A Three-phase Test Circuit Design for High Voltage Circuit Breaker Based on Modeling

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الخلاصة

تعتمد أنظمة الطاقة الكهربائية الحديثة في أدائها الى حداً كبير على عمل قواطع الدورة الكهربائية. يستخدم قاطع الدورة الكهربائية في أكتشاف اضطرابات الشبكة الكهربائية و لحماية الاجهزة الحساسة و المعدات غالية الثمن مثل المولدات و المحولات و غيرها من الاجهزة. لذا فأنها يجب ان تعمل ضمن سماحية ضيقة جداً خصوصاً في الشبكة الكهربائية التي تعمل تحت شروط خطاء دائرة القصر . ان تقييم كفاءة عمل قاطع الدورة امر مهم لأثبات قدرته على ايقاف تيارات الخطأ، خصوصاً تيارات دائرة القصر و لتحسين موثوقية الشبكة. تهدف هذه الورقة الى تصميم دائرة اختبار ثلاثية الطور تستعمل لتقييم اداء قاطع الدورة ذات الفولتية العالية تحت شرط خطاء دائرة القصر بأستخدام المحاكاة. بهذه الطريقة سيتم التغلب على صعوبات الاختبارات العملية كونها لا تحتاج الى قدرة كهربائية عالي من مصادر حقيقية و لها مرونة غير محدودة لضبط قيم عناصر دائرة الاختبار و غير خطرة و اقتصادية.

الكلمات المفتاحية

تيار الدائرة القصيرة، تيار القطع، نظام التشغيل، تيار الحقن، الحقن.

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which is true for any ϵ.

Therefore

$$\omega_{|v|}(f,\frac{1}{n})_p \le c(p)\frac{1}{n^{|v|}}\left\{\sum_{k=1}^{n} k\left(d_p(f,R_k^\sigma(d))\right) + \|f\|_p\right\}$$

To know the number of neurons in the hidden layer, we must choose n smallest integer larger than ϵ^{-1} . So we must choose $m_n \ge \min_{(c<\epsilon)} n^d$, where

$$c = c(p)\omega_v(f,\frac{1}{n})_p \qquad \square$$

3.2. Theorem let K be a compact subset of R^d and $f \in L_p(K)$. Then, $d_p(f,R_n^\sigma(d) = O(n^{-\alpha})$,if and only if $f \in Lip(\alpha)$, where $Lip(\alpha)=\{f:\omega_r(f,t) = O(t^\alpha, \alpha \in (0,r]\}$

Proof: first let us assume $d_p(f,R_n^\sigma(d))=O(n^{-\alpha})$. From Theorem3.1 we have

$$\omega_r(f,\frac{1}{n})_p \le c(p)\frac{1}{n^r}(\sum_{k=1}^{n} kd_p(f,R_k^\sigma(d)) + \|f\|_p)$$

$$\le c(p)\frac{1}{n^r}(\sum_{k=1}^{n} k\frac{1}{n^\alpha} + \|f\|_p)$$

$$= c(p)\frac{1}{n^r}(\frac{n(n+1)}{2}\frac{1}{n^\alpha} + \|f\|_p)$$

$$= c(p)\frac{1}{n^r}(\frac{1}{n^{\alpha-2}} + \|f\|_p$$

$$\le c(p)(\frac{1}{n^{2\alpha-2}} + \frac{1}{n^\alpha})$$

$$\le c(p)\frac{1}{n^\alpha}.$$

Now for the opposite side we have, for $f\in Lip(\alpha)$, that

$\omega_r(f,\frac{1}{n})_p = O(\frac{1}{n^\alpha})$ .Using Theorem2.3 to have

$$d_p(f,R_n^\sigma(d)) \le c(p,d)\omega_r(f,\frac{1}{n})_p$$

$$\le c(p,d)\frac{1}{n^\alpha} \qquad \square$$

3.3. Theorem let K be a compact subset of R^d and $\in L_p(K)$. if

$$d_p(f,R_n^\sigma(d)) \le (1+\frac{1}{n})^2 d_p(f,R_{n+1}^\sigma(d))$$

Then

$$\omega_r(f,\frac{1}{n})_p \le c(p)\{d_p(f,R_n^\sigma(d)) + \frac{1}{n^2}\|f\|_p\}.$$

And

$$\omega_r(f,\frac{1}{n})_p \le c(p)\frac{1}{n^2}\|f\|_p \le d_p(f,R_n^\sigma(d))$$

$$\le \left(\frac{1}{2}+\frac{\pi^2}{4}\sqrt{d}\right)\omega_r(f,\frac{1}{n})_p.$$

Proof:

Using Theorem 1.3, we have

$$\omega_r(f,\frac{1}{n})_p \le c(p)\frac{1}{n^2}(\textstyle\sum_{k=1}^{n} kd_p(f,R_k^\sigma(d)) + \|f\|_p).$$

Then using proposition2.4 with

$A_n=\omega_r(f,1/n)_p$ and $B_k=d_p(f,R_k^\sigma(d))$ and $E=\|f\|_p$ we get

$$\omega_r(f,\frac{1}{n})_p \le c(d_p(f,R_n^\sigma(d)) + n^{-2}\|f\|_p) \le c(p)(d_p(f,R_n^\sigma(d)) + n^{-2}\|f\|_p)$$

$$\le c(p)((1+1/n)\, d_p(f,R_{n+1}^\sigma(d))+n^{-2}\|f\|_p) .$$

This completes the proof $\square$

3.4. Theorem If K is a compact subset of R^d and

$$\omega_r(f,\frac{1}{n})_p \le c(p)n^{2s-r}d_p(f,R_1^\sigma(d)) + c(p)n^{r-s}d_p\left(f,R_{[n^\delta]}^\sigma(d)\right) + c(p)\frac{1}{n^r}\|f\|_p$$

Proof . In Theorem 3.2 we have

$$\omega_r\left(f,\frac{1}{n}\right)_p \le c(p)\frac{1}{n^r}(\sum_{k=1}^{n} k\, d_p(f,R_k^\sigma(d)) + \|f\|_p)$$

$$= c(p)\frac{1}{n^r}(\textstyle\sum_{k=1}^{[n^\delta]-1} k\, d_p(f,R_k^\sigma(d)) + \sum_{k=[n^\delta]}^{n} k\, d_p(f,R_k^\sigma(d)) + \|f\|_p)$$

$$\le c(p)\frac{1}{n^r}(d_p(f,R_1^\sigma(d))\sum_{k=1}^{[n^\delta]-1} k + d_p\left(f,R_{[n^\delta]}^\sigma(d)\right)\sum_{k=[n^\delta]}^{n} k + \|f\|_p)$$

$$= c(p)(d_p(f,R_1^\sigma(d))\frac{1}{n^r}\frac{([n^\delta]-1)[n^\delta]}{2} + d_p\left(f,R_{[n^\delta]}^\sigma(d)\right)\frac{1}{n^r}\frac{n(n+1)}{2} + \frac{1}{n^r}\|f\|_p$$

$$\le c(p)n^{2s-r}d_p(f,R_1^\sigma(d)) + c(p)n^{r-s}d_p\left(f,R_{[n^\delta]}^\sigma(d)\right) + c(p)\frac{1}{n^r}\|f\|_p$$

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$$d_p(f, R_n^\sigma(d)) \le c(p,d)\omega_r(f, \tfrac{1}{n})_p.$$

2.4. Proposition [4] assume that for the nonnegative sequences $\{a_n\}$, $\{b_n\}$, satisfied $b_k \le (1+\frac{1}{k})^p b_{k+1}$, and the inequality

$$a_n \le Cn^{-2}\{\sum_{k=1}^{n} kb_k + \in\}$$

Holds for n∈N, then one has

$a_n \le C(b_n + n^{-2}\in)$.

Here C≥1 is a constant and ∈ is a constant independent of n, k

3. The main results

In this article we introduce our main results

3.1. Theorem let K be a compact subset of R^d and $f\in L_p(K)$. Then there is a nearly exponential type of forward neural network with hidden components number $m_n \ge \min_{(c<\in)} n^d$, where $c = c(p)\omega_v(f,\frac{1}{n})_p$. And ∈ N , such that

$$\omega_v(f,\frac{1}{n})_p \le c(p)\frac{1}{n^v}(\sum_{k=1}^{n} k.\, d_p(f, R_k^\sigma(d)) + \|f\|_p)$$

Proof we have

$$\|L(V_n,f)\|_p = \left\|(2\pi)^{-d}\int_{[-\pi,\pi]^d} f(x-t)V_n(t)dt\right\|_p$$

$$= (\int_{[-\pi,\pi]^d}\left|(2\pi)^{-d}\int_{[-\pi,\pi]^d} f(x-t)V_n(t)dt\right|^p dx)^{\frac{1}{p}}$$

$$\le c(p)\|f\|_p\left(\frac{1}{(2\pi)^d}\int_{[-\pi,\pi]^d} V_n(t)dt\right) = c(p)\|f\|_p$$

Therefore

$$\left\|D^{|v|}L(V_n,f)\right\|_p \le c(p)\|D^v f\|_p \quad (1)$$

Then using Bernstein inequality we get

$$\left\|D^{|v|}L(V_n,f)\right\|_p \le c(p)n^{|v|}\|f\|_p \quad (2)$$

Now let, $A_n = \frac{1}{n^{|v|}}\left\|D^{|v|}L(V_n,f)\right\|_p$, $B_n = \|L(V_n,f)-f\|_p$. Then using (1)and(2) to get

$$A_n = \frac{1}{n^{|v|}}\left\|D^{|v|}L(V_n,f)\right\|_p \text{for } n > k \ge 1$$,

we have

$$A_n = \frac{1}{n^{|v|}}\left\|D^{|v|}\, L(V_n, L(V_k,f) - L(V_k,f) + f)\right\|_p$$

$$A_n \le c(p)\frac{1}{n^{|v|}}(\left\|D^{|V|}L(V_n, L(V_k,f)\right\|_p + \left\|D^{|v|}L(V_n, f - L(V_k,f)\right\|_p)$$

$$\le c(p)\frac{1}{n^{|v|}}\left\|D^{|v|}L(V_k,f)\right\|_p + c(p)\frac{1}{n^{|v|}}n^{|v|} \le c(p)\left(\frac{k}{n}\right)^{|v|}A_k + c(p)B_k$$

Then for p=|v| in Lemma2.1, we get

$$A_n \le c(p)n^{-|v|}(\textstyle\sum_{k=1}^{n} k^{|v|-1}B_k + A_1),$$

and

$$\left\|D^{|v|}L(V_n,f)\right\|_p \le c(p)n^{-|v|}(\sum_{k=1}^{n} k^{|v|-1}\|L(V_k,f)-f\|_p + \|f\|_p)$$

Then for n≥|v|, there is a natural number m satisfy n / |v| ≤ m ≤ n.

Then

$$\|f - L(V_m,f)\|_p \le \|f - L(V_k,f)\|_p \qquad \frac{n}{|v|} \le k \le n.$$

Then using definition of K-functional to obtain

$$K_{|v|}(f, t^{|v|}) = \inf_{D^{|m|}g\in L_p^{|m|}}\left\{\|f-g\|_p + t^{|v|}\sup_{|m|=|v|}\left\|D^{|m|}g\right\|_p\right\},$$

and

$$K_{|v|}(f,\frac{1}{n^{|v|}})_p \le \|f - L(V_m,f)\|_p + \frac{1}{n^{|v|}}$$

$$\le \frac{c(p)}{n^{|v|}}\sum_{\frac{n}{|v|}\le k\le n} k\|f - L(V_n,f)\|_p$$

$$+ \frac{c(p)}{n^{|v|}}\left(\sum_{k=1}^{n} k^{|v|-1}\|L(V_k,f)-f\|_p + \|f\|_p\right)$$

$$\le \frac{c(p)}{n^{|v|}}\left(\sum_{k=1}^{n} k\|L(V_k,f)-f\|_p + \|f\|_p\right)$$

$$\le \frac{c(p)}{n^{|v|}}\left(\sum_{k=1}^{n} k\, d_p(f, P_k(d)) + \|f\|_p\right)$$

Then using Theorem2.2 to obtain

$$\omega_{|v|}(f,\tfrac{1}{n})_p \le c(p)\tfrac{1}{n^{|v|}}\{\textstyle\sum_{k=1}^{n} k\big(d_p(f,R_k^\sigma(d)) + \epsilon\big) + \|f\|_p\}$$

1. Introduction

In [3,5,6], the authors proved inverse theorems for the approximation by neural networks of continuous functions on R^d using the 1st order modulus of continuity. There is a natural question can we improve the above estimates interms of the k th order modulus of smoothness for k variate functions in L_p spaces for <1 ? in this article we answer this question.

Let N be the set of nonnegative integers numbers, R^dbe the d-dimensional Euclidean space (d≥1), $x=(x_1,x_2,\ldots,x_3)\in R^d$, R_n^σ (d)the set of all polynomials of the form

$\sum_{\lambda\in l(N\cup\{0\})^d} a_\lambda\, \sigma(-\lambda x+b_\lambda)(l>0)$.

, σ:R→R, and let|k|th order partial derivatives of fas

$$(d \geq 1), x = (x_1, x_2, \ldots, x_3) \in [2]$$

A Korovkin's kernel $u_n(x)$, defined by

$$\sum_{\lambda\in l(N\cup\{\cdot\})^d} a_\lambda \sigma(-\lambda x + b_\lambda)(l > \cdot),$$

where $u_n(x) \in T_n(1)$, $u_n \geq 0$ and $1/2\pi \int_{-\pi)}^{\pi} u_n(x)\,dx=1$, where $T_n(1)$ is the space of all triangular trigonometric polynomials of degree less than n, $t_n(x)$ =arc cos(nx), $t_n(x)$ is called Chebyshev polynomial. Define the d-product of $u_n(x)$ as follows

$$V_n(x_1,x_2,\ldots,x_d) = \overbrace{u_n(x)\times u_n(x)\times \ldots \times u_n(x)}^{d\ times} \epsilon T_n(d)$$

, also $V_n \geq 0$, $(2\pi)^{-d}\int_{(-\pi,\pi)^d} V_n(x)dx = 1$. [4]

We can define the K -functional as follows :

$$K_r(f,t^r) = \inf_{D^{|m|}g\in A.C.loc}\left\{\|f-g\| + t^r \sup_{|m|=r}\left\|D^{|m|}g\right\|\right\}$$

where g∈A.C.loc means that gis |m| times differentiable and $D^{|m|}$ g is continuous in the finite set [5]. Bernstein inequality can be written as

$$\left\|P_n^k\right\|_p \leq c(p)n^k\|P_n\|_p.$$

1.1. Definition [4] let Q be metric space with metric d then if $f \in L_p(Q)$, given a direction $e\in R^d$, the rth order Symmetric difference of f defined by

$$\Delta_h^r f(x) = \sum_{i=0}^{r}(-1)^{r-i}\binom{r}{i} f\left(x+\left(\frac{r}{2}-i\right)he\right)$$

and ,the rth modulus of smoothness of a function f have the form

$$\omega_r(f,t)_p = \sup_{x\pm\frac{he}{2}\in Q,|h|\leq t} \|\Delta_h^r f(x)\|_p.$$

2. Auxilary results

In this section we shall introduce some results that we need in our proof of the main result.

2.1. Lemma [7] a positive sequences $\{a_n\}$, $\{b_n\}$, if (p>0), and

$$a_n \leq \left(\frac{k}{n}\right)^p a_k + b_k (1 \leq k \leq n)\forall n \in \mathbb{N} \quad (1)$$

Then

$$a_n \leq C_p n^{-p}\{\textstyle\sum_{k=1}^n k^{p-1}b_k + a_1\}\,. (2)$$

2.2. Theorem [7] If $f\in L_p(R^d)$,

$$K_r(f,t^r)_p = \inf_{D^{|m|}g\in L_p^{(m)}} \|f-g\|_p + \underset{|m|=r}{t^r}\left\|D^{(m)}g\right\|_p,$$

Then

$$c(p)K_r(f,t)_p^r \leq \omega_r(f,t)_p \leq c(p)K_r(f,t^r)_p,$$

where c(p) is a positive constant depending on p, and it may different from one line to other.

2.3. Theorem [1] Let $f\in L_p([0,1]^d)$and n∈N, then there is a nearly exponential type of forward neural networks , and let R_n^σ (d)as defined above, its number of hidden layer components is

$M_n \geq \min_{C<\varepsilon}(n+1)^d$,

(where $C = c(p,d)\omega(f,\frac{1}{n})_p$), n is any integer satisfy

Abstract

In this paper we introduce a lower bound estimates for approximation by neural networks in L_p spaces for p<1.

Keywords

Neural networks, modulus of smoothness, direct theorem.