تقدير دالة الموثوقية الضبابية لبيانات التوزيع (Topp-Leone) باستخدام قيمة الوسيط كعامل ضبابي

اسامة عبد العزيز كاظم القريشي
وزارة التربية – العراق
سما سعدي الهاشمي
معهد الادارة الرصافة

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Estimation of the fuzzy reliability function for distribution data (Topp-Leone) using the median value as a fuzzy factor

اسامة عبد العزيز كاظم القريشي*

وزارة التربية – العراق

سما سعدي الهاشمي

معهد الادارة الرصافة

Osama Abdulazeez Kadhim Al-Quraishi*

Ministry of Education Iraq

Sama Saadi Ali Alhashimi

Rusafa Management Institute, Baghdad

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المستخلص:

تعد الموثوقية الضبابية في الإحصاء من الادوات المهمة المستخدمة في مختلف جوانب الحياة والتي تستخدم في حالة وجود عدم يقين في دقة وضوح البيانات مما يؤدي إلى التعامل مع مشاهدات غامضة معبر عنها بأرقام غامضة، إن استخدام طرق التقدير الكلاسيكية لتقدير معالم النموذج الإحصائي غير امثل لمثل هذه المشاهدات، وتتمثل مشكلة البحث في وجود أرقام ضبابية تعرف بأوقات تشغيل جهاز الرنين المغناطيسي حتى العطل في مستشفى الرفاعي. ويهدف البحث إلى تقدير دالة الموثوقية الضبابية للنوع الأول من البيانات الخاضعة للرقابة لتوزيع (Topp-Leone) لجهاز الرنين المغناطيسي في مستشفى الرفاعي للفترة (1-3-2022، حتى 1-3-2024) من خلال بعض طرق التقدير دالة الامكان الاعظم، التقديرات الجزئية وطريقة تقدير مقترحة هي معدل التكرار ومن ثم اختيار أفضل موثوقية بناءً على أقل انحراف معياري، وقد وجد أن الموثوقية الضبابية لجهاز السونار هي (79%) لكل (42) يومًا.

الكلمات المفتاحية : المعولية الضبابية، توزيع توب-ليون ، معدل التكرار، بيانات مراقبة من النوع الاول.

Abstract:

Fuzzy reliability in statistics is one of the important methods used in various aspects of life that are used in the event of questionable data accuracy, which leads to dealing with ambiguous observations expressed in ambiguous numbers. The use of classical estimation methods to estimate the parameters of the statistical model is not suitable for such observations, and the research problem is represented by the presence of fuzzy numbers known as the operating times of the magnetic resonance device until failure at Al-Rifai Hospital. The research aims to estimate the fuzzy reliability function for the first type of censored data for the Top Lion distribution of the magnetic resonance device at Al-Rifai Hospital for the period (1-3-2022, until 1-3-2024) through some estimation methods maximum likelihood function, partial estimates, Proposed method for estimating the frequency rate and choosing the best reliability based on the least Std. Deviation, and it was found that the fuzzy reliability of the sonar device is (79%) for every (42) days

Keywords: Fuzzy reliability , Topp - Leone distribution , frequency ratio , censored data type

Corresponding Author: Osama Abdulazeez Kadhim

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Introduction:

Fuzzy reliability in statistics is an important topic, and is used in many aspects of life. The term fuzzy is one of the theories that has become popular in recent years because of its ability to deal with biological phenomena whose variables are measured by periods and not by points. An example of this is the fruit category. It is clear that apples, bananas, oranges, etc. belong to that category, and it is clear that eagles, water, mountains, etc. do not belong to the fruit group, while we find that tomatoes, eggplants, etc. have uncertainty about their belonging to the fruit group. Groups that are in doubt about belonging to the fruit category are classified within the fuzzy group, which is considered a new start away from the classical pattern, because it facilitates the way to deal with topics that have fuzzy observations that are suspected of being accurate. The research problem was represented by the presence of inaccurate data about the times of the MRI device's operation until its failure due to the lack of documentation by the device's workers accurately, which makes it data that has a vague quality. Therefore, classical estimation methods are not suitable for estimating the parameters of the statistical distribution, as the estimation of the reliability function depends on the accuracy of the data used in estimating the parameters of the statistical distribution. The research aims to estimate the fuzzy reliability function of the first type censored data subject to the Top-Lyon distribution of the Al-Rifai Hospital MRI device through some estimation methods (maximum likelihood function, partial estimates,) and Proposed estimation method frequency ratio, choosing the best estimation method based on the Standard deviation. Recently, many researches have been published on reliability function estimations using different methods and different failure distributions, including: In 2014, researchers Ghitany, M. E& et al [5], They studied some measures related to the reliability of this distribution, including the hazard rate, the inverse hazard rate, the average remaining life, the expected idle time, and their random arrangements. & In 2015 , researcher Awji [3], estimated the fuzzy reliability for three periods, the first period from 1986 to 2013, the second period from 2013 to 2033, while the third period from 2033 to 2066, for the construction of Mosul Dam. Through the results obtained, it was found that the fuzzy reliability of Mosul Dam at the present time reaches approximately 0.5, while the fuzzy reliability of Mosul Dam will reach approximately 0.1 after twenty years, meaning that the actual risk of collapse will be in 2033, In the year (2019) [4] ,researchers Bantan

& et al created a new family of continuous distributions called the second type of the Topp-Leone-G family. Using an additional shape parameter, a special model was presented for the family based on the inverse exponential distribution, applied to hypothetical and real data, and the result of the work was superior to the proposal on other known models. , In 2016, AL-Shomrani& et al[1], proposed a family of Top-Lyon distributions, The distribution function for the new family, moments and hazard rate were presented., In the year (2016) [12], the researchers Rezaei & et al presented a new set of distributions based on the distribution width with one parameter for the Topp-Leone distribution. They studied the mathematical properties of that distribution and estimated the parameters using the maximum likelihood method. They reached the first generalization proposal for the Topp-Leone distribution, which contains the hazard rate and can be used to model life span. ,In 2023, researchers Muhammad & Abdul Hussein [9], studied the fuzzy reliability of failure times for data on 70 engines in the South Baghdad-Zafaraniya power station, using the failure distribution (Topp Leone- Kumaraswamy), and the researchers reached an estimated average engine operating period until failure of 5.03, which is equivalent to six days and seven hours,

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1- The theoretical aspect

1-1 : Reliability function:

Reliability is defined as the probability of a machine operating successfully without failure during a certain period of time (0, t), meaning that the machine continues to operate after time t has passed, and (t>0) is written mathematically as follows: (Khamis, Ahmed,2016:p292)

$$R(t) = pr\ (T > t\ ) \qquad \ldots (1)$$

Whereas :

t: The machine running time is equal to zero or greater than zero.

T: Random variable of machine operating times until failure occurs..

The mathematical formula for the reliability function is written as follows:

$$R(t_i) = \int_{t_i}^{Max(t_i)} f(u)du \qquad \ldots (2)$$

Properties of the reliability function:

1- $(0 \ \leq R(t_i) \leq 1)$
2- A function inversely proportional to time$[R(t_1\ ) > R(t_2) > \cdots R(Max(t)]$

1-2 :Fuzzy reliability:

Fuzzy sets theory is one of the most important theories that was developed in 1965 by the scientist ZadehIt contributed to the study of a special type of data sets in which each element is characterized by having a degree of belonging that falls within the closed period [0,1], which is determined using a specific belonging function that is consistent with the nature of the data being dealt with. The fuzzy reliability of any vehicle is the fuzzy probability of the vehicle continuing to operate successfully for a future period of time, and the degree of belonging (α) is determined according to a specific belonging function

The reliability function in the continuous distribution is written mathematically as follows :(Nea'ama , Mahdi Wahab & Hafedh Ali Mithe,2021:p735-739)

$$R(k_i t_i) = \int_{t_i}^{Max(t_i)} f(k_i u_i)du \qquad \ldots (3)$$

Whereas:

k_i: Probability value between (0,1)

1-3: type- I -censored Data:

This type is called data Time censored Data ,The censored time is fixed (t_0) and predetermined and varies from one experiment to another for all sample data (sample units) subject to testing, When testing the life of n

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units at zero time, we will watch the work of the sample units until the end of the pre-determined fixed time, meaning the life experience (test) stops, The units that failed the test are m units, and m is a random variable that we cannot know or determine until after the end of time (t_0), and (n-m) is the number of units remaining after time (t_0),The maximum likelihood function for observation data of the first type is: (Glob, Ismail Hadi & shafiq Balsam Mustafa,2013,p319,320).

$$L = \frac{n!}{(n-m)!}\prod_{i=1}^{m} f(t,\theta)[R(t_0)]^{n-m} \qquad \dots(4)$$

Whereas:

$R(t_0)$: Reliability function at time (t_0)

$f(t,\theta)$: Failure density function

$(n-m)$: The number of units remaining after time (t_0)

1-4:the Topp-Leone distribution:

One of the failure models is of great importance in the field of reliability. Interest and research have increased in estimating the optimality of the reliability system for this distribution to indicate the operational life of a number of devices and equipment by representing them with a single function to know the extent of the efficiency of this equipment and its ability to work for long periods of time, Then evaluate these machines and equipment for future planning and development, as the (Topp-Leone) distribution is a continuous distribution of life mentioned by researchers Chester W. Topp and Fred C. Leone when they published their research in the Journal of the American Statistical Association in 1955, It was discovered by Nadarajah and Kotz in 2003 .(Rezaei ,Sadegh & Sadra , Behnam Bahrami&Morad, Alizadehb Saralees Nadarajahc,2016,p2-3)

The probability density function (p.d.f) of the random variable T distributed with a Topp-Leone distribution is: (Al-Shomrani , Ali& Arif, Osama& A Shawky & Hanif, Saman& Shahbaz ,Muhammad Qaiser,2016,p444)

$$\boldsymbol{f}(t_i,\theta) = 2\theta {t_i}^{\theta-1}(1-t_i)(2-t_i)^{\theta-1} \quad \dots(5) \quad 0 < t < 1 \,,\, \theta > 0$$

Whereas: θ : Shape parameter

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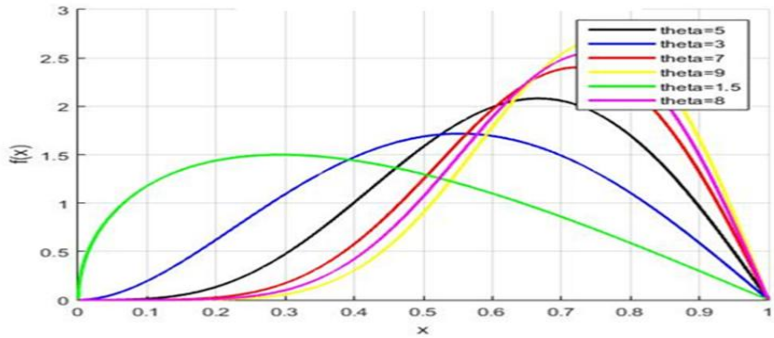

Figure (1) PDF function curve for Topp-Leone distribution

The cumulative distribution function has the following formula:

$$F(t_i) = \int_{o}^{t} 2\theta u^{\theta-1}(1-u)(2-u)^{\theta-1} du$$

$$F(t_i) = t_i^{\theta}(2-t_i)^{\theta} \quad ...(6)$$

,

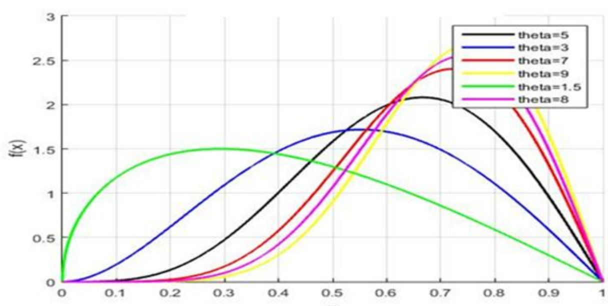

Figure (2) CDF curve of the Topp-Leone distribution

The reliability function is calculated as follows:

$$R(t) = 1 - F(t)$$

$$R(t_i) = 1 - \left[t_i^{\theta}(2-t_i)^{\theta}\right] \quad ...(7)$$

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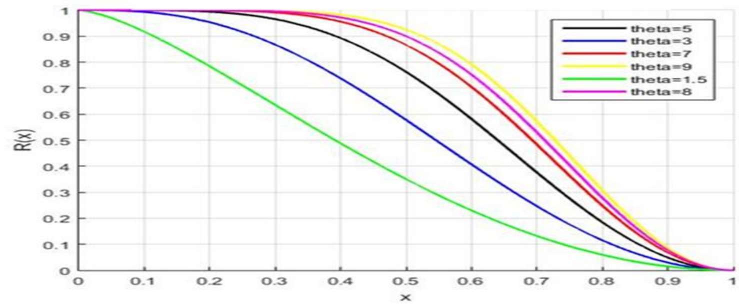

Figure (3) The reliability function curve of the Topp-Leone distribution

The Fuzzy reliability function is calculated as follows:

$$\mathrm{R}(k_i\ \mathrm{t}_i) = 1 - \left[k_i \mathrm{t}_i{}^{\theta}(2 - k_i \mathrm{t}_i)^{\theta}\right] \quad \text{...(8)}$$

1-5 :Estimation methods:

1- Maximum Likelihood Method :

This method is considered one of the important and commonly used estimation methods in estimation because the estimators of the maximum likelihood method possess a set of good properties, including adequacy and consistency sometimes, It has greater accuracy than other methods, as the sample size increases. as it is unbiased when The sample size is large [3], and the goal of this method is to find estimated values for the parameters that we want to estimate by making the maximum likelihood function of the random variables as large as possible, and the greatest likelihood function is symbolized by the symbol (L).(Al-Taie,Abdul Hussein Habib& Al-Qurashi Osama Abdul Aziz Kazim,2019,p210)

If the random variable (T) has a probability density function, as in equation (4), then the Maximum Likelihood function for the independent random variables T_1, T_2,.......T_3 is:(Kamar,Saifaldin Hashim & Aboud Hussam Najm,2016,p534-535)

$$L(T_1, T_2, \dots, T_n, \theta) = f\left(t_{1,}, \theta\right) . \mathrm{f}(t_2, \theta) \dots \mathrm{f}(t_n, \theta)$$

$$\therefore L = \prod_{i=1}^{n} f(t_i\,, \beta, \theta) \quad \dots (9)$$

Since the maximum likelihood function for observational data of the first type is:

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$$L = \frac{n!}{(n-r)} \prod_{i=1}^{m} f(t_i, \theta) \ [1 - F(t_r)]^{m-r} \quad ...(10)$$

By substituting the probability density function of the Topp-Leone distribution into the maximum likelihood function, we obtain the following equation:

$$L = \frac{n!}{(n-r)} \prod_{i=1}^{m} \boldsymbol{2\theta t_i{}^{\theta-1}(1-t_i)(2-t_i)^{\theta-1}} \quad [R(t_m)]^{n-m} \quad ...(11)$$

By substituting the reliability function and assuming k=n!/((n-r)) we get the following:

$$= K\,(2\boldsymbol{\theta})^m \prod_{i=1}^{m} \boldsymbol{t^{\theta-1}(1-t)(2-t)^{\theta-1}} \quad \left[t^{\theta}(2-t)^{\theta}\right]^{n-m}$$

$$= K\,(2\boldsymbol{\theta})^m \prod_{i=1}^{m} \boldsymbol{t^{\theta-1}} \quad \prod_{i=1}^{m}(\boldsymbol{1-t}) \quad \prod_{i=1}^{m}(\boldsymbol{2-t})^{\boldsymbol{\theta-1}} \ \prod_{i=1}^{m} \ \left[t^{\theta}(2-t)^{\theta}\right]^{n-m} \quad ...(12)$$

$$\begin{aligned} lnLf(t,\theta) = mkln(2\theta) + (\theta-1)\sum_{i=1}^{m}\ln(t) + \sum_{i=1}^{m} ln(1-t) \\ + (\theta-1)\sum_{i=1}^{m} ln(2-t) + \theta(n-m)\sum_{i=1}^{m} ln\,t \\ + \theta(n-m)\sum_{i=}^{m} ln\,(2-t) \end{aligned}$$

$$lnLf(t,\theta) = mkln(2\theta) + (\theta\sum_{i=1}^{m} ln(t) - \sum_{i=1}^{m} ln(t)) + \sum_{i=1}^{m} ln(1-t) + (\theta\sum_{i=1}^{m} ln(2-t) - \sum_{i=1}^{m} ln(2-t)) + \theta(n-m)\sum_{i=1}^{m} ln\,t + \theta(n-m)\sum_{i=1}^{m} ln\,(2-t) \quad ...(13)$$

$$\frac{\partial lnLf(t,\theta)}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^{m} ln(t) + \sum_{i=1}^{m} ln(2-t) + \ (n-m)\sum_{i=1}^{m} ln\,t + (n-m)\sum_{i=1}^{m} ln\,(2-t) = 0$$

$$\frac{-n}{\theta} = \sum_{i=1}^{n} ln(t) + \sum_{i=1}^{n} ln(2-t) + (n-m)\sum_{i=1}^{m} ln\,t + (n-m)\sum_{i=1}^{m} ln\,(2-t)$$

$$\hat{\theta} = \frac{-n}{(\sum_{i=1}^{m}\ln(t) + \sum_{i=1}^{m} ln(2-t)) + (n-m)\sum_{i=1}^{m} lnt + (n-m)\sum_{i=1}^{m} ln(2-t)} \quad ...(13)$$

2- **Frequency Ratio :(Suggested method)**

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The estimators by this method depend on solving $\frac{F(t_{i+1},\theta)}{F(t_i,\theta)}$ and equal any non-parametric estimater like $\left(\frac{\frac{3}{8}i}{n+2}\right)$ It is written as follows:

$$\frac{F(t_{i+1},\theta)}{F(t_i,\theta)} = \frac{\frac{3}{8}i}{n+2} \quad ...(14)$$

Substituting the cumulative distribution function into the Topp-Leone distribution in Equation 6 we get:

$$\frac{t_i{}^{\theta}(2-t_{i+1})^{\theta}}{t_i{}^{\theta}(2-t_i)^{\theta}} = \frac{\frac{3}{8}i}{n+2} \quad ...(15)$$

$$(\frac{3}{8}i)\ t_i{}^{\theta}(2-t_i)^{\theta} = (n+2)\ t_i{}^{\theta}(2-t_{i+1})^{\theta}$$

$$ln(\frac{3}{8}i) + \theta \ln t_i + \theta ln(2-t_i) = ln(n+2) + \theta ln\ t_i + \theta \ln(2-t_{i+1})$$

$$\theta[ln(2-t_i) - \ln(2-t_{i+1})] = ln(n+2) - ln(\frac{3}{8}i)$$

$$\hat{\theta} = \frac{\ln(n+2) - ln(\frac{3}{8}i)}{[ln(2-t_i) - \ln(2-t_{i+1})]} \quad ....(16)$$

3- **partial estimates method :**

This method depends on(CDF) , Where pi is the estimator of the(CDF) , The estimators at which the function is at its minimum are found as follows: (Nea'ama , Mahdi Wahab & Hafedh Ali Mithe, 2021,p724-744)

$$F(x,\theta) = x^{\theta}(2-x)^{\theta} \quad ...(17)$$

We equate the estimator p_i with the cumulative distribution function and the following:

$$p_i = x^{\theta}(2-x)^{\theta} \quad ...(18)$$

Taking the logarithm of both sides of the equation as follows:

$$ln\, p_i = \theta\, ln\, x + \theta\, ln(2-x)$$

$$ln\, p_i - \theta\, ln\, x - \theta\, ln(2-x) = 0$$

By squaring both sides of the equation and taking the sum, the equation will be as follows:

$$\sum_{i=1}^{n}[ln\, p_i - \theta\, ln\, x - \theta\, ln(2-x)]^2 = 0 \quad ...(19)$$

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/4.2.2025.15

We derive with respect to the parameter θ :

$$2\sum_{i=1}^{n}[\ln p_i-\theta\ln x-\theta\ln(2-x)]\,[-\ln x-\ln(2-x)]=0$$

Divide by 2:

$$\sum_{i=1}^{n}[\ln p_i-\theta\ln x-\theta\ln(2-x)]\,[-\ln x-\ln(2-x)]=0$$

Let

$$P_i=\left(\frac{i}{n+1}\right)$$

$$\hat{\theta}=\frac{\sum_{i=0}^{n}\ln p_i \ln x-\ln p_i\ln(2-x)}{\sum_{i=0}^{n}[\ln x^2+2\ln x\ln(2-x)+\ln(2-x)^2]}\qquad\ldots(20)$$

2 - The practical side :

2-1 : Data collection :

A sample consisting of (62) observations was provided representing the operating time until failure, starting from 1-3-2022 until 1-3-2024 for the sonar device

The ultrasound device is an ultrasound imaging device that uses high-frequency sound waves that can capture live images of the inside of the human body. It is a technology somewhat similar to radar, which enables the doctor to diagnose problems with various organs of the human body, blood vessels and tissues.

Most ultrasound examinations are performed using an ultrasound device outside the body, although some involve placing a device inside the body. Ultrasound imaging is safe, and there are no known risks associated with it yet. Ultrasound is characterized by not using any type of radiation in imaging, unlike all other medical imaging methods. Ultrasound is used for the purposes of diagnosis, treatment, and follow-up after treatment as well, and to guide surgeons in some surgical procedures such as taking samples and others.

There are several types of sonar devices, including:

1-External sonar.

2-Internal sonar.

3- Binoculars or binoculars.

Our study relied on the quality of the external sonar device to collect data.

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Table (1) Sonar operating times in days until failure

27	20	29	21	22	30	26	22	26	20
28	35	36	24	32	26	23	11	19	26
22	21	16	16	27	17	39	31	23	27
24	28	16	25	23	21	33	29	23	21
28	42								

2-2 : goodness of fit test :

To find out whether the data follow the Tobe-Lyon distribution or not, the Goodness of Fit test It was used according :.to the hypothesis below

H_0: the data are distribution (Topp-Leone)

H_1: the data are not distribution (Topp-Leone)

$$x_c^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \qquad \dots (21)$$

Using Matlab, the value (P-Value = 0.2532) appeared, which is greater than the significance level (0.05), so we accept the null hypothesis, i.e. the data are distributed according to the Topp-Leone distribution.

2-3 : Calculating fuzzy reliability function (k=median=0.5).

Using MATLAB, the values of the fuzzy reliability function were obtained.

Table No. (2) represents the values of the fuzzy reliability.

i	Day(t_i)	$\widehat{R}_{MLE}$	$\widehat{R}_{PCE}$	$\widehat{R}_{PROP}$
1	11	0.96	0.99	0.98
2	16	0.95	0.99	0.96
3	16	0.95	0.99	0.96
4	16	0.95	0.99	0.96
5	17	0.94	0.96	0.91
6	19	0.92	0.93	0.90
7	20	0.91	0.89	0.88
8	20	0.91	0.89	0.88

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/4.2.2025.15

9	21	0.89	0.85	0.86
10	21	0.89	0.85	0.86
11	21	0.89	0.85	0.86
12	21	0.89	0.85	0.86
13	22	0.86	0.83	0.84
14	22	0.86	0.83	0.84
15	22	0.86	0.83	0.84
16	23	0.83	0.82	0.81
17	23	0.83	0.82	0.81
18	23	0.83	0.82	0.81
19	23	0.83	0.82	0.81
20	24	0.82	0.79	0.78
21	24	0.82	0.79	0.78
22	25	0.80	0.76	0.74
23	26	0.78	0.75	0.73
24	26	0.78	0.75	0.73
25	26	0.78	0.75	0.73
26	26	0.78	0.75	0.73
27	27	0.75	0.73	0.70
28	27	0.75	0.73	0.70
29	27	0.75	0.73	0.70
30	28	0.73	0.70	0.76
31	28	0.73	0.70	0.76
32	28	0.73	0.70	0.76
33	29	0.71	0.66	0 .72
34	29	0.71	0.66	0 .72
35	30	0.70	0.66	0 .72
36	31	0.68	0.66	0 .72
37	32	0.66	0.62	0.69
38	33	0.65	0.85	0.66
39	35	0.62	0.55	0.64
40	36	0.59	0.54	0.62
41	39	0.55	0.49	0.58
42	42	0.51	0.44	0.45
	Std. Deviation	.113.	.123.	.257

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Whereas :

t_i: Days of sonar operation until failure.

$\hat{R}_{MLE}$: Reliability function values calculated using maximum likelihood function estimates

$\hat{R}_{PCE}$: Reliability function values calculated using the proposed method estimates recurrence averag

$\hat{R}_{PROP}$: Reliability function values calculated using the Partial estimation method

By looking at Table No. (2) we notice

1- that the maximum likelihood method is the best in estimating the fuzzy reliability function, because the standard deviation of the values of the fuzzy reliability function is smaller than other estimation methods.
2- The proposed method (**Frequency** rate) came in second place in terms of preference.
3- The values of the reliability function decrease with time, and this is what was mentioned alongside the theory.
4- The reliability of the sonar device appeared to be decreasing, and the sonar device can be relied upon at a rate of (96%) when it works for (11) days, and when the device works for (16) days the reliability will decrease and become (95%), and when the device works for (17) days the reliability becomes (94%) and thus the reliability begins to gradually decrease to reach (51%) when the device works for (42) days.
5- Fuzzy reliability of the sonar device is (79%) per (42) days

Conclusions:

1- the maximum likelihood method appeared to be the best, compared to other methods.
2- The proposed method (Frequency rate) came in second place in terms of preference.
3- Fuzzy reliability of the sonar device is (79%) per (42) days.

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Conflicts of Interest

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