



## Resonance Tunneling Current Density of Quantum Well in a-Si/Si<sub>1-x</sub>Gex Super-lattice

Rusul Adnan Al-Wardy

Department of Physics, College of Science, Al-Mustansiriyah University, Baghdad, Iraq.

e-mail: dr.rusuladnan @uomustansiriyah.edu.iq

Received Date: 8 / 10/ 2017

Accepted Date: 9 / 2 / 2018

### الخلاصة

ان الغاية من هذا العمل هو تقديم دراسة نظرية حول كثافة تيار تنفيق الالكترونات عبر منظومة من الشبكة الفائقة لشبه موصل. والمنظومة التي تم أخذها بنظر الاعتبار هي (a-Si/Si<sub>1-x</sub>Gex) وضمن مدى الطاقة  $E > E_g$  ارتفاع حاجز الجهد ( $V_0$ ) ومن هكذا نظام تم تعيين اعتماد كثافة تيار تنفيق الالكترونات على عدد الحواجز (N) والكسر المولي (x). لقد أظهرت نتائج هذا العمل بأنها ليس فقط تضيء المفهوم الفيزيائي لفكرة التنفيق الرنيني في تركيب الحاجز الكهربائي بل أنها تعدت ذلك الى كيفية تصميم النبائط الكمية الإلكترونية والمستندة على تراكيب التنفيق المتعدد الحواجز.

### الكلمات المفتاحية

علوم النانو، الشبكة الفائقة، كثافة تيار التنفيق الرنيني



## Abstract

The purpose of this work is to report a theoretical study for resonance tunneling current density ( $J$ ) of electrons tunneling through semiconductor super-lattice system. The system that we have considered is (a-Si/Si<sub>1-x</sub>Gex) hetero-junction super-lattice within the energy range of  $E < \text{the potential barrier height } (V_0)$ . From such system, we determined the ( $J$ ) dependency on number of barriers ( $N$ ) and the mole fraction ( $x$ ). The results shown in this work not only shed new light on the physics of resonant tunneling in electric barrier structure but are also helpful in designing quantum devices based on multi-barrier tunneling structures.

## Keywords:

Nano-science, Super-lattice, Resonance tunneling current density.



## 1. Introduction

The multi-barrier potential profile needed for resonant tunneling may be realized using semiconductor by variation of doping in the same semiconductor material, but the most successful approach is to use heterojunction consisting of different types of semiconductor materials [1]. A good combination of heterojunction devices is two materials of similar lattice constant, but different in energy gap ( $E_g$ ) [2]. The basic example of an artificial low-dimensional structure is an epitaxial grown QW, which is formed by two heterojunctions or three layers of materials such that the middle layer which has a lowest ( $E_c$ ) for an electron well or the highest ( $E_v$ ) for a hole well. It is well known that a (QW) structure has a finite number of bound states (ground state plus excited states) for electrons and holes. In quantum physics, properties are dominated by the wave-nature of electrons (or holes), related to the quantum mechanical wave function (eigen function) which has a specific energy (eigenvalue) associated with it. Recently, extensive research works have been carried out on heterojunctions between amorphous silicon (a-Si), and crystalline semiconductors, because of their use in many semiconductor devices, such as metal-amorphous silicon FETs, solar cells. However, the physics of amorphous-crystalline heterojunctions is not clearly understood yet [3]. On the other hand, very little work has been done on the (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) heterojunction, although (a-Si) exhibits a series of important properties

(e.g. wide optical band gap, mechanical strength) and (SiGe) is the most well-known semiconductor material, so that the (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) heterojunction could be very useful in many semiconductor devices [4]. In (1969), research on quantum structures was initiated with a proposed of an “engineered” semiconductor super-lattice (SL) by Esaki and Tsu [5]. The (SL) consider an alternating thin layers of two or more materials with different equilibrium lattice constants [6], coherent layers on nanometer thickness scale may be deposited by molecular-beam epitaxial or metal-organic vapor deposition, so thus building up a super periodic structure on layer scale [7]. When electrons are confined within a semiconductor thin film with a thickness of the order of the de Broglie wavelength, the wave nature of the electrons becomes important [8].

In classical mechanics, carriers are completely confined by the potential wells, only those carriers with excess energy higher than the barriers can escape. But in quantum mechanics point of view, a particle can tunnel through a potential barrier of height higher than that of the incident particle. The necessary conditions for tunneling are: (1) Occupied energy states exist on the side from the electron tunnels. (2) Unoccupied energy states exist at the same energy level on the side to which the electron can tunnel. (3) The tunneling potential barrier height is low and the barrier width is small enough that there is a finite tunneling probability,



and (4) the momentum is conserved in the tunneling process [2]. Tunneling through a double barrier was first solved in the Wentzel-Kramers-Brillouin (WKB) approximation by David Bohm in (1951)[9].

In this paper, we have considered the tunneling current density of ground state level ( $J_0$ ) in the (MBS) as a comprehensive manner by using the energy of incident particles is less than ( $V_0$ ) with applied bias voltage ( $V_{bias}$ ). In addition, we have examined the dependence of resonant tunneling current density on various factors like the number of barriers ( $N$ ) and the mole fraction ( $x$ ) of the system.

## 2.Theory

It will be better, if we consider a model to understand the tunneling of electron through multi barrier semiconductor hetero-junction. In such model, we have a super-lattice structure contains alternately semiconductor hetero-junction of (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) for ( $x=0.2$  to 0.8). These two materials have similar band structure, but different energy gap, where the (Si<sub>1-x</sub>Ge<sub>x</sub>) has the small gap to form the well, and (a-Si) has the large energy gap to form the barrier. In this structure, the barriers thickness ( $b$ ), thickness of well ( $a$ ) and the super-lattice period ( $c$ ) are related with each other ( $c=a+b$ ). If these systems consist of ( $N$ ) barriers then one will find ( $N-1$ ) wells as shown in Fig. (1):

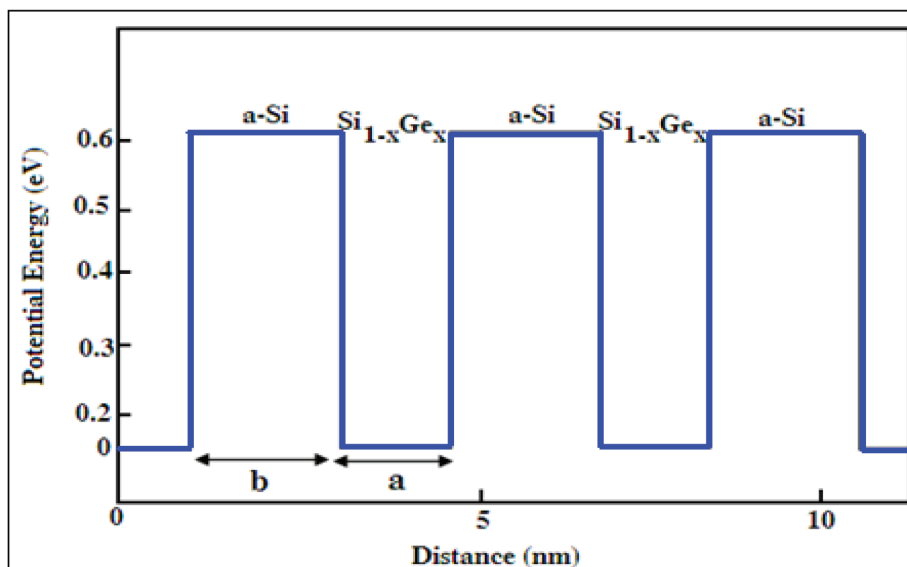


Fig. (1): Scheme of the periodic potential of (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) super-lattice.



Suppose an electron of energy ( $E$ ) and mass ( $m^*$ ) is incident from the left of an arbitrary shaped, one-dimensional, continuous potential  $V(x)$ , in order to calculate the probability of transmission through the double barriers structure in the presence of an applied field, dividing the barrier into steps of infinitesimal width [10], as shown in Fig. (2).

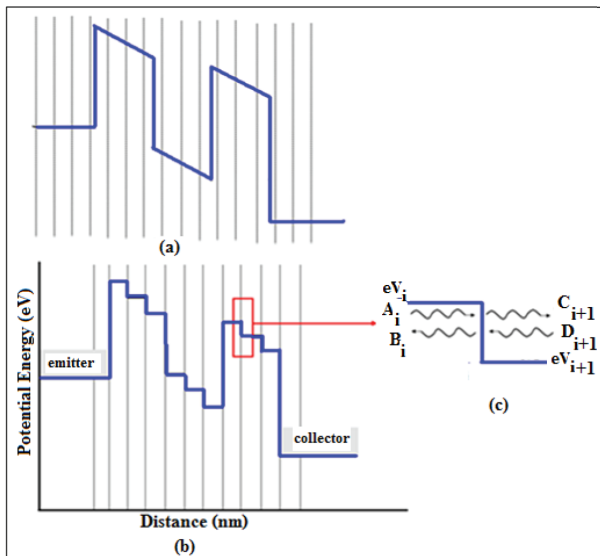


Fig. (2):- (a) Divide the active region into  $N$  slices, (b) The potential through the (RTD) conduction band is quantized into small steps for the transmission matrix calculation, (c) In region ( $i$ ) the potential energy is ( $eV_i$ ), and in region ( $i+1$ ) the potential energy is ( $eV_{i+1}$ ) [11].

To solve the transmission probability, the Propagation-Matrix Method (PMM) is used, this method has among its advantages computational simplicity and good accuracy. The transmission probability  $T(E_i)$  is given by [12]:

$$T(E_i) = \frac{m^*_i}{m^*_{i+1}} \frac{k_{i+1}}{k_i} \frac{|C|^2}{|A|^2} \dots \dots \dots (1)$$

Where  $m^*_i$  and  $m^*_{(i+1)}$  are the effective

electron masses in the regions ( $i$ ) and ( $i+1$ ), respectively,  $k_i$  and  $k_{(i+1)}$  are the electron wave numbers in the regions ( $i$ ) and ( $i+1$ ), respectively,  $C$  and  $A$  are the transmission coefficients for an electron entering the left barrier into the collector.

If electrons ( $n$ ) in the system with area ( $A$ ), the current density ( $J$ ) in the system:

$$J = \frac{I}{A} = ne\bar{V} \dots \dots \dots (2)$$

$$\bar{V} = \frac{1}{h} \cdot \frac{dE_i}{dk_i} \dots \dots \dots (3)$$

$$n = \int_0^\infty D(E) \cdot k_B T \cdot \frac{\partial P(E)}{\partial E} dE \dots \dots \dots (4)$$

$$\frac{\partial P(E)}{\partial E} = -P(E) \frac{(1 - P(E))}{k_B T} \dots \dots \dots (5)$$

$$n = \int_0^\infty D(E) \cdot P(E) \cdot (1 - P(E)) dE \dots \dots \dots (6)$$

$$J = \frac{ek_B T}{2\pi^2 h^3} \cdot \int_0^\infty P(E) \cdot (1 - P(E)) dE \frac{dE_i}{dk_i} \dots \dots \dots (7)$$

The net current ( $J$ ) is due to the electrons going from the left-hand side with energy ( $E = E_i + (\hbar^2 k_i^2)/(2m^*)$ ) and from the right-hand side with energy ( $E' = E + eEl = E + eV$ ), where  $E^-$  is the electric field and  $l$  is the distance between the contacts on the two sides, ( $J$ ) is obtained by Esaki formula [13]:

$$J = \frac{em^* k_B T}{2\pi^2 h^3} \left( 1 - e^{\frac{-eV}{k_B T}} \right) \int_0^\infty T(E_i) \cdot e^{\frac{(E_i - E_i)}{k_B T}} dE_i \dots \dots \dots (8)$$



### 3 .Results and Discussion:

Fig. (3) Shows the J-V characteristics of (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) super-lattice with (x=0.5,a=b=1.2nm,N=7). The net tunneling current, at zero applied voltage, is zero, as shown in Fig. (3) region (a), when a bias voltage is applied (V<sub>bias</sub>=0.1 V) to the structure, a small current flow. The current increases strongly as

(J<sub>p</sub>=4.558\*10<sup>(-3)</sup> A/nm<sup>2</sup> ) at peak voltage(V<sub>p</sub>=0.9 V), see region (b), when the energy of the incident electron coincides with energy one of confined states. Further increase of the bias voltage detunes the resonance and current decreases sharply as

(J<sub>v</sub>=5.654\*10<sup>(-4)</sup> A/nm<sup>2</sup> ) at valley voltage (V<sub>v</sub>=1.2 V), see region (c) Fig. (3), and creates the negative differential resistance (NDR). Still with further increase of the voltage (V<sub>bias</sub>=1.3 V), the excess current starts to dominant, as region (d) of Fig. (3).

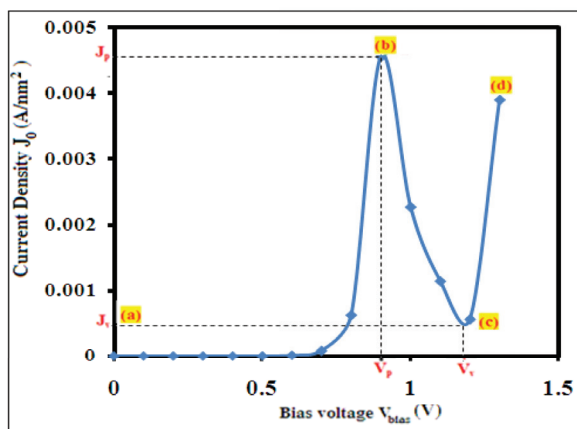


Fig. (3): J-V characteristics of (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) super-lattice with (x = 0.5, a = b = 1.2nm, N = 7).

Fig. (4) corresponds to the electrical characteristics which are shown in Fig. (3). The Fermi-level is constant across the junction and no filled states exist (electrons) on either side of the junction, while below the Fermi-level there are no empty states available on either side of the junction, as shown in Fig. (4a). Therefore, the net tunneling current at zero applied voltage is zero. Physically, the current maxima occur at certain voltages such that the resonant energies (E<sub>0</sub> state) approach the Fermi energy of the electrons at the left of the barrier [14], see Fig. (4b). The decrease in current with the increase in applied voltage results from the fact that this band of common energies decreases, see Fig. (4c). The decreasing current after the peak gives rise to the negative differential resistance (NDR) [15, 16]. Still with further increase of the voltage, the excess current start to dominant, see Fig. (4d), whenever the resonant energies (E<sub>1</sub> state) approach the Fermi energy of the electrons at the left of the barrier.

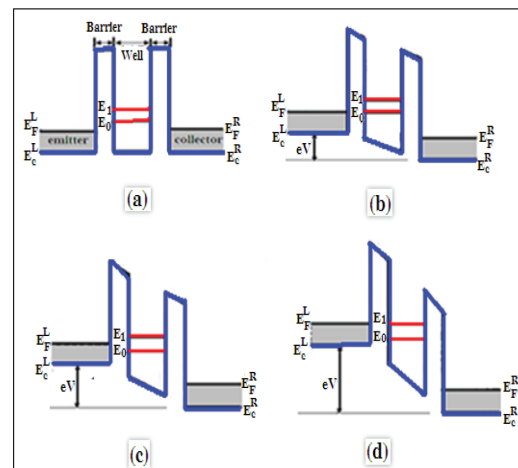


Fig. (4): Energy band diagram of (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) super-lattice with (x = 0.5, a = b = 1.2nm, N = 3) under various biases, (a) near zero bias, (b) Resonant tunneling through (E<sub>0</sub>state) at V<sub>bias</sub> = 0.9V, (c) (E<sub>0</sub>state) below (E<sub>F</sub>) at V<sub>bias</sub> = 1.2V, (d) Resonant tunneling through (E<sub>1</sub>state) at V<sub>bias</sub> = 1.3V .



Fig. (4) corresponds to the electrical characteristics which are shown in Fig. (3). The Fermi-level is constant across the junction and no filled states exist (electrons) on either side of the junction, while below the Fermi-level there are no empty states available on either side of the junction, as shown in Fig. (4a). Therefore, the net tunneling current at zero applied voltage is zero. Physically, the current maxima occur at certain voltages such that the resonant energies ( $E_o$  state) approach the Fermi energy of the electrons at the left of the barrier [14], see Fig. (4b). The decrease in current with the increase in applied voltage results from the fact that this band of common energies decreases, see Fig. (4c). The decreasing current after the peak gives rise

to the negative differential resistance (NDR) [15, 16]. Still with further increase of the voltage, the excess current start to dominant, see Fig. (4d), whenever the resonant energies ( $E_1$  state) approach the Fermi energy of the electrons at the left of the barrier.

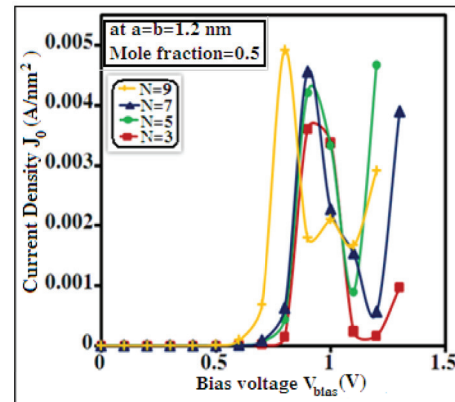


Fig. (5): The current density ( $J_0$ ) versus bias voltage ( $V_{bias}$ ) by varying number of barriers.

Table (1): the values of ( $J_o$ ) for system (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) with ( $x=0.5$ ,  $a=b=1.2$ nm) and varying bias voltage from (0.1,0.5,0.9)V,  $N=3,5,7,9$ .

	$J_0(A/nm^2)$			
a-Si/Si <sub>1-x</sub> Ge <sub>x</sub> Superlattice ( $x = 0.5$ , $a = b = 1.2$ nm)	$N = 3$	$N = 5$	$N = 7$	$N = 9$
At $V_{bias} = 0.1$ V	$1.98 \cdot 10^{-10}$	$3.26 \cdot 10^{-10}$	$4.78 \cdot 10^{-10}$	$5.35 \cdot 10^{-10}$
At $V_{bias} = 0.5$ V	$4.61 \cdot 10^{-7}$	$1.31 \cdot 10^{-6}$	$1.56 \cdot 10^{-6}$	$1.75 \cdot 10^{-6}$
At $V_{bias} = 0.9$ V	$3.61 \cdot 10^{-3}$	$4.22 \cdot 10^{-3}$	$4.56 \cdot 10^{-3}$	$4.92 \cdot 10^{-3}$

Also shown that ( $J_o$ ), at ( $V_{bias}=V_o$ ) versus bias voltage ( $V_{bias}$ ) for (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) super-lattice structure with ( $N=3, a=b=1.2$ nm) and different in mole fraction of the composite about ( $x=0.2, 0.4, 0.5$  and  $0.6$ ). It is clear that when the mole fraction of the composite increases the resonant

Fig. (6) shows the resonant tunneling current density for ground state level ( $J_o$

) versus bias voltage ( $V_{bias}$ ) for (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) super-lattice structure with ( $N=3, a=b=1.2$ nm) and different in mole fraction of the composite about ( $x=0.2, 0.4, 0.5$  and  $0.6$ ). It is clear that when the mole fraction of the composite increases the resonant





tunneling current density ( $J_0$ ) will decrease, this behavior comes in agreement with the results in experiment by S. Mukherjee et al [18], and the values of ( $J_p$ ) for each ( $x$ ) will be determined as the corresponding value of bias voltage ( $V_{bias}$ ) that is equal to the barrier high ( $V_0$ ). The explanation of this behavior is expected when increasing the mole fraction of (Ge) it will cause an increase in the barrier height ( $V_0$ ) due to poor tunneling through the barriers, because the necessary conditions for tunneling occur when potential barrier height is low and the energy gap of quantum wells. See Table (2).

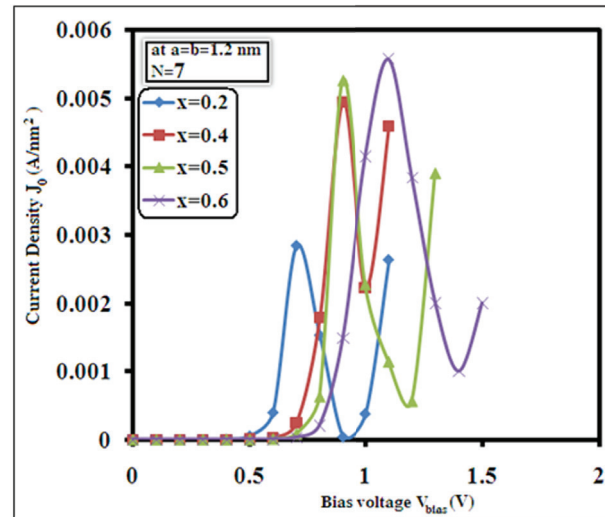


Fig. (6): The current density ( $J_0$ ) versus bias voltage ( $V_{bias}$ ) by varying the mole fraction of the composite.

Table (2): the values of the resonant tunneling current density ( $J_0$ ) for system (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) with ( $N=7$ ,  $a=b=1.2\text{nm}$ ) and varying bias voltage from (0.1,0.5,0.9)V,  $x=0.2,0.4,0.5,0.6$ .

	$J_0 (\text{A}/\text{nm}^2)$			
a-Si/Si <sub>1-x</sub> Ge <sub>x</sub> Superlattice ( $N=3, a=b=1.2\text{nm}$ )	$x=0.2$	$x=0.4$	$x=0.5$	$x=0.6$
At $V_{bias}=0.1\text{ V}$	$1.89 \cdot 10^{-8}$	$1.54 \cdot 10^{-9}$	$4.78 \cdot 10^{-10}$	$1.70 \cdot 10^{-10}$
At $V_{bias}=0.5\text{ V}$	$5.53 \cdot 10^{-5}$	$4.75 \cdot 10^{-6}$	$1.55 \cdot 10^{-6}$	$5.38 \cdot 10^{-7}$
At $V_{bias}=0.9\text{ V}$	$5.04 \cdot 10^{-3}$	$4.97 \cdot 10^{-3}$	$4.55 \cdot 10^{-3}$	$1.49 \cdot 10^{-3}$

Also the transmission coefficient ( $T(E)$ ) will be decrease so ( $J_0$ ) will decrease as Eq. (8).

Table (3) is put to illustrate how ( $J_p$ )

Table (3): the values of ( $J_p$ ) for system (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) with ( $N=7$ ,  $a=b=1.2\text{nm}$ ) at ( $V_{bias}=V_0$ ).

a-Si/Si <sub>1-x</sub> Ge <sub>x</sub> Superlattice ( $N=3, a=b=1.2\text{nm}$ )	$(V_{bias}=V_0)V$	$J_p (\frac{\text{A}}{\text{nm}^2})$
$x=0.2$	0.709	$2.84 \cdot 10^{-3}$
$x=0.4$	0.841	$4.95 \cdot 10^{-3}$
$x=0.5$	0.901	$5.26 \cdot 10^{-3}$
$x=0.6$	0.957	$5.58 \cdot 10^{-3}$





#### 4. Conclusions

In this work, a theoretical model for computation of resonant tunneling current density ( $J_o$ ) has been studied to (a-Si/Si<sub>1-x</sub>Ge<sub>x</sub>) with (x=0.2 to 0.8) super-lattice hetero-junction structure. We have found that when increasing the number of barriers, ( $J_o$ ) will increase. while ( $J_o$ ) will be decreased with the increasing of a mole fraction (x). Also, We have found that that the effect of increasing the bias voltage will shift the quantized levels inside the well and the energy levels, which are tunable by some voltage, would exhibit peak current-voltage (I-V) characteristic.

#### References

- [1] W. Li, H. Liu, S. Wang, S. Chen and Z. Yang, "design of high performance Si/SiGe heterojunction tunneling FETs with a T-shaped gate", nanoscale research letters 12:198, (2017). <http://doi.org/10.1186/s11671-017-1958-3>.
- [2] Sze S.M., and Kwok K.Ng, "Physics of Semiconductor Devices", Third Edition, John Wiley & Sons. Inc. (2007).
- [3] Georgoulas, Magafas N. L., and Thanailakis A, "A study of a-Si/c-Si (n) Isotype Heterojunctions" Active and Passive Elec. Comp Vol.16:p.55-64. (1993).
- [4] A. Ramesh, "boron delta-doping dependence on Si/SiGe resonant interband tunneling diodes grown by chemical vapor deposition", IEEE Transactions on electron Devices, vol.59, no.3 (2012).
- [5] Esaki L. and Tsu R., "super-lattice and negative differential conductivity in semiconductors", IBM J. Res. Develop.14:61-5, (1970).
- [6] C. L. Roy, "Tunneling of Electrons Through Semiconductor Super lattices", Bull. Master. Sci., Vol.25, No.6, P.469-471, (2002).
- [7] Kittel C., "Introduction to Solid State Physics", 8<sup>th</sup> edition, John Wiley & Sons., Inc., (2004).
- [8] Cressler J. D., "Silicon Hetero structure Devices", Taylor & Francis Group, LLC, (2008).
- [9] Bohm D., "Quantum Theory", Prentice-Hall, New York, (1951).
- [10] J. Singh, "Electronic and Optoelectronic Properties of Semiconductor Structures", Cambridge University Press, Ch.1-3, (2003).
- [11] Zozoulenko I.V., "High performance Si/Si<sub>1-x</sub>Ge<sub>x</sub> Resonant Tunneling Diodes", IEEE Electron Device Letters, Vol.22, No.4, (2001).
- [12] Levi A.F.J., "Applied Quantum Mechanics", University of Southern California, A. Levie, (2003).
- [13] Sun J. P., Haddad G.I., "Resonant Tunneling Diodes: Models and Properties", Proceeding of the IEEE, Vol.86, No.9, (1998).



- [14] Tsu R. and Esaki L., "Tunneling in a finite Super lattice", Appl. Phys. Lett. Vol.22, No.11, P.562-564, (1973).
- [15] Y. C. lin, R. K. Ghosh, "Atomically thin resonant tunnel diodes built from synthetic van der Waals hetero-structures", nature communication DOI:10.1038/ncomms8311, (2015).
- [16] A. J. Trindadea and L. Pereira," bulk hetero-junction organic solar cell area-dependent parameter fluctuation", international journal of photo energy, volume 2017 (2017), <http://doi.org/10.1155/2017/1364152>.
- [17] F. Sattari, "calculation of current density for grapheme super lattice in a constant electric field", J. Theor. Appl. Phys. (2015) 9:81-87. <http://doi.org/10.1007/s40094-015-0165-9>.
- [18] S. Mukherjee et al, " theoretical computation of transmission coefficient of double quantum well triple barrier structure in presence of electric field", international journal of soft computing and engineering (IJSCE) ISSN:2231-2307, volume 1, (2011).