The Complete Basis Set of The Orthonormal Vector Polynomials in A unit Annular Circular Pupil

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الخلاصة

تم في هذا البحث اشتقاق المجموعة الكاملة لمتعددات الحدود المتجهة العيارية والمتعامدة لفتحة حلقية مساحتها الخارجية تساوي وحدة واحدة بإيجاد أولا مجموعة الدوال المتعامدة والتي تمثل كميات متجهة يمكن الحصول عليها بأخذ انحدار دوال زرنيك الحلقية PZ ، ويمكن الحصول على التعامد باستخدام طريقة كرام شمت للتعامد. العلاقة بين هذه المتعددات ومتعددات حدود زرنيك للفتحة الدائرية وانحدارات متعددات حدود زرنيك الدائرية تم ايجادها ايضا في هذا البحث.

بعد ذلك، ولإكمال المجموعة، تم اضافة مجموعة متممة من الدوال والتي لها انحدار يساوي صفر، وهي ما يطلق عليها بالالتفاف أو الدوران.

الكلمات المفتاحية

متعددات الحدود المتجهة العيارية، انحدار دوال زرنيك الحلقية zp، طريقة التعامد ل (كرام شمدت).

Abstract

In this paper, a complete set of the orthonormal vector polynomials in a unit annular pupil were derived by finding, first a set of orthogonal functions that represent vector quantities which can be generated from the gradients of annular Zernike polynomials ZP, and the orthogonality is made by MATLAB code using Gram Schmidt orthogonalization method. A relation of these polynomials with the circular ZP and circular ZP gradients are represented also in this work.

Then, to complete the basis, a complementary set of functions were added that have zero divergence, those which are defined locally as a rotation or curl.

Keywords

Orthonormal vector polynomials, Annular Zernike polynomials ZP, Gram Schmidt orthogonalization method .

object tracking", IEEE Trans. Patt. Analy. Mach. Intell,Volume 25, Issue 5, pp. 564–575, May (2003).

[5] DucPhuCHAU,Francois BREMOND and Monique THONNAT, "Object Tracking in Videos: Approaches and Issues", The International Workshop "Rencontres UNS-UD" (RUNSUD), Danang, Vietnam, 18 April (2013).

[6] Yilmaz, A., Javed, O., and Shah "Object tracking: A survey" ACM computing surveys, vol.38, article 13, (2006).

[7] Xi Li, Zhang, shen "surveys of appearance models in visual object tracking" ACM transactions on technology, arXiv :1303.4803v1, (2013).

[8] S. J. Rajput, S.D.Oza, "A New Algorithm for Tracking of Multiple Moving Objects" international Journal of Scientific Engineering and Technology, Volume No.2, Issue No.7, pp : 691-693,(2013).

[9] N.G.Chitaliya , A.I.Trivedi ,"Novel block matching algorithm using predictive motion vector for video object tracking based on color histogram"3rd International Conference, Pages 81-85 IEEE, (2011).

[10] David H. Douglas, Thomas K. Peucker. "Algorithms for the reduction of the number of points required to represent a digitized line or its caricature", Cartographica: The International Journal for Geographic Information and Geovisualization, vol. 10, no. 2, pp.112-122, (1973).

[11] Santiago Garrido, Dolores Blanco, Luis Moreno and Fernando Mart´ın, "Improving RRT motion trajectories using VFM" IEEE International Conference on Mechatronics, 978-1-4244-4195-2/09/$25.00, Málaga, Spain, April (2009).

[12] Bingbing Ni, Pierre Moulin, Xiaokang Yang and Shuicheng Yan, "Motion Part Regularization: Improving Action Recognition via Trajectory Group Selection", Proceedings of the IEEE Computer Society on Computer Vision and Pattern Recognition, pp. 3698-3706, (2015).

[13] Dr. Saad Talib Al-jebori and Israa Hadi Ali, "Clustering of Movie Film Objects Behavior Using Graph Mining", dissertation, (2013).

[14] Qiang Wu, Fatima A. Merchant, Kenneth R. Castleman, "Microscope Image Processing", Elsevier Inc, (2008).

[15] Tinku Acharya, Ajoy K. Ray, "Image Processing Principles and Applications", John Wiley & Sons, (2005).

[16] Ian H. Witten, Eibe Frank, and Mark A. Hall, "Data Mining Practical Machine Learning Tools and Techniques", Morgan Kaufmann, (2011).

7	141	280	4	141	280	4	1 =4/4
8	143	272	29	143	272	29	1 =29/29
9	142	243	14	142	243	14	1 =14/14
10	143	229	8.5	143	229	8.5	1 =8.5/8.5
11	145	212	8.7	145	212	8.7	1 =8.7/8.7
12	148	186	____	148	186	____	____
13	148	172	4	148	172	4	1 =4/4
14	149	176	1	149	176	1	1 =1/1
15	146	173		146	173		**Sum**= 12.18333333
Avg.= 0.870238095 \ **Percentage** = 87%							

4. Conclusion

Tracking can be defined as the problem of estimating an object trajectory in the image plane as it moves around the scene. In other words, a tracker appoints constant labels to tracked objects in various frames of video. The tracking operation aims to get the trajectory of an object over time depending on the location of the object in all frames, our interest is to determine the new points in the trajectory, and the negligence of the other points of the trajectory, this is done depending on the enhancement method which we previously referred to in the paper. The goal of the segmentation is to identify the objects as quick as possible. Enhancement process is essential in this paper, whereas in this paper the goal of enhancement is approximation the study object trajectory in movie from the typical trajectory of other object and this object in the same or other movie film by used Enhancement Method to obtain enhancement trajectory this trajectory may be approximate or same the typical trajectory, also, this method shows good results between (87% - 100%) . In video tracking scenario, an object can be defined as anything that is of interest for further analysis. For instance, boats on the sea, fish inside an aquarium, vehicles on a road, planes in the air, people walking on a road, or bubbles in the water are a set of objects that may be important to track in a specific domain. Objects can be represented by their shapes and appearances.

5. References

[1] Himani S. Parekh, Darshak G. Thakore and Udesang K. Jaliya "A Survey on Object Detection and Tracking Methods" International Journal of Innovative Research in Computer and Communication Engineering, Vol.2, Issue2, February (2014).

[2] Tiesheng Wang, Irene Y.H. Gu, Mats Viberg, Zhongping Cao and Nuan Song "TRACKING MOVING OBJECTS IN VIDEO USING ENHANCED MEAN SHIFT AND REGION-BASED MOTION FIELD", 15th European Signal Processing Conference (EUSIPCO), Poznan, Poland, pp. 307-311, September (2007).

[3] Yilmaz, A., Javed, O. and Shah "Object tracking: A survey" ACM computing surveys, vol.38, article 13, (2006).

[4] Comaniciu, D., Ramseh,V. and Meer, P.,"Kernel-based

Table (3): The percentage before enhancement between typical and study trajectories.

.No	Typical trajectory			Study trajectory			Percentage
	X	Y	|Slope|	X	Y	|Slope|	
1	136	344	15	136	344	16	0.9375 =15/16
2	137	329	24	137	328	1	0.041666667 =1/24
3	138	305	15	138	329	5	0.333333333 =5/15
4	139	290	1.6	139	334	19	0.08421053=19/ 1.6
5	144	282	1	138	315	2	0.5=1/2
6	139	277	1.5	139	317	5.5	0.272727273=1.5/5.5
7	141	280	4	137	306	1	0.25 =1/4
8	143	272	29	138	305	14	0.48275862 =14/29
9	142	243	14	139	291	1.8	0.12857143 =1.8/14
10	143	229	8.5	144	282	1	0.11764706 =1/8.5
11	145	212	8.7	139	277	1.5	0.17241379=1.5/8.7
12	148	186	———	141	280	4	———
13	148	172	4	143	272	29	0.13793103 =4/29
14	149	176	1	142	243	14	0.07142857 =1/14
15	146	173		143	229		**Sum=** 3.530188306
Avg.= 0.252156308 \ **Percentage** = 25%							

Table (4): The percentage after enhancement between typical and enhancement trajectories.

No.	Typical trajectory			Enhancement trajectory			Percentage
	X	Y	|Slope|	X	Y	|Slope|	
1	136	344	15	136	344	15	1 =15/15
2	137	329	24	137	329	14	0.583333333=14/24
3	138	305	15	138	315	25	0.6 =15/25
4	139	290	1.6	139	290	1.6	1 =1.6/1.6
5	144	282	1	144	282	1	1 =1/1
6	139	277	1.5	139	277	1.5	1 =1.5/1.5

7	145	140	1	145	140	1	1 =1/1
8	150	135	1	150	135	1	1 =1/1
9	155	130	1.4	155	130	1.4	1 =1.4/1.4
10	160	123	0.6	160	123	0.6	1 =0.6/0.6
11	165	120	1	165	120	1	1 =1/1
12	170	115	1.5	170	115	1.5	1 =1.5/1.5
13	180	100	1.5	180	100	1.5	1 =1.5/1.5
14	200	70	1.5	200	70	1.5	1 =1.5/1.5
15	220	100	1	220	100	1	1 =1/1
16	240	120		240	120		**Sum=15**
Avg.=		\ **Percentage =** 100%					

Case 2:

Applying the proposed method on ready trajectory to prove correct of the method. the step of running a program in language C# 2010 as follows:

Step (1):

Fig. (12) illustrated the typical trajectory of the object but Fig. (13) illustrated the study trajectory of other object.

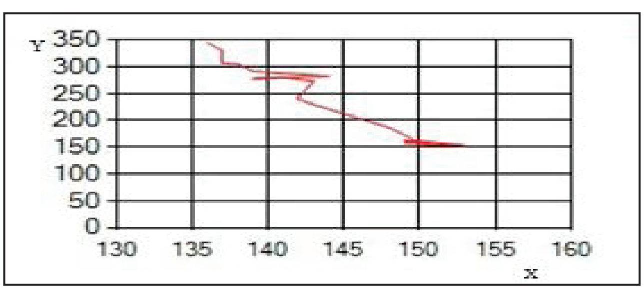

Fig. (12): The typical object trajectory

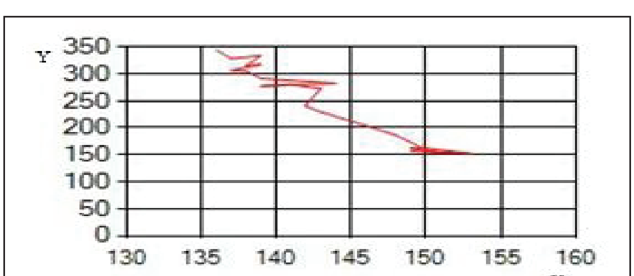

Fig. (13): The study object trajectory

Step (2):

For enhancing (approximation) the trajectory of object illustrated above in fig. (13) based on the typical trajectory of the object in fig. (12) can be apply the flowchart in Fig. (4) to obtain on the Enhancement Trajectory. Fig. (14) illustrated Enhancement Trajectory. Table (3) and Table (4) illustrate the percentage before and after the enhancement of trajectory.

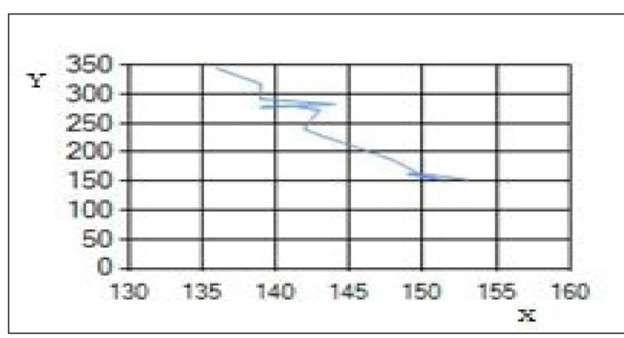

Fig. (14): Enhancement object trajectory

Table (1): The percentage before enhancement between typical and study trajectories.

No.	Typical trajectory			Study trajectory			Percentage
	X	Y	|Slope|	X	Y	|Slope|	
1	105	100	2	105	100	1.5	0.75 =1.5/2
2	110	110	1	115	115	1	1 =1/1
3	120	120	0.5	120	120	3	0.166666667 =0.5/3
4	130	125	3	130	150	0	0 =0/3
5	135	140	2	135	150	0	0 =0/2
6	140	150	2	140	150	0	0 =0/2
7	145	140	1	145	150	5	0.2 =1/5
8	150	135	1	140	125	5	0.2 =1/5
9	155	130	1.4	135	100	0.83	0.592857143=0.83/1.4
10	160	123	0.6	147	90	0.71	0.845070423=0.6/0.71
11	165	120	1	140	95	0.75	0.75 =0.75/1
12	170	115	1.5	160	80	0.25	0.166666667=0.25/1.5
13	180	100	1.5	180	75	0.25	0.166666667=0.25/1.5
14	200	70	1.5	200	70	1.5	1 =1.5/1.5
15	220	100	1	220	100	1	1 =1/1
16	240	120		240	120		**Sum**= 6.837927566
Avg.=	**\ Percentage =** 45%						

Table (2): The percentage after enhancement between typical and enhancement trajectories.

.No	Typical trajectory			Enhancement trajectory			Percentage
	X	Y	|Slope|	X	Y	|Slope|	
1	105	100	2	105	100	2	1 =2/2
2	110	110	1	110	110	1	1 =1/1
3	120	120	0.5	120	120	0.5	1 =0.5/0.5
4	130	125	3	130	125	3	1 =3/3
5	135	140	2	135	140	2	1 =2/2
6	140	150	2	140	150	2	1 =2/2

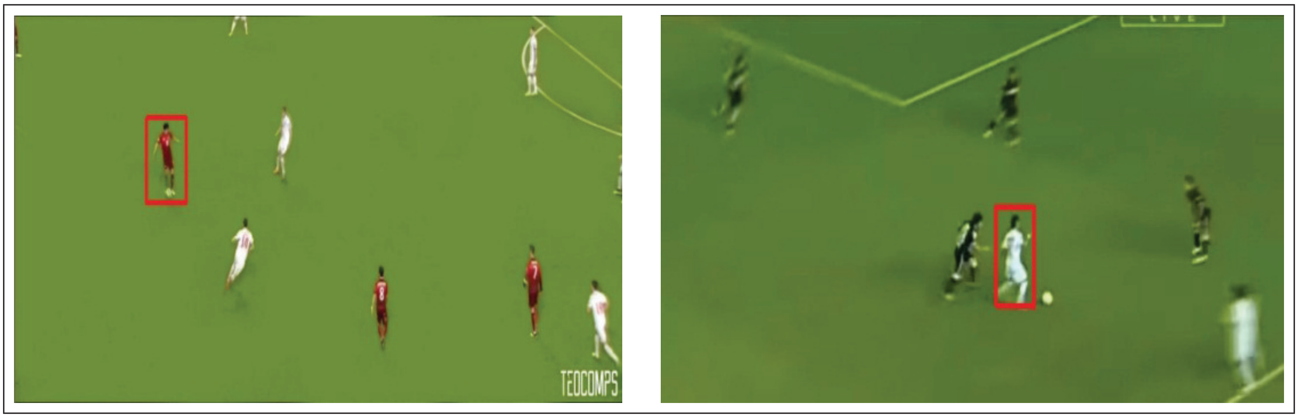

Fig. (8): Determine The Object

Step (3):

Extract the trajectory of target object by applying segmentation and using template to determine one object only in first frame as illustrated in Fig. (8) that object surrounded by red boundary, calculate center points in each frame for that object by use equation 2 and storing that points for represent object trajectory. Figs. (9 and 10): illustrated the typical and study trajectory.

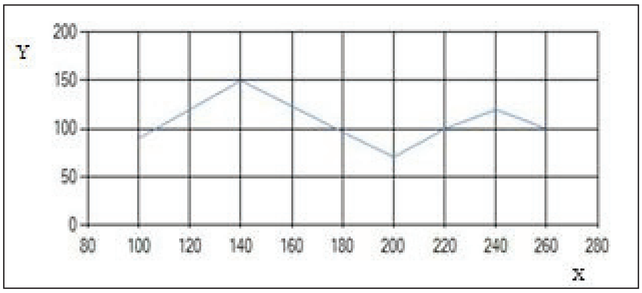

Fig. (9): The typical object trajectory

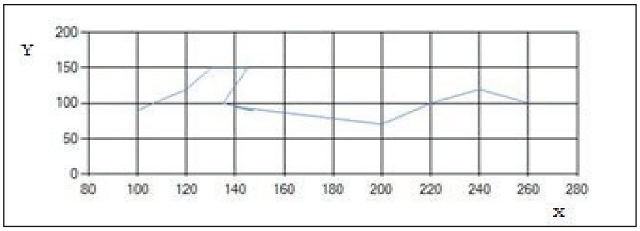

Fig. (10): The study object trajectory

Step (4):

For obtaining the enhancement object trajectory apply the flowchart in Fig. (4) to approximation the trajectory in fig. (10) from fig. (9) as illustrated in Fig. (11), that mean the result of the flowchart is enhanced to the study object trajectory in Fig. (10). Table (1) and table (2) illustrate the percentage before and after the enhancement of trajectory.

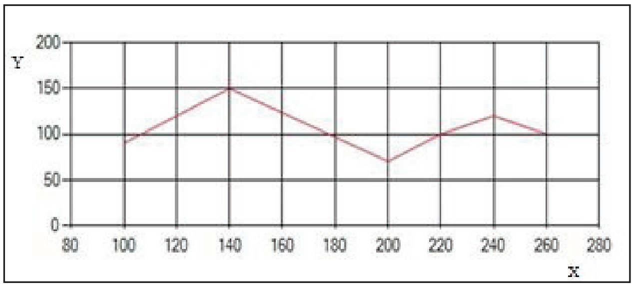

Fig. (11): Enhancement object trajectory

3. Experiments and Results

Case 1:

To test the performance of the proposed method, we take a video (.AVI) format and fractionate video into frames sequence, each frame call still image, after that extract the object trajectory and apply the enhancement method. The step of running a program in language C# 2010 as follows:

Step(1):

Display two videos (.AVI) format as illustrated in Figs. (5 and 6).

Fig. (5): View typical video

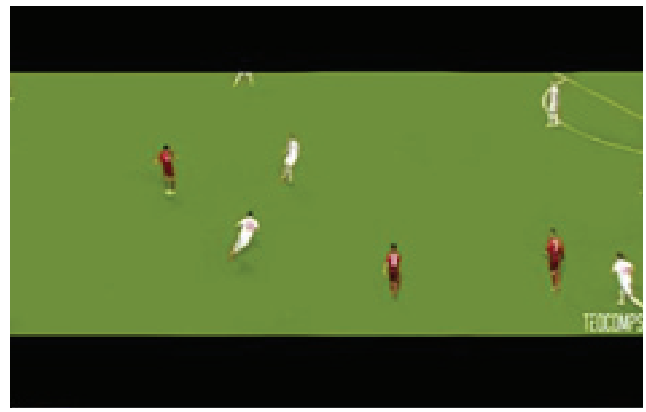

Fig. (6): View study video

Step(2):

Split the two videos into sequence frames, call Video frames as illustrated in Fig. (7).

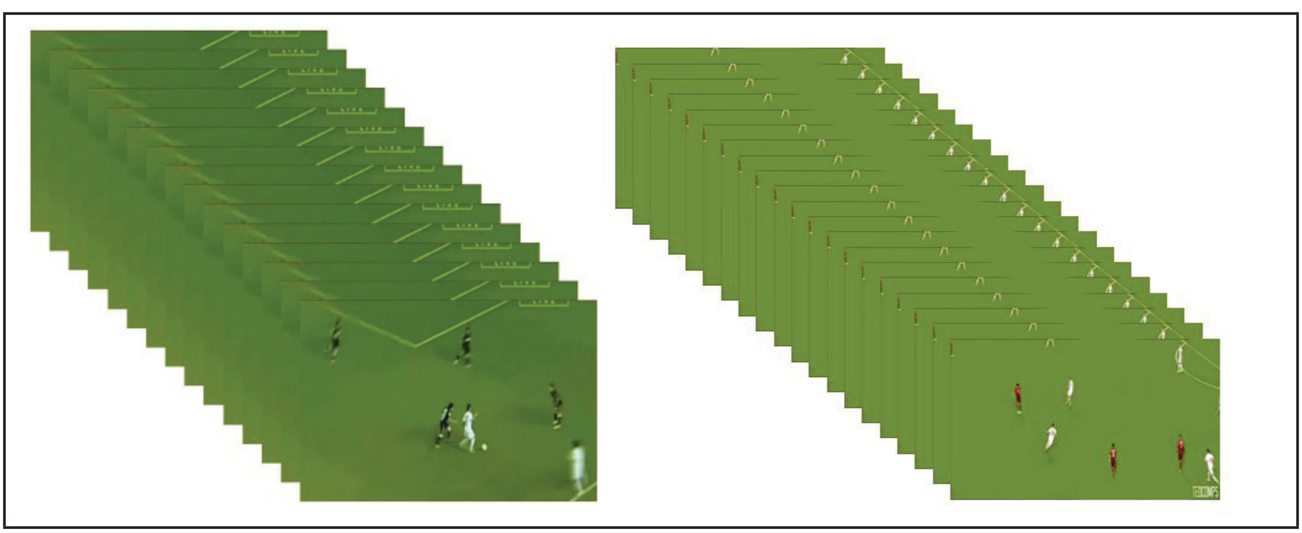

Fig. (7): Fractionate video into sequence Frames

2.4. Enhancement processing:

In this step obtain enhancement trajectory of the object based on a new method called Enhancement (approximation) method, this method take the typical trajectory (TT) of the target object in first video and studied trajectory (ST)of the certain object in second video and store the start and the end point of the TT as the new start and last point of the enhancement trajectory (EN), after that, working the scan to both of the trajectories to compute and store the distance between each point of TT and ST, after obtained the distance, first from the second distance (current distance) to compare the current distances that found between the second (current) and next point of the TT and ST of an object, if the distance of the ST is equal or greater than distance of the TT then store the current point of the ST as the new point in EN and become the next point of the ST is current point and take the next distance of the TT and ST, else summation the next distance of ST with the previous distance of ST and check if the result of the summation is equal or greater than the distance of the TT then store the next point of the ST as the new point in EN, repeat the same operation to reach to the final point, after that, Taking into account the existence of other cases, such as if the length of ST finished but not reached the length of TT to the end then in this case store all remained points of TT in ET else stop and finally draw the output trajectory (Enhancement Trajectory). Enhancement Method to obtain enhancement trajectory this trajectory may be approximate or same the typical trajectory. Showing more details in flowchart of the enhancement method in Fig. (4).

To calculate distance between each points in the typical trajectory and calculate distance between each points in the study object trajectory must work scan to the two trajectories for compute the distance between each point by using the Euclidean Distance law. The Euclidean Distance law formula can be defined as follows [16]:

$$\text{dist.} = \sqrt{(\boldsymbol{X2} - \boldsymbol{X1})^2 + (\boldsymbol{Y2} - \boldsymbol{Y1})^2} \qquad \ldots(3)$$

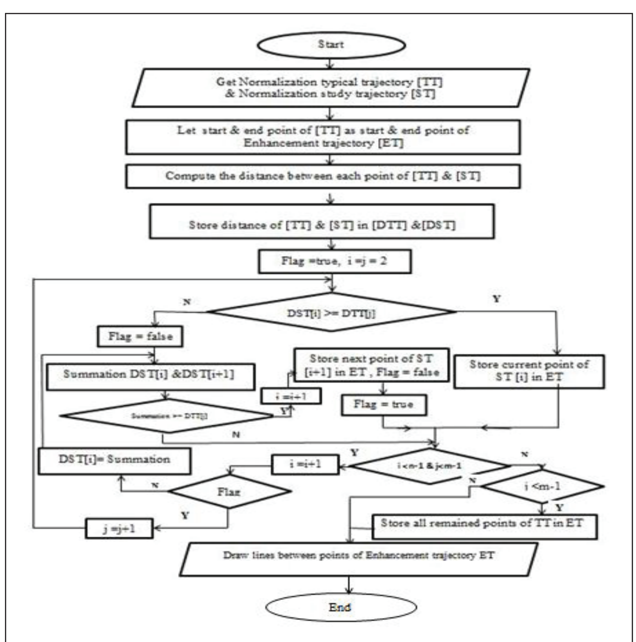

Fig. (4): Flowchart of the enhancement method

2.3. Extract typical and studied trajectories:

Tracking the object to extract trajectory of that object from the video file, must be applied the segmentation after that determine certain object only first frame, calculate the center point for that object and store the center points in all frames for representing these centers as trajectory of that object. Need two parameters X and Y to represent any center point (X, Y).

2.3.1. Segmentation:

A segmentation process in order to be able to identify the objects only in first frame and make it easier to extract features of each object such as orientation, center, perimeter, area, texture and aspect ratio. These features are a useful at the same time requires the use in the next stages. Here we use (region growing [13], the idea of Image segmentation is to find regions that refer to the all objects. The region growing method of segmentation is looking for the objects that either a degree of homogeneity that within themselves or to own much of the variance with the objects on the border. Measures could include the homogeneity and contrast features such as the gray level and color and texture. One of the stages of the growing region algorithm is collect the relevant neighboring pixels that belong to the same object.

2.3.2. Determine certain object:

Tracking target object only in the first frame, for determining specific object we use a template or ID of that object. Template (matching the template with all object found in the first frame), that template indicates that object. ID of that object determined after applied the region growing algorithm.

2.3.3. Calculate center point for that object:

For compute the center point of the object must compute the area of an object.

• Size : The area of an object can be computed using equation 1 as follows [14]:

$$A = \sum_x \sum_y O(x, y) \quad ...(1)$$

Where, O (x, y) refers to the pixel object. Area is meant to collect the total number of pixels of one object.

• Centers: The location of an object usually refers to the object's center and can be computed as the center points using equation 2 as follows [15]:

$$X_c = \left(\sum_x \sum_y (x\, O(x, y))\right)/A$$
$$Y_c = \left(\sum_x \sum_y (y\, O(x, y))\right)/A \quad ...(2)$$

Where X_c and Y_c are the centroid coordinates of an object, A refers to the object's area.

2.3.4. Store the center points in all frames as object trajectory

The last step, using center points of the target object from each frame of the typical and studied video which was calculated and save and then draw lines between those points for obtain on the trajectory. Object trajectory illustrated in Fig. (3).

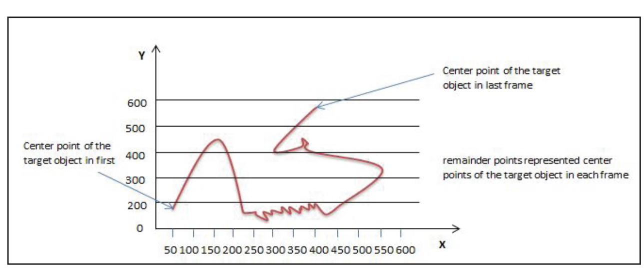

Fig. (3): Object trajectory

of motion parts are generated, and in the second step an objective function was used to select an appropriate term or label for each motion action [12].

The goal of improve the trajectory is approximation the object trajectory to typical trajectory.

In this paper the proposed method , first track object in each frame and determine an object only first frame after that from the two video is extract the two trajectories of the specific objects by extract the center of the objects in each frame that found in video in order to represent these centers as trajectories of the objects, from the first video we obtain the typical trajectory and from the second video we obtain the trajectory of the study object. The next step enhancement the second trajectory call study object trajectory of specific object based on the first trajectory call typical trajectory of specific object using the new method.

2. Proposed method

The proposed method illustrated in Figure 1 which explain the block diagram:

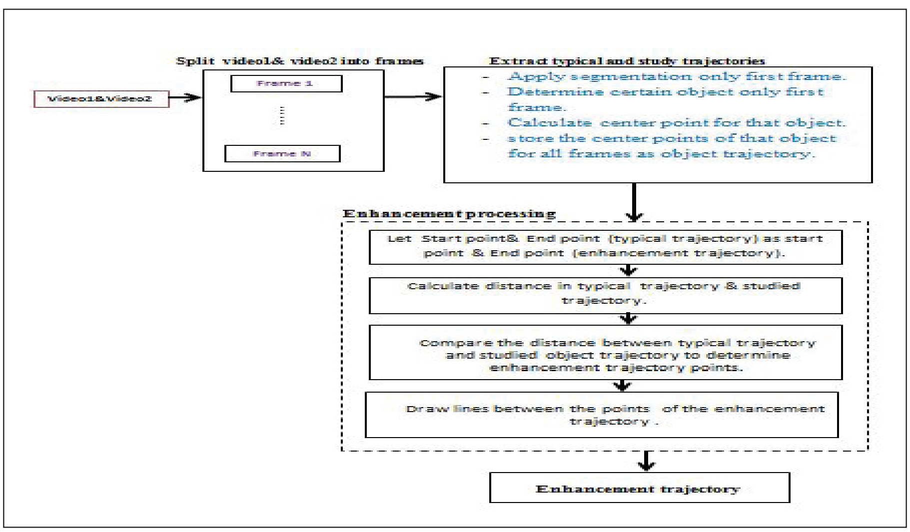

Fig. (1): Block diagram of proposed method

As we show in the above figure we can describe each step as follows:

2.1. Video1&Video:

Input video that (.AVI) extension, but if the extension of a video is not (AVI), then you must convert the video from any format to (AVI) format.

2.2. Split the video1&video2 into frames:

Convert the video with (.AVI) extension into a series of frames (still image type is BMP), video frames contain sequences of still images, Fig. (2). illustrates the Video frames.

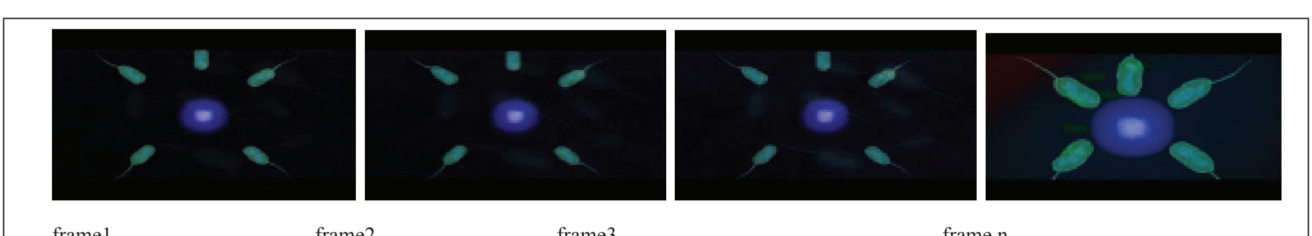

Fig. (2): Video frames

1. Introduction:

Videos are in fact the sequence of images, each of which is called the frame, displayed in a fast-paced enough so that the human eye can be aware of the continuity of its content. It is clear that all image processing techniques can be applied to individual frames. Besides, the contents of the consecutive two frames are usually closely related [1], while there has been a growing interest in object detection and tracking video due to, for example, applications of multimedia and virtual reality and video surveillance [2].

Tracking the trajectory of the object is an important topic in computer vision. In the video images tracking a moving object taken a significant amount of interest in computer vision, establishment correspondence between objects or parts of object in sequence frames and then extracts temporal information related to objects such as track, position, speed and so on are the goal of the object detection and tracking. a great and difficult task is detection the track of the objects in frame by frame in special video.

In video tracking, tracking objects can be complex due to: loss of information caused by projection of the 3D world on a 2D image, noise in images, complex object motion, articulated nature of objects, partial and full object occlusions, complex object shapes, scene illumination changes, and real-time processing requirements [3].

Selecting the right features plays a critical role in video tracking. In general, the most desirable property of a visual feature is its uniqueness so that the objects can be easily distinguished in the feature space. Feature selection is closely related to the object representation. For example, color is used as a feature for histogram-based appearance representations, while for contour-based representation, object edges are usually used as features. In general, many tracking algorithms use a combination of these features.

Usually, tracking with the passage of time made up of identical mobile objects in a series of frames. Categories of tracking (point tracking, kernel tracking [4], shapes matching [3], and silhouette tracking [5].

Once we can simplify object tracking by placing restrictions on the movement and/or the appearance of the object, for example, tracing algorithm almost all assume that the movement of the object is smooth with sudden changes. The problem can be simplified by having a prior knowledge about the number and the size of objects, or the object's appearance and shape [6] [7].

Method for detection in addition to tracking of various moving objects under static background, such method is derived from motion detection, background subtraction using efficient edge detection such as canny method through edge difference among consecutives frames [8].

The algorithm block matching using motion vector predictive tracking the object has been suggested. Color histogram is used to similar standards for motion tracking [9].

To improve the trajectory of object movement tracking there are many methods for example Douglas-Peucker [10] and Voronoi Fast Marching Method (VFM) [11], etc.

Bingbing Ni et al., in 2015 presented "Motion Part Regularization: Improving Action Recognition via Trajectory Group Selection" recognized and labeled an action using local features of a trajectory which are related to motion parts, in the first step candidates

Abstract

In this research, a new framework is proposed to enhance the studied trajectory of certain object based on typical trajectory by using a specific algorithm. The proposed method consists of two stages, the first stage, extract the two trajectories of the specific objects from the two videos, by take the center points of the target object in each frame of typical video to represent these center points as a typical trajectory of the target object. And taking the center points of the other object in each frame of studied video to represent these center points as studied trajectory of the object. The second stage enhancement method, this method shows a good result in the aspect of enhancement to approximate the studied trajectory based on a typical trajectory. Consequently, the enhancement trajectory will be obtained, this trajectory may be either approximate or the same typical trajectory. The percentage of enhancement increased from (25%- 45%) to percentage between (87%- 100%).

Keywords

video tracking, segmentation, enhancement trajectory, studied trajectory.