أنساق الجسم الصلد لاهتزاز عارضة تيمو شنكو راقدة فوق اساس مرن

الدكتور بهاءالدين حسين عباس

قسم الهندسة الميكانيكية

جامعة البصرة

الخلاصة

يقدم هذا البحث طريقة تحليلية فريدة لايجاد ترددات انساق الجسم الصلدلاهتزازعارضة سميكة ترقد فوق اساس مرن النتائج متفقة تماما مع نتائج طريقة الاجزاء المحددة .

تم التوصل الى أن تردد نسق الجسم الصلد الدوراني يتناقص بنسبة $\frac{1}{(1 + 12R)}$ حيث تمثل R وسيط القصور الذاتي الدوراني كما تم تحديد مناطق الاستقرار الحركي الخاصة بالنسقين الانتقالي والدوراني للاهتزاز .

ON THE RIGID BODY MODES OF VIBRATION OF TIMOSHENKO BEAM RESTING ON AN ELASTIC FOUNDTION

by

Dr . Baha A.H. Abbas
B. Sc. (Eng .) M. sc., Ph. D., MASME
Department of Mechanical Engineering
College of Engineering ,
University of Basrah
Basrah

https://doi.org/10.30684/etj.1983.1.5
2412-0758/University of Technology-Iraq, Baghdad, Iraq

On the Rigid Body Modes of Vibration of Timoshenko Ream Resting on an Elastic Foundation.

by
Dr. B.A.H. ABBAS
Department of Mechanical Engineering,
College of Engineering,
University of Basrah,
Basrah, Iraq.

Abstract

An analytical method is presented for the determination of the frequencies of the rigid body mo[illegible] of vibra[illegible] of a Timoshenko beam resting on an elastic foundation. The results obtained show excellent agre[illegible]nent with th[illegible] obtined from a finite element analysis. It is concluded that the frequency of the rotational ri[illegible]d body mode[illegible] diminished by the ratio $\frac{1}{(1 + 12R)}$ where R is the rotary inertia parameter. A dynamic [illegible]ability analysis [illegible] a free free Timoshenko beam resting on an elastic foundation and subjected to periodic axial loads is carrie[illegible] out. The regions of dynamic instability associated with the translational and rotational rigid body modes [illegible] determined.

1. Introduction :

The effect of elastic foundation on the frequencies of the rigid body modes of vibration of Bernoulli-Euler an[illegible] Timoshenko beams has been investigated by Abbas and Thomas (1,2) using a finite element analysis. For the ca[illegible] of a free-free slender beam with no elastic support two zero eigenvalues were obtained representing the transla[illegible]nal and rotational rigid body modes of vibration. As the elastic foundation becomes stiffer the zero eigenvalu[illegible] disappear and two equal freuency values are obtained. Abbas and Thomas (1,2) also noted that for a free-fre[illegible] Timoshenko beam resting on an elastic foundation, the two frequencies of the rigid body modes are not equ[illegible]. The frequency associated with the rotational rigid body mode is reduced while that associated with the translatio[illegible] mode remains unchanged. This reduction can be attributed to the effect of rotary inertia and shear deformatio[illegible] the thick beam.

In this paper, an attempt is made to explain the nature of this effect by the use of a novel analytical metho[illegible]. The results obtained by the present analytical method are compared with the results obtained by the finite ele[illegible] analysis.

Furthermore a dynamic stability analysis (3,4) is carried out to determine the regions of dynamic instab[illegible] associated with these two rigid body modes of vibration.

II . Analysis :

The dynamic system of a free-free beam resting on an elastic foundation can be simulated by the sim[illegible] two degrees of freedom system shown in Figure (1). The differential equations of motion of this system are unc[illegible]pled and can be written as :

2.1. Translational motion :

$$m\ddot{x} = -\frac{k}{2}x - \frac{k}{2}x \quad \ldots (1)$$

where m is the mass of the beam . k is the stiffness of the foundation . Equation(1) gives

$$\omega_n = \sqrt{\frac{k}{m}} \quad \ldots (2)$$

where ω_n is the natural frequency of free vibration .

2.2. Rotational motion :

$$I\ddot{\theta} = -\frac{k}{8}l^2\theta - \frac{k}{8}l^2\theta \quad \ldots (3)$$

where I is the moment of inertia of the system and is equal to $\frac{ml^2}{4}$
Equation (3) gives

$$\omega_n = \sqrt{\frac{k}{m}} \quad \ldots (4)$$

It is seen from equations (2) and (4) that the frequencies of the translational and rotational modes of vibration are equal and can be written as one equation . Using the following notations :

$$m = \rho A l \qquad \rho, \lambda \quad \ldots (5)$$

$$\gamma = \frac{k_f l^4}{\pi^4 EI} \quad \ldots (6)$$

$$k_f = \frac{k}{l} \quad \ldots (7)$$

$$\lambda = \rho A l^4 \omega_n^2 / EI \quad \ldots (8)$$

where ρ is the mass density . A is the cross-sectional area and l is the length of the beam : γ is the elastic foundation constant .k_f is the foundation stiffness per unit length. EI is the flexural rigidity and λ is the frequency parameter. Equation (2) or (4) becomes

$$\omega_n = \frac{\gamma\pi^4 EI}{A l^4} \quad \ldots (9)$$

Equation (9) can be written as

$$\sqrt{\lambda} = \pi^2 \sqrt{\gamma} \qquad \dots (10)$$

The results obtained by the present analytical method are shown in Table 1 . Results obtained by the finite element method are also shown in Table 1 .

2.3. Effect of Rotary Inertia

The effect of rotary inertia on the rotational rigid body mode of vibration is investigated in this section by relating this effect to the change in the moment of inertia of the beam due to the transverse dimensions.

The moment of inertia I_t of a thick prismatic bar shown in Figure (2) about a centroidal axis normal to the bar is given by

$$I_t = \frac{m}{12} (l^2 + t^2) \qquad \dots (11)$$

where m is the mass, l is the length and t is the depth of the beam.

Expression (11) may be simplified for a long prismatic bar (Bernoulli - Euler bar) whose transverse dimensions are small compared with the length. In this case t^2 may be neglected and the moment of inertia I_b becomes

$$I_b = \frac{m\, l^2}{12} \qquad \dots (12)$$

By introducing the rotary inertia parameter

$$R = \frac{I_a}{A\, l^2} \qquad \dots (13)$$

where I_a is the second moment of the area of the cross - section and is given by

$$I_a = \frac{b\, t^3}{12} \qquad \dots (14)$$

expression (13) can be written as

$$R = \frac{t^2}{12\, l^2} \qquad \dots (15)$$

Expression (11) becomes

$$I_t = \frac{m\, l^2}{12} (1 + 12\, K) \qquad \dots (16)$$

It is expected therefore that the natural frequency of the rotational rigid body mode is diminished by the ratio $1 / (1 + 12R)$. The results obtained using this new relationship are shown in Table (2) and compared with the results obtained from a finite element analysis usinga six element idealization. The values used for the shear coeff icient k_c, and Poisson's ratio ν are 0.85 and 0.3 respectively The Timoshenko beam element and the method of solution are given in reference (2).

2.4. Dynamic stability analysis

A dynamic analysis of free - free Bernoulli - Euler and Timoshenko beams resting on elastic foundations and subjected to periodic axial loads are carried out, using the method of analysis given in referense (2). The two regions of dynamic - instability associated with the translational and rotational rigid body modes are determined and shown in Figures (3) and (4).

In Figure (3), the periodic axial load is taken as

$$P_{(t)} = \alpha P^*_e + \beta P^*_e \cos \omega t$$

where

$P^*_e = 9.870\ EI / l^2$ fundamental static buckling load of a Bernoulli - Euler beam with zero elastic foundation constant.

$p_e = 22.380 \sqrt{EI / \rho A l^4}$ fundamental natural frequency of Bernoulli Euler beam with zero elastic foundation constant.

ω = disturbing frequency

$\alpha = 0.5$

In Figure (4), the periodic axial load is taken to be

$$P_{(t)} = \alpha P^*_t + \beta P^*_t \cos \omega t$$

where

$P^*_t = 8.361\ EI / l^2$ fundamental static buckling load of a Timoshenko beam with zero elastic foundation constant.

$p_t = 18.214 \sqrt{EI / \rho A l^4}$ fondamental natural frequency of a Timoshenko beam with zero elastic foundation constant.

ω = disturbing frequency

$\alpha = 0.5$

III. Discussion

Table 1 shows the variation of the square root of the frequency parameter of the rotational and translational rigid body modes of vibration with the elastic foundation constant for a free - free Bernoulli - Euler beam. Results obtained from finite element analysis using six - element idealization are also shown in Table 1. The comparison of these two results shows excellent agreement.

Table 2 shows the variation of the square root of the frequency parameter of the rotational rigid body mode of vibration with the elastic foundation constant for a free-free Timoshenko beam with $\nu = 0.3$, $k_c = 0.85$ and $\sqrt{R} = 0.08$. The frequency of the translational mode of vibration is the same as that of a Bernoulli-Euler beam given in Table 1. It is seen that the frequency of the rotational rigid body mode is diminished. The results obtained from a finite element analysis using six-element idealization are also shown in Table 2. Comparison of these results indicates that the analytical method presented in this paper, for the first time, is correct.

Figure 3. shows the regions of dynamic instability associated with the rotational and translational rigid body modes of a free-free Bernoulli-Euler beam. It is seen that the region associated with the translational rigid body mode degenerates into a vertical line and hence has no effect on the stability characteristics of the beam.

Figure 4. shows the regions of dynamic instability associated with the rotational and translational rigid body modes of a free-free Timoshenko beam with $k_c = 0.85$, $\nu = 0.7$ and $\sqrt{R} = 0.08$. It is seen that as the rotary inertia effect increases, the width of the region associated with the rotational rigid body mode is increased thus making the structure more sensitive to periodic forces .

IV. Conclusions :

An analytical method for the determination of the frequencies of the rigid body modes of vibration of a Timoshenko beam resting on an elastic foundation is presented . It is concluded that the frequency of the rotational rigid body mode is diminished by the ratio $1/(1+12R)$ where R is the rotary inertia parameter.

From the dynamic stability analysis it is concluded that the translational rigid body mode has no effect on the stability characteristics of the beam. As the rotary inertia parameter increases the width of the region associated with the rotational rigid body mode is increased thus making the beam more sensitive to periodic forces .

References

(1) B.A.H. ABBAS, Mechanical Vibration and Dynamic Stability of Complex Structures by Finite Element Method. Ph. D. thesis, University of Surrey, England, 1977.

(2) B.A.H. ABBAS and J. THOMAS, Dynamic Stability of Timoshenko Beams Resting on an Elastic Foundation. . Journal of Sound and Vibration 60, 33-44, 1978.

(3) V.V. BOLOTIN, The Dynamic Stabilit y of Elastic Systems, Holden-Day, 1964.

(4) S.P. TIMOSHENKO a ad J.M.GERE, Theory of Elastic Stability, McGraw- Hill, New York, 1961.

γ	$\lambda^{\frac{1}{2}}$ present analytical method	$\lambda^{\frac{1}{2}}$ Finite element method
0	0	0
1	9.8696	9.8532
2	13.9577	13.9392
3	17.0947	17.0808
4	19.7392	19.7327
5	22.0691	22.0545

Table 1. Natural frequency parameters of translational and rotational rigid body modes of vibration of a free-free Benoulli-Euler beam resting on an elastic foundation.

γ	$\lambda^{\frac{1}{2}}$	
	Present analytical method	Finite element method
0	0	0
1	9.5058	9.5111
2	13.4351	13.4506
3	16.4441	16.4736
4	18.9756	19.0221
5	21.2009	21.2673

Tableb2. Natural frequency parameters of rotational rigid body mode of vibration of a free-free Timoshenko beam with $\sqrt{R} = 0.08$ resting on an elastic foundation.

CAPTIONS

Figure 1

(a) Simplified two degrees of freedom system

(b) Translational mode of vibration

(c) Rotational mode of vibration

Figure 2

Thick prismatic beam

Figure 3

Regions of dynamic instability associatec with the two rigid body modes of a free-free Bernoulli-Euler beam

⊡ rotational mode; ⊙ translational mode

△ fundamental mode.

Figure 4

Regions of dynamic instability associated with the two rigid body modes of a free-free Timoshenko beam ($\sqrt{R} = 0.08$ $k_c = 0.85$. $\nu = 0.3$).

⊡ rotational mode: ⊙ translational mode

△ fundamental mode.

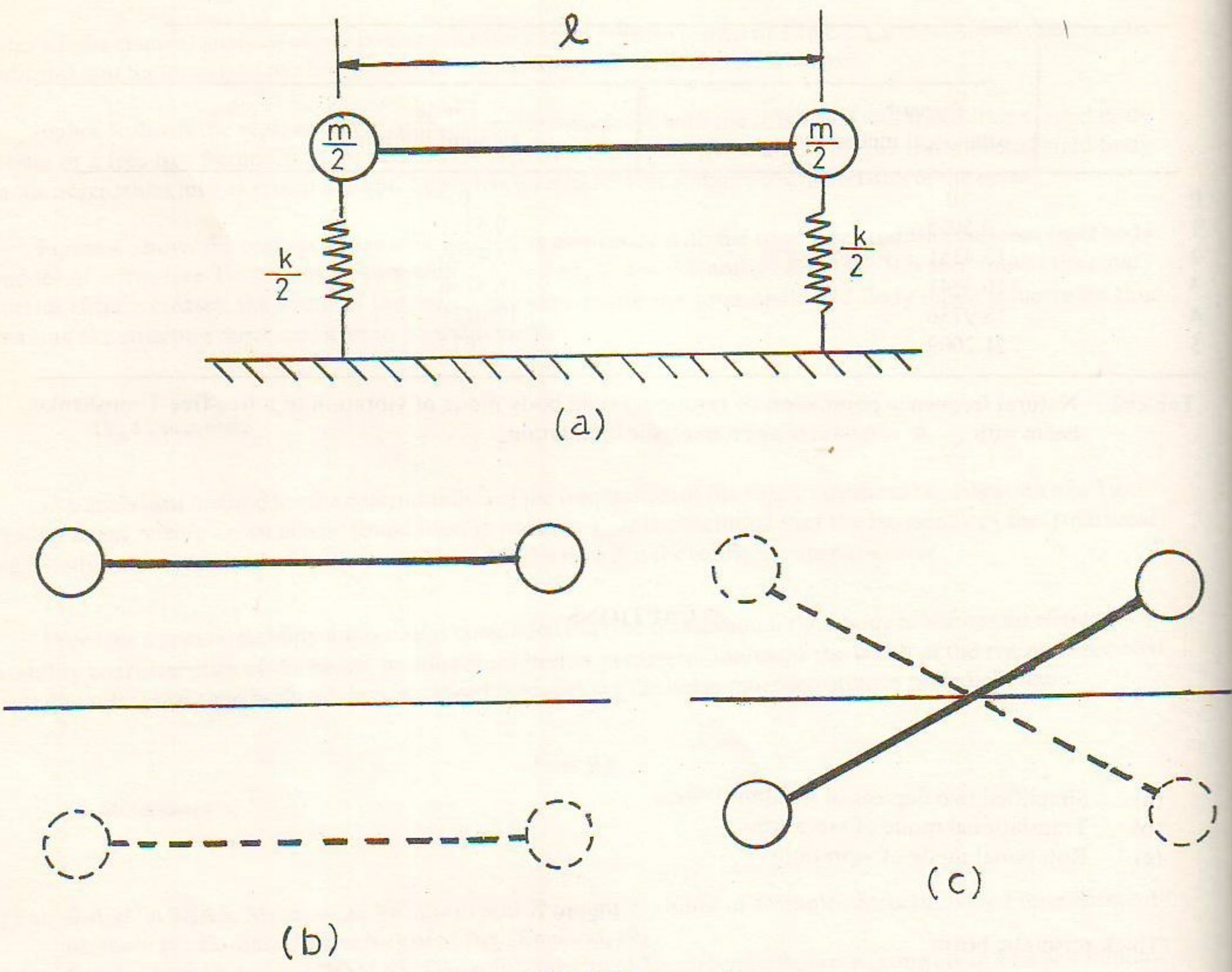

FIGURE 1

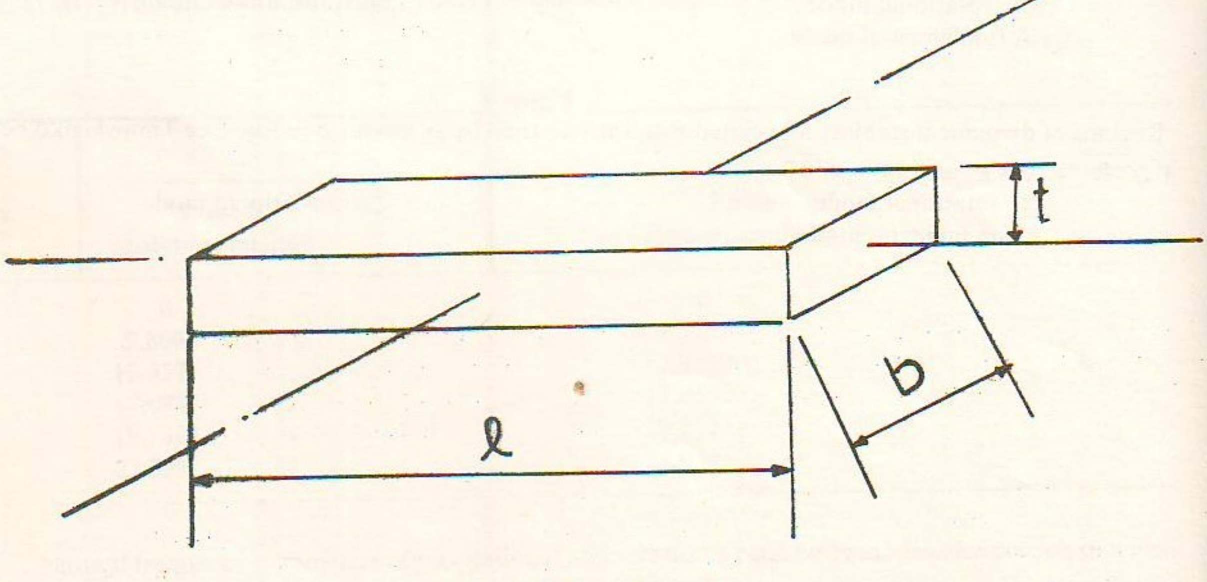

FIGURE 2

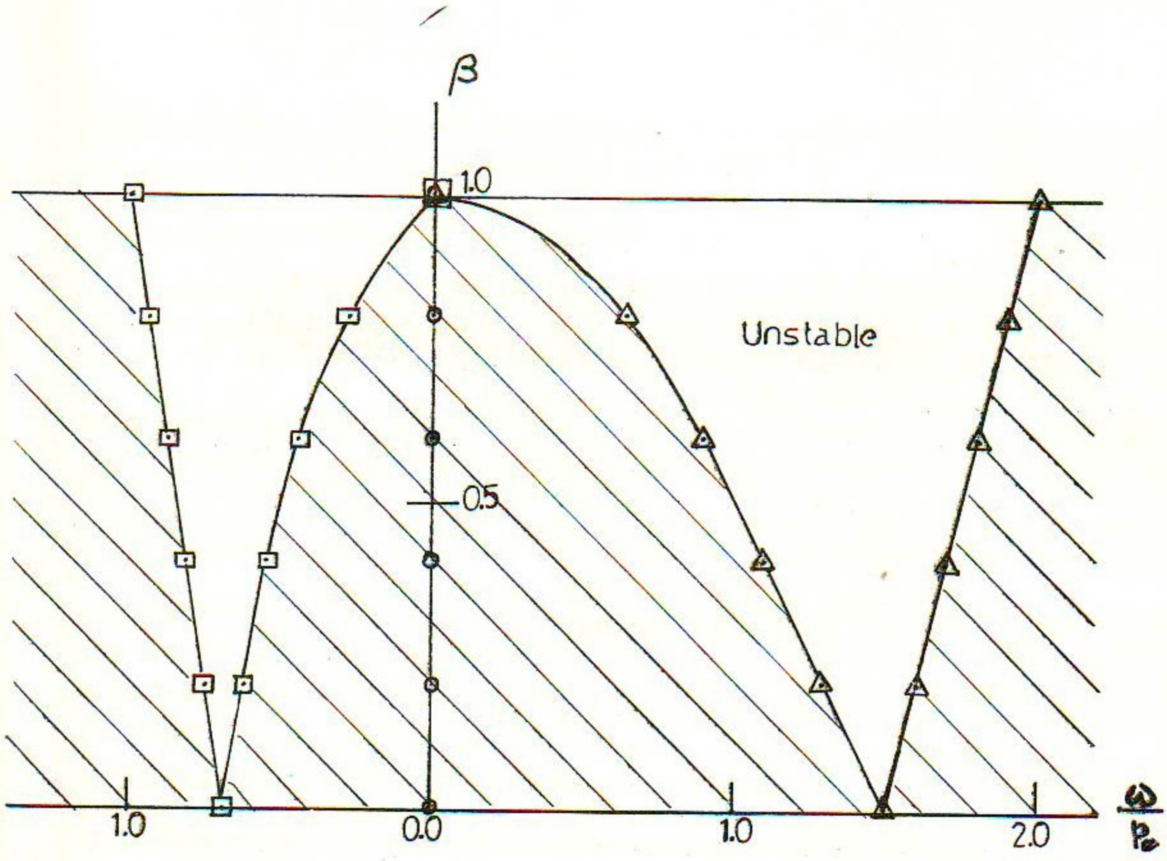

FIGURE 3

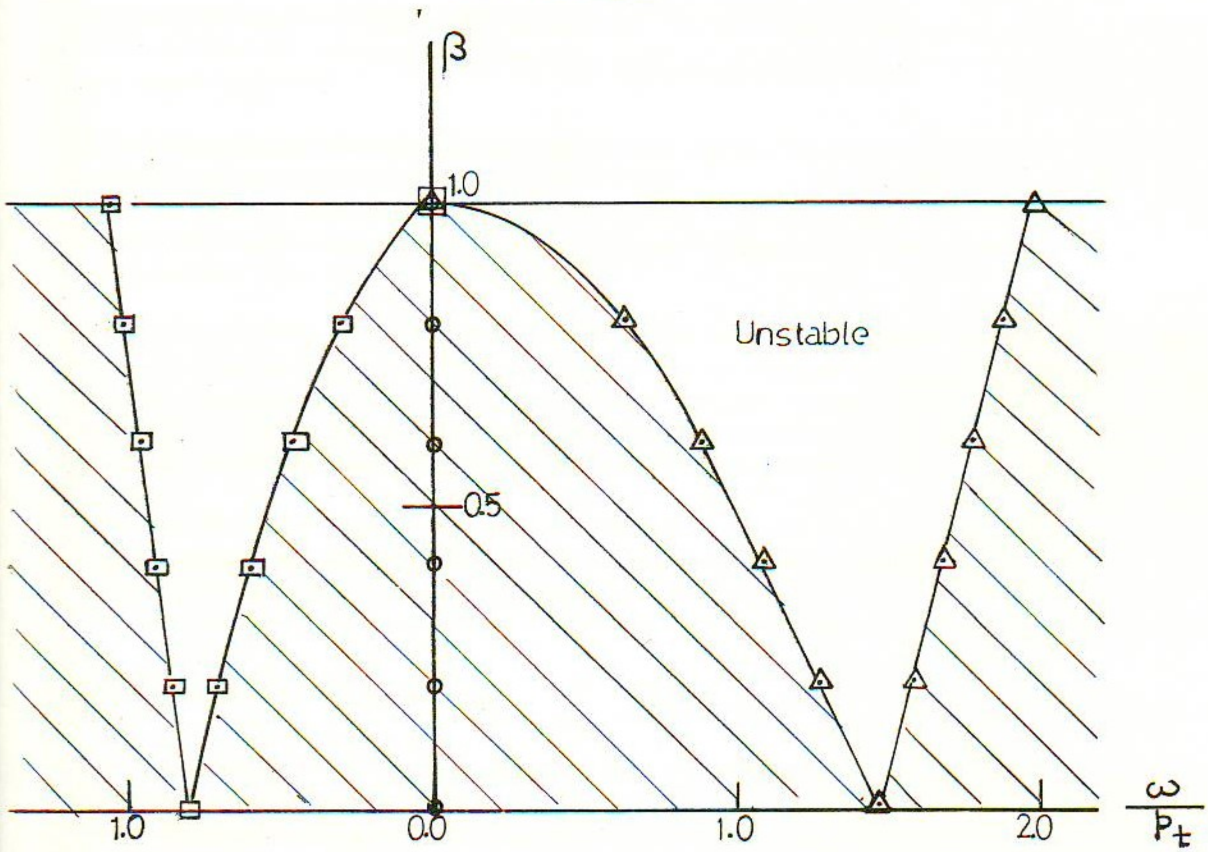

FIGURE 4