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Nihimat Iyebuhola Aleshinloye
Department of Mathematics and Statistics, Kwara State University, Malete P.M.B. 1530, Ilorin, Nigeria

Samuel Adewale Aderoju
Department of Mathematics and Statistics, Kwara State University, Malete P.M.B. 1530, Ilorin, Nigeria

Alfred Adewole Abiodun
Department of Statistics, University of Ilorin, P.M.B. 1515, Ilorin, Nigeria

Bako Lukmon Taiwo
Department of Mathematics and Statistics, Kwara State University, Malete P.M.B. 1530, Ilorin, Nigeria

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ORIGINAL STUDY

A New Generalized Gamma-Weibull Distribution and Its Applications

Nihimat I. Aleshinloye [a], Samuel A. Aderoju [a,*], Alfred A. Abiodun [b], Bako L. Taiwo [a]

[a] Department of Mathematics and Statistics, Kwara State University, Malete, P.M.B. 1530, Ilorin, Nigeria
[b] Department of Statistics, University of Ilorin, P.M.B. 1515, Ilorin, Nigeria

Abstract

In this paper, a New Generalized Gamma-Weibull (NGGW) distribution is developed by compounding Weibull and generalized gamma distribution. Some mathematical properties such as moments, Rényi entropy and order statistics are derived and discussed. The maximum likelihood estimation (MLE) method is used to estimate the model parameters. The proposed model is applied to two real-life datasets to illustrate its performance and flexibility as compared to some other competing distributions. The results obtained show that the new distribution fits each of the data better than the other competing distributions.

Keywords: Weibull, Generalized gamma, Moments, Hazard function, Goodness-of-fit

1. Introduction

Lifetime models have witnessed some improvement due to their rapid expansion concerning methodology, theory, fields of application and constant research for decades [1]. Some of the researchers (see [2–6]) have revealed that many classical statistical distributions can be made more flexible to capture several real-life problems by the introduction of an additional parameter using various methods.

Parametric statistical methods used for modelling data require that such data follow some specific statistical distributions. In practice, however, many classical statistical distributions have been observed to be inadequate in modelling certain kinds of datasets. Weibull distribution is an important lifetime distribution and has extensive applications in survival and reliability studies. The distribution has undeniable popularity in probability and statistics due to its versatility in modelling real-world data [7]. In recent years, it has been observed that many well-known distributions used to model such datasets do not offer enough flexibility to provide adequate fit. Hence, researchers and authors continue to work on distributions that can produce more flexible models and as such, they have introduced distributions that are the generalization of the existing ones. However, the search and need for more flexible distributions continue.

Lindley distribution [8], which is a two-component mixture of exponential and gamma distributions. This is one of the foundation distributions in the context of Fiducial and Bayesian Statistics. Many authors have studied the Lindley distribution in great detail and have attempted to modify it using various methods to come up with other distributions that have been known to be more flexible in terms of their properties.

The Lindley distribution was however discovered to have some setbacks hence [9], studied the distribution and its applications in great detail. Their study revealed that there are some occasions in which the Lindley distribution may not be suitable from a theoretical or application point of view. Therefore [10], proposed Power Lindley distribution using the power transformation technique. They observed that the distribution is more flexible.

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* Corresponding author.
E-mail address: samueladeroju77@gmail.com (S.A. Aderoju).

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[11] proposed two-parameter Rama distribution (TPRD) which was coined from the Lindley distribution. This distribution is a mixture of exponential (θ) and gamma $(4, \theta)$ distributions. The two-parameter Rama distribution yielded a better fit than some of the existing distributions [3]. introduced a two-parameter distribution called the Samade Probability Distribution (SPD), which is a mixture of exponential (θ) and gamma $(4, \theta)$ distributions. The distribution was applied to real-life datasets and compared with other distributions to determine its performance. The distribution was found to provide a better fit compared to some contemporary distributions.

[12] introduced a Weibull-Generalized Gamma (W-GG) distribution, which is obtained by compounding Weibull and generalized gamma distributions. The model contains some existing lifetime distributions as its special cases such as the Weibull, Lindley, and exponential distributions, among others. The model is capable of modelling various shapes of ageing and failure criteria.

[13] proposed a distribution called the Nwipke Distribution. The distribution is a one-parameter distribution that was derived using a two-component additive mixture of gamma $(\theta, 6)$ and exponential distributions. They also applied the distribution to real-life data and compared its goodness-of-fit with some existing distributions. Other models have been developed over time [14]. extended Hamza distribution, which was introduced earlier by [15]. [2] studied the Lomax-Kumaraswamy distribution and detailed some insightful qualities of the model [6]. introduced a power generalized Akash distribution and studied the properties and application of the model to real-life data [16]. introduced Maxwell–Weibull Distribution with its properties discussed. The model flexibility in Modeling Lifetime datasets was extensively discussed [17]. proposed a novel trigonometric version of the Weibull model called a new alpha power cosine-Weibull (NACos-Weibull) distribution. Certain distributional properties of the NACos-Weibull model were derived and the estimators of the model were derived by implementing the maximum likelihood approach. They demonstrated the effectiveness of the NACos-Weibull model through applications from the hydrological and engineering sectors.

The aim of this paper is to introduce a New two-parameter lifetime distribution, which is expected to bring more flexibility in modeling survival data. The rest of the paper is organized as follows: The materials and method are presented in Section 2, where the New Generalized Gamma-Weibull Distribution (NGGW) is derived and studied in detail. Some mathematical properties of the distribution are presented in Section 3. In section 4, the parameters estimation of the NGGW distribution, using the method of maximum likelihood is presented. The proposed model is applied to data sets in Section 5. Concluding remarks are given in Section 6.

2. Materials and method

Consider the pdf of a kth component additive mixture distribution for a random variable X defined as

$$f(x,\theta)=\sum_{j=1}^{k} f_j(x,\theta_j)\omega_{j,} \sum_{j=1}^{k}\omega_j=1, \tag{1}$$

where θ_j is the vector of parameters for the mixture model and $\omega_{j,}$ is the mixture proportion.

Theorem 1: Suppose a random variable X follows a New Generalized Gamma-Weibull distribution denoted as $X|\theta,\lambda \sim NGGW(\theta,\lambda)$, then the pdf of X is given as:

$$f_1(x;\theta,\lambda)=\frac{\lambda\theta^3}{12(\theta^2+10)}\left(\lambda\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^{\lambda}}; \quad x>0,\theta>0,\lambda>0 \tag{2}$$

Proof 1: Recall the pdf of a kth component additive mixture distribution given in (1), then the pdf in (2) is a two-component mixture of Weibull (θ,λ) and Generalized Gamma $(6,\theta,\lambda)$ with the mixing proportion, $\pi=\frac{\theta^2}{\theta^2+10}$ and can be written as:

$$f_1(x;\theta,6,\lambda)=\pi g_1(\theta,\lambda)+(1-\pi)g_2(6,\theta,\lambda),$$

where

$$g_1(x;\ \theta,\lambda)=\lambda\theta x^{\lambda-1}e^{\theta x^{\lambda}}, and$$

$$g_2(x:6,\theta,\lambda)=\frac{\lambda\theta^6x^{5\lambda}x^{\lambda-1}e^{\theta x^{\lambda}}}{120}$$

This gives

$$f_1(x|\theta,\lambda)=\frac{\theta^2}{\theta^2+10}\left(\lambda\theta x^{\lambda-1}e^{-\theta x^{\lambda}}\right)+\frac{10}{\theta^2+10}\left(\frac{\lambda\theta^6x^{5\lambda}x^{\lambda-1}e^{\theta x^{\lambda}}}{120}\right)$$

$$=\frac{120\theta^3\lambda+10\theta^6\lambda x^{5\lambda}}{120(\theta^2+10)}x^{\lambda-1}e^{-\theta x^{\lambda}}$$

$$=\frac{12\theta^3\lambda+\theta^6\lambda x^{5\lambda}}{12(\theta^2+10)}x^{\lambda-1}e^{-\theta x^{\lambda}}$$

$$= \frac{\lambda\theta^3}{12(\theta^2+10)}\left(\lambda\theta^3 x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}$$

Therefore, the pdf given in (2) is the probability density function of the NGGW distribution.

Note that when λ assumes the value of 1, the NGGW distribution reduces to the One-Parameter Nwipke distribution proposed by [13].

2.1. Validity check of the New Generalized Gamma-Weibull distribution

Theorem 2: Suppose a random variable X follows a New Generalized Gamma-Weibull distribution denoted $X|\theta,\lambda \sim TwWGG(\theta,\lambda)$, then the pdf of X is a true pdf if it satisfies the condition that

$$\int_{-\infty}^{\infty} f_1(x;\theta,\lambda)dx = 1$$

Proof 2: Since $x\in R$, and $f_1(x;\theta,\lambda)$ to be $\in (0,\infty)$, then

$$\int_{-\infty}^{\infty} f_1(x;\theta,\lambda)dx = \int_0^{\infty}\frac{\lambda\theta^3}{12(\theta^2+10)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}dx$$

$$= \frac{\lambda\theta^3}{12(\theta^2+10)}\int_0^{\infty}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}dx$$

$$= \frac{\lambda\theta^3}{12(\theta^2+10)}\left(\frac{12(\theta^2+10)}{\lambda\theta^3}\right)$$

$$= 1$$

Hence, the pdf of the random variable X is a true pdf.

Fig. 1 presents the shapes of the pdf of the proposed NGGW distribution for varying values of θ and λ while Fig. 2 presents the shapes of its cdf for varying values of the parameters.

Consequently, The cdf of the NGGW distribution is given as

$$F_1(t;\theta,\lambda) = \int_{t=0}^{x} f_1(t;\theta,\lambda)dt$$

$$= \int_{t=0}^{x}\frac{\lambda\theta^3}{12(\theta^2+10)}\left(\lambda\theta^3t^{5\lambda}+12\right)t^{\lambda-1}e^{-\theta t^\lambda}dt$$

The graphs of the CDF of NGGW are given below.

$$\therefore F_1(x;\theta,\lambda) = \left[1-\left(1+\frac{\theta^5x^{5\lambda}+5\theta^3x^{3\lambda}(4+x^\lambda)+60x^\lambda\theta(\theta x^\lambda+2)}{12(\theta^2+10)}\right)e^{-\theta x}\right];\ if\ x,\theta,\lambda>0 \quad (3)$$

3. Mathematical properties of then NGGW distribution

Some of the mathematical properties of the NGGW distribution are derived and presented in this section. The properties include the moments, reliability analysis, entropy and the order statistics.

3.1. Moments

The rth moment of a random variable X with the NGGW distribution is obtained by:

$$E(x^r) = \mu_r' = \int_0^{\infty} x^r f_1(x;\theta,\lambda)dx \quad (4)$$

$$= \int_0^{\infty} x^r\left[\frac{\lambda\theta^3}{12(\theta^2+10)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}\right]dx$$

$$= \frac{\lambda\theta^3}{12(\theta^2+10)}\left[\int_0^{\infty} x^r\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}dx\right]$$

$$= \frac{\lambda\theta^3}{12(\theta^2+10)}\left[\int_0^{\infty}\theta^3x^{6\lambda+r-1}e^{-\theta x^\lambda}dx+\int_0^{\infty}12x^{r+\lambda-1}e^{-\theta x^\lambda}dx\right]$$

$$= \frac{\lambda\theta^3}{12(\theta^2+10)}\left[\frac{\theta^3\Gamma(r+6\lambda)!}{\lambda\theta^{r+6\lambda}}+12\frac{12\Gamma(r+\lambda)!}{\lambda\theta^{r+\lambda}}\right]$$

$$= \frac{\lambda\theta^3}{12(\theta^2+10)}\left[\frac{\theta^3\Gamma(r+6\lambda-1)!}{\lambda\theta^{r+6\lambda}}+12\frac{12\Gamma(r+\lambda-1)!}{\lambda\theta^{r+\lambda}}\right], \quad (5)$$

where r is a positive integer.

From (5), we can express the first four moments as $(r=1,2,3,4)$ as follows:

$$\mu_1' = \frac{\theta^{-1/\lambda}\left(\Gamma\left[6+\frac{1}{\lambda}\right]+12\theta^2\Gamma\left[\frac{1}{\lambda}\right]\right)}{12(10+\theta^2)\lambda}$$

$$\mu_2' = \frac{\theta^{-2/\lambda}\left(\Gamma\left[6+\frac{2}{\lambda}\right]+12\theta^2\Gamma\left[\frac{2+\lambda}{\lambda}\right]\right)}{12(10+\theta^2)}$$

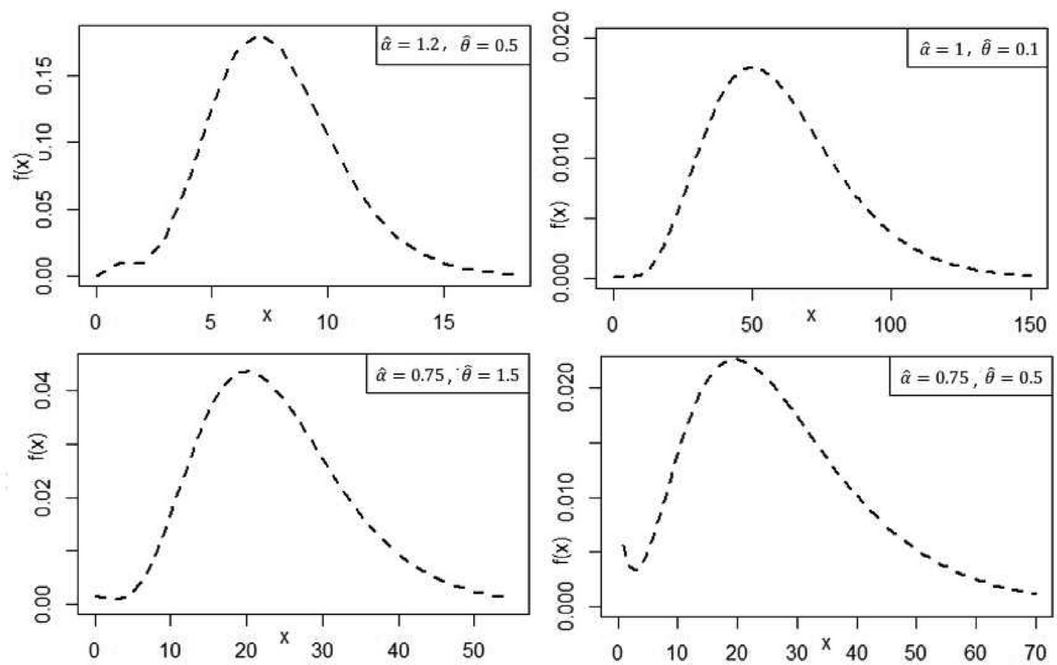

Fig. 1. Graph of the PDF of NGGW Distribution for varying values of parameters.

$$\mu_3' = \frac{\theta^{-3/\lambda}\left(\Gamma\left[6+\frac{3}{\lambda}\right] + 12\theta^2\Gamma\left[\frac{3+\lambda}{\lambda}\right]\right)}{12\left(10+\theta^2\right)}$$

$$\mu_4' = \frac{\theta^{-4/\lambda}\left(\Gamma\left[6+\frac{4}{\lambda}\right] + 12\theta^2\Gamma\left[\frac{4+\lambda}{\lambda}\right]\right)}{12\left(10+\theta^2\right)}$$

3.2. Renyi's entropy

The Renyi's Entropy of NGGW is given as

$$R_H(x;\theta,\lambda) = \frac{1}{1-p}\log\int_0^\infty \left[f_1(x|\theta,\lambda)\right]^p dx$$

$$R_H(x;\theta,\lambda) = \frac{1}{1-p}\log\int_0^\infty \left[\frac{\lambda\theta^3}{12\left(\theta^2+10\right)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}\right]^p dx$$

$$= \frac{1}{1-p}\log\int_0^\infty \left[\frac{\lambda\theta^{3p}}{\left[12\left(\theta^2+10\right)\right]^p}\left(\theta^3x^{5\lambda}+12\right)^p x^{p\lambda-p}e^{-\theta p x^\lambda}dx\right]$$

$$= \frac{1}{1-p}\log\int_0^\infty \frac{\lambda\theta^{3p}}{\left[12\left(\theta^2+10\right)\right]^p}\left(1+\frac{\theta^3x^{5\lambda}}{12}\right)^p x^{p\lambda-p}e^{-\theta p x^\lambda}dx$$

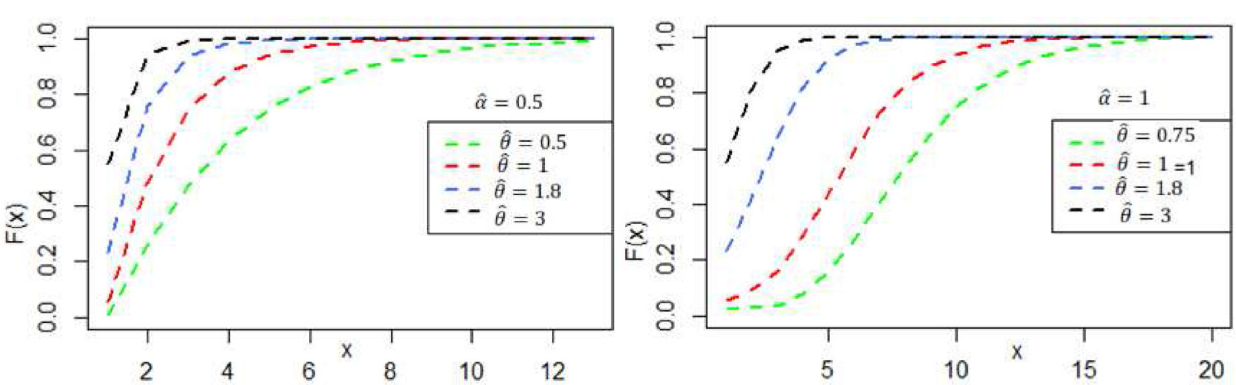

Fig. 2. Graph of the CDF of the NGGW Distribution for varying values of parameters.

$$=\frac{1}{1-p}log\frac{\lambda\theta^{3p}}{\left[12\left(\theta^2+10\right)\right]^p}\sum_{j=1}^{\infty}\binom{p}{j}\left(\frac{\theta^3}{12}\right)^j\int_0^{\infty}x^{5\lambda j+1}e^{-\theta p x^{\lambda}}dx$$

$$=\frac{1}{1-p}log\frac{\lambda\theta^{3p}}{\left[12\left(\theta^2+10\right)\right]^p}\sum_{j=1}^{\infty}\binom{p}{j}\left(\frac{\theta^3}{12}\right)^j\frac{\Gamma(5\lambda j+1)}{(\theta p)^{5\lambda j+1}} \quad (6)$$

3.3. Order statistics

Let X_1, X_2, …,X_n denote the random sample from NGGW distribution, then the density function of the kth order statistics, denoted as $f_{x(k)}(x;\theta,\lambda)$ is given by

$$f_{x(k)}(x;\theta,\lambda)=\frac{n!}{(k-1)!(n-k)!}f_1(x)[F_1(x)]^{k-1}[1-F_1(x)]^{n-k}$$

where $f_1(x)$ and $F_1(x)$ are the pdf and cdf defined in (2) and (4) respectively

Hence, the density function of the kth order statistics of the *NGGW* distribution becomes

$$f_{x(k)}(x;\theta,\lambda)=\frac{n!\lambda\theta^3}{(k-1)!(n-k)!12\left(\theta^2+10\right)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^{\lambda}}\left[1-\left(\frac{\theta^5x^{5\lambda}+5\theta^3x^{3\lambda}\left(4+t^{\lambda}\right)+60x^{\lambda}\theta(\theta x^{\lambda}+2)}{12\left(\theta^2+10\right)}\right)e^{-\theta x}\right]^{k-1}$$

$$\left[\left(\frac{\theta^5x^{5\lambda}+5\theta^3x^{3\lambda}\left(4+t^{\lambda}\right)+60x^{\lambda}\theta(\theta x^{\lambda}+2)}{12\left(\theta^2+10\right)}\right)e^{-\theta x}\right]^{n-k}, \quad (7)$$

where $n\neq 0$.

Particularly, the density functions of the first and nth order statistics can be easily derived from (7) to give respectively

$$f_{x(1)}(x;\theta,\lambda)=\frac{n!\lambda\theta^3}{(k-1)!(n-k)!12\left(\theta^2+10\right)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^{\lambda}}\left[\left(\frac{\theta^5x^{5\lambda}+5\theta^3x^{3\lambda}\left(4+t^{\lambda}\right)+60x^{\lambda}\theta(\theta x^{\lambda}+2)}{12\left(\theta^2+10\right)}\right)e^{-\theta x}\right]^{n-1}$$

and

$$f_{x(n)}(x;\theta,\lambda)=\frac{n!\lambda\theta^3}{(k-1)!(n-k)!12\left(\theta^2+10\right)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^{\lambda}}\left[1-\left(\frac{\theta^5x^{5\lambda}+5\theta^3x^{3\lambda}\left(4+t^{\lambda}\right)+60x^{\lambda}\theta(\theta x^{\lambda}+2)}{12\left(\theta^2+10\right)}\right)e^{-\theta x}\right]^{n-1} \quad (8)$$

3.4. Reliability analysis

The reliability analysis of a distribution is determined by the survival and hazard rate function of the distribution. These are obtained as follows.

The Survival, S(x) function of the NGGW distribution is

$$S(x;\theta,\lambda)=1-F_1(x;\theta,\lambda)$$

Using (3), this implies

$$S(x;\theta,\lambda)=1-\left[1-\left(1+\frac{\theta^5x^{5\lambda}+5\theta^3x^{3\lambda}(4+x^{\lambda})+60x^{\lambda}\theta(\theta x^{\lambda}+2)}{12\left(\theta^2+10\right)}\right)e^{-\theta x}\right]$$

$$=\left(1+\frac{\left(120x^{\lambda}\theta+60x^{2\lambda}\theta^2+20x^{3\lambda}\theta^3+5x^{4\lambda}\theta^4+x^{5\lambda}\theta^5\right)}{12\left(10+\theta^2\right)}\right)e^{-\theta x^{\lambda}} \quad (9)$$

Similarly, the hazard rate function, h(x), of the NGGW distribution is obtained as

$$h(x;\theta,\lambda)=\frac{f_1(x;\theta,\lambda)}{1-F_1(x;\theta,\lambda)}=\frac{f_1(x;\theta,\lambda)}{S(x;\theta,\lambda)}$$

Using (2) and (9), we have

$$h(x;\theta,\lambda)=\frac{\frac{\lambda\theta^3}{12\left(\theta^2+10\right)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^{\lambda}}}{\left(1+\frac{\left(120x^{\lambda}\theta+60x^{2\lambda}\theta^2+20x^{3\lambda}\theta^3+5x^{4\lambda}\theta^4+x^{5\lambda}\theta^5\right)}{12\left(10+\theta^2\right)}\right)e^{-\theta x^{\lambda}}}$$

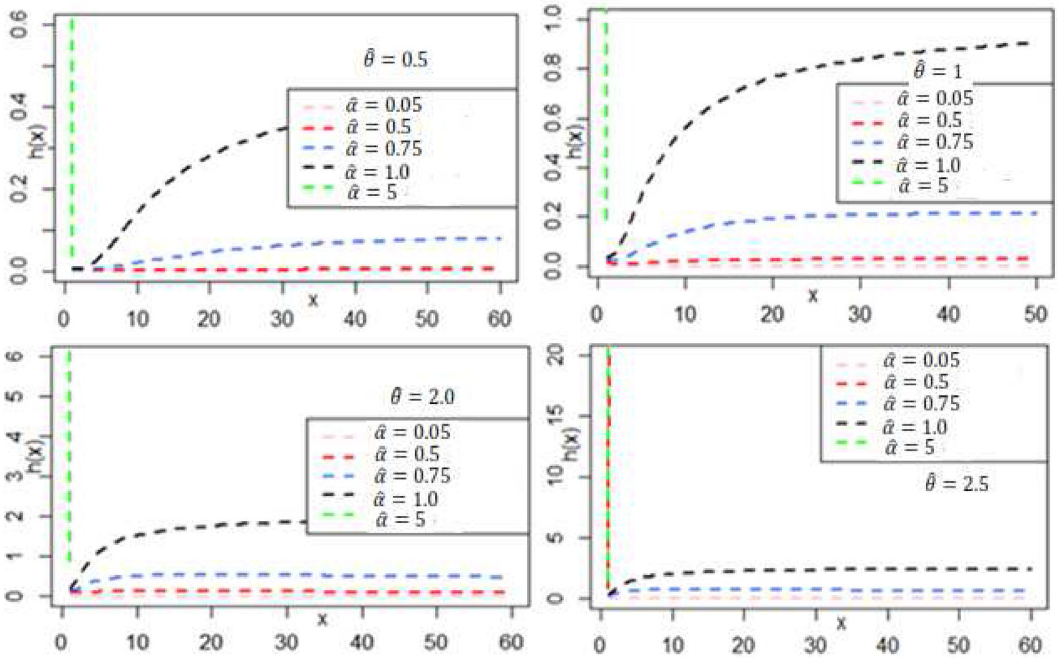

Fig. 3. Graph of the hazard function of the NGGW Distribution for varying values of parameters.

$$= \frac{x^{\lambda-1}\theta^3\left(12+x^{5\lambda}\theta^3\right)\lambda}{120x^{\lambda}\theta+60x^{2\lambda}\theta^2+20x^{3\lambda}\theta^3+5x^{4\lambda}\theta^4+x^{5\lambda}\theta^5+12\left(10+\theta^2\right)} \quad (10)$$

Fig. 3 presents the behavioural patterns of the hazard function of the proposed NGGW distribution at varying values of θ and λ.

4. Estimation and goodness-of-fit test

The parameters of the NGGW distribution are estimated using the method of maximum likelihood.

4.1. Maximum likelihood estimation

Let $X_1, X_2, \ldots, X_n$ be the random sample from the NGGW distribution with parameters θ and λ..

Then the likelihood function is given by

$$L(\theta,\lambda)=\prod_{i=1}^{n} f_1(x_i;\theta,\lambda),$$

This gives

$$L(\theta,\lambda)=\prod_{i=1}^{n}\frac{\lambda\theta^3}{12(\theta^2+10)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^{\lambda}},$$

The log-likelihood function is given as

$$l(\theta,\lambda)=nlog\lambda\theta^3-n\,log\left(12\left(\theta^2+10\right)\right)+\sum_{i=1}^{n}log\left(\theta^3x_i^{5\lambda}+12\right)+\sum_{i=1}^{n}log\left(x_i^{\lambda-1}\right)-\sum_{i=1}^{n}log\left(\theta x_i^{\lambda}\right)$$

Differentiating the log-likelihood function partially with respect to the associated parameters, we have

$$\frac{\partial l(\theta,\lambda)}{\partial\theta}=-nx^{\lambda}+\frac{3n}{\theta}-\frac{2n\theta}{10+\theta^2}+\frac{3nx^{5\lambda}\theta^2}{12+x^{5\lambda}\theta^3} \quad (11)$$

$$\frac{\partial l(\theta,\lambda)}{\partial\lambda}=\frac{n}{\lambda}+nLog[x]-nx^{\lambda}\theta Log[x]+\frac{5nx^{5\lambda}\theta^3Log[x]}{12+x^{5\lambda}\theta^3}, \quad (12)$$

where $\theta>0$ and $\lambda>0$

The Maximum Likelihood Estimates (MLEs), $\widehat{\theta}, and\ \widehat{\lambda}$ can be obtained by equating (11) and (12) to zero and solving simultaneously. However, analytical expressions for $\widehat{\theta}\ and\ \widehat{\lambda}$ are not feasible. Hence, we computed the MLEs numerically using R software [18].

4.2. Models' selection criterion

To compare the performance of the proposed distribution with some existing distributions, we use goodness-of-fit statistics: -2loglikelihood (-2InL), Akaike Information Criterion (AIC), Corrected Akaike Information Criterion (CAIC), and Bayesian Information Criteria (BIC). Note that the preferred model is the one with the least value of the -2lnL, AIC, CAIC and BIC. These statistics can be computed as follows

$$CAIC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

$$BIC = -2\,ln(L) + klog(n)$$

where L is the likelihood and k is the number of parameters in the model.

5. Applications

Two datasets were used as detailed below to evaluate the applications of the NGGW distribution. We compared the goodness-of-fit of the proposed distribution with some existing distributions, including the Exponential Distribution (ED), Two-Parameter Odoma Distribution (TPOD), Two-parameter Sujatha Distribution (TPSD), Two-parameter Rama Distribution (TPRD) Three-parameter Weighted Lindley Distribution (WLD), Three-parameter Power Weighted Lindley Distribution (PWLD), Three-parameter Generalized Gamma Distribution (GGD) with parameters $((\theta, \alpha, \beta))$, Nwipke, Rama, Exponential and Lindley distributions.The competing distributions considered are presented in Table 1.

Dataset 1: The first date set which was reported by [13] is the length of time (in years) that 81 randomly selected Nigerian graduates stayed without a job before being employed by the Universal Basic Education Commission (UBEC) in 2011. The data set is as given below:

2, 5, 7, 5, 6, 7, 7, 6, 6, 9, 9, 6, 6, 7, 5, 4, 5, 2, 9, 8, 5, 9, 6, 6, 7, 2, 8, 3, 6, 6, 2, 8, 5, 7, 4, 5, 6, 8, 8, 9, 3, 7, 6, 2, 6, 8, 9, 7, 6, 6, 9, 5, 9, 5, 5, 5, 3, 9, 8, 6, 6, 6, 7, 9, 4, 4, 6, 9, 7, 8, 8, 9, 4, 6, 3, 5, 4, 7, 6, 6, 5.

Dataset 2: The second data which is reported in [24], represents the lifetime data relating to the time (in months from 1st January 2013 to 31st July 2018) of 120 patients who were diagnosed with hypertension and received at least one treatment-related to hypertension in the hospital where death is the event of interest. The data set is as given below:

45, 37, 14, 64, 67, 58, 67, 55, 64, 62, 9, 65, 65, 43, 13, 8, 31, 30, 66, 9, 10, 31, 31, 31, 46, 37, 46, 44, 45, 30, 26, 28, 45, 40, 47, 53, 47, 41, 39, 33, 38, 26, 22, 31, 46, 47, 66, 61, 54, 28, 9, 63, 56, 9, 49, 52, 58, 49, 53, 63, 16, 67, 61, 67, 28, 17, 31, 46, 52, 50, 30, 33, 13, 63, 54, 63, 56, 32, 33, 37, 7, 56, 1, 67, 38, 33, 22, 25, 30, 34, 53, 53, 41, 45, 59, 59, 60, 62, 14, 57, 56, 57, 40, 44, 63.

The results of NGGW and the competing distributions for the datasets are presented in Table 2 and Table 3. As observed in the tables, the NGGW has the lowest -2log-likelihood among the models considered, which indicates that the proposed model provides the best fit for the given datasets among all the models. Note that selection by least -2log-likelihood will also be equivalent to selecting the best model by AIC, BIC and CAIC as well, among models with an equal number of parameters. The model with the lowest information criteria is the best.

Table 1. Some of the selected competing models for Comparison.

Distribution	PDF	Authors
Two-Parameter Odoma (TPOD)	$f(x;\alpha,\beta) = \frac{\alpha^5}{2(\alpha^5\beta + \alpha^3 + 24)}(2x^2 + \alpha x^2 + 2\alpha\beta)e^{-\alpha x}$	[19]
Two-Parameter Sujatha (TPSD)	$f(x;\theta,\alpha) = \frac{\theta^3}{\theta^3 + \alpha\theta + 2\alpha}(\theta + \alpha x + \alpha x^2)e^{-\theta x}$	[20]
Two-Parameter Rama (TPRD)	$f(x\vert\alpha,\theta) = \frac{\theta^4}{\alpha\theta^3 + 6}(\alpha + x^3)e^{-\theta x}$	[11]
Three-Parameter Weighted Lindley (TWLD)	$f(x;\theta,\alpha,\beta) = \left[\frac{\theta^{\alpha+1}x^{\alpha-1}(\beta + x)}{(\beta\theta + \alpha)\Gamma(\alpha)}\right]e^{-\theta x}$	[21]
Three-Parameter Generalised Lindley (TGLD)	$f(x;\theta,\alpha,\beta) = \left[\frac{\theta^{\alpha+1}x^{\alpha-1}(\alpha + \beta x)}{(\theta + \beta)(\Gamma(\alpha + 1))}\right]e^{-\theta x}$	[22]
Generalised Gamma (GGD)	$f(x;\theta,\alpha,\beta) = \left[\frac{\beta\theta^{\alpha}x^{\alpha-1}}{(\theta + \beta)(\beta\Gamma(\alpha))}\right]e^{-\theta x}$	[23]
Nwipke	$f_{(x;\theta)} = \frac{\theta^3}{12(\theta^2 + 10)}(\theta^3x^5 + 12)e^{-\theta x}$	[13]
Lindley	$f(t;\beta) = \frac{\beta^2}{\beta + 1}(1 + t)e^{-\beta t}$	[8]

Table 2. Goodness-of-fit for NGGW distribution for dataset 1 (n = 81).

Distribution	MLE Estimates			-2InL	AIC	BIC	CAIC
	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\lambda}$				
TWLD	1.3320	7.4310	2.4750	348.0436	354.0436	361.2269	354.3553
TGLD	2.2589	12.7769	3.8157	2462.094	2456.094	2448.910	2455.782
GGD	0.0071	1.3991	2.7980	338.3836	344.3836	351.5669	344.6952
NGGW	**0.1803**	**1.8815**	**-**	**272.2451**	**276.2451**	**281.034**	**276.399**
TPRD	0.6451	2.7951	_	366.8598	370.8598	375.6487	371.0137
TPOD	0.1875	3.4932	_	461.1573	465.1573	469.9462	465.3111
TPSD	0.4611	18.9823	_	385.0256	389.0256	393.8145	389.1795
Nwipke	0.9450	_	_	361.4103	363.4103	365.8048	363.4610
Lindley	0.2910	_	_	418.5780	420.5780	422.9725	420.6287

Table 3. Goodness-of-fit for NGGW distribution for dataset 2 (n = 120).

Distribution	MLE Estimates			-2InL	AIC	BIC	CAIC
	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\lambda}$				
TWLD	0.0902	3.0292	9.8465	926.0821	932.0821	940.0440	932.3197
TGLD	0.2946	11.8342	5.0040	2164.1440	2158.1400	2150.1800	2157.9100
GGD	0.5709	1.8372	1.5605	917.7972	923.7972	931.7591	924.0348
NGGW	**0.0518**	**1.2599**	**-**	**913.2065**	**917.2065**	**922.5144**	**917.3241**
TPRD	0.0916	381.2336	_	918.9262	922.9262	928.2341	923.1080
TPOD	0.1199	0.0050	_	2433.8760	2437.8760	2443.1840	2438.0580
TPSD	0.0693	0.0071	_	928.0211	932.0211	937.3291	932.2030
Nwipke	0.1411	_	_	929.2422	931.2422	933.8961	931.3019
Lindley	0.0464	_	_	946.9942	948.9942	951.6481	949.0539

6. Conclusion

A new two-parameter distribution, called New Generalized Gamma-Weibull (NGGW) has been introduced and studied in this paper. The model is a mixture of Weibull and Generalized Gamma distributions. Its mathematical properties which include the first four moments, coefficient of variation, reliability function, Renyi's entropy measure and distribution of order statistics have been successfully derived. The maximum likelihood estimation of its parameters and its application to real-life data are discussed. Results from the applications of the model to two real-life datasets revealed that the proposed model gives a satisfactorily better fit than competing models, having the smallest values of the AIC, CAIC, and BIC. The NGGW is therefore suitable for application in various lifetime data sets with different shapes. The distributions can be developed further to incorporate covariates and censoring.

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