Dynamic Analysis of Isolated Machine Foundation by Finite Element Method

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Abstract
In this work, the study is made upon the dynamic behaviour of isolated foundations supporting vibrating machinery considering the effect of soil-structure interaction. The finite element procedure is used in simulating of the machine foundation for its capability in modeling and its compatibility with the computer software. The foundation is discretized using plate elements having the capability of sustaining forces in three directions as well as bending . The soil-structure interaction is modeled using Winkler type foundation having moduli in three directions. The modeling and analysis are done by the NASTRAN computer software. The effect of the base foundation and the stiffness of isolated springs are considered in the analysis.

التحليل الديناميكي لاسس المكائن المعزولة بطريقة العناصر المحددة

الخلاصة
يهتم البحث بدراسة التصرف الديناميكي لأسس المكائن المعزولة بنوابض و الساندة للمكائن المهتزة مع الاخذ بنظر الاعتبار التداخل بين التربة والأساس . أستخدمت طريقة العناصر المحددة في تمثيل الاساس والتربة . حيث تم تمثيل الاساس باعتماد عناصر ثنائية البعد مستطيلة الشكل لها القابلية باخذ القوى بثلاث اتجاهات بالاضافة الى عزوم الانحناء . أما التربة فقد تم تمثيلها باعتماد النموذج وينكلر المرن بمعاملات لرد فعل التربة بثلاث اتجاهات.

Introduction

The problem associated with the design of foundations to resist dynamic loadings, either from supported machinery or from external sources, still requires special solutions dictated by local soil conditions and environment. In special cases and due to environment conditions, it may be necessary to limit the vibration amplitude at the foundation base to a much lower values than those usually allowed for static analysis. This requirement may not be practical to achieve even by selection of largermass or base area of the foundation. In such cases, use of an inertia block and spring absorbers are recommended [2]. Also spring absorbers are used to prevent excessive propagation of vibration to adjacent structures. The finite element procedure is used in the simulation of the machine foundation for its capability in modeling and its compatibility with the computer software. A lot of research had been developed for the analysis of soil structure interaction by the finite element method. Severn [6] reported the use finite element method for solving mat foundations. He presented a

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derivation of stiffness matrix of a plate, which includes the effect of an elastic foundation. Wegmuller A.W. et al [7] used a finite plate bending element in the discretization of mat foundation and used discrete springs in representing Winkler foundation. They used this model in analyzing a papermachine mat foundation. Abul-Ella and Novak [1] presented a plate element in which the element stiffness matrix was modified so as to incorporate the supporting medium consisting of piles and soil. They used this type of element for the dynamic analysis of pile supported frame foundation. In such generalization, the stiffness matrix was achieved by using the same displacement function as the plate alone and then adding the contribution from the piles.

In this paper, the study is made upon the dynamic behaviour of isolated foundation (by means of isolated springs) supporting vibrating machinery by considering the effect of soil-structure interaction. The foundation is discretized by using plate elements having the capability of sustaining forces in three directions as well as bending [4]. The soil-structure interaction is modeled using Winkler type foundation with subgrade reaction having moduli in three directions. The modeling and analysis are done by the NASTRAN computer software.

Element Stiffness Matrix:

The element stiffness matrix of the 4-nodded plate element in membrane and bending action can be easily derived by using the principle of virtual work. The adopted element has the ability of sustaining forces and bending moments in three directions. Accordingly, the stiffness matrices of plane stress and plate bending elements are superposed [5]. The resulted stiffness matrix is given in Appendix.

Element Mass Matrix:

The simplest form of mathematical model for inertia properties of structural elements is the lumped-mass representation. In this idealization concentrated masses are placed at the nodal points at which the only translational displacements are defined. The elements of matrix adopted are as listed in Appendix.

Damping of Soil:

Damping in a soil-structure system consists of two components, geometric damping and material damping. Geometric component is a measure of energy radiated away from the immediate region of the foundation. Material damping within the soil is a measure of energy lost as a result of hysteresis effects. In this work, a damping ratio with a value of (ξ=0.15) with respect to the critical damping is adopted.

Dynamic Forces:

The machine manufacturers generally furnished data concerning the unbalanced forces. For the case of centrifugal machine that used in this study the forces are generated by unbalanced rotating mass. The used dynamic forces are given in Appendix.

Modulus of subgrade reaction:

The determination of moduli of subgrade reaction is generally carried out by formulas for spring constants as given in table (1).

Case Study:

The problem of a centrifugal machine resting on inertia block and isolated from the foundation by isolator springs is considered. The detail of isolator spring is shown in Fig.(2) and the modeling of such isolator is given in Fig.(3). The information adopted in the analysis is listed in table (3). The soil consists of soft silty clay having shear modulus of ($25MN/m^2$) and Poisson's ratio equal to (0.35). The foundation slab is to be represented by plate elements and the soil by elastic foundation springs.

The data about the isolators used in the problem are listed in table (4). Also, the layout of the foundation and inertia block is shown in Fig.(4).

The foundation is discretized into (140) rectangular plate elements and the inertia block is discretized into (108) rectangular plate elements. The mass of machine is modeled using mass elements having acceleration in three directions and lumped at level of machine centroid. The whole modeling of the system is given in Fig.(5). The soil reactions are taking according to table (1).

Results and Discussion:

The main parameters adopted in this study are the foundation thickness and the isolator spring stiffness. Fig.(6) shows the lower twelve mode shapes of the system for the case of foundation thickness equal to (0.8m) and isolator stiffness K_x=30,K_y=K_z= 15(MN/m). It can be seen that these modes are of coupled rigid motions of two bodies. This indicates that the stiffness built in both the foundation and the inertia block are greater than those built in the soil and the isolated springs. Fig.(7) shows small amplitude values due to the effect of isolators.

Fig.(8) shows that the increase in foundation thickness have a negligible effect on the natural frequencies of the lower modes. However, the natural frequencies of higher modes are decreased with increasing in foundation thickness. Also ،a convergence can be seen between the natural frequencies of the higher modes and those of the lower modes when the foundation thickness is increased. This indicates that the lower modes are corresponding to the response of the inertia block and the machine where the smallest relative stiffness built in the system are those of isolated springs. Fig.(9) shows that the natural frequencies is increased with increasing in stiffness of isolators which characterize their motion even those of lower modes.

Fig.(10) shows that the foundation thickness has a negligible effect on the amplitudes. Fig.(11) shows that the amplitudes increase with the increase in the isolator stiffness.

Conclusions:

From the results of the present work, the following points may be concluded:

1. The finite element representation is used for considering the elastic behaviour of foundation in the dynamic analysis. It is also effective in coupling of all expected modes of vibration and it can be used for solving different shapes of foundations.

2. The variation of the base thickness has a negligible effect on the lower natural frequencies. However, the natural frequencies of the higher modes are decreased with the increase of foundation thickness.
3. The natural frequencies are increased with increasing of isolator stiffness.
4. The variation of the base thickness has a little effect on the amplitudes of the vibration with high operating frequency machine.
5. The amplitude increases with increasing in stiffness of isolator springs for the foundation with high operating frequency machine.

References:

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6. Severn R"،.The Solution of Foundation Mat Problems by Finite Element Method", Structural Engineer, Vol.44,No.6,(1966).
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Table (1): Spring constants for rectangular footing [2].

Motion	Spring constant	Notation
Vertical	$k_v = \frac{G}{1-\mu}\beta_z\sqrt{BL}\eta_z$	G: Soil shear modulus. μ :Poisson's ratio of the soil. β :Geometry factor.
Horizontal	$k_h = 2(1+\mu)G\beta_x\sqrt{BL}\eta_x$	η :Embedment coefficient. L, B: Footing length and width.

Table (2): Embedment coefficients for spring constants [2]

Motion	Coefficient	Notation
Vertical	$\eta_z = 1+0.6(1-\mu)(H/r_o)$	H: depth of foundation embedment below the grade.
Horizontal	$\eta_x = 1+0.55(2-\mu)(H/r_o)$	r_o: equivalent radius for rectangular foundation = $\sqrt{BL/\pi}$.

Table (3): Data used in the case study.

Data	Values
Total mass of the machine, (kg).	18000
Center of gravity height of the machine above the foundation base, (m).	1
Working frequency, Hertz.	40
Unbalanced centrifugal force, (kN).	5
Length of the inertia block, (m).	5
Width of the inertia block, (m).	2.5
Thickness of the inertia block, (m).	0.2
Length of footing, (m).	8.4
Width of footing, (m).	6
Thickness of footing, (m)	0.8

Table (4): Data about the isolators used in the problem.

Data	Values
No. of isolator springs.	24
No. of isolator springs along the longitudinal direction.	9
No. of isolator springs along the lateral direction.	4
Interior distance of isolator from the edge of inertia block, in (m).	0.2
Height of isolator, in (m).	0.15
Stiffness of the isolator, in (MN/m):	
K_x.	30
K_y.	15
K_z.	15

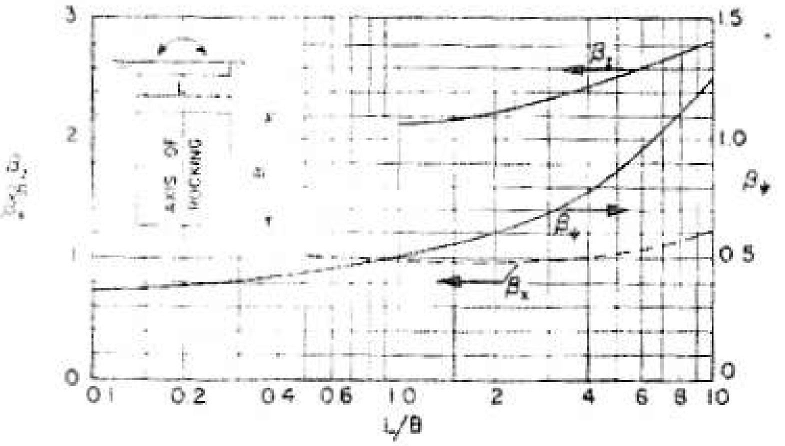


Fig.(1): Geometry factor for rectangular footings [2].

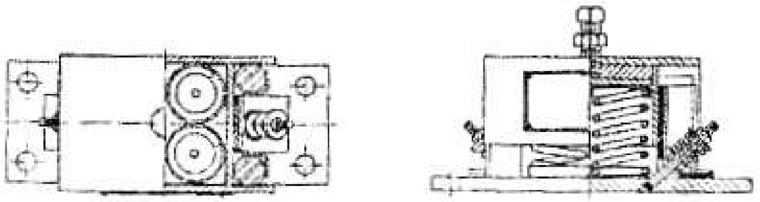


Fig.(2): Isolated spring used with the machine foundation.

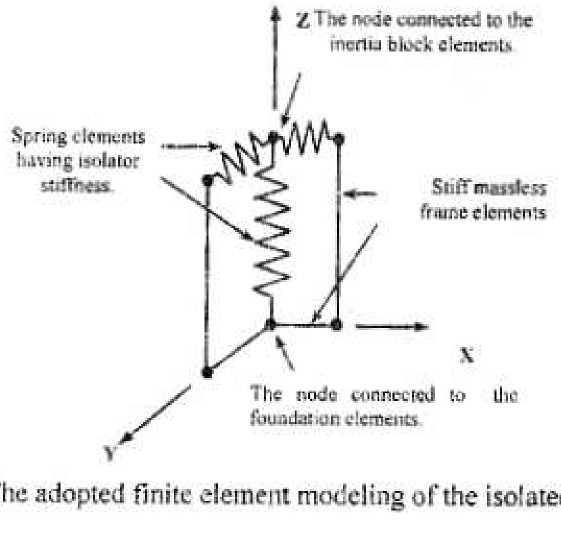


Fig.(3): The adopted finite element modeling of the isolated springs [4].

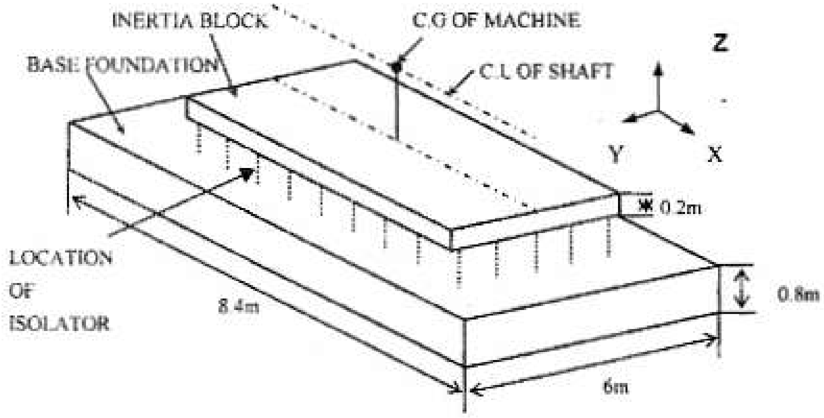
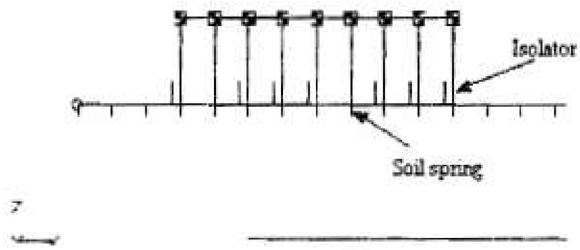
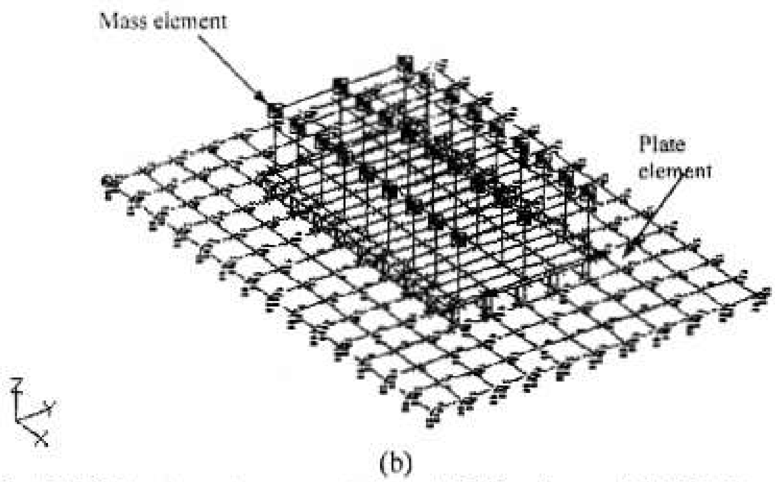


Fig.(4): Layout of the isolated foundation.



(a)



(b)

Fig.(5):Finite element representation ;(a) Side view and (b) 3D view.

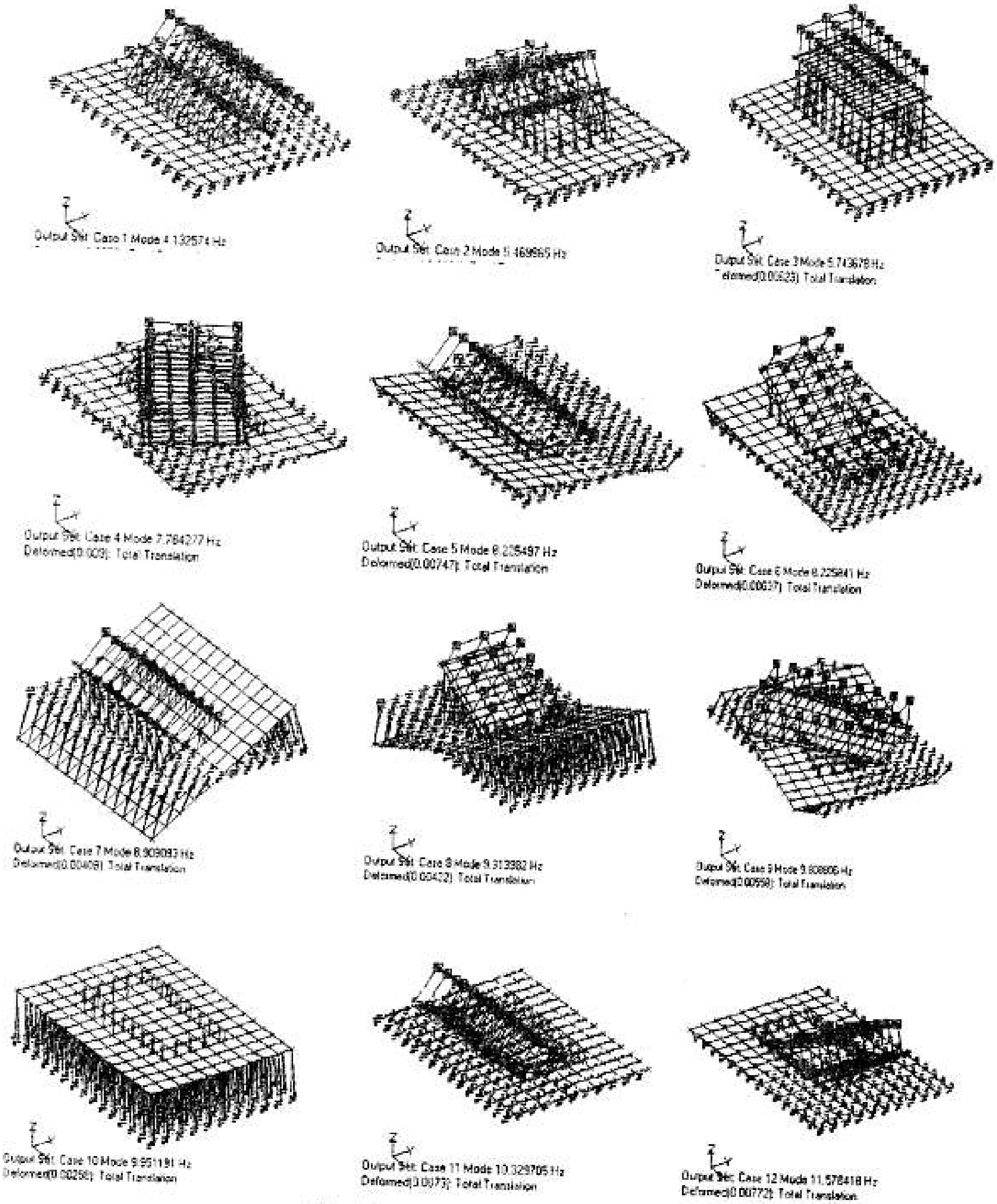


Fig.(6): First twelve mode shapes.

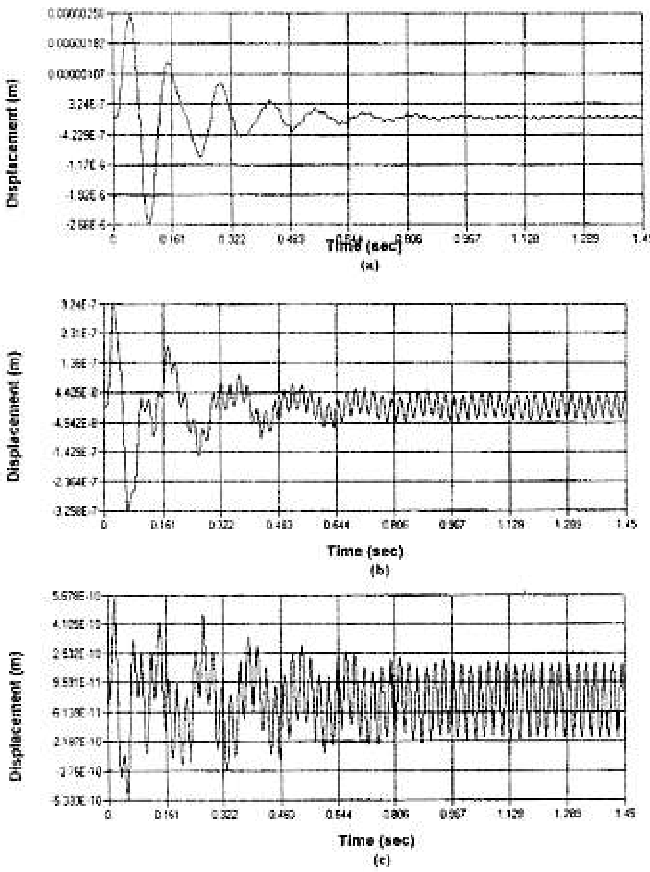


Fig.(7): Time-history curves at the corner of base foundation in
(a)Vertical, (b) Lateral and (c) Longitudinal directions.

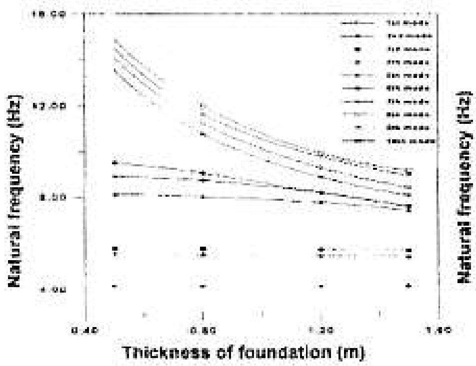


Fig.(8):Variation of natural frequency with thickness of footing .

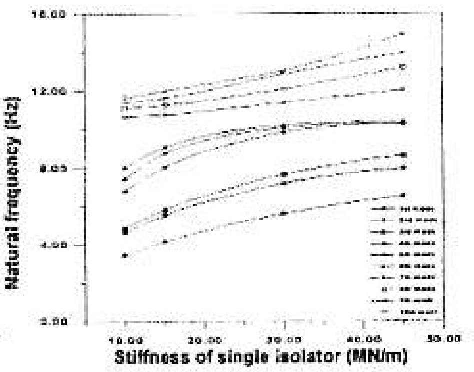


Fig.(9):Variation of natural frequency with stiffness of isolator .

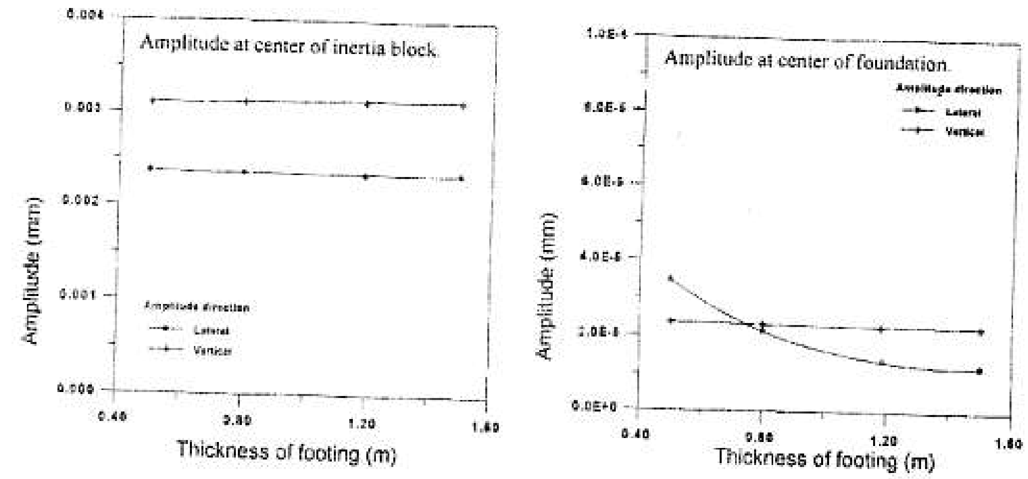


Fig.(10):Variation of amplitude with thickness of footing.

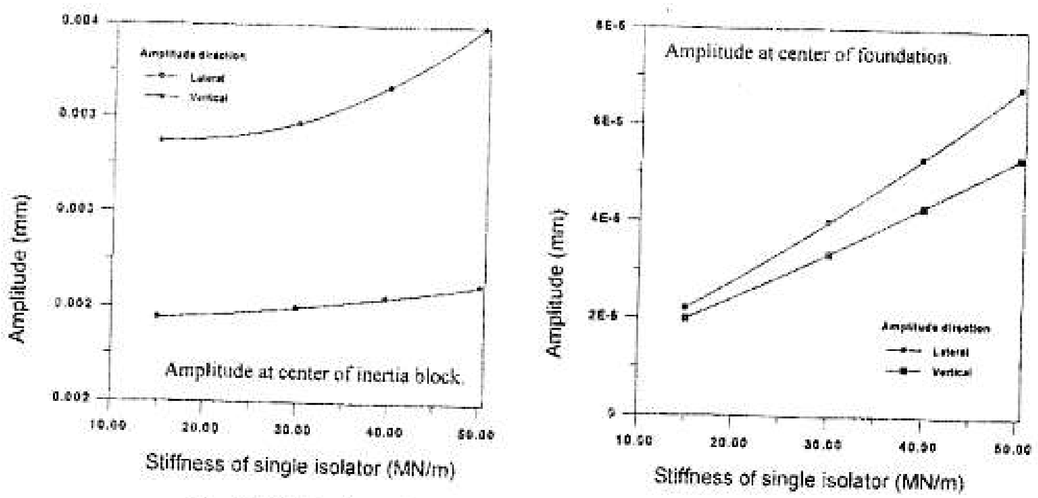


Fig.(11):Variation of amplitude with the stiffness of isolator .

Appendix:

The stiffness matrix of the plate element that used in the analysis is given by applying the principle of superposition for both plane-stress stiffness matrix K^p, and plate bending stiffness matrix K^b. Fig.(a1) shows both plane stress and plate bending elements.The total stiffness matrix is now made up from the following submatrices [6]:

$$K=\left[\begin{array}{cc:ccc:c} \multicolumn{2}{c:}{} & 0 & 0 & 0 & 0 \\ \multicolumn{2}{c:}{K^p} & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & & & & 0 \\ 0 & 0 & & K^b & & 0 \\ 0 & 0 & & & & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

where;

$$K^p=\frac{Et}{12(1-\mu)}\begin{bmatrix} 4\gamma+2(1-\mu)\gamma^{-1} & & & & & & & \\ \frac{3}{2}(1-\mu) & 4\gamma^{-1}+2(1-\mu)\gamma & & & & & \text{Symmetric} & \\ 2\gamma-2(1-\mu)\gamma^{-1} & -\frac{3}{2}(1-3\mu) & 4\gamma+2(1-\mu)\gamma^{-1} & & & & & \\ \frac{3}{2}(1-3\mu) & -4\gamma^{-1}+(1-\mu)\gamma & -\frac{3}{2}(1+\mu) & 4\gamma^{-1}+2(1-\mu)\gamma & & & & \\ -2\gamma-(1-\mu)\gamma^{-1} & -\frac{3}{2}(1+\mu) & -4\gamma+(1-\mu)\gamma^{-1} & -\frac{3}{2}(1-3\mu) & 4\gamma+2(1-\mu)\gamma^{-1} & & & \\ -\frac{3}{2}(1+\mu) & -2\gamma^{-1}-(1-\mu)\gamma & \frac{3}{2}(1-3\mu) & 2\gamma^{-1}-2(1-\mu)\gamma & \frac{3}{2}(1+\mu) & 4\gamma^{-1}+2(1-\mu)\gamma & & \\ -4\gamma+(1-\gamma)\gamma^{-1} & \frac{3}{2}(1-3\mu) & -2\gamma-(1-\mu)\gamma^{-1} & \frac{3}{2}(1+\mu) & 2\gamma-2(1-\mu)\gamma^{-1} & -\frac{3}{2}(1-3\mu) & 4\gamma+2(1-\mu)\gamma^{-1} & \\ -\frac{3}{2}(1-3\mu) & 2\gamma^{-1}-2(1-\mu)\gamma & \frac{3}{2}(1+\mu) & -2\gamma^{-1}-(1-\mu)\gamma & \frac{3}{2}(1-3\mu) & -4\gamma^{-1}+(1-\mu) & -\frac{3}{2}(1+\mu) & 4\gamma^{-1}+2(1-\mu)\gamma \end{bmatrix}$$

$$K^b=\frac{Et^3}{12(1-\mu^2)ab}\left[\begin{array}{c:c} K_1 & \text{Symmetric} \\ \hdashline K_2 & K_3 \end{array}\right]$$

where;

$$K_2=\begin{bmatrix} -2(\gamma^2+\gamma^{-2})+\frac{1}{5}(14-4\mu) & [-\gamma^2+\frac{1}{5}(1-\mu)]b & [\gamma^2-\frac{1}{5}(1-\mu)]a & -2(2\gamma^2-\gamma^{-2})-\frac{1}{2}(14-4\mu) & [-\gamma^2+\frac{1}{5}(1+4\mu)]b & [2\gamma^2+\frac{1}{5}(1-\mu)]a \\ [\gamma^{-2}-\frac{1}{5}(1-\mu)]b & [\frac{1}{3}\gamma^{-2}+\frac{1}{15}(1-\mu)]b^2 & 0 & [-\gamma^{-2}+\frac{1}{5}(1+4\mu)]b & [\frac{2}{3}\gamma^{-2}-\frac{4}{15}(1-\mu)]b^2 & 0 \\ [-\gamma^2+\frac{1}{5}(1-\mu)]b & 0 & [\frac{1}{3}\gamma^2+\frac{1}{15}(1-\mu)]a^2 & -[2\gamma^2+\frac{1}{5}(1-\mu)]a & 0 & [\frac{2}{3}\gamma^2-\frac{1}{15}(1-\mu)]a^2 \\ -2(2\gamma^2-\gamma^{-2})-\frac{1}{5}(14-4\mu) & [\gamma^{-2}-\frac{1}{15}(1+4\mu)]b & [2\gamma^2+\frac{1}{5}(1-\mu)]a & -2(\gamma^2+\gamma^{-2})+\frac{1}{5}(14-4\mu) & [\gamma^{-2}-\frac{1}{5}(1-\mu)]b & [\gamma^2-\frac{1}{5}(1-\mu)]a \\ [\gamma^{-2}-\frac{1}{5}(1-4\mu)]b & [\frac{2}{3}\gamma^{-2}-\frac{4}{15}(1-\mu)]b^2 & 0 & [-\gamma^{-2}+\frac{1}{5}(1-\mu)]b & [\frac{1}{3}\gamma^{-2}+\frac{1}{15}(1-\mu)]b^2 & 0 \\ -[2\gamma^2-\frac{1}{5}(1-\mu)]a & 0 & [\frac{2}{3}\gamma^2-\frac{1}{15}(1-\mu)]a^2 & [-\gamma^2+\frac{1}{5}(1-\mu)]a & 0 & [\frac{1}{3}\gamma^2+\frac{1}{15}(1-\mu)]a \end{bmatrix}$$

and

$$K_3=\begin{bmatrix} 4(\gamma^2+\gamma^{-2})+\frac{1}{5}(14-4\mu) & & & & & \\ -[2\gamma^{-2}+\frac{1}{5}(1+4\mu)]b & [\frac{4}{3}\gamma^{-2}+\frac{4}{15}(1-\mu)]b^2 & & & \textbf{Symmetric} & \\ [2\gamma^2+\frac{1}{5}(1+4\mu)]a & -\mu ab & [\frac{4}{3}\gamma^2+\frac{4}{15}(1-\mu)]a^2 & & & \\ 2(\gamma^2-2\gamma^{-2})-\frac{1}{5}(14-4\mu) & [2\gamma^{-2}+\frac{1}{5}(1-\mu)]b & [\gamma^2-\frac{1}{5}(1+4\mu)]b & 4(\gamma^2+\gamma^{-2})+\frac{1}{5}(14-4\mu) & & \\ -[2\gamma^{-2}+\frac{1}{5}(1-\mu)]b & [\frac{2}{3}\gamma^{-2}-\frac{1}{15}(1-\mu)]b^2 & 0 & [2\gamma^{-2}+\frac{1}{5}(1+4\mu)]b & [\frac{4}{3}\gamma^{-2}+\frac{4}{15}(1-\mu)]b^2 & \\ [\gamma^2-\frac{1}{5}(1+4\mu)]a & 0 & [\frac{2}{3}\gamma^2-\frac{4}{15}(1-\mu)]a^2 & [2\gamma^2+\frac{1}{5}(1+4\mu)]a & \mu ab & [\frac{4}{3}\gamma^2+\frac{4}{15}(1-\mu)]a^2 \end{bmatrix}$$

$$K_1=\begin{bmatrix} 4(\gamma^2-\gamma^{-2})-\frac{1}{5}(14-4\mu) & & & & & \\ [2\gamma^{-2}+\frac{1}{5}(1+4\mu)]b & [\frac{4}{3}\gamma^{-2}+\frac{4}{15}(1-\mu)]b^2 & & & & \\ -[2\gamma^2+\frac{1}{5}(1+4\mu)]a & -\mu ab & [\frac{4}{3}\gamma^2+\frac{4}{15}(1-\mu)]a^2 & & \textbf{Symmetric} & \\ 2(\gamma^2-2\gamma^{-2})+\frac{1}{5}(14-4\mu) & -[2\gamma^{-2}+\frac{1}{5}(1-\mu)]b & [-\gamma^2+\frac{1}{5}(1+4\mu)]b & 4(\gamma^2+\gamma^{-2})+\frac{1}{5}(14-4\mu) & & \\ [2\gamma^{-2}+\frac{1}{5}(1-\mu)]b & [\frac{2}{3}\gamma^{-2}-\frac{1}{15}(1-\mu)]b^2 & 0 & -[2\gamma^{-2}-\frac{1}{5}(1+4\mu)]b & [\frac{4}{3}\gamma^{-2}+\frac{4}{15}(1-\mu)]b^2 & \\ [-\gamma^2+\frac{1}{5}(1+4\mu)]a & 0 & [\frac{2}{3}\gamma^2-\frac{4}{15}(1-\mu)]a^2 & -[2\gamma^2+\frac{1}{5}(1+4\mu)]a & \mu ab & [\frac{4}{3}\gamma^2-\frac{4}{15}(1-\mu \end{bmatrix}$$

where; $\gamma=\frac{b}{a}$ (μ) Poisson's ratio, a and b are the length and width of the plate respectively.

The element mass matrix for plate element is given by:

$$[M^e] = \frac{\rho At}{4}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where; ρ, A and t are density ,area and thickness of the plate respectively.

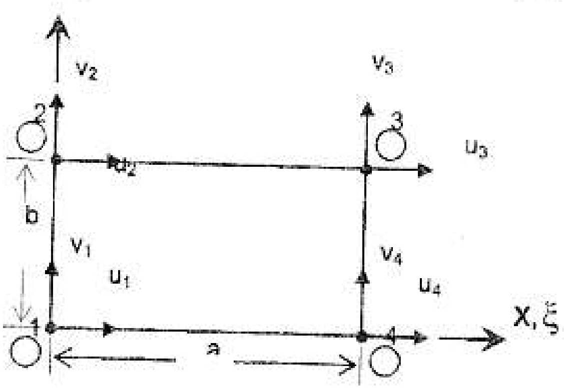
Dynamic forces:

Fig.(a2) shows a typical rotating mass type in which single mass m_e is placed on a rotating shaft at eccentricity e from the axis of rotation.The components of the unbalanced forces are given by [2]:

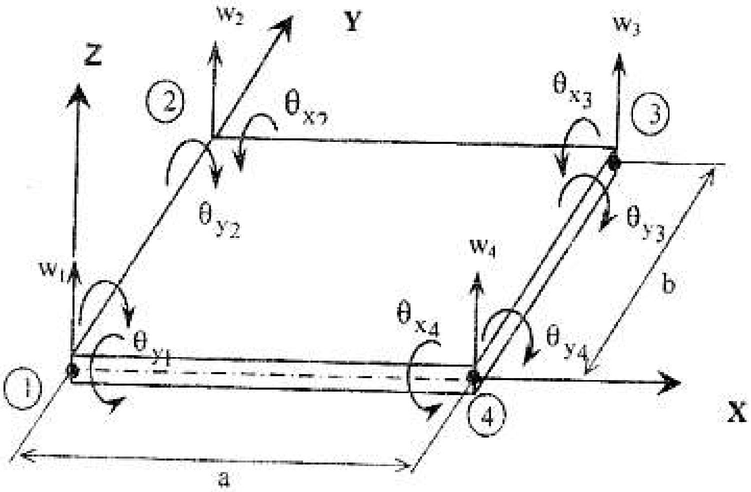
$$F_x = F_o \times \cos \varpi t$$
$$F_y = F_o \times \sin \varpi t$$

in which :

$$F_o = m_e \times e \times \varpi^2$$



Plane stress element action.



Bending element action.

Fig(a1): The used plate element that subject to 'in plane' and 'bending' actions

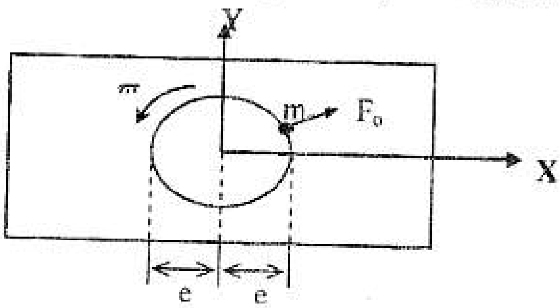


Fig.(a2): Unbalanced force of centrifugal machine.