The Weighted Transmuted Pareto Distribution

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الخلاصة

ان توزيع الحياة يلعب دورا مهما في مجالات الحياة المختلفه كالأحصاءات الحياتية، تحليل المعولية او البقاء،... الخ. وفي هذا البحث حاولنا المساهمة في ايجاد توزيع حياة مقترح وهو توزيع باريتو المحول الموزون مع مناقشة بعض الخواص.

الكلمات المفتاحية

توزيع باريتو المعزول ، التوزيع الموزون ، دالة التوزيع التراكمي ، وظيفة الموثوقية.

Abstract

The lifetime distributions play important role in many real life fields such as the biostatistics, reliability and survival analysis, so we try to contribute infinding anew proposed lifetime distribution, weighted transmuted Pareto distribution, and discuss some of its statistical properties.

Keywords

Transmuted Pareto distribution, Weighted distribution, Cumulative distribution function, Reliability function.

1. Introduction

The statistical analysis depends on the statistical model, or distribution, represented the life phenomena under study. However, the models available do not fit data of many important and practical problems. So that a non-linear parametric model may be recommended. The weighted, transmuted, weighted transmuted distributions are from these non-linear parametric distributions.

Shaw & Buckley (2009) [3] used the rank transmutation map RTM, a tool for the construction of new families of non-Gaussian distributions. They used it to modulate a given base distribution for the purposes of modifying the moments, in particular the skew and kurtosis. They introduced the quadratic rank transmutation map (QRTM) that has been used by many authors to introduce different new important distributions. The transmuted Pareto was derived by F. Merovciaand L. Pukab

2.distribution:

The weighted distribution is considered as a good tool for modeling statistical data when these data can not fit the standard distributions because the damage, missing caused to the original observation resulting in a reduced value, or adoption of a sampling procedure which gives unequal chances to the units in the original. Fisher (1934) [1] introduced this concept and Rao (1965) [2] developed it.

Some recent searches of weighted distributions studied by many authors like:

K. K. Das and T. Deb Roy (2011) [4] who introduced the length-biased Weighted Generalized Rayleigh distribution and some of its properties,

X. Shi, B. O. Oluyede and M. Pararai (2012) [5] derived a new class of weighted generalization of the Rayleigh distribution, and discussion some of properties,

K. abed al-kadim and A.F. Hantoosh (2013) [6] constructed the Double weighted distribution, and Double weighted Exponential distribution.

K.A. Mir, A. Ahmed and J. A. Reshi (2013) [7] studied a new class of Length-biased beta distribution introduced the first kind, and estimation its parameters,

N. I. Rash wan (2013) [8] presented a new weighted distribution which is known as the double weighted rayleigh distribution and some of its properties,

A. Ahmad, S.P Ahmad and A. Ahmed (2014) [9] constructed a new weighted distribution which is known as the Double Weighted Rayleigh Distribution, and discussion statistical properties of this distribution ,

K Abed Al-Kadim and N. A. Hussein (2014) [10] introduce new class of length-biased of weighted exponential and Rayleigh distributions. And they studied some of its statistical properties with application.

P. Seenoi, T. Supapakorn and W. Bodhisuwan (2014) [11] introduce a length-biased of the exponentiated inverted Weibull distribution.

The aim of this paper is to propose and

study a generalization of the Pareto distribution using the weighted distribution, that is obtain a larger class of flexible parametric distribution. In this paper, the transmuted Pareto distribution derived by Faton M., Llukan P. (2014) [12], we study the a new proposed distribution, the weighted transmuted Pareto distribution, and discuss some of its statistical properties. It may use in applications of reliability, actuarial science, economics, finance and telecommunications. This paper is organized as follows. Section 1 contains introduction. The weighted transmuted Pareto distribution is introduced in section 2, including the cumulative distribution function (cd f), pdf, hazard and reverse hazard functions and some of its properties. In section 3, the conclusions are presented.

3. Mainresul:

3.1. The Weighted Transmuted Pareto Distribution(WTPD):

3.1.1. Definition (WTPD):

Let the weight function and the transmuted Pareto distribution be $w(x)\geq 0$ and $f_{TPD}(x)$ respectively. Then the weighted transmuted Pareto density function $g_{WTPD}(x)$ is obtained as:

$$g_{WTPD}(x) = \frac{w(x) f_{TPD}(x)}{w_D} \quad (1)$$

Where $w(x)=x$,is called the weight , it is a normalizing factor obtained to make the total probability equal to unity that is chosen such that choosing

$0 < \omega = E[w(X,\beta)] < \infty$.

$$w_D = E[w(x)] = \int_{x_m}^{\infty} x \frac{\alpha x_m^{\alpha}\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{x^{\alpha+1}} = \frac{\alpha x_m[2\alpha-1-\lambda]}{(\alpha-1)(2\alpha-1)} \quad (2)$$

where the pdf of transmuted Pareto distributionis:

$$f_{TPD}(x) = \frac{\alpha x_m^{\alpha}\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{x^{\alpha+1}},\ x > 0\ , |\lambda| \leq 1, \alpha > 0 \quad (3)$$

So that

$$g_{WTPD}(x) = \frac{x_m^{\alpha-1}(\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{x^{\alpha+1}(2\alpha-1-\lambda)}, x > 0, |\lambda| \leq 1, \alpha > 0 \quad (4)$$

Which is similar to pdfof WTPD with weight $(\alpha-1)(2\alpha-1)/((2\alpha-1-\lambda))$ for $|\lambda|\leq 1,\alpha>0$

And with slightly difference.

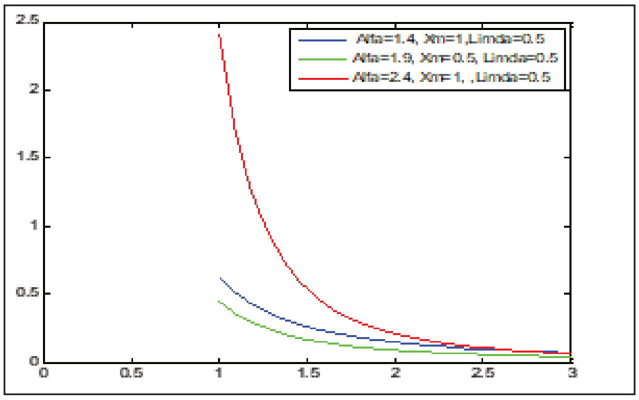

Fig. (1) The pdf of WTPD for different values of λ,α

In Fig. (1)we note as the shape of the pdf of WTPD, it is decreasing function along the increasing x and at different values of λ,α .

3.1.2. Remark

In this remark we prove that $g_{WTPD}(x)$ is a pdf as follows:

$\int_{x_m}^{\infty} g_{WTPD}(x)dx = \int_{x_m}^{\infty} (x_m^{\alpha-1})\ x^{-\alpha}(\alpha-1)(2\alpha-1)(1-\lambda+2\lambda(x_m/x)^{\alpha})/((2\alpha-1-\lambda))\ dx$

$$= \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)}\left[(1-\lambda)x_m^{\alpha-1}\int_{x_m}^{\infty} x^{-\alpha}dx + 2\lambda x_m^{\alpha-1}\int_{x_m}^{\infty} x^{-\alpha}\left(\frac{x_m}{x}\right)^{\alpha} dx\right]$$

$$= -\frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)}\left(\frac{-2\alpha+1+2\alpha\lambda-\lambda-2\alpha\lambda+2\lambda}{(\alpha-1)(2\alpha-1)}\right)$$

$$= -\frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)}\left(\frac{-2\alpha+1+\lambda}{(\alpha-1)(2\alpha-1)}\right) = 1 \quad \blacksquare$$

Acumulative distribution function (cd f) of weighted transmuted Pareto distribution is given as:

$$G_{WTPD}(x) = 1 - \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} x_m^{\alpha-1} x^{-\alpha+1}\left[\frac{(1-\lambda)}{(\alpha-1)} + \frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x}\right)^{\alpha}\right] \quad (5)$$

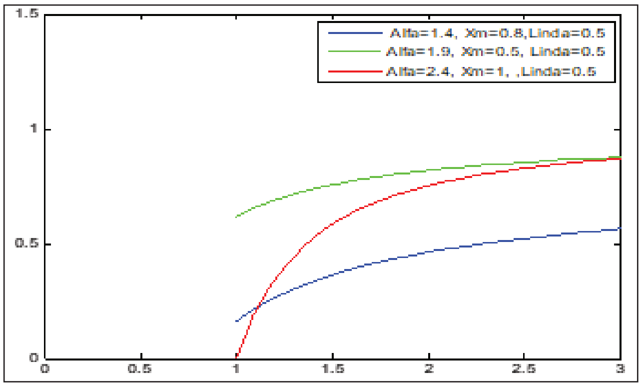

Fig. (2) The cdf of WTPD for different values of λ,α

Fig. (2) shows as the shape of the cdf of WTPD, it is increasing function along increasing x and at different values of λ,α

3.2. Reliability Analysis:

3.2.1. The Reliability Function (RF):

The reliability function of weighted transmuted Pareto distribution is given as:

$$R_{WTPD}(x;\alpha,\lambda) = \frac{(\alpha-1)(2\alpha-1)x_m^{\alpha-1}}{(2\alpha-1-\lambda)x^{\alpha-1}}\left[\frac{(1-\lambda)}{(\alpha-1)} + \frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x}\right)^{\alpha}\right] \quad (6)$$

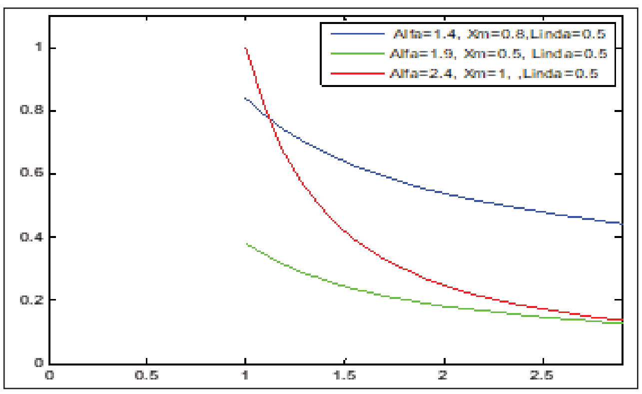

Fig. (3) The RF of WTPD for different values of λ,α

In Fig. (3)we note that the shape of the RF of WTPD, it is the inverted of the shape of cdf function along increasing x and at different values of λ,α

3.2.2. The Hazard Rate Function (HF):

The hazard rate function of (WTPD) is

$$h_{WTPD}(x;\alpha,\lambda) = \frac{g_{WTPD}(x;\alpha,\lambda)}{R_{WTPD}(x;\alpha,\lambda)} = \frac{\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{x\left[\frac{(1-\lambda)}{(\alpha-1)}+\frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x}\right)^{\alpha}\right]} \quad (7)$$

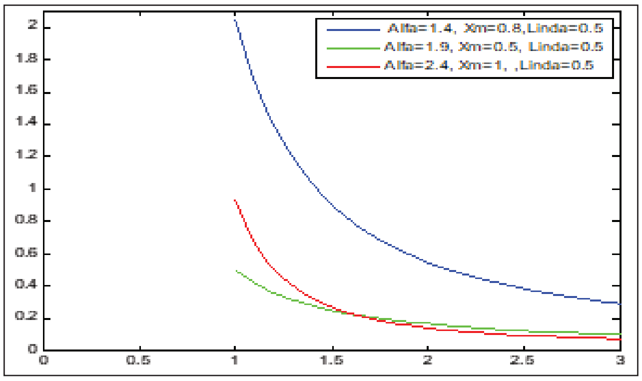

Fig. (4) The HF of WTPD for different values of λ,α

In Fig. (4)we note that the shape of the HF of WTPD, it is the similar to the shape of pdf function along increasing x and at different values of λ,α but with some differences.

3.2.3. The Reverse Hazard Function:

The reverse hazard function of WTPD(x;α,λ), is defined as

$$H_{WTPD}(x;\alpha,\lambda) = \frac{g_{WTPD}(x;\alpha,\lambda)}{G_{WTPD}(x;\alpha,\lambda)}$$

$$= \frac{x_m^{\alpha-1}x^{-\alpha}(\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{\left[(2\alpha-1-\lambda)-x_m^{\alpha-1}x^{-\alpha+1}\left((1-\lambda)(\alpha-1)+2\lambda(2\alpha-1)\left(\frac{x_m}{x}\right)^{\alpha}\right)\right]} \quad (8)$$

3.2.4. The Limit:

Proposition (1)

The limits of the pdf and its hazard function of WTPD as x→x_m and as →∞ , respectively, are equal.

Proof

$$\lim_{x\to x_m} g_{WTPD}(x,x_m,\alpha,\lambda) = \lim_{x\to x_m} \frac{x_m^{\alpha-1}x^{-\alpha}(\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)}$$

$$= \frac{x_m^{-1}(\alpha-1)(2\alpha-1)(1+\lambda)}{(2\alpha-1-\lambda)}$$

The limit of weighted transmuted density function of Pareto distribution when x→∞ is

$$\lim_{x\to\infty} g_{WTPD}(x,x_m,\alpha,\lambda) = \lim_{x\to\infty} \frac{x_m^{\alpha-1}x^{-\alpha}(\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)}$$

$$= \frac{x_m^{\alpha-1}\infty^{-\alpha}(\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{\infty}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)} = 0$$

The limit of weighted transmuted hazard rate function of Pareto distribution when x→x_m is:

$$\lim_{x\to x_m} h_w(x,x_m,\alpha,\lambda) = \frac{\left(1-\lambda+2\lambda\left(\frac{x_m}{x_m}\right)^{\alpha}\right)}{x_m\left[\frac{(1-\lambda)}{(\alpha-1)}+\frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x_m}\right)^{\alpha}\right]}$$

$$= \frac{(1+\lambda)(\alpha-1)(2\alpha-1)}{x_m(2\alpha-1-\lambda)}$$

$$= \lim_{x\to x_m} g_{WTPD}(x,x_m,\alpha,\lambda)$$

The limit of hazard rate function of weighted transmuted Pareto distribution when x→∞ is:

$$\lim_{x\to\infty} g_w(x,x_m,\alpha,\lambda) = \frac{\left(1-\lambda+2\lambda\left(\frac{x_m}{\infty}\right)^{\alpha}\right)}{\infty\left[\frac{(1-\lambda)}{(\alpha-1)}+\frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{\infty}\right)^{\alpha}\right]}$$

$$= 0 = \lim_{x\to\infty} g_{WTPD}(x,x_m,\alpha,\lambda) \blacksquare$$

3.2.5. The mode:

Take the natural logarithm to two sides of the pdf of the WTPD

$$ln g_{WTPD}(x) = ln \frac{x_m^{\alpha-1}x^{-\alpha}(\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)}$$

$$ln g_w(x) = (\alpha-1)ln x_m - \alpha ln x + ln(\alpha-1) + ln(2\alpha-1) + ln\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right) - ln(2\alpha-1-\lambda)$$

$$\frac{\partial ln g_w(x)}{\partial x} = \frac{-\alpha}{x} - \frac{2\alpha\lambda x_m^{\alpha}x^{-\alpha-1}}{\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}$$

$$\frac{-\alpha}{x} - \frac{2\alpha\lambda x_m^{\alpha}x^{-\alpha-1}}{\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)} = 0$$

$$x_{mode} = \left(\frac{\lambda-1}{4\lambda x_m^{\alpha}}\right)^{-1/\alpha} \quad (9)$$

3.3. Moment and Moment Generated Function:

3.3.1. The Moments:

Proposition(2) If X has the WTPD(x;α,λ) with |λ| ≤ 1,then the rth moment of X is

$$E(X^r) = \frac{(\alpha-1)(2\alpha-1)(2\alpha-(1+\lambda)(1+r))}{(2\alpha-1-\lambda)(\alpha-r-1)(2\alpha-r-1)} x_m^r (10)$$

For $\alpha > \frac{1+\lambda}{2}, \alpha > r+1, \alpha > \frac{1+r}{2}$.

Especially, $\mu = \frac{(2\alpha-1)(\alpha-1-\lambda)}{(2\alpha-1-\lambda)(\alpha-2)} x_m$

$$\sigma^2 = \frac{(2\alpha-1)}{(2\alpha-1-\lambda)} x_m^2 \left(\frac{(\alpha-1)(2\alpha-3(1+\lambda))}{(\alpha-3)(2\alpha-3)} - \frac{(2\alpha-1)(\alpha-1-\lambda)^2}{(2\alpha-1-\lambda)(\alpha-2)^2}\right) (11)$$

Proof

$$E(X^r) = \int_{x_m}^{\infty} x^r \frac{x_m^{\alpha-1}x^{-\alpha}(\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)} dx$$

$$= \frac{(\alpha-1)(2\alpha-1)(2\alpha-(1+\lambda)(1+r))}{(\alpha-r-1)(2\alpha-\lambda-1)(2\alpha-r-1)} x_m^r$$

We note that

$E(X^r) \to \infty$ as $x \to \infty$ and at $\alpha \le \frac{1+\lambda}{2}$, or $\alpha \le (r+1)$, or $\alpha \le \frac{1+r}{2}$ for $r\epsilon Z^+$.

Therefor $E(X^r)$ does not exist for $\alpha \le \frac{1+\lambda}{2}$, or $\alpha \le (r+1)$, or $\alpha \le \frac{1+r}{2}$ for $r\epsilon Z^+$,only.

Now ifr=1,then the mean

$$\mu = E(X) = \frac{(\alpha-1)(2\alpha-1)\big(2\alpha-2(1+\lambda)\big)}{(2\alpha-1-\lambda)(\alpha-2)(2\alpha-2)}x_m = \frac{(2\alpha-1)(\alpha-1-\lambda)}{(2\alpha-1-\lambda)(\alpha-2)}x_m$$

$$E(X^2) = \frac{(\alpha-1)(2\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(2\alpha-1-\lambda)(\alpha-3)(2\alpha-3)}x_m^2$$

$$\sigma^2 = E(X^2) - [E(X)]^2$$

$$= \frac{(2\alpha-1)}{(2\alpha-1-\lambda)}x_m^2\left(\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{(2\alpha-1)(\alpha-1-\lambda)^2}{(2\alpha-1-\lambda)(\alpha-2)^2}\right) \blacksquare$$

Proposition (3)

If X has the WTPD(x;α,λ) with |λ|≤1 , then the rth central moment about the mean μ is defined as

$$\mathrm{E}(X-\mu)^r = \sum_{j=0}^{r} C_j^r\,(-\mu)^{r-j}\,\frac{(\alpha-1)(2\alpha-1)\big(2\alpha-(1+\lambda)(1+j)\big)}{(2\alpha-1-\lambda)(\alpha-j-1)(2\alpha-j-1)}x_m^j \quad (12)$$

And the coefficients of variation, skewness, kurtosis are as respectively

$$CV. = \frac{\sqrt{\frac{(2\alpha-1)}{(2\alpha-1-\lambda)}\left(\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{(2\alpha-1)(\alpha-1-\lambda)^2}{(2\alpha-1-\lambda)(\alpha-2)^2}\right)}}{\frac{(2\alpha-1)(\alpha-1-\lambda)}{(2\alpha-1-\lambda)(\alpha-2)}} \quad (13)$$

$$CS. = \frac{\frac{1}{(\alpha-2)}\left[\frac{(\alpha-1)\big(\alpha-2(1+r)\big)}{(\alpha-4)} - 3\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)(2\alpha-1)(\alpha-1-\lambda)}{(\alpha-3)(2\alpha-3)(2\alpha-1-\lambda)} + 4\frac{(2\alpha-1)^2(\alpha-1-\lambda)^3}{\big((2\alpha-1-\lambda)(\alpha-2)\big)^2}\right]}{\left(\frac{(2\alpha-1)}{(2\alpha-1-\lambda)}\right)^2\left[\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{(2\alpha-1)(\alpha-1-\lambda)^2}{(2\alpha-1-\lambda)(\alpha-2)^2}\right]^{3/2}} \quad (14)$$

$$CK. = \frac{\frac{w_2 x_m^4}{w_3}\left[\frac{w_1\big(2\alpha-5(1+\lambda)\big)}{(\alpha-5)(2\alpha-5)} - 4\frac{w_1 w_2\big(\alpha-2(1+r)\big)w_4}{w_3(\alpha-4)(\alpha-2)^2} + 6\left(\frac{w_2 w_4}{w_3(\alpha-2)}\right)^2\frac{w_1\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - 3\frac{(w_2)^3(w_4)^4}{(w_3)^3(\alpha-2)^4}\right]}{\left[\frac{w_2}{w_3}\left(\frac{w_1\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{w_2(w_4)^2}{w_3(\alpha-2)^2}\right)x_m^2\right]^2} \quad (15)$$

Where

$$w_1 = (\alpha-1), w_2 = (2\alpha-1), w_3 = (2\alpha-1-\lambda), w_4 = (\alpha-1-\lambda)$$

Proof

$$E(X-\mu)^r = \int_{x_m}^{\infty}(x-\mu)^r\,\frac{x_m^{\alpha-1}x^{-\alpha}(\alpha-1)(2\alpha-1)\left[1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right]}{(2\alpha-1-\lambda)}dx$$

$$= \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)}\left[(1-\lambda)x_m^{\alpha-1}\int_{x_m}^{\infty}(x-\mu)^r x^{-\alpha}dx + 2\lambda x_m^{2\alpha-1}\int_{x_m}^{\infty}(x-\mu)^r x^{-2\alpha}dx\right]$$

$$= \sum_{j=0}^{r} C_j^r\,(-\mu)^{r-j}\,\frac{(\alpha-1)(2\alpha-1)\big(2\alpha-(1+\lambda)(1+j)\big)}{(2\alpha-1-\lambda)(\alpha-j-1)(2\alpha-j-1)}x_m^j$$

Then 1st ,2nd, 3rdcentral moments about the mean μ are defined as

$$\mathrm{E}(X-\mu)^2 = \frac{(2\alpha-1)}{(2\alpha-1-\lambda)}\left(\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{(2\alpha-1)(\alpha-1-\lambda)^2}{(2\alpha-1-\lambda)(\alpha-2)^2}\right)x_m^2 = \sigma^2$$

$$E(X-\mu)^3 = E(X^3 - 3X^2\mu + 3X\mu^2 + \mu^3)$$

$$= \frac{(2\alpha-1)x_m^3}{(2\alpha-1-\lambda)(\alpha-2)}\left[\frac{(\alpha-1)\big(\alpha-2(1+r)\big)}{(\alpha-4)} - 3\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)(2\alpha-1)(\alpha-1-\lambda)}{(\alpha-3)(2\alpha-3)(2\alpha-1-\lambda)} + 4\frac{(2\alpha-1)^2(\alpha-1-\lambda)^3}{\big((2\alpha-1-\lambda)(\alpha-2)\big)^2}\right]$$

$$E(X-\mu)^4 = E(X^4 - 4\mu X^3 + 6\mu^2X^2 - 4\mu^3X + \mu^4)$$

$$= \frac{(2\alpha-1)x_m^4}{(2\alpha-1-\lambda)}\left[\frac{(\alpha-1)\big(2\alpha-5(1+\lambda)\big)}{(\alpha-5)(2\alpha-5)} - 4\frac{(\alpha-1)(2\alpha-1)\big(\alpha-2(1+r)\big)(\alpha-1-\lambda)}{(2\alpha-1-\lambda)(\alpha-4)(\alpha-2)^2} + 6\left(\frac{(2\alpha-1)(\alpha-1-\lambda)}{(2\alpha-1-\lambda)(\alpha-2)}\right)^2\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - 3\frac{(2\alpha-1)^3(\alpha-1-\lambda)^4}{(2\alpha-1-\lambda)^3(\alpha-2)^4}\right]$$

So that

CV=σ/μ

$$= \frac{\sqrt{\frac{(2\alpha-1)}{(2\alpha-1-\lambda)}\left(\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{(2\alpha-1)(\alpha-1-\lambda)^2}{(2\alpha-1-\lambda)(\alpha-2)^2}\right)}}{\frac{(2\alpha-1)(\alpha-1-\lambda)}{(2\alpha-1-\lambda)(\alpha-2)}}$$

$$CS = \frac{E(X-\mu)^3}{\sigma^3}$$

$$= \frac{\frac{1}{(\alpha-2)}\left[\frac{(\alpha-1)\big(\alpha-2(1+r)\big)}{(\alpha-4)} - 3\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)(2\alpha-1)(\alpha-1-\lambda)}{(\alpha-3)(2\alpha-3)(2\alpha-1-\lambda)} + 4\frac{(2\alpha-1)^2(\alpha-1-\lambda)^3}{\big((2\alpha-1-\lambda)(\alpha-2)\big)^2}\right]}{\left(\frac{(2\alpha-1)}{(2\alpha-1-\lambda)}\right)^2\left[\frac{(\alpha-1)\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{(2\alpha-1)(\alpha-1-\lambda)^2}{(2\alpha-1-\lambda)(\alpha-2)^2}\right]^{3/2}}$$

$$CK = \frac{E(X-\mu)^4}{\sigma^4}$$

$$= \frac{\frac{w_2 x_m^4}{w_3}\left[\frac{w_1\big(2\alpha-5(1+\lambda)\big)}{(\alpha-5)(2\alpha-5)} - 4\frac{w_1 w_2\big(\alpha-2(1+r)\big)w_4}{w_3(\alpha-4)(\alpha-2)^2} + 6\left(\frac{w_2 w_4}{w_3(\alpha-2)}\right)^2\frac{w_1\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - 3\frac{(w_2)^3(w_4)^4}{(w_3)^3(\alpha-2)^4}\right]}{\left[\frac{w_2}{w_3}\left(\frac{w_1\big(2\alpha-3(1+\lambda)\big)}{(\alpha-3)(2\alpha-3)} - \frac{w_2(w_4)^2}{w_3(\alpha-2)^2}\right)x_m^2\right]^2}$$

We suppose that:

$$w_1 = (\alpha - 1), w_2 = (2\alpha - 1), w_3 = (2\alpha - 1 - \lambda), w_4 = (\alpha - 1 - \lambda)$$

3.3.2. Moment Generating Function:

Proposition (4)

If a X random variable is distributed WTPD(x;α,λ) with |λ| ≤ 1, then the moment generating function of X is

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{(\alpha-1)(2\alpha-1)(2\alpha-(1+\lambda)(1+r))}{(2\alpha-1-\lambda)(\alpha-r-1)(2\alpha-r-1)} x_m^r \quad (16)$$

Proof: Using the moment generating function, then

$$M_x(t) = \int_{x_m}^{\infty} e^{tx} g_{WTPD}(x; \alpha, \lambda)\, dx$$

$$= \int_{x_m}^{\infty} e^{tx} \frac{x_m^{\alpha-1} x^{-\alpha} (\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)} dx$$

$$= \int_{x_m}^{\infty} \sum_{r=0}^{\infty} \frac{(tx)^r}{r!} g_{WTPD}(x; \alpha, \lambda)\, dx$$

Since $e^{tx} = \sum_{r=0}^{\infty} \frac{(tx)^r}{r!}$

$$M_x(t) = \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} \sum_{r=0}^{\infty} \frac{t^r}{r!} \left[\int_{x_m}^{\infty} x_m^{\alpha-1} x^{r-\alpha} \left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right) dx\right]$$

$$= \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} \sum_{r=0}^{\infty} \frac{t^r}{r!} \left[(1-\lambda)\int_{x_m}^{\infty} x_m^{\alpha-1} x^{r-\alpha} dx + 2\lambda \int_0^{\infty} x_m^{2\alpha-1} x^{r-2\alpha} dx\right]$$

$$= \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} \sum_{r=0}^{\infty} \frac{t^r}{r!} \left[(1-\lambda)\frac{x_m^r}{\alpha-r-1} + 2\lambda \frac{x_m^r}{2\alpha-r-1}\right]$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{(\alpha-1)(2\alpha-1)(2\alpha-(1+\lambda)(1+r))}{(2\alpha-1-\lambda)(\alpha-r-1)(2\alpha-r-1)} x_m^r \blacksquare$$

3.4. Order Statistic:

Proposition (5)

Let $X_1, \ldots, X_n$ denote a random sample from a WTPD distribution, then the pdf of the rth order statistic is given by

$$f_{i,n}(x) = \frac{n!}{(i-1)!(n-i)!} g_{WTPD}(x; \alpha, \lambda)(G_{WTPD}(x; \alpha, \lambda))^{i-1}[1 - G_{WTPD}(x; \alpha, \lambda)]^{n-i} \; ()$$

And the pdfs of the minimum, the maximum and the median respectively are defined as follows

If i=1we have the pdf of the minimum

$$f_{1,n}(x)$$

$$= n \frac{x_m^{\alpha-1} x^{-\alpha} (\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)} \left[\frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} x_m^{\alpha-1} x^{-\alpha+1} \left(\frac{(1-\lambda)}{(\alpha-1)} + \frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x}\right)^{\alpha}\right)\right]^{n-1}$$

If i=n we have the pdf of the maximum

$$f_{n,n}(x) = n \frac{x_m^{\alpha-1} x^{-\alpha} (\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)} \left(1 - \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} x_m^{\alpha-1} x^{-\alpha+1} \left(\frac{(1-\lambda)}{(\alpha-1)} + \frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x}\right)^{\alpha}\right)\right)^{n-1}$$

And if i=m+1we have the pdf of the median

$$f_{m+1,n}(x) = \frac{n!}{m!\,(n-m-1)!} \frac{x_m^{\alpha-1} x^{-\alpha} (\alpha-1)(2\alpha-1)\left(1-\lambda+2\lambda\left(\frac{x_m}{x}\right)^{\alpha}\right)}{(2\alpha-1-\lambda)} \left(1 - \frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} x_m^{\alpha-1} x^{-\alpha+1} \left(\frac{(1-\lambda)}{(\alpha-1)} + \frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x}\right)^{\alpha}\right)\right)^{m} \left(\frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)} x_m^{\alpha-1} x^{-\alpha+1} \left[\frac{(1-\lambda)}{(\alpha-1)} + \frac{2\lambda}{(2\alpha-1)}\left(\frac{x_m}{x}\right)^{\alpha}\right]\right)^{n-m-1}$$

4. Conclusions:

We has found new lifetime distribution, weighted transmuted Pareto, which is similar to pdf of WTPD with weight $\frac{(\alpha-1)(2\alpha-1)}{(2\alpha-1-\lambda)}$ for $|\lambda| \le 1, \alpha > 0$; and with slightly difference.

The limit of hazard rate function of weighted transmuted Pareto distribution equal to the limit of the pdf of WTPD as x→x_m and as x→∞ respectively. It has single mode. And we can find its variance using the central moments about the origin, and about the mean directly. Also we can find its the central moments about the origin using the moment generating function.

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