Data Envelopment Analysis using Stochastic Frontier Analysis and Bootstrap Confidence Intervals

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ORIGINAL STUDY

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ABSTRACT

Evaluating company growth potential has moved away from traditional financial focused ratios and ratios analysis that has origins in the early twentieth-century economics. However, these conventional methods might not be accurate in measuring such efficient factors as this combined proposed framework of Data Envelopment Analysis (DEA) and improved mathematical models do. The present research focuses on the prospect of growth of companies through evaluating the performance of 40 DMUs in terms of efficiency DEA and MMTs. DEA is used to determine the efficient DMUs while SFA underline the factors such as investment on research and development, effective marketing strategies and qualified human resource as the determinants of efficiency. Bootstrap Confidence Intervals (BCIs) are employed for the purpose of improving the efficiency scores precision. The integration of DEA with MMTs presents useful information on the identification of the key drivers of the growth and the effective formulation of efficient and competitive strategies for the managers and policymaker. The study emphasizes the need to apply these combined methodologies as valid in the growth evaluation of companies with implications to strategic decisions. This proposed framework can be used in finance, marketing, healthcare, and operations management to compare the current organizational effectiveness and future development.

Keywords: Data envelopment analysis, Mathematical modelling, Growth potential, Empirical investigation, Stochastic frontier analysis, Bootstrap confidence intervals

1. Introduction

In the context of the farm machinery market's expansion, one of the remaining challenges is the insufficient efficiency of Russian manufacturing goods (the proportion of domestic firms' goods is not higher than 26%). This development becomes the primary challenge concerning developing a successful structure for forming a program for manufacturing rival farm machinery firms. A company's capacity to succeed in a specific product marketplace is determined by an amalgamation of corporate practices and the level of competition for its goods [1]. Efficiency is a key metric in assessing business success since it measures how well a company can create optimal outcomes using the least labour, money, and equipment. Examining corporate effectiveness has grown critical for improving processes and developing novel methods to stay competitive. Because there is no hypothetical limit that might be utilized as a standard, efficiency within companies may be evaluated as an analogous notion [2]. The transition from a business-focused to a professionally operated and created organization, in addition to the introduction of an additional group of skilled professionals, organizational structures, assets (HR, monetary, technology, and a few more), and their impact on the company's development, were all thoroughly studied.

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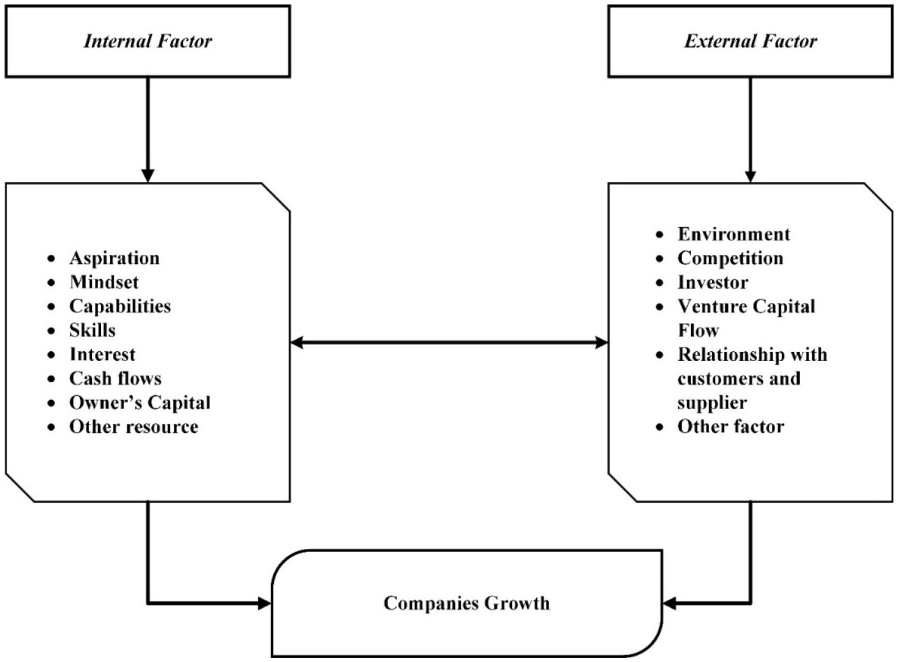

Fig. 1. Factors that influence the company's growth.

Several investigators have highlighted problems with development in different phases of the organizational life cycle and suggested responses and development strategies [3]. Numerous research investigations have been conducted on the significance of both inside and outside organizational assets from the standpoint of resources. The verification, appropriation, and mobilization of these assets and their impact on the company's development have been thoroughly investigated. Fig. 1 depicts internal and external variables influencing a business's development [4].

Efficiency evaluation techniques are classified into three types: ratio evaluation, parametric evaluation, and non-parametric evaluation. When these strategies are contrasted, they each have benefits and drawbacks. The primary distinction between parametric and non-parametric techniques is in the fundamental presumptions used to estimate the frontier of effectiveness [5–7]. Accordingly, the most often utilized parametric and non-parametric methodologies for study are stochastic frontier analysis (SFA) and data envelopment analysis (DEA). DEA is a strategy for analyzing the efficiency and viability of decision-making units (DMUs) in terms of proportional variance in inputs or outputs that uses methods based on linear programming [8]. The data envelopment analysis technique first appeared by [9], and for this purpose, it is referred to as the CRR approach. Convex optimization techniques for high dimensional data clustering analysis has been examined by Yacoub et al. [10].

The CRR simulation accepts an equilibrium return of scale manufacturing function, and this simulation was subsequently modified to include an independent exchange to scale (VRS) efficiency measurement approach, additionally referred to as the BCC framework, based on the results obtained by Romano and Molinos-Senante [11], the CCR and BCC models are classified into two types: input-oriented models and output-related systems. The input-oriented aims to minimize the use of inputs given an initial number of results, whereas the output orientation aims to maximize output for a given number of resources.

However, all research used the panel SFA model with disturbance and inefficiencies factors separated. [11] recently employed a modelled instance to handle the panel data structure with dependant variable elements, but without making any inferences about the predicted efficiency. Accounting for dependence is often a desirable feature since the effectiveness of an individual DMU at a given moment may be affected by the fact that the DMU was 'lucky,' as stated by the random noise component. For instance, if the DMU was unfortunate at certain times, it would try to make up for it by increasing efficiency, resulting in a dependency on noise and effectiveness [12]. The regulator makes supplementary ad hoc changes to the DEA framework. After calculating cost savings with DEA, the baseline expense effectiveness is calculated using the mean cost inefficiencies greater than 55%. The baseline cost was assessed to be 79% in the most recent TRC. Furthermore, a Bootstrap model suggested by Simar and Wilson [13] and [14] establishes confidence ranges for cost-effectiveness. Finally, the DEA cost efficiencies, ranges of confidence, and standard cost-effectiveness are added together to generate final cost efficiencies ranging from 37% to 119%. The regulator contends that efficient firms, i.e., DSOs with cost efficiencies of 100% assessed using the DEA approach, should be compensated for being completely effective [15, 16]. As a result, their final efficiency may be larger than 100%. This ad hoc approach also raises the minimal value of efficiency in costs. The government agency also claims that the ad hoc methods account for potential variables not present in the DEA framework. One could argue that the ad hoc approach just permits DSOs to pass the effectiveness frontier [17].

The market for the purchase of farm machinery has a number of constraints when expanding in Russia due to the low efficiency of domestic manufacturing goods; the share of products that meet the needs of local companies was 26% at most. This inefficiency raises a big loophole in the overall competitiveness of Russian firms to compete globally, therefore the need to establish a strong method of assessing and improving the manufacturing issue. For example, basic evaluation of organizational performance by means of financial figures and their ratios overlook the interactions of various factors that affect growth possibilities. Past literature has discussed numerous factors of organizational effectiveness, and there has been little cohesive subsequently linking efficiency analysis to a strategic factor for growth. This research seeks to address this gap by using DEA and mathematical modelling tools such as SFA and Bootstrap Confidence Intervals to decompose the determinants of growth in the farm machinery industry. Not only does this work provide valuable prescription for managers and policy makers by increasing the measurement and understanding of the efficiency of decision-making units and the critical success factors that underpin it, it also provides a rich vein of theoretical thought for academics interested in the application of Operation Research techniques to manufacturing strategy and the understanding of resource utilisation. The use of these other advanced analytical tools helps in the computation of potential growth rate, which will help in arriving at good decisions on how firms in Russia can improve their competitiveness in the market place of the world. The key contribution of the proposed framework are:

- The proposed framework utilizes a robust dataset derived from the farm machinery market, encompassing various performance metrics and operational variables of domestic manufacturing firms.
- By integrating Data Envelopment Analysis (DEA), Stochastic Frontier Analysis (SFA), and Bootstrap Confidence Intervals, the framework offers a comprehensive approach to evaluate company growth potential, enhancing the accuracy and reliability of the analysis.
- DEA functions within the framework to assess the efficiency of decision-making units (DMUs) by comparing input-output ratios, identifying firms that optimally utilize resources, and establishing benchmarks for performance improvement.
- SFA operates in the framework to model the production frontier, incorporating random disturbances to account for external variables and uncertainties, thus providing a clearer estimation of firms' growth potential and resource utilization efficiency.

The remaining sections of this article are organized as follows: A brief summary of related studies is provided in Section 2. Section 3 of the study describes the study methodology and architecture of Data Envelopment Analysis using Stochastic Frontier Analysis and Bootstrap Confidence Intervals proposed. The results of the evaluation and subsequent discussion are presented in Section 5. Section 6 discusses the findings of the proposed model and its future applications.

2. Related works

In Dooranov et al. [18] proposed research on assessing and developing an inventively engaged enterprise's potential to export using economic and mathematical modelling. The research is focused on creating recommendations for enhancing the assessment and promoting the ability to export a creatively engaged firm. The authors evaluated the basics and characteristics of creatively engaged enterprises' activities and their impact on their internationalization efforts. The carried out conceptual and methodical research enabled the researchers to demonstrate their understanding of the primary elements of the company's export abilities; additionally, to evaluate and evaluate the exporting capacity of a creatively active company, research suggested a financial and mathematical structure, an aspect whose significance involves taking consideration of the hidden possibility of export, which has significant consequences for the economy. The authors developed an approach of labour resource stimulation and regulatory tools with a strong societal effect to boost the export capability of a creatively engaged firm. It is crucial to remember that there continues to be a shortage of thorough studies with particular suggestions on monetary policy approaches and methodology for implementing them in substantially expanding and intensifying export-related operations and improving the standing of national exporters in the global market.

Afzal et al. [19] suggested an approach to Data Envelopment Analysis (DEA), and the TOBIT framework was used to conduct a qualitative study of the National Innovation System (NIS). This work aims to look into the input-output elements of national innovation structures and develop a robust effectiveness assessment using the DEA Bootstrapping method. Most past NIS investigations are descriptive, with a lack of emphasis on complicated analysis. A recent study assessed the technological effectiveness of 20 developing and industrialized economies' achievements in innovation. The present investigation makes a major contribution by employing the DEA Bootstrap approach, in which researchers rank nations using bias-corrected prediction in addition to traditional DEA performance. The effectiveness ratings generated by this approach show which countries are considered innovative leaders because the history of invention is economical, given both constant and variable returns when measured in transforming originality supplies into imaginative products. Researchers propose several important policy consequences that may be gleaned from these innovators. The Tobit paradigm is then used to clarify inefficiencies. According to the Tobit regression approach, insufficient economies. DEA CRS related to a technology effective might have been modified within three main parameters: The high school admittance ratio, labour force development as a percentage of overall population growth, and domestic financial growth by the industrial sector as a percentage of GDP.

Nguyen et al. [20] suggested an approach to Optimal Math Model is used to solve challenging issues by companies in Vietnam that manufacture and process agricultural commodities. During the corona outbreak, various harvesting and producing farming goods firms in Vietnam had various challenges in the demand for agricultural commodities, even though they found themselves unwilling to sell. As a result, the businesses are becoming increasingly tough. Many businesses are not sufficiently resilient to resume manufacturing. Thus it is vital to find alternatives to get through this challenging phase. In this study, the writer utilized contemporary statistical approaches, together using the grey technique, to forecast upcoming company leads over companies alongside a framework of Super-slacks-based-measure performance (Super-SBM) to assist companies in selecting the ideal vendors in the supply chain to achieve their commercial objectives. According to the suggested strategy, the chosen option (AG6 paired with AG10) should be carried out later on to help stabilize results and enhance efficiency, allowing both sides to enhance the product's quality, accomplish company objectives, and contribute to equitable growth. It is vital to expand this research, in conjunction with these aspects and other mathematical models, to give shareholders a fuller picture, allowing them to arrive at better judgments and build their enterprises and advancement in society and the economy. Considering this research's boundaries, numerous additional supply chain elements influence cooperation, including weather, seasonality, and government restrictions. Another disadvantage is that the research investigation only looks at mathematical models.

Horváthová et al. [21] suggested a method for evaluating the financial condition of a business; the outcomes of a Data Envelopment Analysis framework and a Logit model are compared. This study is concerned with evaluating the financial condition of a corporation using specified mathematics and statistical approaches. Assessing financial well-being and predicting company demise is a hotly debated topic in Slovakia and worldwide. This research aimed to develop a DEA model and compare its estimated efficiency to that of the model known as logit. The study was conducted on several enterprises in the industry of heat delivery in Slovakia. The researcher chose relevant financial variables as drivers of insolvency for our sample of enterprises. A correlation matrix, a univariate logit approach, and associated empirical research were used to determine the markers. The research used a pair of primary models in the present study: the BCC version of the DEA approach, which was handled in DEA Frontier programs, and the model known as logit, which was handled in the Statistic program. The researcher evaluated the estimated efficiency of the built models utilizing error types I and II. The paper's principal finding is that the DEA approach is a suitable option for measuring the financial stability of enterprises based on the sample examined. Unlike the model created by logit, the outcomes of this method are not affected by any presumptions made.

Emrouznejad et al. [5] suggested an approach to data envelopment analysis-based mathematical framework for dynamic efficiency. In this research, the researcher presents a data envelopment analysis (DEA)-based approach to analyzing the comparative effectiveness of units executing manufacturing processes with inter-temporally variable input-output levels. Capital supplies, which affect the output over multiple manufacturing times, are one source of intertemporal dependency among inputs and final product values. Conventional or static DEA cannot assess these measurements. It presupposes that input-output connections are contemporary because the amount of output noticed over a specific period merely results from the input values reported within the identical interval. The model described in the study solves the issue of inter-temporal inputs and outputs dependency by analyzing both utilizing input-output channels drawn through operation units throughout time. For example, the researcher contrasts the dynamic versus static framework findings for a group of UK academics. According to the article, dynamic models capture performance better than static models. An educational effectiveness assessment application shows how static efficiency may become extremely volatile in the face of inter-temporal impacts, but the dynamic framework catches these implications effectively. Ahmed [22] examined the constant-stress partially accelerated life testing for Weibull inverted exponential distribution with censored data.

Muhammad [23] seeks to the implementation of BI is necessary for organizations if they are to achieve a better strategic position and improved competitiveness in the present highly competitive global market. Literature review revealed that BI plays the part of converting large volumes of unprocessed data into structured information that can be used to make decisions that Hood a firm's performance. BI crucial activities, important technologies, and tools help to analyze data from internal sources and markets and find concealed relationships. This capability enables the generation of timely and accurate reports and graphs to enhance operational efficiency and timely decisions. BI can enable organisations to predict market trends, analyse opportunities and threats for their business and create a feedback loop with performance metrics or KPI for processes improvement. The use of historical data as well as predictive costs and analytics improves the forecast enabling firms to implement their strategies well. Further, BI facilitate cross functional implementation because it provides accurate information across the organization and different departments work towards achieving a common vision. Summarizing, strong BI capacities are critical success factors for organizations willing to build endurance and obtain competitive advantage in the new conditions of a globally big data environment. This framework integrates Data Envelopment Analysis (DEA) and mathematical modelling in order to provide an advanced investigation of potential growth in firms.

A synthesis of literature points to a few weaknesses that have accompanied previous research on increasing the company's growth prospects. Firstly, while many papers undertake an analysis of patterns, there is no application of economic and mathematical modeling to arrive at actionable insights for firms. Further, the frameworks tend to fail in integrating the dynamics involved in export capability and financial performance, thereby limiting more realistic evaluations. Besides, some works like only describe mathematical models and exclude external factors like market conditions and regulations that determine business performance. To overcome these limitations of the study, the proposed framework combines DEA with SFA and Bootstrap Confidence Interval. Such an approach makes performance evaluations more stable to random fluctuations and variations of the input output relationship over time. Due to the integration of historical information and data analytics the framework improves the accuracy of the prognosis and, thus, assists in changing the approaches in a timely manner. Additionally, is enables the comprehension of an organization operational efficiency and its export potential to maintain a constant state of evolution. In conclusion, this extensive model proposes practical guidelines, and based on the goal of allowing organizations to optimally operate in a constantly evolving market environment.

3. Proposed data envelopment analysis (DEA) and mathematical SFA modeling for a company's growth framework

Numerous major processes are included in the approach for using Data Envelopment Analysis (DEA) and mathematical modeling to assess a company's growth prospects. First, data on businesses, including input and output variables, is gathered. The data has been pre-processed by resolving missing values, outliers and normalizing variables for comparability. The DEA assessment is then performed to analyze each company's efficiency in utilizing inputs to produce outputs and to provide benchmarks for future growth potential. Mathematical modeling tools, such as Stochastic Frontier Analysis (SFA), are used to evaluate the potential for expansion by calculating production frontiers which allow for unpredictability and external variables. An empirical inquiry verifies the results through statistical examination and compares potential growth projections to business outcomes. The findings are evaluated to identify major growth factors and areas for development. The process is strengthened further by sensitivity analysis and validation. This complete methodology combines DEA, mathematical modeling, and empirical research to assess growth potential and influence company decision-making.

3.1. Data collection

In this research, consistent with a secondary 7+ Million Company Dataset [24], we use a data collection process, and it is necessary to clarify the required inputs and outputs to assess the 40-DMUs efficiency and further expansion. This investigation also included lists of exposure identified relevant to various sectors. To ensure that the selected papers fits the subject of study for this research and estimate the total number of works of interest, some phrases were selected and an introduction to an organization for each approach was reconstructed in different industrial industries. To gather all the above information, make use of only reliable secondary sources like database of official statistics, researching reports, annual reports, and research papers. Obtain all the necessary data for each DMU specified and ensure compatibility of the variable and units used in the data collected. The various of control budgets that include numerical inputs such as labor, inventory, fixed assets, and capital are then scaled appropriately for the units and scales of measurement. The attributes that take on distinct categories, like industry categories as well as market shares, are often transformed through one hot or label encoding.

3.2. Dataset pre-processing

Data normalization is a very important step in data preprocessing especially when analyzing the 7+ Million Company Dataset and especially when considering efficiency of the forty DMUs. Normalization also facilitates the comparability of the numerical inputs (including labour, inventories, the fixed assets and capital) that link organizational units at different scales. Here, each of the numerical features are normalized and altered to a similar scale while retaining range differences of the values. Some of the normalization methods are the Z-score normalization. Standardization is one of the processes used under this method, and it shifts features' values by the mean and the standard deviation represented in the Eq. (1).

$$X_{norm} = \frac{X - \mu}{\sigma} \tag{1}$$

where μ is the mean of the feature and σ sigma is the standard deviation. It is used to equally balance all the input variable in the efficiency evaluation process thus making sure that no large scale variable controls the study result. This is particularly the case in DEA where efficiency scores are prone to relative expressions of inputs and outputs. This method normalizes the data making a ratio which keeps an equitable measure of the DMU's; the labor in hours is compared with capital in dollars.

3.3. Data envelopment analysis

According to the range of inputs and effects, the corresponding efficiency of 40-DMUs is determined by the scientific method called DEA [25]. The mathematical framework based on Data Envelopment Analysis (DEA), as illustrated in the above case is used to compare each decision making unit with other decision making units in order to determine the efficiency amount of 40 Decision-Making Units (DMUs). DEA finds out the combination of weight for inputs and outputs that makes a DMU have the highest efficiency score and none of them should be above the efficiency frontier of one. This is realised using the fractional programming model expressed by Eq. (2) which is then converted into a linear programming problem for practical optimisation as depicted by Eq. (3).

$$max\ m_0 = \frac{\sum_{b=1}^{o} b_o y_{bd0}}{\sum_{a=1}^{i} a_i x_{ad0}} \leq 1,\ d = 1, \ldots t \tag{2}$$

$$\sum_{b=1}^{o} b_o y_{bd0} - \sum_{a=1}^{i} a_i x_{ad0,} \leq 0,\ d = 1, \ldots t \tag{3}$$

The two closes of this LP problem; represented by Eqs. (4) and (5) contain Dual variables, which are the measures of input and output slack. The efficiency score, θ, estimated by this dual model reveals relative operating performance of a DMU, with the value of one denotes the efficient frontier.

$$\sum_{d=1}^{t} \mu_d x_{ad} + s_a^- = \theta x_{ad0}\ a = 1, \ldots i \tag{4}$$

$$\sum_{d=1}^{t} \mu_d y_{bd} + s_b^+ = \theta y_{bd0}\ b = 1, \ldots o \tag{5}$$

If a DMU is deemed inefficient, it can improve its performance by adjusting its inputs and outputs according to the improvement targets derived from the dual equations Eqs. (6) and (7).

$$x'_{ad0} = \theta^* x_{ad0} - s_a^-,\ a = 1, \ldots i \tag{6}$$

$$y'_{bd0} = \theta^* y_{bd0} - s_b^+,\ b = 1, \ldots o \tag{7}$$

These are called input oriented adjustment where the idea is to minimize inputs while at the same time increasing the outputs in order to improve the efficiency. It is especially useful where management has better control over some inputs and applying this approach will be useful in performance assessment and efficiency enhancement of operations. Input-oriented efficiency is stressed in the study because it enables measurement of a DMU's efficiency in use of inputs while keeping up the outputs. Furthermore, the idea of a benchmark, including DMUs with the efficiency score of one, called a peer group, is proposed to allow for measuring inefficiency and applying considerations for performance enhancement [26].

3.4. Stochastic frontier analysis (SFA)

The frontier notion has been central to the methodologies for gauging efficiency suggested over the past ten years. Effective units operate on the cost or production frontier. In contrast, inefficient units operate either below (in the case of the manufacturing frontier) or above (in the case of the cost frontier). The underlying stochastic theory of the frontier cost (or generation) function indicates that any deviation from the conventional microeconomics expenditure (or production) function may be accounted for only by random disturbances and inefficiencies. The composite error component gives the variance in the stochastic frontier model. The entirely random component represents the impact of factors other than the production of the equipment under discussion. As a result, one significant advantage of stochastic frontier over DEA is that it removes the impact of factors other than inefficient behaviors, minimizing the potential upward bias of inefficiencies from determinism approaches [27, 28]. The m input slack factors of n decision-making unit in arranging one is utilized as the secondary variables of set up two, which are declined into the abilities of three free factors, counting natural components, factual commotion, and administration variables, by developing the SFA relapse demonstrate. As a result, the expression is as follows:

$$U_{mn} = a^m\left(x_n;\alpha^m\right) + t_{mn} + \vartheta_{mn} \tag{8}$$

In Eq. (8) the slack variable is denoted as U_{mn} in which the decision unit on the input m is mentioned as n and the difference among the ideal and real input, which is stated in Eq. (9):

$$u_{mn} = b_{mn} - B_m\gamma \tag{9}$$

Eq. (9) the growth potential element of slack variable is denoted as $q^m(Z_n,\delta^m)$ and normally, $q^m(Z_n,\delta^m) = \ x_n, \delta X_n$ which is represented as the k-dimensional variable. The parameter to be assessed is represented as δ^m. The combined error term is mentioned as $t_{mn} + \vartheta_{mn}$. The half-normal, exponential, and truncated-normal distributions are typical distributional forms for technical inefficiencies. The second component is a two-sided element with a zero mean and variance. Since the degree of inefficiency is determined from the regression residuals the bootstrap method can be utilized for estimating the frontier function. The stochastic frontier production function for panel data was developed and presented in Eq. (10):

$$A_{mn} = exp\left(b_{mn}\delta + t_{mn} - \vartheta_{mn}\right) \tag{10}$$

where b_{mn} denotes manufacturing at the n-th observation (n = 1,2,. . .,N) for the m-th firm (m = 1,2,. . .,N); it x is a (1xk) vector of values of recognized functions related to manufacturing inputs and other explicable variables associated with the m-th firm at the n-th observing; Deltas ais vector of unidentified variables to be determined. ϑ_{mn} are considered to have the t_{mn} (0), t_{mn} s are non-negative random factors related to technical inefficiency of production, and are thought to be separately dispersed, such that t_{mn} can be calculated by truncation (at zero) of the normal distribution with mean u_{mn} and variance 2; x is a vector of variables of explanation connected with technical ineffectiveness of manufacturing firms over time. The approaches presented in this work can also be used in several enhanced versions of this model [29, 30].

3.5. Bootstrap confidence interval

Although the DEA approach has many advantages, it is not without limitations. Production ratings are susceptible to sample selection errors because efficiency ratings are computed using the piece-wise frontier. Furthermore, the efficiency score of a DMU is calculated by its performance compared to the efficient DMUs detected inside the sample, or "reference set," rather than the entire population. As a consequence, the DEA's efficiency figures are sometimes inflated. Furthermore, the number of efficient DMUs that use DEA appears to increase with the number of elements, and the method only generates point estimates for efficiency evaluations instead of uncertainty ranges, making the DEA outcomes less reliable.

The bootstrapped DEA approach addresses these constraints by generating effective scores from reproduced sets of information and offering confidence ranges for efficiency ratings [6]. Since in the bootstrapping procedure applied in this study, replicas were developed, through the smooth bootstrap method suggested by Simar and Wilson [12], what

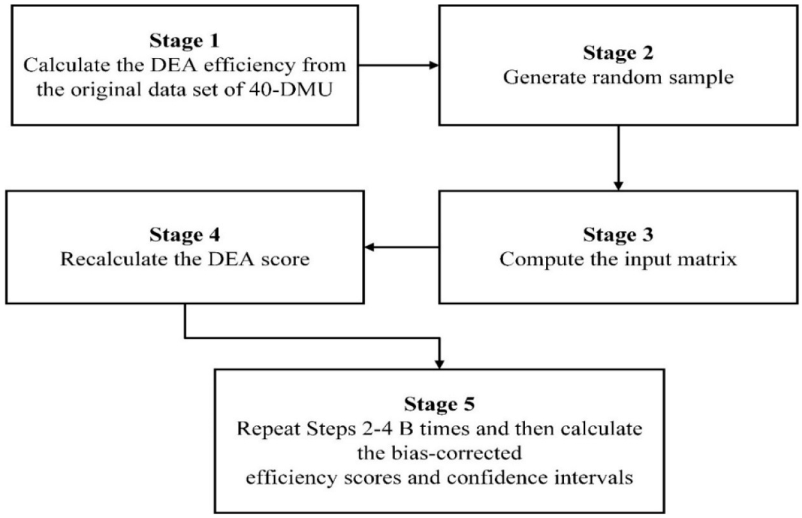

Fig. 2. Bootstrapped DEA score.

this method does is to smooth the empirical distribution of the DEA scores before resampling, a move which greatly enhances the estimation, especially near the frontier or boundary of unity. Evidently, it is common to run between 1000 and 2000 bootstrap replications for the purpose of convergence. The resampling procedure entails bootstrapping which entails sampling with replacement, estimating DEA efficiency from the resampled data and then using the efficiency scores to build the confidence intervals and reduce bias. This detailed approach to bootstrapping provides not only a more convincing interpretation of DEA efficiency measures but also organizes the results that are closer to those obtained with SFA, where the stochastic character of the data is taken into consideration. Utilizing Monte Carlo re-enactment based on the observational examining dissemination, a reliable appraise of A, alluded to as $\bar{A}$, can be recreated to surrender a vector of comparing proficiency scores $\bar{A}^*$with the taking after relationship mentioned in Eq. (11):

$$(\bar{\phi}^* - \hat{\phi})|\bar{\mathrm{A}} \approx (\bar{\phi}^* - \hat{\phi})|\mathrm{A} \tag{11}$$

where $\hat{\phi}$ is the original DEA effectiveness rating vector and is the genuine, undetermined field of efficacy ratings. [21] allows us to calculate the bias of the effectiveness score via DEA ($\hat{\phi}$). The association above exists because of $\bar{\phi}^* > \bar{\phi} > \phi^*$. According to Simar and Wilson [10], the evaluation of may be computed in Eq. (12) follows:

$$\hat{\phi}^* = 2\hat{\phi} - \bar{\phi}^* \tag{12}$$

Use the median amount if the theoretical assignment of b is biased. The general phases associated with bootstrapped DEA scores are depicted in Fig. 2. Step 2 is crucial in the bootstrapping process. Simar and Wilson [10] propose using the smooth bootstrap approach to enhance the estimate of efficiency ratings around the upper bound of 1. Resampling around 1000 and 2000 times is the usual practice [31–33].

4. Result and discussion

4.1. Analysis of dataset normalization to different input variables for a few DMUs

Table 1 presents a comparative analysis of the dataset before and after normalization, focusing on four key input variables: labour hours, inventory USD, fixed assets USD and capital USD for five Decision making Units DMU. In part (a) the first raw data illustrates how large the differences are between the scales of the input variables. For example, labor hours are between 400 to 800, and capital between 900 000 & 2000 000 USD differentiating the DMUs. Such variations in scale may have severe consequences and cause biased results within efficiency evaluations if left unnoticed. Hence normalization is used to standardize all the values so that none of them dominates the analysis more than others. Normalized data are shown in part (b) of the table where all the input variables are scaled within the range, 0 to 1. For instance, considering labour hour, DMU4 has 400 and is made 0 while DMU2 has 800 and is made 1.

Likewise, with other inputs and measures, such as inventory or capital, which were also large, the data is standardized by scaling. The fixed assets variable demonstrates that DMU3 has got 3,00,000 USD

Table 1. Before and after normalization of dataset.

(a) Before Normalization				
DMU	Labor (Hours)	Inventory (USD)	Fixed Assets (USD)	Capital (USD)
DMU1	500	50,000	2,00,000	10,00,000
DMU2	800	75,000	2,50,000	20,00,000
DMU3	600	60,000	3,00,000	15,00,000
DMU4	400	55,000	1,80,000	9,00,000
DMU5	700	65,000	2,20,000	12,00,000
(b) After Normalization				
DMU	Labor (Norm)	Inventory (Norm)	Fixed Assets (Norm)	Capital (Norm)
DMU1	0.33	0	0.25	0.05
DMU2	1	1	0.7	1
DMU3	0.5	0.33	1	0.65
DMU4	0	0.2	0	0
DMU5	0.83	0.5	0.4	0.15

in fixed assets and then it is normalized to 1 and DMU4 has 1,80,000 USD fixed assets and thus it is normalised to 0. This normalization process is very important to avoid having a DEA model that has its basis skewed by the raw magnitude differences of the input data in the actual evaluation of the efficiency of these DMUs. By bringing all inputs to the same scale, normalization facilitates a fair comparison among DMUs, allowing the analysis to focus on the efficiency derived from the relationship between inputs and outputs rather than being influenced by the absolute values of the inputs. As a result, the normalized data provides a more balanced and accurate basis for assessing the efficiency and performance of the DMUs, leading to more reliable and interpretable outcomes in the study.

4.2. Analysis of efficient decision-making units

It defines factors which were considered as hall marks of efficient decision-making units through the Table 2 below. It begins with a Labor Efficiency model that indicates productivity at 1,500 units per 100 hours. The Average Inventory Turnover Ratio is 4 suggesting the number of cycles per year that inventory is sold and restocked. The Average Value of Fixed Assets is $3,000,000 and the Annual Capital Investment is $500,000. The table shows Input Utilization and points out that these units' require 20% less labor and 15% less capital in comparison with the standard. Operational Practices which involve the integration of lean management practices, which seek to reduce operational costs without reducing on service delivery. The Industry Sector is defined as growth sectors, technology included. Last of all, the Efficiency Score (DEA) stands at 0.85 with DEA representing Data Envelopment Analysis; this is a methodology with which one evaluates the efficiency of these units. This integrated framework gives an efficient dimension to the strategic units in making operations and financial decisions in changing environments.

Table 2. Efficiency of DMU.

Characteristic	Efficient DMUs
Labor Efficiency (Output/Hours)	1,500 units/100 hours
Average Inventory Turnover Ratio	4 times/year
Average Value of Fixed Assets	$3,000,000
Annual Capital Investment	$500,000
Input Utilization	20% less labor & 15% less capital
Operational Practices	Lean management techniques
Industry Sector	Growing sectors (tech)
Efficiency Score (DEA)	0.85

Prior investigations have used a variety of inputs and outputs to assess the effectiveness of an organization. The most commonly utilized parameters are performance as the output. The input parameters are variable berth length, terminal regions, warehousing ability, and transportation technology. Even though labour is a key input factor in manufacturing concepts, it is frequently difficult to get. Furthermore, due to the vital role that the firm plays in port management, the capacity of technology for information and communication frequently affects the production of modern terminals. Depending on these frequently used factors, accessibility to data, and the additional factor.

4.3. Analysis of DEA efficiency score

The following Figs. 3 and 4 show the DEA efficiency score. The DEA (Data Envelopment Analysis) efficiency score assesses the relative efficiency of 40-Decision-Making Units (DMUs) or firms in generating outputs by utilizing their available resources. The effectiveness score is determined by contrasting each

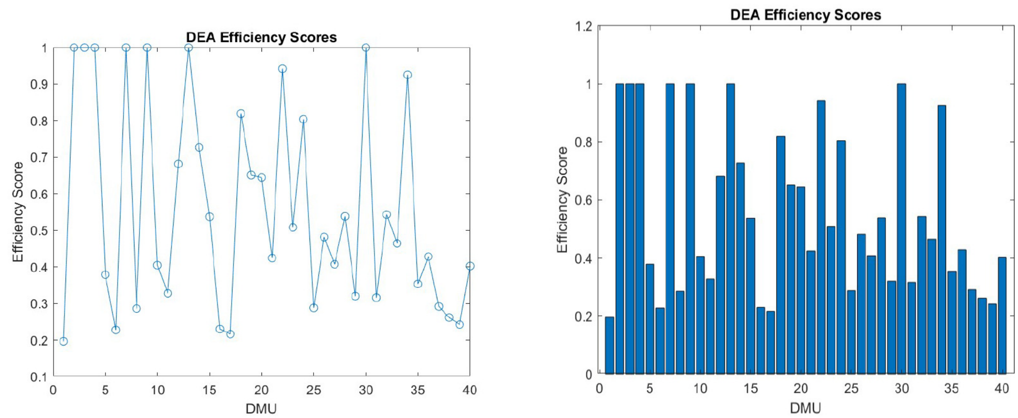

Fig. 3. DEA efficiency score.

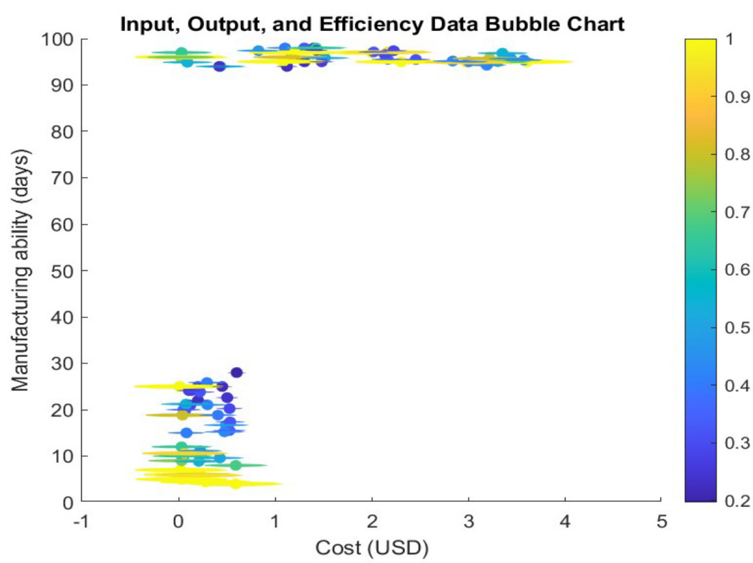

Fig. 4. Statistic of input, output and efficiency data.

DMU's performance to a built benchmark representing the most efficient DMUs in the information set. The DEA efficiency score goes from 0 to 1, with 1 representing entire efficiency and 0 representing total inefficiency.

A DMU with an efficiency score of 1 is deemed functioning at the cutting edge of efficiency, utilizing all of its assets effectively and producing the most potential outputs considering its inputs. The first portion of the study examines the companies' operational effectiveness. Table 1 contains descriptive information about the inputs and outcomes. 30 days, $60 million in cost, 1.1 billion dollars in income, and a 98.9 percent delivery rate are typical managerial ability values. The standard deviation of income is quite big. This could be due to cost differences between businesses. There are a total of 40 DMUs being evaluated Fig. 3 illustrates the efficient output.

4.4. Statistical variable measures

For instance, the average income of 1.12 is below two units of figures suggesting that most firms earn relatively little, although the delivery performance rate of 93.66 % is relatively high but characterized by high volatility. Standard Deviation (SD) generally defines the amount of spread or dispersion of each variable. The use of 0.15 coefficient of variation in cost indicates considerable variation in cost among firms; whereas a coefficient of variation of 9.71 in manufacturing capability shows variability in production times. Coefficients of variation for income are 0.86, implying large fluctuations in firms' revenues across the industry; while delivery percentage standard deviation is only 2.04, suggesting less variability in delivery performance by firms.

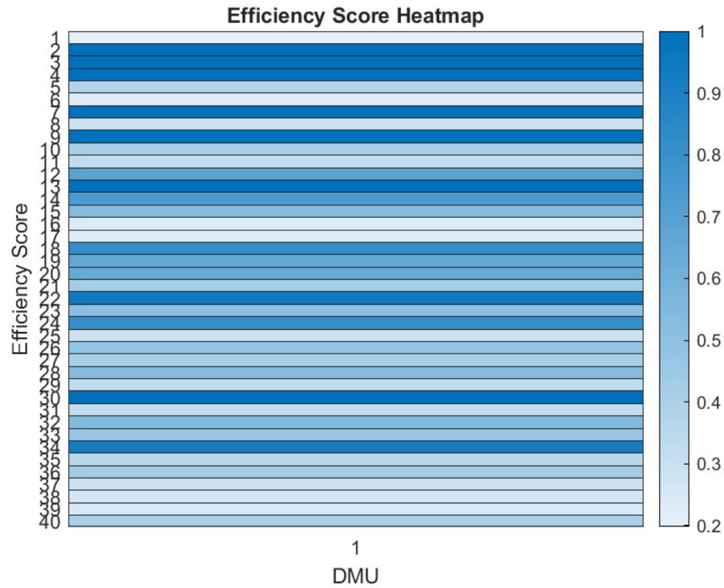

Fig. 5. Efficiency score heat map.

Table 3. Statistical variables.

Description	Cost **(USD)**	Manufacture capability(days)	Income **(USD)**	Delivery (Percentage)
Max	0.60	30	3.8	98.9
Mini	0.02	6	0.022	96
Average	0.05	23.68	1.12	93.66
SD	0.15	9.71	0.86	2.04

It gives some initial notion of the dataset's characteristics and, more specifically, disparities in the businesses' operating performance and financial performance. These summary statistics, when linked to SFA, assist in an explanation of variations in input costs, production, income and delivery method for the component of firm efficiency profiles and identification of areas for improvement. The Table 3 presents a detailed summary of key statistical measures for four variables: Cost (USD), Manufacturing Capability (days), Income (USD), and Delivery Percentage.

The maximum (Max) values represent the highest recorded Fig. 4 in the dataset, indicating the peak performance or cost within the observed firms. For example, the maximum cost of production is 0.60 USD, while the maximum income is 3.80 USD. These values may signify firms operating at different scales or efficiency levels. The minimum (Min) values, such as a cost as low as 0.02 USD and a delivery percentage of 96%, highlight the lower bounds of performance, possibly reflecting more efficient or resource-constrained firms. The average (Mean) values, such as an average cost of 0.05 USD and an average manufacturing capability of 23.68 days, offer insights into the central tendency of the data. These averages are crucial for understanding the typical performance across firms.

Fig. 5 shows the heat map of efficiency scores for a collection of 40 Decision Making Units (DMUs) and provides a succinct visual depiction of the efficiency scores. The heat map shows DMUs on the y-axis along with efficiency ratings on the x-axis, with each cell indicating the efficiency score of a DMU. The efficiency level is shown by the colour intensity within each cell, with deeper colors indicating better efficiency and lighter colors indicating lesser efficiency. This visualization enables rapid detection of trends, clusters, and outliers in efficiency scores, assisting in assessing the relative performance of the DMUs and identifying areas for improvement or best practices. The efficiency score heat map is useful for benchmarking, performance evaluation, resource allocation, and process optimization decision-making.

4.5. DMUs distribution of efficiency scores

The density plot or percentile may be used to analyse the distribution of efficiency scores, providing insight into the density and variability of values. Fig. 6 represents the density plot depicts the probability of witnessing a specific efficiency score, whereas the histogram shows the frequency or count of scores within defined intervals or bins. Understanding the focus of scores, identifying potential outliers, and assessing the general efficacy of the DMUs or firms can be accomplished by studying the distribution's shape, central tendency, and spread. Asymmetric distribution with a tail towards lower efficiency scores

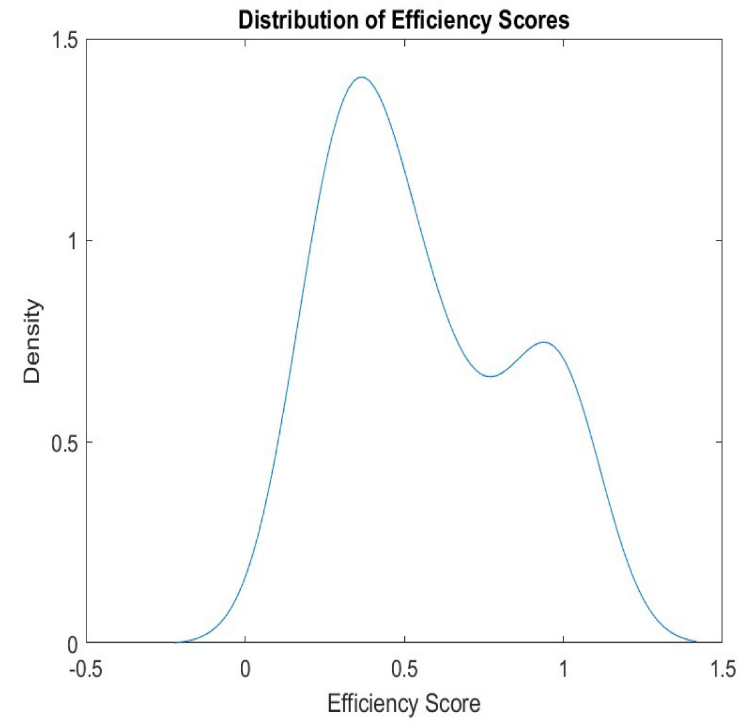

Fig. 6. Distribution efficiency score.

indicates an overabundance of highly efficient entities, whereas a positively skewed distribution shows a tail towards higher efficiency scores. A multimodal distribution suggests the presence of distinct groups with varying levels of efficiency as shown in the Fig. 6.

Table 4 presents results on efficient inputs, outputs, cost targets, relative effectiveness scores, and ranks. DMU 13's managerial abilities must be reduced by 34.5 percent, from 30 to 7.85 days. Similarly, managerial competency (7.55 days) and cost (0.02 billion USD) are DMU 40s effective target inputs.

The parameters cost, production capability and delivery rate are coupled with specific coefficients that describe their impact on the outcome variable, while the intercept reflects the model's constant term. Analysing the parameter estimations and their confidence intervals allows a better understanding of the model's linkages and relevance.

Figs. 7 and 8 show the performance chart and efficiency performance. Labour expenses, material costs, resource consumption, and production capacity are efficient input variables in a 40-DMU scenario. Productivity, quality, environmental effect, and innovation are output variables. A mean and standard deviation graph can be used to analyse the data. The mean is the average value of the variables across the 40 DMUs, and it indicates the central tendency.

The standard deviation measures the variability or dispersion of data around the mean, demonstrating the degree of diversity or consistency among the DMUs. By presenting the mean and standard deviation on a graph, one can visualize the average performance and performance dispersion over the 40 DMUs, assisting in identifying outliers, trends, and overall patterns in the data.

Table 5 and Fig. 9 above show the parameter estimation with bootstrap confidence interval. The parameter estimation with bootstrap confidence intervals at a 95% confidence level provides insights into the values and uncertainty surrounding the parameters: cost, manufacturing capability, delivery rate, and intercept. The estimated values are the point estimates generated from the estimation method, while the bootstrap confidence intervals provide the range of plausible values for each parameter.

The 95% confidence level indicates that the true population parameter is 95% likely to fall inside the stated interval. These confidence intervals assist in assessing the precision and reliability of parameter estimations by correcting for variability using the bootstrap resampling technique. The parameters cost, production capability and delivery rate are coupled with specific coefficients that describe their impact on the outcome variable, while the intercept reflects the model's constant term. Analyzing the parameter esti-

Table 4. Efficient input and output variables for 40-DMU.

	Actual inputs		Actual outputs		Efficient original inputs		Efficient original outputs	
DMU. No	Cost (USD)	Manufacturing ability (days)	Revenue (USD)	Delivery rate (Percentage)	Cost (USD)	Manufacturing ability (days)	Revenue (USD)	Delivery rate (Percentage)
1	0.8	28	1.42	98.5	0.02	3.00	2.02	98.7
2	0.56	4	3.5	95.6	0.11	10.00	0.46	95.1
3	0.05	25	1.24	97.2	0.53	9.81	0.34	97.3
4	0.04	5	0.022	96.3	0.04	5.00	1.54	96.5
5	0.06	20	0.44	94	0.02	6.74	0.52	94.6
6	0.22	26	1.14	94.4	0.04	4.20	2.06	94.7
7	0.12	8	2.4	95.5	0.08	9.01	0.52	96.8
8	0.13	22	1.18	96.5	0.44	9.14	0.24	97.0
9	0.06	8	2.1	97.6	0.18	9.18	1.06	98.9
10	0.09	16	1.6	98.9	0.49	9.25	0.48	98.8
11	0.08	22	1.38	98.9	0.02	3.00	2.02	95.4
12	0.57	10	3.4	95.3	0.14	10.00	0.46	93.00
13	0.01	24	1.22	97.4	0.54	9.82	0.38	95.00
14	0.05	10	0.026	96.5	0.05	5.03	1.52	94.00
15	0.08	11	0.46	94.6	0.06	6.77	0.58	92.00
16	0.22	25	0.48	94.6	0.14	7.24	2.12	93.00
17	0.47	22	1.8	95.8	0.58	9.03	0.56	98.00
18	0.23	8	1.18	96.9	0.39	6.22	0.38	96.00
19	0.04	14	0.04	97.8	0.55	9.54	0.14	95.00
20	0.05	11	1.48	98.5	0.68	2.13	1.36	94.00
21	0.7	29	1.42	98.4	0.02	3.00	2.02	98.00
22	0.58	6	3.8	95.6	0.12	10.00	0.47	95.00
23	0.02	24	1.22	97.8	0.53	9.81	0.33	96.00
24	0.04	6	0.022	96.5	0.06	5.00	1.52	98.00
25	0.06	20	0.44	94.3	0.04	6.74	0.51	94.00
26	0.26	25	1.14	94.2	0.06	4.20	2.08	98.00
27	0.13	6	2.4	95.2	0.08	9.01	0.56	94.00
28	0.03	7	2.2	97.5	0.17	9.18	1.06	94.6
29	0.07	15	1.2	98.6	0.49	9.25	0.48	95.8
30	0.08	20	1.32	98.7	0.02	3.00	2.02	96.9
31	0.58	8	3.6	95.1	0.12	10.00	0.46	97.8
32	0.08	25	1.24	97.3	0.54	9.82	0.37	98.5
33	0.04	9	0.026	96.5	0.05	5.03	1.51	98.4
34	0.03	10	0.44	94.6	0.02	6.77	0.57	95.6
35	0.21	22	0.46	94.7	0.14	7.24	2.11	97.8
36	0.22	6	1.18	96.8	0.38	6.22	0.34	96.5
37	0.02	12	0.04	97.0	0.54	9.54	0.12	94.3
38	0.05	10	1.44	98.9	0.68	2.13	1.34	94.2
39	0.7	28	1.44	98.8	0.02	3.00	2.00	95.2
40	0.58	4	3.8	95.4	0.11	10.00	0.45	94.6

Table 5. Parameter estimates with a bootstrap confidence interval.

Factors	Parameter estimation	Bootstrap interval
Cost	−0.453	[0.318,0.598]
Manufacturing Ability	−0.788	[0.655, −0.915]
Delivery rate	−0.222	[−0.399, −0.057]
Intercept	−2.154	[−1.983, −2.335]

mations and their confidence intervals allows a better understanding of the model's linkages and relevance.

4.6. Analysis of SFA model estimation for each bootstrap sample

On a Stochastic Frontier Analysis (SFA) Fig. 10, bootstrap estimation entails resampling the data to estimate the uncertainty associated with efficiency scores. First, the data is fitted to the SFA model, yielding efficiency scores that represent the relative performance of decision-making units (DMUs). The bootstrap resampling technique is then used, which involves randomly choosing samples from the original dataset with replacement.

The SFA model is re-estimated for each bootstrap sample, yielding fresh efficiency ratings. This procedure is repeated several times to distribute efficiency scores for each DMU. Measures such as mean, standard deviation, confidence intervals, and percentile ranks can be produced by analyzing these distributions, providing insights into the uncertainty and variability of the efficiency estimations.

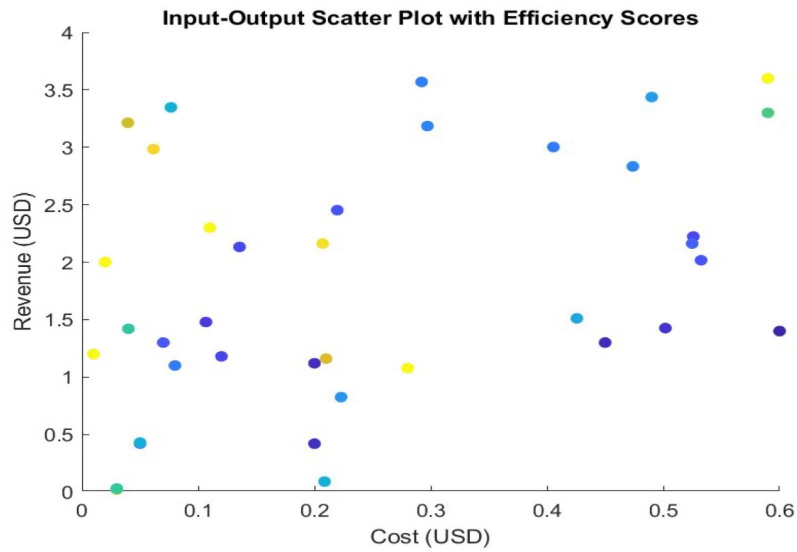

Fig. 7. Scatter plot with an efficiency score.

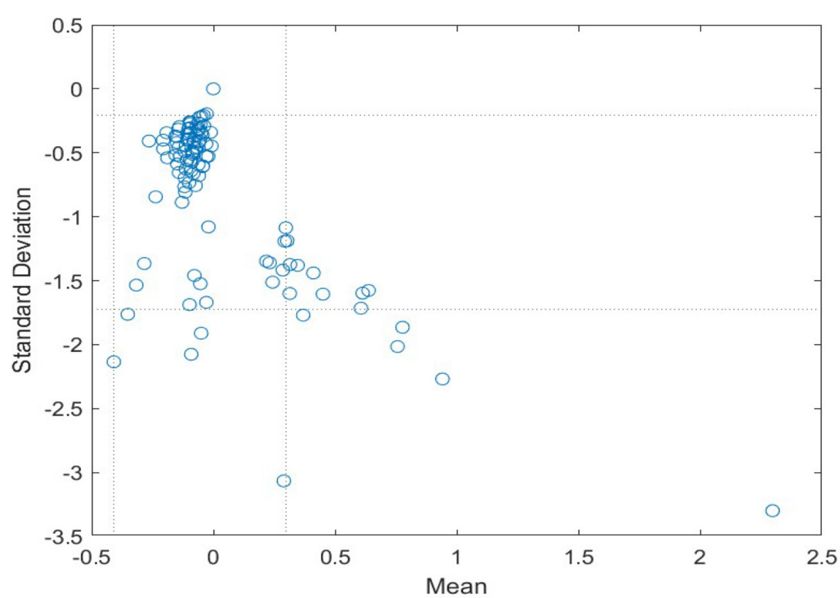

Fig. 8. Performance chart.

Using bootstrap estimation improves the understanding of efficiency ratings by incorporating robustness.

4.7. Discussion

The results demonstrate the critical role of dataset normalization in ensuring accurate efficiency evaluations using DEA. Before normalization, significant disparities in scale among input variables like labor hours and capital led to potential biases in efficiency scores. Post-normalization, the data for the DMUs became more comparable, eliminating scale-based distortions and facilitating a fairer assessment of efficiency. The normalized data allowed for a clearer analysis of how effectively DMUs utilize their inputs to generate outputs. The DEA efficiency scores, visualized through heat maps and scatter plots, highlighted variations in performance, with some DMUs achieving near-optimal efficiency while others exhibited room for improvement. These findings underscore the importance of normalization in DEA studies for reliable and interpretable efficiency analysis.

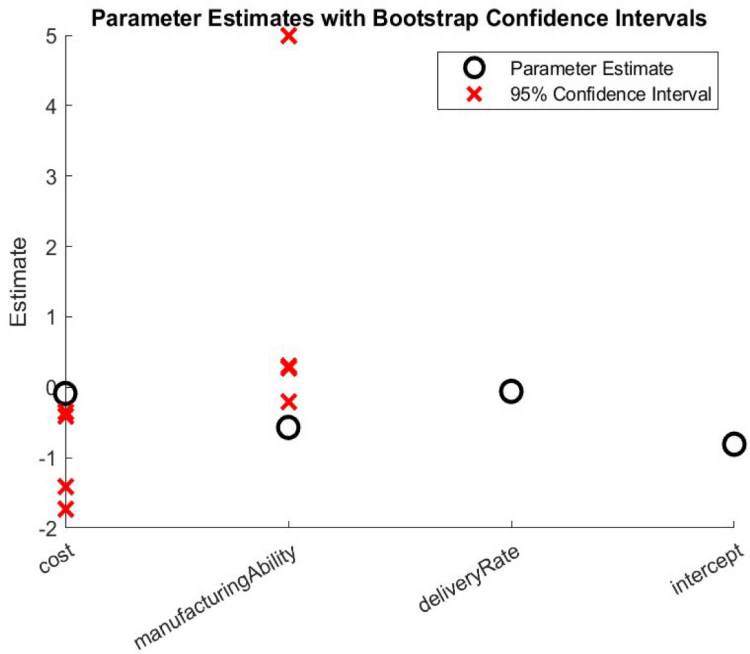

Fig. 9. Parameter estimation with a bootstrap confidence interval.

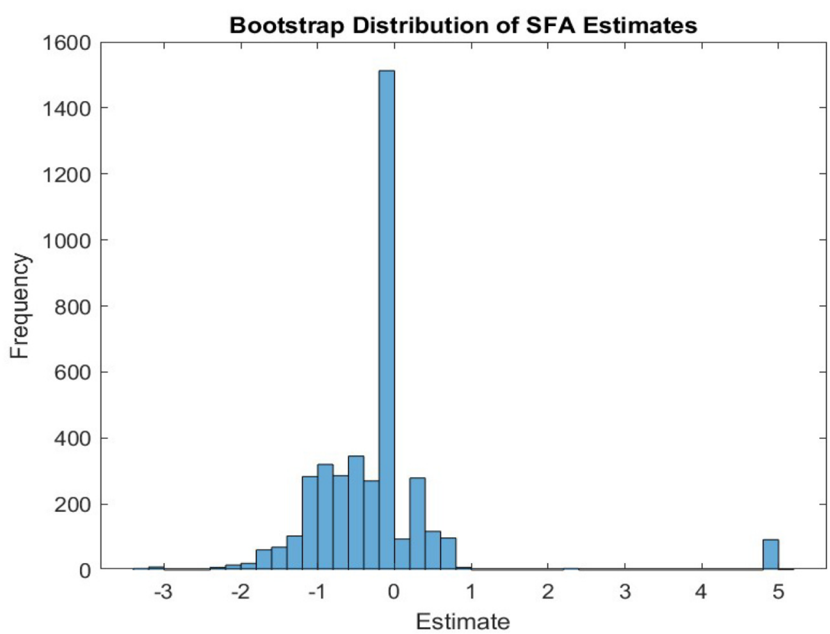

Fig. 10. Bootstrap estimation of SFA.

5. Conclusion and future works

DEA, SFA, and Bootstrap Confidence Intervals form a useful theoretical background and conceptual tool designed for assessing growth potential of a company. DEA compares efficiency rates, and SFA reserves specific growth opportunities by separating inefficiency from noise. To the managers, these methods present practical suggestions regarding resource utilization and performance enhancement. Through Bootstrap confidence intervals, managers can be better placed to understand the uncertainty of its growth estimates hence arrive at more accurate decisions. They should secure the maintenance of efficiency-enhancing technologies together with regular improvement of production procedures through these superior analytical tools to advance standard managers' sustainable growth. The outlined approach of synergy creates important findings about how resources are used, reveals development potential, and helps to make tactical decisions that will contribute to the company's success. By using these methods, the academics and practitioners tend to know about more value in the in growth potential, providing assistance into the development of business evaluation processes and enhancing the firm's capability to develop sustainable growth. Future research could apply the DEA-SFA-BCIs framework to industries like healthcare, financial services, and manufacturing to assess efficiency in resource allocation, risk management, and productivity. This approach could uncover industry-specific efficiency drivers, offering tailored recommendations for operational improvement. Expanding its application across diverse sectors would enhance the framework's versatility and impact.

Conflicts of interest

The author declares that he has no conflicts of interest to report regarding the present study.

Data availability

Data sharing is not applicable to this article as no data sets were generated during the current study.

Ethical approval

This article does not contain any studies with human participants performed by the author.

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