AL BAHIR

Quarterly Refereed Journal
for Natural and Engineering Sciences

Issued by
Al-`Abbas Holy Shrine
International Al-`Ameed Centre for Research and Studies

Licensed by
Ministry of Higher Education
and Scientific Research

Third Year, Fifth Volume, Issue 9 and 10
Ramadhan, 1438, June 2017

Secretariat General
of Al-'Abbas
Holy Shrine

Al-Ameed Interna-
tional center
for Research and Studies

Print ISSN: 5721 – 2312
Online ISSN: 0083 – 2313
Consignment Number in the Housebook and Iraqi
Documents: 1996, 2014
Iraq - Holy Karbala

Mobile: +964 760 235 5555
+964 771 948 7257
http://albahir.alkafeel.net
Email: albahir@alkafeel.net

Publication Conditions

Inasmuch as Al-`Bahir- effulgent- Abualfadhal Al-`Abbas cradles his adherents from all humankind, verily Al-Bahir journal does all the original scientific research under the conditions below:

1. Publishing the original scientific research in the various scientific sciences keeping pace with the scientific research procedures and the global common standards; they should be written either in Arabic or English .
2. The research should not be published before under any means .
3. The research should adhere the academic commonalties; the first page maintains the title, researcher name /names, address, mobile number under condition that the name, or a hint , should never be mentioned in the context and keywords should be written in Arabic and English as there is an abstract in Arabic and English.
4. The Research studies should be delivered to us either via Journal website http://albahir.alkafeel.net , after filling the two standard format the first with the name of the researcher and the second without in Word .
5. The page layout should be (2)cm .
6. The font should be of (16 bold),Time New Roman, subtitles of (14 bold) and also the context.
7. The space should be single, indentation should not be, as 0 before, 0 after and no spacing, as 0 before, 0 after.
8. There should be no decoration and the English numeral should proceed to the last text.
9. Any number should be between two brackets and then measurement unit, for instance: (12) cm .
10. All sources and references should be mentioned at the end of the article and categorized in conformity with Modern Language Association (MLA) , for instance :

Name of Author/ Authors, Journal Name Volume Number (Year) pages from - to.

Similarly done in the Arabic article withy a proviso that superscript should be employed.

11. There should be a caption under a diagram in 10 dark , Arabic and English; for instance:

Title or explanation; number of the Fig.
Similarly done with tables.
12. Diagrams , photos and statics should be in colour with high resolution without scanning.
13. The marginal notes, when necessary, should be mentioned at the end of the article before the references.
14. Wherever there is the word " figure" should be abbreviated as Fig. and table should be Table.
15. The pages never exceed 25 pages.
16. The Formulae should be written in Math Type.
17. All the ideas and thoughts reveal the mindset of the researcher not the journal and the article stratification takes technical standards.
18. All the articles are subject to :
a- The researcher is notified that his paper is received within 14 days in maximum.
b- The article is to be sent to the researcher as soon as it does not meet the requirement of the publication conditions.
c- The researcher is notified that his article is accepted.
d- The articles need certain modification , as the reviewers state, are sent to the researchers to respond in a span of a month from the date of dispatch.
e- The researcher is to be notified in case the article is rejected.
f- The researcher is to be granted an edition containing his article.
19. Priorities are given in concordance to :
a. The articles participated in the conferences held by the publication institute.
b. The date of receiving.
c. The date of acceptance.
d. The importance and originality of the article.
e. The diversity of the fields the articles maintain in the meant edition.
20. The researchers should appeal to the modifications the language and scientific reviewers find in the articles.
21. The researcher should fill the promise paper having the publication rights of the Scientific Al-Bahir Journal and adhering to integrity conditions in writing a research study.

In the Name of Allah
Most Compassionate, Mort Merciful

Edition Word

O Allah, my Lord

Cast felicity in me , facilitate my cause and unknot my tongue to perceive my speech , thanks be upon Him the Evolver of the universe and peace be upon Mohammad and his immaculate and benevolent progeny .

A fledged edition of Al-Bahr , peer reviewed scientific journal, embraces a constellation of research studies pertinent to engineering and natural sciences we do hope to overlap a scientific gap the specialists observe as an academic phenomenon worth being under the lenses of the researchers, that is why there is diversity in the studies to meet the requirements of the journal readership . For the journal, now, comes to the fore , at the efforts of the editorial and advisory boards and the researchers who strain every sinew to publish in Al-Bahr, to be global as to be published in an international publishing house in line with the global scientific journals.

On such an occasion we do pledge the promise of fealty and loyalty to those who observe our issues with love and heed in the International Al-`Ameed for Research and Studies , Department of Cultural and Intellectual Affairs in the Holy Al-`Abbas Shrine and the strenuous endeavour to cull whatever invigorates the scientific interaction and academic research in Iraq and worldwide to create a new generation keeping pace with the development of the current scientific phase and to lay the hands of the researchers, nationwide and worldwide, upon the desired missions.

Thanks be upon Him ,the Evolver ad infinitum .

Consultation and Editaril

Fully Stable Semimodules

Asaad Mohammad Ali Husain, Haider Abbas Abdul AL-Ameer
Department of Mathematics, College of education for pure science, University of Babylon, Iraq

Received Date: 6 / 4 / 2015
Accepted Date: 18 / 7 / 2016

الخلاصة

نقدم في هذا العمل مفهوم شبه الموديول الجزئي المستقر وشبه الموديول تام الاستقرارية وندرس الشروط التي نحتاجها لنحصل على خصائص وصفات مشابهه كما في الموديولات.

الكلمات المفتاحية

شبه الموديول الجزئي الدوري، شبه الموديول الجزئي المستقر، شبه الموديول تام الاستقرارية.

Abstract

In this work, the concept of stable sub semi module and fully stable semi module will be introduced and studied, investigating the conditions which need to get properties and characterizations similar or related to the case in modules.

Keywords

Cyclic sub semi module, Stable sub semi module, Fully stable semi module.

1. Introduction and Preliminaries

In this work the concept of fully stable module, that was introduced and studied in [1], will be converted to semi modules, investigating characterizations, properties and examples. Let M be an R-module, and N be a sub module of M, then N is called invariant if $f(N) \subseteq N$, for each R-endomorphism f of M [2]. A sub module N of an R-module M is called stable if $g(N) \subseteq N$ for each R-homomorphism g of N into M, and M is said to be fully stable if each sub module of M is stable. A ring R is called fully stable if it is fully stable R-module [1].

A semi ring is a set R together with two binary operations, addition and multiplication, such that

(i) addition and multiplication are associative,

(ii) addition is commutative,

(iii) the distribution law holds, that is, if r, s, t $\in R$ then r(s + t) = rs + rt and (r + s)t = rt + st,

(iv) there is an additive identity element (denoted 0) and a multiplicative identity element (denoted 1),

(v) 0 . r = r . 0 = 0 for all r $\in R$.

The semi ring R is said to be commutative if its multiplication is commutative [3]. A (left) semi module M over a semi ring R is a commutative additive semi group which has a zero element, together with a mapping from $R \times M$ into M (sending (r, m) to rm) such that(r + s)m = rm + sm, r(m + p) = rm + rp, r(sm) = (rs)m and $0m = r0_M = 0_M$ for all m, p $\in M$ and r, s $\in R$,[4]. A subset N of the R-semi module M is called a sub semi module of M if a, b $\in N$ and r $\in R$ implies that a + b $\in N$ and ra $\in N$ [5], in this case N itself is an R-semi module. The concepts of homomorphism, kernel, image are defined similar to the case in modules.

It is known that a module is duo if each of its sub modules is invariant, that is, for each sub module N of M and for each endomorphism f of M, it follows that $f(N) \subseteq N$ [6]. Analogously, duo semi module can be defined. A semi module M is said to be duo if each sub semi module of M is invariant. A left R-semi module is said to be simple if it has no non-zero proper sub semi modules [7]. A semi module M is said to be semi simple if it is a direct sum of its simple sub semi modules [8]. A semi module M is cancellable if for all m, m', m'' $\in M$, m + m' = m + m'' $\Rightarrow$ m' = m'' [9]. If M is a left R-semi module then its left annihilator is $L_R(M) = \{r \in R: rm = 0$ for every element $m \in M\}$ [10]. Let N be a sub semi module of an R-semi module M, then $(N : M)$ is defined as

$(N : M) = \{ r \in R | rM \subseteq N\}$. Clearly $(N : M)$ is an ideal of R. The annihilator of M is defined as $(0 : M)$ and is denoted by $ann_R(M)$, too [5]. A left R-semi module M is called cyclic if M can be generated by a single element, that is $M = (m) = Rm = \{rm \mid r \in R\}$ for some $m \in M$ [11]. A semi ring R is called a regular semi ring if for each a $\in R$, there exists an element r $\in R$ such that a = ara [12]. An element r in a semi ring R is said to be nilpotent if there exists apositive integer n (depending on r), such that $r^n = 0$ for r $\in R$ [13].

2. Fully Stable Semi module

In what follows, R will stand for a semi ring with zero and identity.

2.1. Definition:

Let M be a semi module over a semi ring R. A sub semi module N of M is said to be stable if $f(N) \subseteq N$ for each homomorphism $f: N \rightarrow M$.

M is called fully stable if each sub semi module N of M is stable. We note that the semi ring is called fully stable if it is a fully stable semi module over itself.

2.2. Definition:

A semi module M is said to be duo if each sub semi module of M is invariant.

2.3. Remark:

Any fully stable semi module is duo.

Proof: Assume that M is fully stable, let N be a sub semi module of M and $f: M \rightarrow M$ be any homomorphism. Consider $g = f|N : N \rightarrow M$, by assumption $g(N) \subseteq N$, it follows that $f(N) \subseteq N$, that is N is invariant, thus M is duo.

But the converse is not true. For example, let $M = Z^+$ be a semi module over R, where $R = (Z^+, +, \cdot)$ is a commutative semi ring, then any sub semi module of M is of the form $Rn = \{ rn \mid r \in R \}$,with n $\in M$, let $f: M \rightarrow M$ be a homomorphism, then $f(Rn) = Rf(n) \subseteq Rn$, then Rn is invariant, hence M is duo. Now let f: $R2 \rightarrow M$, defined by $f(r2) = r3$, then $f(r_1 2 + r_2 2) = f((r_1 + r_2)2) = (r_1 + r_2)3 = r_1 3 + r_2 3 = f(r_1 2) + f(r_2 2)$.

$f(s(r2)) = f((sr)2) = (sr)3 = s(r3) = sf(r2)$, where $r_1, r_2, s, r \in R$. It follows that f is a homomorphism. Note that $f(2) = 3 \notin R2$, then $R2$ is not stable. Therefore, M is not fully stable.

2.4. Remark:

If R is a semi ring and $_RM$ is a duo semi module over R, then, for each endomorphism f of M, and for each x $\in M$, there exists r $\in R$ such that $f(x) = rx$.

Proof: Since $f(Rx) \subseteq Rx$, then $f(x) \in Rx$, hence $f(x) = rx$ for some r $\in R$. □

2.5. Examples and Remarks:

(a) Consider the semi ring $R = (N, +, \cdot)$, where $x + y = \max\{x, y\}$, $x \cdot y = \min\{x, y\}$, $\forall$ x , y $\in N$, and let A be the left semi module R over itself, then the proper sub semi modules of A are of the form $(I_n, +) = \{1, 2, \dots, n\} \subseteq A$, as we will be shown in the following :

$(I_n, +)$ is a commutative semi group, and for each r $\in R$ and m $\in I_n$, if $r \leq m$, it follows that $r \cdot m = r \in I_n$, if $r > m$, then $r \cdot m = m \in I_n$, thus I_n are sub semi modules of A for each n.

Now assume that J is any proper sub semi module of A, then:

Case1:

J has no greatest element, then $\forall$ r $\in R$, there exists b $\in J$ such that b > r, then r · b = min{r, b} = r $\in J$ which is a contradiction, that is $R \subseteq J$, which implies $J = A$, (not possible).

Case2:

If J has a greatest element say n, then $\forall$ r $\in R$, if r < n, then r · n = min{r, n} = r $\in J$, that is $J = \{1,2,\ldots,n\} = I_n$.

To prove A is a fully stable semi module, let $f: I_n \rightarrow A$ be any homomorphism, and m $\in I_n$. Assume that f(m) $\notin I_n$, which means f(m) > n, let r $\in R$ such that m ≤ r < n, then f(r · m) = f(m) > n, while r · f(m) = r < n which is a contradiction. Therefore f(m) $\in I_n$, $\forall$ m $\in I_n$. (i.e.) $f(I_n) \subseteq I_n$, then A is a fully stable.

(b) Any simple semi module is fully stable (trivial).

(c) The concepts of semi simple semi module and fully stable semi module are independent.

(d) The semi module in the example of Remark (2.3.) is not fully stable.

2.6. Lemma:

If M is a cancellable semi module with zero and f is an endomorphism of M, then $f(0) = 0$.

Proof: $0 + f(0) = f(0) = f(0 + 0) = f(0) + f(0)$, then $f(0) = 0$.□

Let I be an ideal of R, then $\text{ann}_M(I) = \{m \in M \mid Im = (0)\}$, it is easy to prove that $\text{ann}_M(I)$ is a sub semi module of M.

2.7. Remark:

Let M be a cancellable R- semi module with zero, where R is a semi ring. If I is an ideal of R, then $\text{ann}_M(I) = \{m \in M \mid Im = (0)\}$ is a stable sub semi module of M.

Proof: Let f: $\text{ann}_M(I) \rightarrow M$ be a homomorphism, and $\forall$ m $\in \text{ann}_M(I)$. Im = (0), that is If(m) = $f(I$m) = f((0)) = (0), hence f(m) $\in \text{ann}_M(I)$, thus $\text{ann}_M(I)$ is a stable sub semi module.□

2.8. Remark:

Let M be an R-semi module. If every cyclic subs emi module of M is stable, then M is a fully stable semi module.

Proof: Let N be a sub semi module of M and f: $N \rightarrow M$ be a homomorphism. If x $\in N$, then Rx is a cyclic sub semi module of M, hence by assumption Rx is stable in M, and so $f(R\text{x}) \subseteq R$x, but Rx $\subseteq N$. Therefore f(x) $\in N$, $\forall$ x $\in N$, that is $f(N) \subseteq N$. Thus N is a stable, it follows that M is fully stable. □

2.9. Corollary:

An R-semi module M is fully stable if and only if every cyclic sub semi module is stable.□

2.10. Corollary:

The sum of any family of stable sub semi modules is stable.

Proof: Let $\{N_i \mid i \in I\}$ be a family of stable sub semi modules of a semi module M. Let

$f: \sum_{i\in I} N_i \to M$, be a homomorphism, and let a $\in \sum_{i\in I} N_i$, then $a = a_1 + a_2 + \ldots\ldots\ldots\ldots + a_n$, where $a_i \in Nk_i$, for some

$i = 1, 2, \ldots\ldots\ldots, n$, and $k_i \in I$, that is, $f|N_i: N_i \to M$ is a homomorphism, $f(a_i) \in Nk_i$ (since each N_i is stable), then $f(a) = f(a_1) + f(a_2) + \ldots + f(a_n) \in \sum_{i\in I} N_i$, that is $f(\sum_{i\in I} N_i) \subseteq \sum_{i\in I} N_i$. Therefore $\sum_{i\in I} N_i$ is a stable.

2.11. Examples and Remarks:

(a) Every regular commutative semi ring is a fully stable semi ring. Let R be a regular commutative semi ring. Let $f: Rm \to R$ be any homomorphism, where $m \in R$. Since R is regular, there exists an element $k \in R$ such that $m = mkm$, thus $f(m) = f(mkm) = mf(km) = f(km)m \in Rm$. It follows that $f(Rm) \subseteq (Rm)$. Hence R is fully stable.

(b) Let $M = Z_8$ be an R-semi module, where R is the semi ring Z_8, then the proper sub semi modules of M are $A_0 = \{0\}$, $A_1 = \{0, 4\}$, $A_2 = \{0, 2, 4, 6\}$. Let $f: A_n \to Z_8$ be a homomorphism, $\forall n = (0, 1, 2)$, note that $|f(A_i)| \le |A_i|$, if $n = 0$, then $f(A_0) = A_0$, and if $n = 1$, then $f(A_1) = A_0$ or $f(A_1) \subseteq A_1$, while if $n = 2$, then $f(A_2) = A_0$ or $f(A_2) = A_1$ or $f(A_2) = A_2$, and hence $f(A_2) \subseteq A_2$. Therefore Z_8 is fully stable. On the other hand 2 in Z_8 is nilpotent, and so not regular. That is Z_8 is not regular.

Note: In the rest of this section, we consider that M is a cancellable semi module.

2.12. Definition:

A semi group $(A, +)$ is said to satisfy the property P, if

$\forall x \neq y \in A$, there exists z in A such that $x + z = y$ or $y + z = x$.

The left module M is said to satisfy the property P, if the semi group $(M, +)$ is satisfy the property P.

2.13. Remark:

Let M be a semi module satisfying the property P. If

$f: M \to M'$ is a homomorphism, where M' is a semi module, then f is

$1 - 1$ if and only if $\ker(f) = \{0\}$.

Proof: The first part, even without, the assumed property by Lemma (2.6) is true. Conversely, assume that f is not $1 - 1$, then $f(x) = f(y)$ for some

$x \neq y$ in M, by the assumed property, there exists z in M such that

$x = y + z$ or $y = x + z$, if $x = y + z$, then $f(x) = f(y) + f(z)$, by cancellablity, then $f(z) = 0$, it is clear that $z \neq 0$, and $\ker(f) \neq \{0\}$.□

2.14. Lemma:

Assume that M is a semi module over a semi ring R, where $(R, +)$ has the property P. If $ann_R(Ra) \subseteq ann_R(Rb)$, then for each

$r_1 \neq r_2 \in R$, $r_1 a = r_2 a \Rightarrow r_1 b = r_2 b$, $\forall$ a, b $\in M$.

Proof: Let $r_1 \neq r_2$, $r_1 a = r_2 a$, and (say, by Property P) $r_1 = t + r_2$, since $r_1 a = r_2 a$, then $(t + r_2)a = ta + r_2 a = r_2 a$. By cancellablity, it follows $t \in ann_R(Ra)$, and then, $t \in ann_R(Rb)$, that is, $r_1 b = (t + r_2)b = tb + r_2 b = r_2 b$, thus $r_1 b = r_2$. □

2.15. Proposition:

Under the conditions of Lemma (2.14), then M is fully stable if and only if for each a , b in M, $b \notin Ra$ implies $ann_R(Ra) \not\subset ann_R(Rb)$.

Proof: Let M be a fully stable semi module, and let $a, b \in M$ such that

$b \notin Ra$, and $ann_R(Ra) \subseteq ann_R(Rb)$. Define $f: Ra \to M$, by $f(ra) = rb$,

$\forall r \in R$, by Lemma (2.14), f is well defined, since $f(ra) = rb$, then

$f(a) = b$. It follows that $b \in Ra$ which is a contradiction.

Conversely, let Ra be a sub semi module of M, and a homomorphism $f: Ra \to M$, such that $f(Ra) \not\subset Ra$, let $b \in Ra$ such that $f(b) \notin Ra$, and let $c \in ann_R(Ra)$, it follows that $ca = 0$, and $cf(a) = f(ca) = f(0) = 0$, that is $ann_R(Ra) \subseteq ann_R(f(b))$ which is a contradiction. Therefore $ann_R(Ra) \not\subset ann_R(f(b))$. □

2.16. Remark:

By Proposition (2.15), M is fully stable if and only if,

$\forall a, b \in M$, $ann_R(Ra) \subseteq ann_R(Rb)$ implies $b \in Ra$.□

Remark (2.16.) leads to another property of fully stable semi module.

2.17. Corollary:

Let M be a fully stable R-semi module. Then for each

a, b in M, $ann_R(Ra) = ann_R(Rb)$ implies $Ra = Rb$.□

3. The Semi ring of Endomorphisms of a Fully Stable Semi module

For any R-semi module M, $End_R(M)$ is the set of endomorphisms of M is a semi ring with respect to the addition and multiplication defined as follows: $f + g = h$ where $h(x) = f(x) + g(x)$ for all $x \in M$, $f \circ g = h$ where $h(x) = f(g(x))$ for all $x \in M$ [14].

3.1. Proposition:

Let R be a commutative semi ring. If M is a fully stable R-semi module, then $End_R(M)$ is a commutative semi ring.

Proof: Let f , g be homomorphisms in $End_R(M)$, and $x \in M$. By Remark (2.4).There exist $r, k \in R$ such that $f(x) = rx$ and $g(x) = kx$, thus $(f \circ g)(x) = f(g(x)) = f(kx) = kf(x) = k(rx) = (kr)x$, and $(g \circ f)(x) = g(f(x)) = g(rx) = rg(x) = r(kx) = (rk)x$, it follows that $(f \circ g)(x) = (g \circ f)(x)$. Hence $End_R(M)$ is commutative.

It is known that if R is a ring then $End_R(R) \cong R$ [15],in the following we prove an analogously result for semi rings.

3.2. Proposition:

If R is a semi ring, then $End_R(R) \cong R$.

The proof is similar to the case in module (see [15]).

Now let $R = Z^+$, then $\text{End}_R(R) \cong R$ which is a commutative semi ring, but ${}_RR$ is not fully stable (see example of Remark (2.3.). That is, the converse of Proposition (3.1.) is not true.

3.3. Definition:

A semi module ${}_RM$ is said to be regular, if every cyclic sub semi module of it is a direct summand.

Note: A direct summand of a semi module is defined in same way as in module [15].

3.4. Proposition:

Let M be a regular semi module. If $\text{End}_R(M)$ is commutative, then M is a fully stable semi module.

Proof: Let $T = Rt$ be any cyclic sub semi module of M, and $f : Rt \to M$ any R-homomorphism. There exists a sub semi module S of M such that $M = Rt \oplus S$. Let $g = f \circ \pi$, where $\pi : M \to Rt$ is the natural projection, it is clear that g is an extension of f to M. Now $g = f \circ \pi \in \text{End}_R(M)$, π can be considered as an element of $\text{End}_R(M)$, a $\in T$, then $g(\pi(a)) = \pi(g(a))$, that is $g(a) = \pi(g(a)) \in T$, it follows that $f(a) \in T$. Therefore $f(T) \subseteq T$, hence M is a fully stable semi module.

Now, by Proposition (3.1.) and Proposition (3.4.), we have the following :

3.5. Corollary:

Let M be a regular R-semi module, where R is commutative semi ring. Then M is fully stable if and only if $\text{End}_R(M)$ is commutative. □

3.6. Proposition:

Let M be a regular R-semi module, where R is commutative, and let $S = \text{End}_R(M)$. If S is a fully stable semi ring, then M is a fully stable semi module.

Proof: Let N be any cyclic sub semi module of M and R-homomorphism $f : N \to M$, now we consider $I = \text{Hom}_R(M,N)$, I is a right ideal of S. Define $h : I \to S$ by $h(g) = f \circ g$ for each $g \in I$. Clearly, $h(g) \in S$, moreover, h is an S-homomorphism. Since S is a fully stable semi ring, then $h(\text{I}) \subseteq I$, that is for each $g \in I$, $f \circ g \in I$ that is $f \circ g : M \to N$. But N is a direct summand of M, then the natural projection π_N of M onto N is in I, hence, $f \circ \pi_N \in I$, that is $f \circ \pi_N : M \to N$, because π_N is onto, then $f : N \to N$ or $f(N) \subseteq N$. Therefore M is a fully stable semi module. □

References

[1] M. S. Abbas., On fully stable modules, Ph.D. Thesis, University of Baghdad. Iraq,(1990).

[2] C.Faith., Algebra: Rings, Modules and Categories I, Springer-Verlag, Berlin,Heidelberg, New York,(1973).

[3] C. Reutenauer. and H. Straubing., Inversion of matrices over a commutative semi ring, Journal of Algebra, Vol. 88,350-360,(1984).

[4] S. E. Atani and F. E. Khalil, On coatomic semi modules over commutative semir ings, Cankaya University Journal of Science and Engineering, Vol. 8,No.

2,189-200,(2011).

[5] H. A. Tavallaee and M. Zolfaghari, On semi prime sub semi modules and related results, J. Indones. Math. Soc., Vol. 19,No. 1, 49-59,(2013).

[6] A. C. Ozcan, A. Harmanci and P. F. Smith, Duo modules, Glasgow, Math. J.,48,533-545,(2006).

[7] K.Pawar, R. Deore and Jalgaon, On normal radicals, International Journalof Pure and Applied Mathematics, Volume72,No.2, 145-157,(2011).

[8] Y.Katsov, T.G. Nam, N.X. Tuyen, On subtractive semi simple Semi rings, Algebra Colloquium, 16 : 3, 415 - 426, (2009).

[9] H. M. J. Al-Thani, Projective semi modules, African Journal of Mathematics and Computer Science, Vol. 4(9),294-299,(2011).

[10] H. M. J. AL-Thani, The Jacobson semi radical over a certain Semi ring, Tamkang Journal of Mathematics, Volume 37, Number 1, 67-76,(2006).

[11] [11] M. K. Dubey and P. Sarohe, On 2-absorbing semi modules, Quasi group and Related Systems, 21,175 −184,(2013).

[12] M. K.Sen and S. K.Maity, Regular additively inverse semi rings, ActaMath. Univ. Comenianae, Vol. LXXV, 1,137–146,(2006).

[13] M. K. Dubey, Prime and weakly prime ideals in semi rings, Quasi group and Related Systems, 20, 197 – 202,(2012).

[14] J. Jezek, T. Kepka and M. Maroti, The endomorphism semi ring of a semi lattice, Semi group Forum, 78, No.(1),21-26,(2009).

[15] F.Kasch, Modules and rings, Academic Press, London(1982).