعة من وحدات اتخاذ القرار (*DMUS*). يُستخدم تحليل مغلف البيانات (*DEA*) لتقييم عملية إنتاج الوحدات التشغيلية؛ واليوم، نظرًا لحاجة جميع الشركات والمؤسسات إلى زيادة كفاءتها وفعاليتها، يُعد تحليل البيانات المتعلقة بها أمرًا بالغ الأهمية، لأنه أحد أساليب تقييم أداء المؤسسات ووحدات اتخاذ القرار. في هذا البحث، تمت دراسة عملية اتخاذ القرارات الجماعية باستخدام علاقات الكفاءة المضاعفة باستخدام مفهوم الكفاءة المتقاطعة لتحليل مغلف البيانات المضاعف والفضاء العشوائي المرجح. وتم اقتراح نموذج مضاعف لتحليل مغلف البيانات (*DEA*). وتم تحليل بعض خصائص النموذج ضمنيًا. [4-1]

1-Introduction

data envelopment analysis (dea) is a method based on mathematical programming for evaluating the efficiency of a set of decision maker units (dmus) . data envelopment analysis (dea) is used to evaluate the production process of operational units ; today , due to the need of all companies and institutions to increase their efficiency and efficiency , analysis of data related to them is very crucial , because data envelopment analysis (dea) is one of the methods for evaluating the performance of organizations and decision making units . in this research , group decision making with multiplicative efficiency relations is studied using the cross - efficiency concept of multiplicative data envelopment analysis and weighted random space . a multiplicative model of data envelopment analysis (dea) is proposed . some features of the model are implicitly analyzed .[1-4]

Necessity and importance of research

Since the calculation of efficiency and the amount of interest on companies, institutions and organizations in the present period is of great importance for the development of societies, so research and development of mathematical models to measure efficiency for the optimal use of resources is necessary. Is.[5-6]

Data envelopment analysis is a non - parametric technique for calculating the performance of a set of decision - making units with multiple inputs and outputs which were presented by Charns et al . [7] . In recent years , performance relationships , widely adopted to integrate expert opinions in the group decision - making modeling , have been studied and studied by researchers [8-9] . Two common performance relationships are used in order to improve the multiplicative efficiency and phase performance relations . The analytical hierarchy process (AHP provides a suitable framework for inferring the results

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using paired comparisons [10] . but in the evaluation using analytic hierarchy analysis , a fundamental problem of compatibility (CR) is the paired comparisons matrix . If the cart 's rate index is less than 0.1 , it can be used , otherwise the results of this method are not accepted [11] .

2- Hierarchical analysis process

In the present era, we face different multi-criteria decision-making in everyday life, from choosing a laptop to choosing a job, etc. For example, when choosing a job, different criteria such as income, social status, creativity, innovation, etc. are suggested that the decision-maker should consider different options according to these criteria, as well as different criteria such as cost, proximity to work, access to shopping centers, etc. that should be the best option in terms of these criteria. [12]

In the industrial arena, sometimes the outcome of decision-making is so important that errors may inflict irreparable losses on us. In macro decisions such as setting the country's annual budget, experts pursue different objectives such as security, education, industrial development, etc. and wish to optimize these goals.[13]

The decision-making process faces many problems with several quantitative and qualitative criteria such as:

- Lack of standards for measuring quality criteria

- Not having units to convert criteria (qualitative and quantitative) to each other

In addition, due to the problems related to the decision-making process with multiple criteria, including complexity and lack of standards, the speed and accuracy of decision making is greatly reduced and makes this process largely dependent on the decision-maker. Therefore, it is necessary to design appropriate techniques or techniques for optimal selection and correct decision making, so in 1980, the AHP technique, meaning the analytical hierarchy process by Thomas-L.H., was designed to solve such a problem, one of the most comprehensive systems designed to make decisions with multiple criteria, since we can:

- formulate the decision-making process;

- Consider different qualitative and quantitative criteria;

-bring decision-making options into the problem;

- Analyze the sensitivity on the criteria and sub-criteria.

In addition, we obtain decision compatibility and incompatibility, which is a privileged feature of this process (incompatibility of a decision shows us the number of errors and errors).

By analyzing the problems and complex problems, this method has found many applications in economic and social problems.

2-1- Decision making using analytical hierarchy process

The implementation of AHP in a decision consists of 3 phases:

1- Hierarchical tree formation

2- Performing pair comparisons

3- Calculation of weights

Phase 1: The formation of a hierarchical tree is a graphical representation of the real complex problem, at the head of which is the general purpose of the problem and at themselves of criteria and options, although there is not a fixed rule for hierarchical drawing, but some people have tried to express a general set of rules in this regard, for example, a hierarchy maybe one of the following:

Target - Criteria - Sub-Criteria - Options

Target - Criteria-Factors - Sub-Factors - Options

At a general glance, it can be said that the way to build a hierarchy depends on the type of decision that needs to be made. For example, if the decision is to choose an option, it is possible to start from the options and show them at the lowest level and at the next level the criteria that are intended for selecting the options |, and at the highest level, the hierarchical goal which is an element, | sometimes the criteria themselves must be analyzed more in part. In such cases, another level (including sub-criteria) is added to the hierarchy, but it is not necessary that all criteria have sub-criteria, in a hierarchy there is no limit to the number of levels, whenever elements of a level cannot be compared with the elements of higher levels, the question arises as to what this element is comparable to in which case another level may be added to the hierarchy. Complete the hierarchy.[14]

Imagine, for example, that among the four cars A, B, C, D, we want to choose one in the choice of cars, three criteria of cost, quality and beauty are considered as follows:

Each of the criteria can also have sub-criteria, such as quality index can have sub-criteria or sub-indicators such as after-sales service and safety.

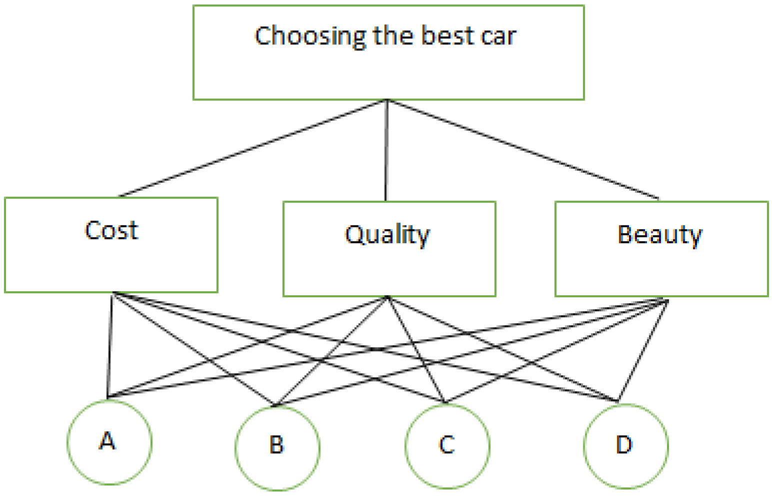
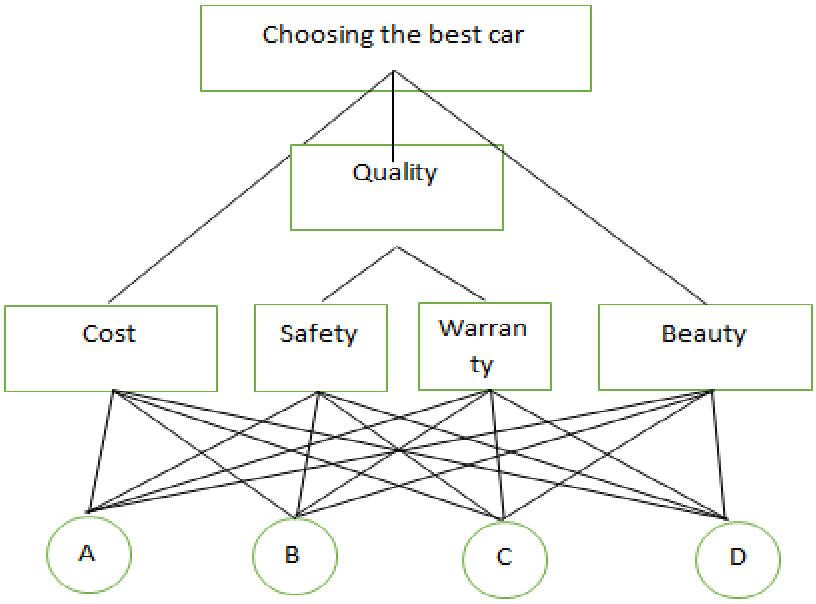


Figure 1: Hierarchical tree choosing the best car



In any case, the AHP technique obtains a score for each component of this tree, whether they are options or criteria, and we call each component of this tree, whether options or criteria or sub-criteria in the term item. [15]

In the AHP method, we get one point for each of the options, and the options are ranked according to the points they have scored, of course, the option that has the most points are the best option to be selected, the method used in AHP to calculate the points is based on the pair comparisons that we describe in the second phase.

Phase 2: Pair comparisons

In AHP, the elements of each level are compared in pairs compared to their respective elements at the higher level and the pair comparison matrix is formed, then the relative weight of the elements is calculated using this matrix, in general, a pair comparison matrix is shown as follows, in which the element's preference is relative to the element.$a_{ij} ij$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Preferences	Numeric value
Quite more importantly.	9
Above	7
Important	5
A little important	3
the same	1

Table 1:Oral Judgments

D	C	B	A	Cost
1	¼	6	1	A
1/5	½	1	1/6	B
1/3	1	2	4	C
1	3	5	1	D

Table 2: Cost Criteria

iCompared to element j, ij the decision-maker will say which of the following states are: identical, slightly important, more important and quite important. In oral opinions used for qualitative indicators, one should consider that if the importance of the element is equal, the importance is equal, and according to this point, it is enough to fill only the high values of the original diameter in the pair comparison matrix, in other words, the values below the original diameter will be inversely high values of diameter. $ij\ nji \frac{1}{n} a_{ij} = \frac{1}{a_{ji}}$

In the previous example, if we want to compare cars based on cost criteria, we first compare cars A with B in this regard, and then A, C, A, D and D, followed by B, B, C and D, and finally C and D (Table 2-2). Also, let's assume that in the second level, the relative importance of the criteria to the target level is measured, and we have extracted the pair comparison matrix 2-5 from the decision-maker's point of view.[16]

D	C	B	A	Quality
1/6	1/5	1	1	A

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1/4	1/3	1	1/1	B
1/5	1	3	5	C
1	5	4	6	D

Table 3: Quality Criteria

In these comparisons, the recipients use oral judgments in such a way that if the element is compared to the element, the decision-maker will say which of the following states are: identical, slightly important, more important and quite important. In oral opinions used for qualitative indicators, one should consider that if the importance of the element is$ijnj$ equal, the importance is $i$equal, and according to this point, it is enough to fill only the high values of the original diameter in the pair comparison matrix, in other $\frac{1}{n}$ words, the values below the original diameter will be inversely high values of diameter, in other words.$a_{ij} = \frac{1}{a_{ji}}$

In the previous example, if we want to compare cars based on cost criteria, we first compare cars A with B in this regard, and then we compare A and C, A and D, followed by B, C, B and D, and finally C and D (Table 2-2). Also, in the second level, the relative importance of the criteria to the target level is measured, we have extracted the pair comparison matrix 2-5 from the decision-maker's point of view, it should be noted that in the case of criteria, we should pay attention to their type of criteria, for example, the cost criterion is a criterion that its low is desirable and the comfort criterion is a kind that its highness is desirable.

Phase 3: Extracting weights from the decision matrix

Weight calculation in the analytical hierarchy process consists of two separate parts:

- The weight of the NESSB.

- Final weight

Relative weight is obtained from the comparison of the pair matrix, while absolute weight is the final rank of each option, calculated by the combination of relative weights.

D	C	B	A	Beauty
1/2	1/5	5	1	A
1/5	1/2	1	1/5	B
1/4	1	2	5	C
1	4	5	2	D

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Table 4: Beauty Criteria

Cost	Beauty	Quality	Goal
3	3	1	Quality
1/2	1	1/3	Beauty
1	2	1/3	Cost

Table 5: Matrix Comparison criteria

2-2-Methods of calculating relative weight

The method of calculating weights from the decision matrix depends on whether or not the decision matrix is compatible, if the condition is in the decision matrix, we say that the decision matrix is compatible, which means that if, for example, the item is twice as preferred as the$a_{ij} \times a_{jk} = a_{ik} ij(a_{ij}) k(a_{jk})$ item, then the item is $i(a_{ik}) kik6$ times more important than the item. However, it may occur when the importance of the item is not 6 times that of the item, and this relationship is not established, and the failure to establish this relationship amounts to heterogeneity or incompatibility, always the decision matrix that is obtained from comparing the options to a quantitative criterion has this property, but in the case of qualitative criteria, if this property is not established, the matrix is incompatible, which is usually the matrix that meets the criteria. Qualitative and verbal opinions are produced incompatible for each type of decision matrix, there is a special method for calculating weights that we express separately.[17]

Extraction of weights from the compatible matrix

In this case, if the criterion has a positive direction (its high is desirable), we normalize the components of an arbitrary column relative to the sum of that column and the weights are obtained.

If the criterion has a negative direction (its low is desirable), we normalize the components of a desired row relative to the sum of that row and the weights are calculated.

Extract weights from incompatible matrix

In this case, the above method cannot be used to extract weights, four main methods are available in calculating weights in the incompatibility state of the decision matrix:

-Approximate methods

- The method of minimum squares

- Special vector method

- Logarithmic least squares method

In this thesis, we only use approximate methods and briefly describe this method below.

2-3-Approximate Methods

Approximate methods include the following four methods:

1- Total row method

2- Column Sum Method

3- Arithmetic mean method

4- Geometric mean method

In the sum of row method, the sum of elements of each row is written in a vector and the resulting vector is normalized. Normalization means dividing all elements into the sum of the elements of that vector in the resulting vector.

In the column sum method, similar to the method we used in the sum of the row, the sum of the elements of each column is written in a vector, but this time the reverse vector considers it and normalizes the resulting vector.

In the arithmetic mean method, the columns of the main matrix are normalized first and then their mean rows are calculated, finally, in the geometric mean method, the geometric average of the rows is calculated and the final vector is normalized.[18]

2-4-Calculate the final weight or eolith of each option

After we have gained the weight of each option relative to each criterion, we calculate the weight of the criterion itself similarly to the target, then the final weight of each option is calculated as follows:

$$w_i = \sum_{j=1}^{n} w_{ij} \times v_j$$

Where the weight is the option relative to the benchmark w_{ij} $ij$and $v_j$the weight of the benchmark.j

Here's an example with a complete description of the description.

Example

Suppose we decide to buy a car and the options for this issue are three cars A, B, C. Also, our selection criteria include price, fuel consumption, convenience and model to solve the problem first, then we calculate the weight and at the end of the final weight we specify each option.[19]

Building hierarchies

As mentioned above, the hierarchy is a graphical representation of the problem (purpose, criteria, and selection options) and this graphical representation

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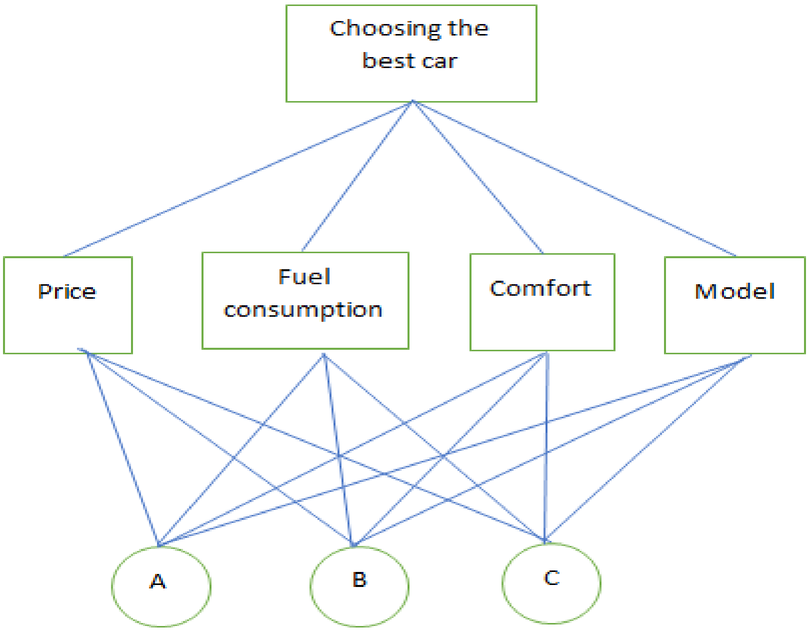
depends on the decision-maker's opinion, assuming that the graphical representation for the desired example is as follows:

Now, from the lowest level, we refine the hierarchy, and in the first step, we form a pair comparison matrix for each criterion separately.

In this example, in terms of convenience, preference A over B is number 2, preference A to C is number 8 and preference B over C is number 6. Thus, preference B over A 1.2, preference of Cover A 1.8 and preference of Cover B is 1.6.

$$\begin{bmatrix} 1 & 2 & 8 \\ \frac{1}{2} & 1 & 6 \\ \frac{1}{8} & \frac{1}{6} & 1 \end{bmatrix}$$



Level 1 – Goal

Level 2 – Criteria

Last Level – Options

Figure 2: Option hierarchy of car selection

In the next step, we use the arithmetic average to determine the weights, i.e., take the following steps:

1- We obtain the sum of each column

2- We divide each element in the pair matrix into its own column sum to normalize the pair matrix.

In this step, we have the following matrix:

$$\begin{bmatrix} \frac{8}{13} & \frac{12}{19} & \frac{8}{15} \\ \frac{4}{13} & \frac{6}{19} & \frac{6}{15} \\ \frac{1}{13} & \frac{1}{19} & \frac{1}{15} \end{bmatrix}$$

3- We calculate the average value of each row in the normalized matrix.

By taking the steps above, the average of the first row is 0.593, the average second rowis0.341 and the average third row is 0.066, so we see that considering the convenience, the A car (weighing0.593NASB) interest car.

Model	Comfort	Consumption	Price	
0.265	0.593	0.087	0.123	Car A
0.655	0.341	0.274	0.320	Car B
0.080	0.066	0.639	0.557	Car C

Table 6: The weight of options relative to the criteria

Model	Comfort	Consumption	Price	
2	2	3	1	Price
4/1	4/1	1	3/1	Consumption
2/1	1	4	2/1	Comfort
2/1	4	2	1	Model

Table 7: Pair comparison matrix of criteria

We perform these steps similarly for other criteria, i.e., pair comparison matrix and normalization matrix for price criterion, fuel consumption and model, and calculate relative weight. To calculate these weights, the decision maker must first compare the cars in pairs to each criterion, forming a normalized matrix and taking an average from each row, assuming the detain table 2-6 is obtained after the calculations. [20]

Now we're scrolling the higher level.

At this level, the criteria should be compared two by two. The paired comparison matrix of criteria is followed (Table 2-7).

And after calculating the matrix and the average of each row, the relative weight of each criterion is obtained (Table 2-8). In this example, we first calculated the weight of the options compared to each criterion, and then we calculated the weight of the criteria according to the goal (choosing the best car), so the final weight of each option can be calculated as follows:

$$w_A = 0.398 \times 0.123 + 0.058 \times 0.087 + 0.218 \times 0.593 + 0.299 \times 0.265 = 0.265$$

And so on:

$$w_B = 0.421$$
$$w_C = 0.314$$

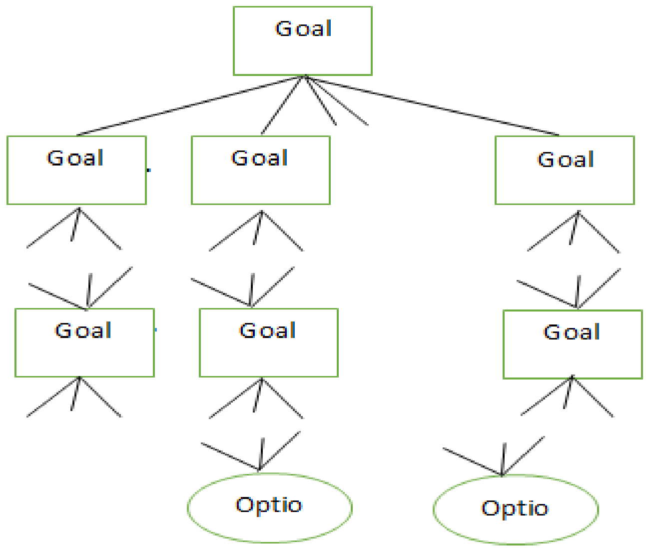
As you can see, the B car is chosen.

0.398	Price
0.085	Consumption
0.218	Comfort
0.299	Model

Table 2-8: Relative weight of each criterion

3-AHP Model Capabilities

One of the capabilities of the AHP model is that the criteria can be multi-layered in the form of general criteria and sub-criteria, and the elements of differentiates not have to be connected or can be changed according to the structure of the problem of these relationships, the general state of a hierarchical tree can be shown as follows:



The use of other people's opinions and thoughts reduces decision-making errorsand improves tasks, considering that decision making in offices and organizations is complicated, it may not be an individual's decision-makers and a group of decisionmakers in AHP, it is possible to determine the decision matrix in AHP only in the case of qualitative criteria depending on the oral opinion and pair comparisons of the decision-makers. Now, if there isa group of decisionmakers who want to comment on the options to a qualitative criterion, AHP allows them to calculate the relative weight of the option using the opinions of all decision makers.[21]

$$a'_{ij} = \left(\prod_{I=1}^{k} a_{ijl}\right)^{\frac{1}{k}},$$
$$I = 1,2,\dots,k,$$
$$i,j = 1,2,\dots,n \quad i \neq j$$

That's the number of decision makers.k

The decision-makers' votes may not be the same if each decision-maker has a greater impact on the votes according to expertise and responsibility, in which case the following relationship will be used:(w_l)

$$a'_{ij} = \left\{\prod_{l=1}^{k} a_{ijl}^{wl}\right\}^{\frac{1}{\sum_{l=1}^{k} wl}}$$

4-Multi-Objective Phase Stochastic Data **Envelopment** Analysis **Model (MOFS-DEA)**

4-1--Original CCR Model

The main data encody analysis model (CCR) presented by Charnes et al. (1978) is as follows:

$$Max \quad Z_0(U,V) = \frac{U^T Y_0}{V^T X_0}$$
$$st.$$
$$\frac{U^T Y_j}{V^T X_j} \leq 1, \quad j = 1,2,\dots,n,$$
$$U,V \geq \varepsilon > 0.$$

In this model, it is $Y_j = (y_{1j},\dots,y_{sj}), X_j(x_{1j},\dots,x_{mj})$ assumed $\mathrm{DMU_j U} = (\mathrm{u_1},\dots,\mathrm{u_s}), \mathrm{V} = (\mathrm{v_1},\dots,\mathrm{v_m})$that n decision-making units use input m to generate output s, and the main idea of this model is to find weights (U,V) in order to maximize the efficiency for which the number of decision-making units should be implemented in order to maximize their efficiency. $DMU_o\varepsilon$

4-2-Proposed MOFS-DEA Model

In the dynamic CCR model, it is assumed that all inputs and outputs of units are constant and definitive values, but in real-world prediction issues, decision makers are usually faced with dynamic environments that due to uncertainty in the nature of these environments, inputs and outputs become random variables, and because decision-makers usually become random variables due to the impact of decision-makers. Environmental factors such as economic and political conditions on them cannotobtain the exact characteristics of these variables, so the phase aspect is added to the random nature of the input and output variables of the units, so in the proposed model of the paper, all future inputs and outputs of units (variable and pseudo-constant) are defined as random phase variables, as follows:

$$X_j^t = \left(x_{ij}^t - a_{ij}^t, x_{ij}^t + b_{ij}^t\right) \quad i = 1, \dots, m \quad j = 1, \dots, n$$
$$Y_j^t = \left(y_{rj}^t - c_{rj}^t, y_{rj}^t + d_{rj}^t\right) \quad r = 1, \dots, s$$
$$K_j^t = \left(k_{lj}^t - e_{lj}^t, k_{lj}^t + f_{lj}^t\right) \quad l = 1, \dots, L$$
$$K_j^{t-1} = \left(k_{lj}^{t-1} - e_{lj}^{t-1}, k_{lj}^{t-1} + f_{lj}^{t-1}\right) \quad l = 1, \dots, L$$

It is assumed and they are. Also, the mean and $K_{lj}^t \sim N\left(\bar{\bar{\mu}}_{lj}^t, \bar{\bar{\sigma}}_{lj}^{2^t}\right), Y_{rj}^t \sim N\left(\bar{\mu}_{rj}^t, \bar{\sigma}_{rj}^{2^t}\right), X_{ij}^t \sim N\left(\mu_{lj}^t, \sigma_{lj}^{2^t}\right)\ K_{lj}^{t-1} \sim N\left(\bar{\bar{\mu}}_{lj}^{t-1}, \bar{\bar{\sigma}}_{lj}^{2^{t-1}}\right)$varia ncesof input and outputvariables are determined through the past performance of DMU and the actual values are positive, which createshigh and low$a_{ij}^t, b_{ij}^t, c_{rj}^t, d_{rj}^t, e_{lj}^t, f_{lj}^t, e_{lj}^{t-1}, f_{lj}^{t-1}$boundsfor phase inputs and outputsand is estimated by experts for the future financial period of the units. According to the definition for inputsand outputs, and also considering that we are faced with a stochastic-fuzzy environment to predict the efficiency of units, the average probability, which plays the same probability role in the random environment and the role of validity in the fuzzy environment, is used in expressing the limitations and the concept of the expected value for the objective function of the proposed model in order to predict the expected efficiency of the units. On the other hand, to use common weights in the proposed model that can overcome the weakness of unrealistic weight distribution in DEA, fuzzy multi-objective programming approach It has been used in modeling. According to the above definitions, the proposed model of fuzzy random data envelopment analysis (MOFS-DEA) with common weights in dynamic environment to predict the efficiency of units will be as follows:

$$Max \quad Z_1^t = E\left[\frac{U^t Y_1^t + \rho^t k_1^t}{V^t X_n^t + \beta^{t-1} K_n^{t-1}}\right]$$
$$\vdots$$
$$\max \quad Z_1^t = E\left[\frac{U^t Y_1^t + \rho^t k_1^t}{V^t X_n^t + \beta^{t-1} K_n^{t-1}}\right]$$
$$st.$$
$$Ch\left\{\left(V^t X_j^t + \beta^{t-1} K_j^{t-1}\right) - \left(U^t Y_j^t + \rho^t k_j^t\right) \geq 0\right\} \geq 1 - \alpha_j^t, j = 1,2, \dots, n,$$
$$U^t, V^t, \rho^t, \beta^{t-1} \geq \varepsilon, \quad t = 1, \dots, T.$$

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In this case, the decision-taking level in relation to the satisfaction of the zam limit for α_j^tDMU$_j$ in the period of time, in the proposed model $(\alpha_j^t \in [0,1))$above, the decision variables of the model in the period t, the common weight of inputs and outputs, i.e., which is common among all decision-making units. In this model, the efficiency of the units as well as the $(U^t, V^t, \rho^t, \beta^{t-1})$-optimalvaluesof inputs and outputsin each period will be dependent on the risk level, in α_j^t general, applying the average probability theory in constraints and mathematical hope in the target functions has caused the difficulty of the calculations of the high model based on the assumption that the inputs and outputsof the proposed model are random triangular fuzzy variables, in the next part the proposed model based on the theories of the average probability theory in the third part will be converted to the random programming model.

Example.

A family with three members wants to establish a criterion weighting scheme for portfolios. Each member employs the pairwise comparison method to provide his/her IMPR, as follows:

$$\tilde{R}^{(1)} = \begin{pmatrix} [1,1] & [1/3, 1/2] & [1/7, 1/4] \\ [2,3] & [1,1] & [1/6, 1/5] \\ [4,7] & [5,6] & [1,1] \end{pmatrix},$$

$$\tilde{R}^{(2)} = \begin{pmatrix} [1,1] & [1/7, 1/3] & [7,9] \\ [3,7] & [1,1] & [5,7] \\ [1/9, 1/7] & [1/7, 1/5] & [1,1] \end{pmatrix}$$

$$\tilde{R}^{(3)} = \begin{pmatrix} [1,1] & [5/3, 4] & [3,6] \\ [1/4, 3/5] & [1,1] & [5/3, 2] \\ [1/6, 1/3] & [1/2, 3/5] & [1,1] \end{pmatrix}$$

Forthe nightof theMonte Carloinstrument, theKof MaterS hasanacceptable rating.

$$A = \begin{pmatrix} 0.0133 & 0.9867 & 0 \\ 0.9867 & 0.0133 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

MaterQ is an indexof modified rank inthe form of A.Y.D.:

$$C = \begin{pmatrix} 0.7400 & 0.0100 & 0.7483 \\ 0.0100 & 0.7400 & 0.9967 \\ 1 & 0.7500 & 0 \end{pmatrix}.$$

Using the base model produces an optimal$y_{12}^* = 1, y_{21}^* = 1, y_{33}^* = 1$solution, the optimal ranking of all variables is determined as follows:

$$x_2(0.9867) > x_1(0.9867) > x_3(1),$$

Like mentioned above, we will have a high weighted index:

$$E(w_1) = 0.3613, E(w_2) = 0.3851, E(w_3) = 0.2535$$

which also supports the decision of x2x1x3. The obtained rank is the same as the results given by Wang and Lin [27]. In Wang and Lin's method [27], the decision makers' weights need to be given in advance, and the interval multiplicative weight vector of three criteria is obtained as w˜ = ([0.8476, 1.2201],[1.0791, 1.4787],[0.6888, 0.7713])T .

5-Conclusions

In this paper, we investigated group decision making through preference relations considering the concepts of efficiency and DEA. A multiplicative DEA model can predict the relative efficiency evaluation for MPRs. Through obtaining the relations and numerical example examined in this research, it was concluded that the assignment problem model can predict the optimal ranking to minimize the total composite rank. The numerical example examined in this research indicated that the results obtained from this paper are superior and advantageous compared to other studies.

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