ii-open Set in Intuitionistic Topological Space

Senia Salim[١] , Hiba O Mousa[٢]

senia.salim٢٧١@st.tu.edu.iq[١], hom_٣٤@tu.edu.iq[٢]

[١,٢] **Dep. Of Mathematics, College of Education for Women, University of Tikrit, Tikrit, Iraq**

Abstract

This paper presented an extension of ii-opren sets to include in intuitionistic space called intuitionistic ii-opren sets. The study also presents some properties of this sets in intuitionistic space and finds the relationship between it and other sets in intuitionistic space which are also known as the intuitionistic iw-closed sets, and intuitionistic iiw-closed sets, with given some basic properties and relationships between these sets and other sets such as intuitionistic w-closed set, and intuitionistic αw-closed sets.

Keywords: intuitionistic space, iw_{is}-closed, iiw_{is}-closed, αw_{is}-closed.

١. Introduction

Open sets and different types of closed sets are pivotal areas in topology, providing the mathematical basis for analyzing complex spaces and describing geometric and functional relationships between them. They emerged as specialized concepts aimed at expanding the traditional understanding of open and closed sets. Among these concepts: $semi$-Open Sets were defined to expand the concept of traditional open sets [١], Closed sets of type w-open set were defined as an extension of the definition of traditional closed sets, where the relationship between internal closure and specific overlaps with open sets was achieved [٢], the α-Open Set concept was introduced to develop the study of sets of a semi-closed nature [٣], and the αw-Closed Set concept was introduced to combine the concepts of α-open and w-closed [٤], and these concepts were expanded to show a new type of sets such as i-Open Set [٥], and ii-Open Set that achieves an additional condition for the i-Open Set [٦], followed by the emergence of the int-Open Set principle that achieves properties specific to internal closure only [٧], Jasim and Sultan presented a study on open subsets of type "Micro-b-open" and continuity of function associated with then [٨], then Ahmed and Jasim followed it with a study on the concept of regular semi-sepra open sets and continuity of functions [٩], then other study delt with the definition and properties of open sets of type Nano βθ-open in nono topology [١٠], a study provided an analysis of open sets of type ζ-open and the continuity of functions associated with them in topological spaces [١١], a study On Soft S_p-Closed and Soft S_p-Open [١٢].

It aims to expand the traditional understanding of open and closed sets with a focus on ii-Open Set to bring them into the concept of intuitive topology. This research also aims to review these sets according to the sequence of their discovery, focusing on their basic properties and mutual relations, and studying their theoretical applications.

٢. Some basic concepts

Definition (٢,١) [٥]: let $(X,\mathcal{T})$ is T.P. space and any subset $\mathcal{P} \in \mathcal{T}$ is called i-open set if ∃ an proper set $G \in \mathcal{T}$ if satisfy $\mathcal{P} \subseteq cl(\mathcal{P} \cap G)$, where $cl(\mathcal{P} \cap G)$, is closure of $\mathcal{P} \cap G$, and $G \neq \emptyset\ or\ X$.

Definition (٢,٢) [٦]: let $(X,\mathcal{T})$ is T.P. space and any subset $\mathcal{P} \in \mathcal{T}$ is called ii-open set if ∃ an proper set $G \in \mathcal{T}$ if satisfy $\mathcal{P} \subseteq cl(\mathcal{P} \cap G)$, and $int(\mathcal{P}) = G$, where $cl(\mathcal{P} \cap G)$, and $int(\mathcal{P})$ are closure of $\mathcal{P} \cap G$, and interior of $\mathcal{P}$ respectively.

Definition (٢,٣) [١]: let $(X,\mathcal{T})$ is T.P. space and any subset $\mathcal{P} \in \mathcal{T}$ is called $semi$-open if $\mathcal{P} \subseteq cl(int(\mathcal{P}))$, where $cl(int(\mathcal{P}))$ is closure-interior of $\mathcal{P}$, and the set of all $semi$-open is denoted X_{semio}.

Definition (٢,٤) [٣]: let $(X,\mathcal{T})$ is T.P. space and any subset $\mathcal{P} \in \mathcal{T}$ is called α-open if $\mathcal{P} \subseteq int\{cl(int(\mathcal{P}))\}$, where $int\{cl(int(\mathcal{P}))\}$ is interior-closure-interior of $\mathcal{P}$, and the set of all α-open is denoted $X_{\alpha o}$.

Definition (٢,٥) [٢], [٥]: let $(X,\mathcal{T})$ is T.P. space and any subset $\mathcal{P} \in \mathcal{T}$ is called w-closed if $\mathcal{P}_{cl} \subseteq R$, where R is $semi$-open of $(X,\mathcal{T})$, and the set of all int-open is denoted X_{wo}.

Definition (٢,٦) [٤],[١٣]: let $(X,\mathcal{T})$ is T.P. space and any subset $\mathcal{P} \in \mathcal{T}$ is called αw-closed if $cl(\mathcal{P}) \subseteq R$, where R is α-open of $(X,\mathcal{T})$, while the complement of $\mathcal{P}$ is called αw-open, and the set of all αw-open is denoted $X_{\alpha wo}$.

Definition (٢,٧) [١٣]: let $(X, \mathcal{T})$ is T.P. space and any subset $\mathcal{P} \in \mathcal{T}$ is called int-open if $\exists G \in X_O$, where $G \neq \emptyset, X$, and X_O is the set of all open sets in $(X, \mathcal{T})$, holds $int(\mathcal{P}) = G$, the complement of $\mathcal{P}$ is called int-closed, and the set of all int-open is denoted X_{into}, respect int-closed is denoted by X_{intc}.

Remark (٢,٨): we can express about definition of ii-open set by (any subset $\mathcal{P} \in \mathcal{T}$ is called ii-open set if $\mathcal{P}$ is i-open set and int-open).

Definition (٢,٩) [٤]: let the set $X \neq \emptyset$, and any $\mathcal{P} \subseteq X$, we say $\mathcal{P}$ is intuitionistic set ($\mathcal{P}_I$) if has form:

$\mathcal{P} = \langle X, \mathcal{P}_١, \mathcal{P}_٢ \rangle$, as $\mathcal{P}_١ \cap \mathcal{P}_٢ = \emptyset$, where $\mathcal{P}_١, \mathcal{P}_٢$ are set of element in $\mathcal{P}$, and set of element not in $\mathcal{P}$ respectively.

Definition (٢,١٠) [٤]: let the set $X \neq \emptyset$, the intuitionistic topology $\mathcal{T}$-I defined as subset of all intuitionistic set in X, satisfy the three Topological Axioms. Thus, the $\mathcal{T}_{is}$ is a framework that combines topological and intuitionistic operations (such as degrees of membership and non-membership). The pair $(X, \mathcal{T}_I)$ is called intuitionistic topology space ($\mathcal{T}$-I).

Remark (٢,١١) [٤]: To any $\mathcal{P} = \langle X, \mathcal{P}_١, \mathcal{P}_٢ \rangle$, and $\mathcal{S} = \langle X, \mathcal{S}_١, \mathcal{S}_٢ \rangle$, are two intuitionistic open sets in $\mathcal{T}$-I, then:

١. $\mathcal{P} \subseteq \mathcal{S} \Leftrightarrow \mathcal{P}_١ \subseteq \mathcal{S}_١ \;\&\; \mathcal{S}_٢ \subseteq \mathcal{P}_٢$

٢. $\mathcal{P}^c =< X, \mathcal{P}_٢, \mathcal{P}_١ >$, where M^c is the complement of OIS M.

٣. $\mathcal{P} \cap V =< X, \mathcal{P}_١ \cap \mathcal{S}_١, \mathcal{P}_٢ \cup \mathcal{S}_٢ >$

٤. $\mathcal{P} \cup V =< X, \mathcal{P}_١ \cup \mathcal{S}_١, \mathcal{P}_٢ \cap \mathcal{S}_٢ >$

٥. $\tilde{X} =< X, X, \emptyset >$

٦. $\tilde{\emptyset} =< X, \emptyset, X >$

٧. $cl(\mathcal{P}^c) = \{int(\mathcal{P})\}^c$.

٨. $int(\mathcal{P}^c) = \{cl(\mathcal{P})\}^c$

٣. Intuitionistic ii-open Set

Definition (٣,١): let $(X, \mathcal{T}_I)$ is $\mathcal{T}$-I and any subset $\mathcal{P} \in \mathcal{T}_I$ is called intuitionistic i-open set (i_I-open) if $\exists$ an proper set $\emptyset \neq X \neq \mathcal{S} = \langle X, \mathcal{S}_١, \mathcal{S}_٢ \rangle \in \mathcal{T}_I$ if satisfy $\mathcal{P} \subseteq$

$cl(\mathcal{P} \cap \mathcal{S})$, i.e. $(X, \mathcal{P}_1, \mathcal{P}_2) \subseteq \{< X, \mathcal{P}_1 \cap G_1, \mathcal{P}_2 \cup \mathcal{S}_2 >\}_{cl} \Longrightarrow \mathcal{P}_1 \subseteq cl(\mathcal{P}_1 \cap \mathcal{S}_1) \& \, cl(\mathcal{P}_2 \cup \mathcal{S}_2) \subseteq \mathcal{P}_2$. The set of all i_I-open in $\mathcal{T}$-I denoted by X_{IiO}.

Example (٣،٢): let $X = \{u, s\}$, and $\mathcal{T} - is = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{u\}, \{s\}\rangle, \langle X, \emptyset, \{s\}\rangle\}$, $\mathcal{T}^{c} - is = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{s\}, \{u\}\rangle, \langle X, \{s\}, \emptyset\rangle\}$, while the power set of X $P(X)_{\mathrm{I}} = \left\{\begin{matrix} \widetilde{\emptyset}, \tilde{X}, \langle X, \{u\}, \{s\}\rangle, \langle X, \{s\}, \{u\}\rangle, \langle X, \emptyset, \emptyset\rangle, \langle X, \{s\}, \emptyset\rangle, \langle X, \{u\}, \emptyset\rangle \\ , \langle X, \emptyset, \{s\}\rangle, \langle X, \emptyset, \{u\}\rangle \end{matrix}\right\}$, finally the $X_{IiO} = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{s\}, \{u\}\rangle, \langle X, \{s\}, \emptyset\rangle, \langle X, \emptyset, \emptyset\rangle, \langle X, \{u\}, \emptyset\rangle, \langle X, \emptyset, \{s\}\rangle\}$

Definition (٣،٢) : let $(X, \mathcal{T}_{is})$ is $\mathcal{T}$-is and any subset $\mathcal{P} \in \mathcal{T}_{is}$ is called intuitionistic ii-open set (ii_I-open) if $\exists$ an proper set $\emptyset \neq X \neq G = \langle X, \mathcal{S}_1, \mathcal{S}_2\rangle \in \mathcal{T}_{is}$ if satisfy $\mathcal{P} \subseteq cl(\mathcal{P} \cap G)$, i.e. $(X, \mathcal{P}_1, \mathcal{P}_2) \subseteq cl\{< X, \mathcal{P}_1 \cap G_1, \mathcal{P}_2 \cup G_2 >\} \Longrightarrow \mathcal{P}_1 \subseteq (\mathcal{P}_1 \cap G_1)_{cl} \& \, cl(\mathcal{P}_2 \cup G_2) \subseteq \mathcal{P}_2$, and $int(\mathcal{P}) = G$, as $int(\mathcal{P}_1) = G_1$ & $int(\mathcal{P}_2) = G_2$. The set of all ii_I-open in $\mathcal{T}$-is denoted by X_{IiiO}.

Example (٣،٤): $X = \{a, b, c\}$, and $\mathcal{T} - is = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \{b, c\}\rangle, \langle X, \{a, b\}, \{c\}\rangle\}$, $\mathcal{T}^{c} - is = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{b, c\}, \{a\}\rangle, \langle X, \{c\}, \{a, b\}\rangle\}$, while the power set of X $P(X)_{\mathrm{IS}} = \left\{\begin{matrix} \widetilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \emptyset\rangle, \langle X, \{b\}, \emptyset\rangle, \langle X, \{c\}, \emptyset\rangle, \langle X, \{a, b\}, \{c\}\rangle, \langle X, \{a, c\}, \{b\}\rangle \\ , \langle X, \{b, c\}, \{a\}\rangle, \langle X, \emptyset, \{a\}\rangle, \langle X, \emptyset, \{b\}\rangle, \langle X, \emptyset, \{c\}\rangle, \langle X, \{a\}, \{b, c\}\rangle, \\ \langle X, \{b\}, \{a, c\}\rangle, \langle X, \{c\}, \{a, b\}\rangle \end{matrix}\right\}$, finally $X_{IiO} = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \{b\}\rangle, \langle X, \{a\}, \emptyset\rangle, \langle X, \emptyset, \emptyset\rangle, \langle X, \{b\}, \emptyset\rangle, \langle X, \emptyset, \{b\}\rangle\}$, and $X_{IiiO} = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \{b, c\}\rangle\}$.

Definition (٣،٥) : let $(X, \mathcal{T}_{is})$ is $\mathcal{T}$-is and any subset $\mathcal{P} \in \mathcal{T}_{is}$ is called intuitionistic $semi$-open set ($semi_I$-open) if $\mathcal{P} \subseteq cl(int(\mathcal{P}))$, and the set of all $semi$-open is denoted X_{IsemiO}.

Definition (٣،٦) : let $(X, \mathcal{T}_I)$ is $\mathcal{T}$-is and any subset $\mathcal{P} \in \mathcal{T}_I$ is called intuitionistic α-open set (α_I-open) if $\mathcal{P} \subseteq int\{cl(int(\mathcal{P}))\}$, and the set of all α_I-open is denoted $X_{I\alpha O}$.

Definition (٣،٧) : let $(X, \mathcal{T}_{is})$ is $\mathcal{T}$-is and any subset $\mathcal{P} \in \mathcal{T}_{is}$ is called intuitionistic w-closed set (w_I-closed) if $cl(\mathcal{P}) \subseteq \mathcal{S}$, where $\mathcal{S}$ is $semi_I$-open i.e. $cl(\mathcal{P}_1) \subseteq \mathcal{S}_1$ & $cl(\mathcal{P}_2) \subseteq \mathcal{S}_2$, and complement of w_I-closed is w_I-open. The set of all w_I-open is denoted X_{IwO}.

Definition (٣،٨) : let $(X,\mathcal{T}_{is})$ is $\mathcal{T}$-is and any subset $\mathcal{P}\in\mathcal{T}_{is}$ is called intuitionistic αw-closed set (αw_I-closed) if $cl(\mathcal{P})\subseteq\mathcal{S}$, where $\mathcal{S}$ is α_I-open i.e. $cl(\mathcal{P}_١)\subseteq\mathcal{S}_١$ & $cl(\mathcal{P}_٢)\subseteq\mathcal{S}_٢$, and complement of αw_I-closed is αw_I-open. The set of all αw_I-open is denoted $X_{I\alpha wO}$.

Definition (٣،٩) : let $(X,\mathcal{T}_{Is})$ is $\mathcal{T}$-is and any subset $\mathcal{P}\in\mathcal{T}_{Is}$ is called intuitionistic int-open (int_I-open) if $\exists\mathcal{S}\in X_{IiiO}$, where $G\neq\emptyset,X$, holds $int(\mathcal{P})=G$, the complement of $\mathcal{P}$ is called int_I-closed, and the set of all int_I-open is denoted X_{IintO}, respect int_I-closed is denoted by X_{IintC}.

Proposition (٣،١٠): in $(X,\mathcal{T}_I)$ is $\mathcal{T}$-I every ii_{is}-open is i_I-open and int_I-open.

Proof: let $(X,\mathcal{T}_I)$ is $\mathcal{T}$-I and a subset $\mathcal{P}\in\mathcal{T}_I$ is ii_I-open, then $\mathcal{P}$ is $\exists$ an proper set $\emptyset\neq X\neq\mathcal{S}=\langle X,\mathcal{S}_١,\mathcal{S}_٢\rangle\in\mathcal{T}_I$ if satisfy

a) $\mathcal{P}\subseteq cl(\mathcal{P}\cap G)$, i.e. $(X,\mathcal{P}_١,\mathcal{P}_٢)\subseteq cl\{<X,\mathcal{P}_١\cap G_١,\mathcal{P}_٢\cup\mathcal{S}_٢>\}\Longrightarrow\mathcal{P}_١\subseteq cl(\mathcal{P}_١\cap\mathcal{S}_١)\&\, cl(\mathcal{P}_٢\cup\mathcal{S}_٢)\subseteq\mathcal{P}_٢$,

b) $int(\mathcal{P})=\mathcal{S}$, as $int(\mathcal{P}_١)=\mathcal{S}_١$ & $int(\mathcal{P}_٢)=\mathcal{S}_٢$,

Follows form condition (a) $\mathcal{P}$ is i_I-open, and from (b) $\mathcal{P}$ is int_I-open.

Remark (٣،١١): the converse of [**Proposition (٣،٨)**] is not true in general, shown in following examples.

Example (٣،١٢): from [**Example (٣،٤)**], then $X_{IiiO}=\{\tilde{\emptyset},\tilde{X},\langle X,\{x\},\{p\}\rangle,\langle X,\emptyset,\{p\}\rangle\}$, hence $\langle X,\{p\},\emptyset\rangle$ is i_I-open but not ii_I-open.

Example (٣،١٣): let $X=\{s,b,y,d\}$, and

$\mathcal{T}-is=\{\tilde{\emptyset},\tilde{X},\langle X,\{s\},\{b,y,d\}\rangle,\langle X,\{s,b\},\{y,d\}\rangle\}$, $\mathcal{T}^{c}-is=\{\tilde{\emptyset},\tilde{X},\langle X,\{b,y,d\},\{s\}\rangle,\langle X,\{y,d\},\{s,b\}\rangle\}$, take $F=\langle X,\{s\},\{b\}\rangle$, then F is int_I-open but not ii_I-open.

Theorem (٣،١٤): In $(X,\mathcal{T}_I)$ is $\mathcal{T}$-Iis every intuitionistic open set is ii_I-open.

Proof. Let $\mathcal{P}\in\mathcal{T}$-$I$, it follows $\mathcal{P}\subseteq cl(\mathcal{P}\cap\mathcal{P})=cl(\mathcal{P})\Longrightarrow\mathcal{P}$ is i_I-open. Also, $\mathcal{P}$ is int_I-open since $int(\mathcal{P})=\mathcal{P}$, since all conditions is holds then $\mathcal{P}$ is ii_I-open.

Corollary (٣،١٥): If $(X,\mathcal{T})$ is a T.P space then $(X,\mathcal{T}_I)$ is $\mathcal{T}$-I.

Proof. Take $\mathcal{P}$ is intuitionistic open, then it is ii_I-open

Therefore, let $x \in \mathcal{P} \Rightarrow x \in cl(\mathcal{P})$ & $x \in cl(G)$, where G is an open set in $(X, \mathcal{T})$ [since $\mathcal{P}$ is intuitionistic open]

- If G is closed, then $x \in G \subseteq \mathcal{P}$ and $\mathcal{P}$ is open.
- If G is not closed and $x \notin G$ then $x \notin cl(G)$

Since $x \in \mathcal{P} \Rightarrow x \in cl(\mathcal{P}) \cap cl(G) \Rightarrow x \notin cl(G)$, this contradiction . Therefore $\mathcal{P}$ is intuitionistic open.

Theorem (٣،١٦): In $(X, \mathcal{T}_I)$ is $\mathcal{T}$-I every α_I-open set is $semi_I$-open.

Proof: take $(X, \mathcal{T}_I)$ is $\mathcal{T}$-I, with $\mathcal{P}$ is α_I-open, since $\mathcal{P} \subseteq int\{cl(int(\mathcal{P}))\}$, and $int\{cl(int(\mathcal{P}))\} \subseteq cl(int(\mathcal{P})) \Rightarrow \mathcal{P} \subseteq int\{cl(int(\mathcal{P}))\} \subseteq cl(int(\mathcal{P}))$, hence $\mathcal{P}$ is $semi_I$-open.

Theorem (٣،١٧): In $(X, \mathcal{T}_I)$ is $\mathcal{T}$-I every α_I-openset is i_I-open.

Proof: take $(X, \mathcal{T}_I)$ is $\mathcal{T}$-I, with $\mathcal{P}$ is α_I-open, hence $\mathcal{P}$ is $semi_I$-open from [**Theorem (٣،١٦)**], because there is an intuitionistic open, namely $G \neq \emptyset, X$, holding $int(\mathcal{P}) \subseteq$ G $\Rightarrow int(\mathcal{P}) \subseteq \mathcal{P} \cap G \Rightarrow \mathcal{P} \subseteq cl(\mathcal{P} \cap G) \Rightarrow \mathcal{P}$ is i_I-open.

Theorem (٣،١٨): In $(X, \mathcal{T}_I)$ is $\mathcal{T}$-I every α_I-openset is ii_I-open.

Proof. take $(X, \mathcal{T}_I)$ is $\mathcal{T}$-I, with $\mathcal{P}$ is α_I-open. Thus, $\mathcal{P}$ is i_I-open from [**Theorem (٣،١٧)**], now we enough to prove for any intuitionistic open, namely $G \neq \emptyset, X$, $int(\mathcal{P}) = G$.

Assuming $int(\mathcal{P}) \neq G \Rightarrow cl(int(\mathcal{P})) \neq cl(G)$, this include to $\mathcal{P} \subseteq cl(\{int(\mathcal{P})\} \cap \mathcal{P} \cap G) \Rightarrow \mathcal{P} \not\subset cl(G)$ [this contradiction] $\Rightarrow$ $\mathcal{P}$ is ii_I-open.

Remark (٣،١٩): in general the converse of theorems { **Theorem (٣،١٦), Theorem (٣،١٧), and Theorem (٣،١٨)}** not true as shown in the following:

Example (٣،٢٠): let $X = \{a, b, c\}$, and

$\mathcal{T}$-I $= \{\tilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \emptyset\rangle, \langle X, \{a, b\}, \emptyset\rangle, \langle X, \{a, b, c\}, \emptyset\rangle\}$, take $F = \langle X, \{a, b\}, \emptyset\rangle$, then F is $semi_I$-open set but not α_I-open.

Example (٣،٢١): let $X = \{s, b, y\}$, and

$\mathcal{T}\text{-}I = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{s\}, \{b, y\}\rangle, \langle X, \{s, b\}, \{y\}\rangle\}$, $\mathcal{T}^c - \mathrm{I} = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{b, y\}, \{s\}\rangle, \langle X, \{y\}, \{s, b\}\rangle\}$, take $F = \langle X, \{b\}, \{y\}\rangle$, then F is i_I-open set but not α_I-open.

Example (٣،٢٢): let $X = \{a, b, c\}$, and

$\mathcal{T}\text{-}I = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{a, b\}, \{c\}\rangle, \langle X, \{a, b\}, \emptyset\rangle\}$, take $\mathcal{P} = \langle X, \{a, b\}, \{c\}\rangle$, then $\mathcal{P}$ is ii_I-open set but not α_I-open.

Theorem (٣،٢٣): In $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$every α_I-openset is int_I-open .

Proof: directly.

The following example demonstrates that the preceding corollary's converse is not generally true.

Example (٣،٢٤): let $X = \{a, b, c\}$, and

$\mathcal{T}\text{-}I = \{\widetilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \{b, c\}\rangle, \langle X, \{a, b\}, \{c\}\rangle, \langle X, \{a, b, c\}, \emptyset\rangle\}$, take $\mathcal{P} = \langle X, \{a, b\}, \{c\}\rangle$, then $\mathcal{P}$ is int_I-open set but not α_I-open.

٤. intuitionistic iw-closed and intuitionistic iiw-closed sets.

We present the concept of intuitionistic $iw-$closed and intuitionistic $iiw-$closed sets in topological space, as well as some of its fundamental characteristics. The evidence supporting this section's primary findings.

Definition (٤،١): let $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$ and any subset $\mathcal{P} \in \mathcal{T}\text{-}I$ is called intuitionistic iw-closed set (iw_I-closed) if $cl(\mathcal{P}_w) \subseteq \mathcal{S}$, where $\mathcal{S}$ is i_I-open i.e. $cl(\mathcal{P}_١) \subseteq \mathcal{S}_١$ & $\mathcal{S}_٢ \subseteq cl(\mathcal{P}_٢)$, and complement of iw_I-closed is iw_I-open. The set of all iw_I-open is denoted X_{IiwO}.

Theorem (٤،٢): In $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$ every iw_I-closed set is w_I-closed.

Proof: take $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$, with $\mathcal{P}$ is iw_I-closed, and $\mathcal{S}$ is any $semi_I$-open in X, as that $\mathcal{P} \subseteq \mathcal{S}$ because [every $semi_I$-open is i_I-open]. Since $\mathcal{P}$ is closed $\Longrightarrow cl(\mathcal{P}_w) \subseteq int(\mathcal{P}) \subseteq \mathcal{S} \Longrightarrow \mathcal{P}$ is w_I-closed.

The following example demonstrates that the preceding corollary's converse is not generally true.

Example (٤،٣): let $X = \{a, b, c\}$, and

$\mathcal{T}\text{-}I = \{\widetilde{\emptyset}, \tilde{X}, \langle X,\{a\},\{b,c\}\rangle, \langle X,\{a,b\},\{c\}\rangle, \langle X,\{a,b,c\},\emptyset\rangle\}$, take $\mathcal{P} = \langle X,\{a,b\},\{c\}\rangle$, then $\mathcal{P}$ is w_I-closed set but not iw_I-closed.

Theorem (٤,٤): In $(X,\mathcal{T}_I)$ is $\mathcal{T}\text{-}I$every iw_I-closed set is αw_I-closed.

Proof: take $(X,\mathcal{T}_I)$ is $\mathcal{T}\text{-}I$, with $\mathcal{P}$ is iw_I-closed, and $\mathcal{S}$ is any α_I-open in X, as that $\mathcal{P} \subseteq \mathcal{S}$ because [every α_I-open is i_I-open]. Since $\mathcal{P}$ is closed $\Longrightarrow cl(\mathcal{P}_w) \subseteq int(\mathcal{P}) \subseteq \mathcal{S} \Longrightarrow \mathcal{P}$ is αw_I-closed.

The following example demonstrates that the preceding corollary's converse is not generally true.

Example (٤,٥): let $X = \{a,b,c\}$, and

$\mathcal{T}\text{-}I = \{\widetilde{\emptyset}, \tilde{X}, \langle X,\{a\},\emptyset\rangle, \langle X,\{a,b\},\{c\}\rangle, \langle X,\{a,b,c\},\emptyset\rangle\}$, take $\mathcal{P} = \langle X,\{a,b\},\{c\}\rangle$, then $\mathcal{P}$ is αw_I-closed set but not iw_I-closed.

Theorem (٤,٦): In $(X,\mathcal{T}_I)$ is $\mathcal{T}\text{-}I$ every i_I-open & iw_I-closed set is w_I-closed.

Proof: take $(X,\mathcal{T}_I)$ is $\mathcal{T}\text{-}I$, with $\mathcal{P}$ is i_I-open & iw_I-closed, and $\mathcal{S}$ is any $semi_I$-open in X, as that $\mathcal{P} \subseteq \mathcal{S}$. We must prove $cl(\mathcal{P}) \subseteq \mathcal{S}$. Since $\mathcal{S}$ is any $semi_I$-open $\Longrightarrow \mathcal{S}$ is i_I-open $\Longrightarrow cl(\mathcal{P}) \subseteq \mathcal{S} \Longrightarrow \mathcal{P}$ is w_I-closed.

Definition (٤,٧): let $(X,\mathcal{T}_I)$ is $\mathcal{T}\text{-}I$ and any subset $\mathcal{P} \in \mathcal{T}_I$ is called intuitionistic iiw-closed set (iiw_{is}-closed) if $cl(\mathcal{P}_w) \subseteq \mathcal{S}$, where $\mathcal{S}$ is ii_I-open i.e. $cl(\mathcal{P}_١) \subseteq \mathcal{S}_١$ & $\mathcal{S}_٢ \subseteq cl(\mathcal{P}_٢)$, and complement of iiw_I-closed is iiw_I-open. The set of all iiw_I-open is denoted X_{IiwO}.

Example (٤,٨): let $X = \{a,b,c,d\}$, and

$\mathcal{T}\text{-}I = \{\widetilde{\emptyset}, \tilde{X}, \langle X,\{a\},\{d\}\rangle, \langle X,\{a,b\},\{c\},d\rangle, \langle X,\{a,b,c\},\{d\}\rangle, \langle X,\{a,b,c,d\},\emptyset\rangle\}$,

then $X_{IiwC} = \widetilde{\emptyset}, \tilde{X}, \langle X,\{a\},\{d\}\rangle, \langle X,\{a,b\},\{c\},d\rangle, \langle X,\{a,b,c\},\{d\}\rangle, \langle X,\{a,b,c,d\},\emptyset\rangle$.

Theorem (٤,٩): In $(X,\mathcal{T}_I)$ is $\mathcal{T}\text{-}I$ every iiw_I-closed set is αw_I-closed.

Proof: take $(X,\mathcal{T}_I)$ is $\mathcal{T}\text{-}I$, with $\mathcal{P}$ is iw_I-closed, and $\mathcal{S}$ is any α_I-open in X, as that $\mathcal{P} \subseteq \mathcal{S}$ because [every α_I-open is ii_I-open]. Since $\mathcal{P}$ is closed $\Longrightarrow cl(\mathcal{P}_w) \subseteq int(\mathcal{P}) \subseteq \mathcal{S} \Longrightarrow \mathcal{P}$ is αw_I-closed.

The following example demonstrates that the preceding corollary's converse is not generally true.

Example (٤,١٠): let $X = \{a, b, c, d\}$, and

$\mathcal{T}\text{-}I = \{\tilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \{d\}\rangle, \langle X, \{a, b, c\}, \{d\}\rangle, \langle X, \{a, b, c, d\}, \emptyset\rangle\}$,take $\mathcal{P} = \langle X, \{a, b\}, \{c\}, d\rangle$, then $\mathcal{P}$ is αw_I-closed set but not iiw_I-closed.

Theorem (٤,١١): In $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$ every iw_I-closed set is iiw_I-closed.

Proof: take $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$, with $\mathcal{P}$ is iw_I-closed, and $\mathcal{S}$ is any ii_I-open in X, as that $\mathcal{P} \subseteq \mathcal{S}$ because [every ii_I-open is i_I-open]. Since $\mathcal{P}$ is closed $\Longrightarrow cl(\mathcal{P}_w) \subseteq int(\mathcal{P}) \subseteq \mathcal{S} \Longrightarrow \mathcal{P}$ is iiw_I-closed.

The following example demonstrates that the preceding corollary's converse is not generally true.

Example (٤,١٢): let $X = \{a, b, c\}$, and

$\mathcal{T}\text{-}I = \{\tilde{\emptyset}, \tilde{X}, \langle X, \{a\}, \emptyset\rangle, \langle X, \{a, b\}, \{c\}\rangle, \langle X, \{a, b, c\}, \emptyset\rangle\}$, take $\mathcal{P} = \langle X, \{a, b\}, \{c\}\rangle$, then $\mathcal{P}$ is iiw_I-closed set but not iw_I-closed.

Theorem (٤,١٣): In $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$ every ii_I-open & iiw_I-closed set is w_I-closed.

Proof: take $(X, \mathcal{T}_I)$ is $\mathcal{T}\text{-}I$, with $\mathcal{P}$ is ii_I-open & iiw_I-closed, we have $cl(\mathcal{P}) \subseteq \mathcal{P} \Longrightarrow cl(\mathcal{P}) = \mathcal{P} \Longrightarrow \mathcal{P}$ is w_I-closed.

Conclusion:

The paper provides an important extension of the traditional concepts of open and closed sets by defining new sets in intuitional topological spaces. It also integrates topological concepts with intuitional topology to develop a framework that combines topological and intuitional operations (such as degrees of belonging and non-belonging). The properties and relationships between open and closed sets of type ii_I-open and other sets are reviewed, providing a deeper understanding of the structural relationships between these types. Several theorems and their proofs are presented to demonstrate the intrinsic properties of open and closed sets of type - in intuitional topological spaces. Cases where the converse propositions are not true are also indicated, demonstrating the complexity of the new concepts and the need for deeper study. This work provides the basis for future studies on the application of intuitional topology in areas such as complex data analysis, artificial intelligence, and the analysis of fuzzy systems.

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