AL BAHIR

Quarterly Refereed Journal
for Natural and Engineering Sciences

Issued by
Al-`Abbas Holy Shrine
International Al-`Ameed Centre for Research and Studies

Licensed by
Ministry of Higher Education
and Scientific Research

Third Year, Fifth Volume, Issue 9 and 10
Ramadhan, 1438, June 2017

Secretariat General
of Al-'Abbas
Holy Shrine

Al-Ameed Interna-
tional center
for Research and Studies

Print ISSN: 5721 – 2312
Online ISSN: 0083 – 2313
Consignment Number in the Housebook and Iraqi
Documents: 1996, 2014
Iraq - Holy Karbala

Mobile: +964 760 235 5555
+964 771 948 7257
http://albahir.alkafeel.net
Email: albahir@alkafeel.net

Publication Conditions

Inasmuch as Al-`Bahir- effulgent- Abualfadhal Al-`Abbas cradles his adherents from all humankind, verily Al-Bahir journal does all the original scientific research under the conditions below:

1. Publishing the original scientific research in the various scientific sciences keeping pace with the scientific research procedures and the global common standards; they should be written either in Arabic or English .
2. The research should not be published before under any means .
3. The research should adhere the academic commonalties; the first page maintains the title, researcher name /names, address, mobile number under condition that the name, or a hint , should never be mentioned in the context and keywords should be written in Arabic and English as there is an abstract in Arabic and English.
4. The Research studies should be delivered to us either via Journal website http://albahir.alkafeel.net , after filling the two standard format the first with the name of the researcher and the second without in Word .
5. The page layout should be (2)cm .
6. The font should be of (16 bold),Time New Roman, subtitles of (14 bold) and also the context.
7. The space should be single, indentation should not be, as 0 before, 0 after and no spacing, as 0 before, 0 after.
8. There should be no decoration and the English numeral should proceed to the last text.
9. Any number should be between two brackets and then measurement unit, for instance: (12) cm .
10. All sources and references should be mentioned at the end of the article and categorized in conformity with Modern Language Association (MLA) , for instance :

Name of Author/ Authors, Journal Name Volume Number (Year) pages from - to.

Similarly done in the Arabic article withy a proviso that superscript should be employed.

11. There should be a caption under a diagram in 10 dark , Arabic and English; for instance:

Title or explanation; number of the Fig.
Similarly done with tables.
12. Diagrams , photos and statics should be in colour with high resolution without scanning.
13. The marginal notes, when necessary, should be mentioned at the end of the article before the references.
14. Wherever there is the word " figure" should be abbreviated as Fig. and table should be Table.
15. The pages never exceed 25 pages.
16. The Formulae should be written in Math Type.
17. All the ideas and thoughts reveal the mindset of the researcher not the journal and the article stratification takes technical standards.
18. All the articles are subject to :
a- The researcher is notified that his paper is received within 14 days in maximum.
b- The article is to be sent to the researcher as soon as it does not meet the requirement of the publication conditions.
c- The researcher is notified that his article is accepted.
d- The articles need certain modification , as the reviewers state, are sent to the researchers to respond in a span of a month from the date of dispatch.
e- The researcher is to be notified in case the article is rejected.
f- The researcher is to be granted an edition containing his article.
19. Priorities are given in concordance to :
a. The articles participated in the conferences held by the publication institute.
b. The date of receiving.
c. The date of acceptance.
d. The importance and originality of the article.
e. The diversity of the fields the articles maintain in the meant edition.
20. The researchers should appeal to the modifications the language and scientific reviewers find in the articles.
21. The researcher should fill the promise paper having the publication rights of the Scientific Al-Bahir Journal and adhering to integrity conditions in writing a research study.

In the Name of Allah
Most Compassionate, Mort Merciful

Edition Word

O Allah, my Lord

Cast felicity in me , facilitate my cause and unknot my tongue to perceive my speech , thanks be upon Him the Evolver of the universe and peace be upon Mohammad and his immaculate and benevolent progeny .

A fledged edition of Al-Bahr , peer reviewed scientific journal, embraces a constellation of research studies pertinent to engineering and natural sciences we do hope to overlap a scientific gap the specialists observe as an academic phenomenon worth being under the lenses of the researchers, that is why there is diversity in the studies to meet the requirements of the journal readership . For the journal, now, comes to the fore , at the efforts of the editorial and advisory boards and the researchers who strain every sinew to publish in Al-Bahr, to be global as to be published in an international publishing house in line with the global scientific journals.

On such an occasion we do pledge the promise of fealty and loyalty to those who observe our issues with love and heed in the International Al-`Ameed for Research and Studies , Department of Cultural and Intellectual Affairs in the Holy Al-`Abbas Shrine and the strenuous endeavour to cull whatever invigorates the scientific interaction and academic research in Iraq and worldwide to create a new generation keeping pace with the development of the current scientific phase and to lay the hands of the researchers, nationwide and worldwide, upon the desired missions.

Thanks be upon Him ,the Evolver ad infinitum .

Consultation and Editaril

Inverse Co-Independent Domination of Graphs

A.A. Omran
Department of Mathematics, College of Education for Pure Science, Babylon University, Babylon, Iraq

Received Date: 4 / 5 / 2016
Accepted Date: 8 / 8 / 2016

الخلاصة

في هذا البحث، نقدم نموذجين جديدين من الهيمنة في الرسوم البيانية التي تسمى تشكيلة المجموعة المستقلة المهيمنة وتشكيلة المجموعة المستقلة المهيمنة العكسية وتناقش في بعض الرسوم البيانية.

الكلمات المفتاحية

الهيمنة في الرسوم البيانية، تشكيلة المجموعة المستقلة المهيمنة، وتشكيلة المجموعة المستقلة المهيمنة العكسية.

Abstract

In this paper, we introduce two new models of domination in graphs which are called co-independent dominating set and inverse co-independent dominating set and they are discussed in some graphs.

Keywords

Domination in graphs, Co-independent dominating set, Inverse co-independent dominating set.

Vol. 5, No. 9 and 10 P. (75-79)E, 2017

1. Introduction

We consider a finite undirected and simple graph G(E,V) with a set V(G) of vertices and a setE(G) of edges. For a vertex v ϵ V(G), theopen neighborhood N(v) of v is the set of vertices adjacent to v, and the closed neighborhood N[v]of v is the set N(v)∪{v}.

A subgraph H of a graph G is said to be induced (or full) if, for any pair of vertices x and y of H, xy is an edge of H if and only if xy is an edge of G. If H is an induced of G with S is a set of its vertices then H is said to be induced by S and denoted by G[S].[2]

The concept of domination was first studied by Ore [6] and C. Berge [3]. A set D ⊆Vis said to be a dominating set of G if every vertex in V -D is adjacent to some vertex in D. The cardinality of a minimum dominating set D is called the domination number of G and is denoted byγ(G).The first one was given independently by Y. Caro and V. Wei [2].An independent set or stable set is a set of vertices in a graph G, where no two of which are adjacent. An independence number denoted by β(G) of a graph Gis the cardinality of a maximum independent set of G.

1.1. Definition, [4] (Complete z-ary trees)

A treeT is a connected graph with no cycles. In a tree, a vertex of degree one is referred to as a pendant (leaf) and a vertex which is adjacent to a pendant is a support vertex. A tree is called a rooted tree if one vertex has been designated the root.

In a rooted tree, the parent of a vertex is the vertex connected to it on the path to the root; every vertex except the root has a unique parent. A child of a vertex v is a vertex of which v is the parent. In a rooted tree, the depth r is the longest length of a path from the root to a vertex v.An internal vertex in a rooted tree is any vertex that has at least one child. A z-ary tree,z $\geq$ 2 is a rooted tree in which every vertex has z or fewer children. A complete z-ary tree($T_{c,z,r}$) is a z-ary tree in which every internal vertex has exactly z children and all pendant vertices have the same depth.

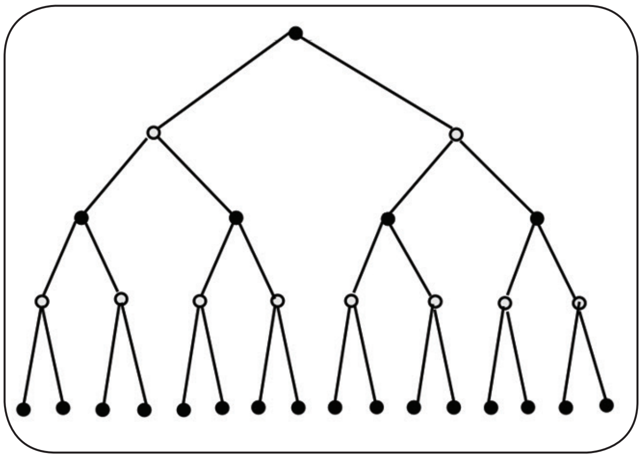

Fig. (1): 1: $T_{c,2,4}$

1.2. Definition [1] (Jahangir graph)

For n and m; m $\geq$ 3 and n $\geq$ 2 the Jahangir graph$J_{n,m}$, is a graph ofnm + 1 vertices consisting of a cycle C_{nm} with one additional central vertex which is adjacent to certain m vertices of C_{nm} where these vertices at distance n in order (sequence) onC_{nm}. Consider v_0 be the center vertex of $J_{n,m}$ and v_1 be one vertex in C_{nm} which is adjacent tov_0, and v_1,v_2,v_3,…,v_{mn} are the other vertices that incident clockwise in C_{nm}.

In this section, we take v_1 is the first vertex adjacent to the center v_0 (see Figure 2) for $J_{4,4}$.

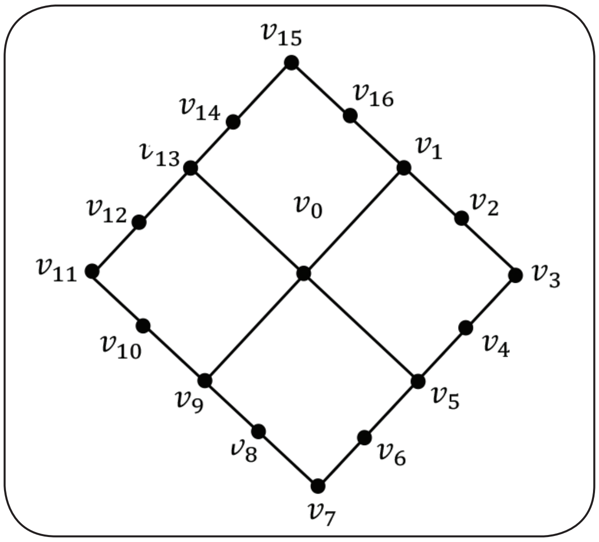

Fig. (2): $J_{4,4}$

Here, we introduce the concept of co-independent domination in graphs. The co-independent domination number for some graphs are determined.

The reader is referred to [5] for survey or results on domination. Any notion or definition of graph which is not found here could be found in [4].

2. Co-independent dominating and inverse co-independent dominating sets

In this section new definitions of domination number are introducedwith some results for these definitions in some graphs are discussed.

2.1. Definition

A dominating set D ⊆V(G) is a co-independent dominating set in G if the complement of D is an independence set. The co-independent domination number of G, denoted by $\gamma_{coi}(G)$, is a minimum cardinality over all co-independent dominating set of G.

2.2. Definition

Let D⊆V(G) be a minimum cardinal of co-independent dominating set in graphG. If V-D contains co-independent dominating set in G, then this set is called an inverse set of D in G and denoted by D^{-1}. The symbol $\gamma_{coi}^{-1}(G)$ refers to the minimum cardinality over all inverse co-independent dominating set in G.

2.3. Proposition

1) $\gamma_{coi}(P_n)=\lfloor n/2\rfloor$.

2) $\gamma_{coi}(C_n)=\lceil n/2\rceil$.

3) $\gamma_{coi}(K_n)=n-1$.

4) $\gamma_{coi}(S_n)=1$.

5) $\gamma_{coi}(K_{n,m})=\min\{n,m\}$.

For a complete z-ary tree G≡T_(c,z,r) with n vertices, have the following co -independence domination number:

2.4. Theorem

$$\gamma_{coi}(G)=\frac{z^{r+1}\left(1-z^{-2\left(\left\lfloor\frac{r-1}{2}\right\rfloor+1\right)}\right)}{z^2-1}.$$

Proof.

Consider $D^{coi}=\cup_{i=0}^{\left\lfloor\frac{r-1}{2}\right\rfloor}$ where D_i is the set of all vertices with depth

r-1-2i in $T_{c,z,r}$, and

E_i={v:v is a vertex of depthr-2i,r-1-2i and r-2-2i inG },i=0,1,…,⌊(r-1)/2⌋. It is clear that D_0 is the minimum dominating and E_0-D_0 is an independent set in G[E_0]. Also D_1 is the minimum dominating set in G[E_1]and E_1-D_1 is an independent set in G[E_1] and so on…. Thus D^{coi} is the co- independent dominating set in G with $|D^{coi}| = \sum_{i=0}^{\lfloor\frac{r-1}{2}\rfloor} z^{r-1-2i}$ .Let's consider that there is a set F of vertices such that|F|<|D^{coi}|, F is not co-independent dominating set in G, since V-F is not an independent set (it contains at least two adjacent vertices). Thus γ_{coi} (G)= $\sum_{i=0}^{\lfloor\frac{r-1}{2}\rfloor} z^{r-1-2i} = \frac{z^{r+1}\left(1-z^{-2\left(\lfloor\frac{r-1}{2}\rfloor+1\right)}\right)}{z^2-1}$. □

2.5. Theorem

$$\gamma_{coi}{}^{-1}(G) = \frac{z^{r+2}\left(1-z^{-2\left(\lfloor\frac{r}{2}\rfloor+1\right)}\right)}{z^2-1}.$$

Proof.

Consider$\left(D^{coi}\right)^{-1} = \cup_{i=0}^{\lfloor\frac{r-1}{2}\rfloor} D_i$, where$D_i$ is the set of all vertices with depth r-2i in$T_{c,z,r}$and

E_i={v:v is a vertex of depthr+1-2i,r-2i and r-1-2i in$T_{c,z,r}$},i=0,1,…,⌊r/2⌋. E_0={v:v is a vertex of depth r and r-1 in $T_{c,z,r}$}. It is clear that D_0 is the minimum co-independent set in G[E_0]. As same the manner in the previous theorem $(D^{coi})^{-1}$ is the minimum dominating set in G where, $(D^{coi})^{-1} \subseteq V - D^{coi}$.

Thus $\gamma_{coi}{}^{-1}(T_{c,z,r}) = \sum_{i=0}^{\lfloor\frac{r}{2}\rfloor} z^{r-2i} = \frac{z^{r+2}\left(1-z^{-2\left(\lfloor\frac{r}{2}\rfloor+1\right)}\right)}{z^2-1}$.

We note that if r≡0(mod 2), then $E_{\lfloor(r-1)/4\rfloor}$ = {v:v is a vertex of depths 1 or 0}. □

For Jahangir J_(n,m) with n≥3,we have the following co- independence domination number:

2.6. Theorem

$$\gamma_{coi}\left(J_{n,m}\right) = \left\lfloor\frac{mn}{2}\right\rfloor + \left\lceil\frac{n}{2}\right\rceil - \left\lfloor\frac{n}{2}\right\rfloor.$$

Proof.

Consider$D = \left\{v_{2i+1}; i = 0,1,\ldots,\left\lfloor\frac{mn}{2}\right\rfloor - 1\right\}$, then we have two cases as follows.

(i) If n is odd, then D is the minimum dominating set and the set of vertices V-D is not independent set in $J_{n,m}$, since v_0 is adjacent to some vertices in V-D. For this reason we add v_0 to D. Therefore D∪{v_0} is adominating set in $J_{n,m}$, and V-(D∪{v_0}) is independent set in $J_{n,m}$, then γ_{coi} ($J_{n,m}$) ≤ |D∪{v_0}|=⌊mn/2⌋+1.If there is a set F of vertices;|F|<⌊mn/2⌋+1, then Fis not co- independent dominating set, since G[V-F]contains at least two adjacent vertices. Thus D∪{v_0} isminimum co-independent dominating set in $J_{n,m}$, and we have γ_{coi} ($J_{n,m}$)=⌊mn/2⌋+1.

(ii) If n is even, then D is minimum dominating set such that the set of vertices

V-D is independent set in $J_{n,m}$, since Dhas no vertex adjacent tov_0, then γ_{coi} ($J_{n,m}$)≤⌊mn/2⌋. Again if F is a set of vertices;|F|<|D|, then Fis not co- independent dominating set, since G[V-F] contains at least two adjacent vertices. Thus D is minimum co-independence dominating set, and γ_{coi} ($J_{n,m}$)=mn/2.

We combine the formulas in (i) and (ii) as one formula for any n, we get

$$\gamma_{coi}\left(J_{n,m}\right) = \left\lfloor\frac{mn}{2}\right\rfloor + \left\lceil\frac{n}{2}\right\rceil - \left\lfloor\frac{n}{2}\right\rfloor. \quad \square$$

2.7. Theorem

There is no inverse co-independent dominating set in J_(n,m).

Proof.

Consider $(D^{coi})^{-1}=V-D^{coi}$, where D^{coi} is a minimum co-independent dominating set in $J_{n,m}$, there are two cases which depend on n as follows.

(i) If n is even there is no any vertex in $(D^{coi})^{-1}$ dominate the vertex v_0 . Thus there is no any dominating set in $J^{n,m}$ such that the vertices of $(D^{coi})^{-1}$contains in $V-D^{coi}$.

(ii) If n is odd then$(D^{coi})^{-1}$is not co-independence dominating set since $V-D^{coi}$ not

independent (there are some vertices adjacent to v_0) and we cannot include v_0 to the set$(D^{coi})^{-1}$ since $v_0 \in D^{coi}$. Thus there is no any dominating set in $J_{n,m}$ such that the vertices of $(D^{coi})^{-1}$ contains in $V-D^{coi}$. □

References:

[1] K. Ali, E. T. Baskoro and I. Tomescu, On the Ramzey number of Paths and Jahangir graph J3,m. 3rd International Conference on 21st Century Mathematics 2007, GC University Lahoor Pakistan, March (2007).

[2] Y. Caro and Z. Tuza, Improved lower bounds on k-independence, J. Graph Theory 15, 99-107, (1991).

[3] C. Berge, Theory of graphs and its Applications, Methuen, London, (1962).

[4] F. Harary, Graph Theory, Addison-Wesley, Reading Mass (1969).

[5] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York (1998).

[6] O. Ore, Theory of Graphs, Amer. Math. Soc. Colloq. Publ., 38 (Amer. Math. Soc., Providence, RI), (1962).