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Forecasting Fertility Rate in Iraq Using Fuzzy Grey Model

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Abstract: Time series forecasting involves examining past data to anticipate future values and this predictive technique relies on relevant historical and present data or information to forecast upcoming values. Moreover, this study aims to predict the fertility rate in Iraq and determine the most suitable time series forecasting model, choosing between the grey model (1,1) and fuzzy grey model (1,1), to achieve accurate predictions of the fertility rate. Where the data took from the website (macrotrends-population-Iraq-fertility rate) for (2005-2023) nineteen periods of time, The analysis showed that the data is trend patterned so it is appropriate to use GM(1,1) and FGM(1,1) models. The GM(1,1) and FGM(1,1) models have been used for forecasting the fertility rate in Iraq to compare and select the appropriate model the researchers depended on mean absolute percentage error (MAPE) and precision rate (p) for both models. It is clear seen that the MAPE value of FGM (1,1) is (1.652471589%) which was less than MAPE value of GM (1,1) for (1.752079612%). Then, based on the critical assessment of precision rates, it is observed that the GM(1,1) model possesses a precision rate of 98.24792039%, whereas the FGM(1,1) model surpasses this with a higher precision rate of 98.34752841%. It is means FGM (1,1) model values more accurate than GM (1,1) model values. FGM (1,1) is strongly suggested for forecast fertility rate in 2024- 2030 in Iraq rate.

Keywords: Grey Model, Fuzzy Grey Model, Mean absolute percentage error, Fertility Rate.

تنبؤ بمعدل الخصوبة في العراق باستخدام النموذج الضبابي الرمادي

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المستخلص: يُعد التنبؤ بالسلاسل الزمنية عملية تحليل البيانات السابقة بهدف توقع القيم المستقبلية، وتعتمد هذه التقنية التنبؤية على البيانات التاريخية والحالية ذات الصلة للتنبؤ بالقيم القادمة. تهدف هذه الدراسة إلى التنبؤ بمعدل الخصوبة في العراق وتحديد أنسب نموذج للتنبؤ بالسلاسل الزمنية، من خلال الاختيار بين النموذج الرمادي

(GM(1,1)) والنموذج الضبابي الرمادي (FGM(1,1))، لتحقيق توقعات دقيقة لمعدل الخصوبة. تم الحصول على البيانات من موقع (macrotrends - population - Iraq - fertility rate) للفترة (٢٠٠٥-٢٠٢٣)، والتي تشمل تسع عشرة فترة زمنية. أظهر التحليل أن البيانات تتبع نمطًا اتجاهيًا، مما يجعل استخدام نماذج GM(١,١) وFGM(١,١) مناسبًا. تم استخدام نماذج GM(١,١) وFGM(١,١) لتوقع معدل الخصوبة في العراق، وللمقارنة بين النموذجين وتحديد الأنسب، اعتمد الباحثون على معيار متوسط نسبة الخطأ المطلق (MAPE) ومعدل الدقة (p) لكلا النموذجين. أظهرت النتائج أن قيمة MAPE لنموذج FGM(١,١) بلغت (١,٦٥٢٤٧١٥٨٩٪)، وهي أقل من قيمة MAPE لنموذج GM(١,١) التي بلغت (١,٧٥٢٠٧٩٦١٢%)وبناءً على التقييم الدقيق لمعدلات الدقة، وُجد أن نموذج GM(1,1) يمتلك معدل دقة يبلغ ٩٨,٢٤٧٩٢٠٣٩٪، في حين أن نموذج FGM(1,1) تفوق عليه بمعدل دقة أعلى بلغ ٩٨,٣٤٧٥٢٨٤١٪. وهذا يعني أن قيم نموذج FGM(1,1) أكثر دقة من قيم نموذج GM(1,1).

الكلمات المفتاحية: النموذج الرمادي، النموذج الضبابي الرمادي، متوسط نسبة الخطأ المطلق، معدل الخصوبة

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Introduction

The fertility rate is a crucial demographic indicator that measures the average number of children born to women during their reproductive years in a specific population or country. It plays a significant role in understanding population dynamics, as it directly influences population growth, age distribution, and overall demographics. Fertility rates are essential for policymakers, healthcare professionals and researchers use fertility rate data to understand demographic trends and plan for the future, as they provide insights into population trends, family planning needs, and resource allocation. Additionally, the study of fertility rates enables a deeper understanding of societal changes, gender roles, and cultural factors affecting birth rates. Accurate and up-to-date fertility rate data are vital for making informed decisions and formulating effective social policies.

The fertility rate in Iraq is a significant demographic metric representing the average number of children born to women during their reproductive years in the country. As a crucial indicator, it plays a vital role in shaping population growth, age distribution, and socio-economic planning. Policymakers, researchers, and organizations rely on fertility rate data to understand population trends, assess healthcare needs, and devise appropriate family planning strategies. The Central Statistical Organization of Iraq (n.d.) likely serves as a reputable source for such demographic data, providing valuable insights into the nation's population dynamics and aiding decision-making processes.

1st: Objective of the research

This study was comparing the grey model (1,1) and fuzzy grey model (1,1), in forecasting the amount of fertility rate in Iraq and secondly to determine the most suitable time series forecasting model for forecast the fertility rate in Iraq during period (2024-2030) to plan for the future years

2nd: Materials and methods

Data classification methods are one of the most widely used in statistical methods (Soran, 2023). Forecasting is a tool or technique used to predict or predict a value in the future by paying attention to relevant data or information, both past data or information or current data or information [3]. In forecasting, there are 2 general methods, namely qualitative and quantitative. The qualitative method is intuitive and is usually done when there is no past data / history, which results in the inability of mathematical calculations. While quantitative methods can be done based on previous data/history so that calculations can be done mathematically. This study employed time series data from 2005 to 2023 to analyze fertility rate in Iraq. The data, sourced from the (macrotrends-population-Iraq-fertility rate) website, specifically the section on fertility rate in Iraq (https://www.macrotrends.net/countries/IRQ/iraq/fertility-rate) (Agriculture, 2023) , were utilized for analysis. The grey model (1,1) and fuzzy grey model (1,1) were employed to forecast fertility rate in Iraq for the years 2024 to 2030.

3rd:

1- Grey model

A. Grey system theory

Grey System theory was initiated developed by (Deng., 1988). As far as information is concerned, the systems which lack information, such as structure message, operation mechanism and behavior document, are referred to as Grey Systems. For example, the human body, agriculture, economy, etc., are Grey Systems. Usually, on the grounds of existing grey relations, grey elements, grey numbers (denoted by ∅) one can identify which Grey System is, where "grey" means poor, incomplete, uncertain, etc. The goal of grey system and its applications is to bridge the gap existing between social science and natural science. Thus, one can say that the Grey System theory is interdisciplinary, cutting across a variety of specialized fields, and it is evident that Grey System theory stands the test of time since 1982. The concept of the Grey System, in its theory and successful application, is now well known in China. The application fields of the Grey System involve agriculture, ecology, economy, meteorology, medicine, geography, industry, etc.

B. Advantage of Grey models

1. Limited Data Requirement: Grey models can work effectively even when there is limited data available. They are capable of handling small data sets, making them valuable for scenarios with insufficient historical data.
2. Simplicity and Ease of Use: Grey models are relatively straightforward and easy to implement. They don't require complex mathematical operations, making them accessible to researchers and practitioners with limited expertise in advanced forecasting techniques.
3. Interpretability: Grey models are transparent, and their forecasting process can be easily understood. This transparency allows analysts to interpret the results and gain insights into the underlying patterns.
4. Robustness to Noise: Grey models are resilient to random noise in the data, which means they can produce reasonably accurate forecasts even in the presence of some level of data irregularity or noise.
5. Short-Term Forecasting: Grey models are particularly useful for short-term forecasting applications. They can provide reliable predictions for a limited time horizon into the future.
6. Small Data Set Forecasting: When traditional statistical methods may not work well due to data limitations, grey models can still generate meaningful forecasts, making them valuable in specific scenarios.

However, it's important to note that every forecasting technique has its limitations, and the choice of the model depends on the specific characteristics of the data and the desired forecasting horizon (Liu, 2010). While grey models offer advantages in certain situations, other sophisticated methods may perform better under different conditions. (Deng., 1988)

C. Grey Theory Steps

The lack of information is named by grey, the Grey system provides an adequate results despite of uncompleted information or small samples under study that is because of the a cumulative generate operator which gives more weights for each observation of the sample. (Pai, 2013)

D. Grey Model

In grey systems theory, GM (m, n) definition of lack information model, where m is the order of the difference formula and n is the number of factors. Grey models can predict the future outputs of systems with relatively high accuracy without a mathematical model of the actual system. The majority of the incisive researchers have fastened their awareness on the GM (1, 1) model in their predictions because of its computational efficiency. (Li, 2015)

The most commonly used grey forecasting model is GM (1, 1), which Indicates that one variable is employed in the model (Javed, 2018). The first order Differential equation is adopted to match the data generated by the Accumulation generating operation (AGO), (Guo, 2005)

The AGO reveal the hidden regular pattern in the system development (Mohammad, 2018). Before the algorithm of GM (1, 1) is described, the raw data series is assumed to be

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \quad (2)$$

Where n is the total number of modeling data. The AGO formation of $X^{(1)}$ is defined as:

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\} \ , n \geq 4 \quad (3)$$

Where:

$$x^{(1)}(1) = x^{(0)}(1)$$

$$x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)} \ (k = 1,2, \ldots, n). \quad (4)$$

The GM(1,1) model can be constructed by establishing a first order differential equation for $x^{(1)}$ (k) as::

$$\frac{dx^{(1)}}{dt} + aX^{(1)} = u \quad (5)$$

Where parameters a and u are called the developing coefficient and grey input, respectively.

In practice, parameters a and u are not calculated directly from Eq. (5). Therefore, the solution of (5) can be obtained by using the least square method. That is,

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{u}{a}\right] e^{-ak} + \frac{u}{a} \quad (6)$$

Where

Where, $\hat{x}^{(1)}(k+1)$ denotes the prediction x at time point k+1 and the coefficients [a,u]T can be obtained by the Ordinary Least Squares (OLS) method:

$$\hat{a} = [a, u]^T = (B^T B)^{-1} B^T Y \quad (7)$$

In that:

$$Y_N = [X^{(0)}(2), X^{(0)}(3), \ldots, X^{(0)}(n)]^T$$

And

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix} \quad (8)$$

Inverse AGO (IAGO) is used to find predicted values of primitive sequence. By using the IAGO:

$$\hat{x}^{(0)}(k+1) = [x^{(0)}(1) - \frac{u}{a}](1 - e^{a})e^{-ak} \quad (9)$$

Therefore, the fitted and predicted sequence $\hat{x}^{(0)}$ is given as:

$$\hat{x}^{(0)} = \left(\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n), \ldots\right) \quad (10)$$

and

$$\hat{x}^{(0)}(1) = x^{(0)}(1) \, , (k = 2,3, \ldots, n).$$

Where

$$\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n)$$

are called the GM (1, 1) fitted sequence while:

$\hat{x}^{(0)}(n+1), \hat{x}^{(0)}(n+2), \ldots,$ are called the GM (1, 1) forecast values

2- Fuzzy Logic

A. Introduction to Fuzzy Logic

Fuzzy time series forecasting is a methodology that incorporates fuzzy logic principles into time series analysis for prediction purposes. Unlike traditional time series forecasting techniques that deal with precise data points, fuzzy time series models accommodate uncertainty and imprecision in the data by representing linguistic terms such as "high," "medium," and "low" instead of precise numerical values. This approach allows for more flexibility in handling vague or incomplete information commonly encountered in real-world datasets.

Fuzzy time series forecasting involves several key steps, including fuzzification of input data, rule generation based on historical patterns, inference using fuzzy logic operators, and defuzzification to obtain crisp forecasts. By capturing the inherent uncertainty in time series data and incorporating human-like reasoning, fuzzy time series models can provide more robust and interpretable forecasts, particularly in domains where data may be noisy or incomplete. This methodology finds applications in various fields such as finance, economics, weather forecasting, and environmental prediction

B. Theoretical Foundation

This section reviews various theoretical concepts relevant in the context of this study. The main topics covered here are fuzzy sets, fuzzy numbers, defuzzification, fuzzy relations, fuzzy aggregation operators, PSO and basic FTS concepts

C. Conventional Sets vs Fuzzy Sets

Conventional set theory relies on the concept of clear distinctions between elements considered either members or non-members of a given set. When queried about an element's presence in a set, the response is binary, offering a definitive "yes" or "no" for all elements. For instance, in the set of tall individuals, each person is either tall or not tall, with no intermediate classification. Conventional sets are typically defined in two manners: explicitly through enumeration or implicitly via a predicate. For example, the finite set A={0, 1, 2, 3} is explicitly listed, while the set of integers greater than 10 is implicitly described and infinite. Regardless of the method, the classification of elements into the set or outside it remains unambiguous.

Fuzzy set theory expands upon the concept of crisp sets by introducing membership degrees to elements within a set, creating a gradual transition from membership to non-membership. Rather than a binary distinction, membership degrees in fuzzy sets are real numbers ranging between 0 and 1. A higher membership degree indicates a stronger association with the set, while a degree of 0 indicates clear non-membership. Elements with degrees between 0 and 1 possess varying degrees of membership. For example, in the context of height, a person's tallness can be more or less pronounced. Unlike conventional set theory, fuzzy set theory allows for nuanced degrees of membership, accommodating scenarios where elements exhibit partial belonging to a set.

D. The Universe of Discourse

In set theory, all elements belong to a universe of discourse or universe set, encompassing all elements considered when forming a set. While the term 'set' is commonly used, sets are essentially subsets of a universe set. In fuzzy set theory, every element in the universe set holds membership to some extent within the fuzzy set, even if it's zero. The collection of elements with non-zero membership constitutes the support of the fuzzy set. The notation U typically denotes the universe set.

E. Fuzzy Subsets

A fuzzy subset A within the universe set U is defined by a membership function (also known as a characteristic function) that assigns a real number in the unit interval to each element in A. Mathematically, this is represented as $\mu_A\colon \mathrm{U} \rightarrow [0,1]$, where $\mu_A(x)$ denotes the degree of membership of element x in the fuzzy set A. Through the membership function, it is determined which elements of U belong to A and which do not. This process of transforming crisp sets into fuzzy sets is termed the extension principle.

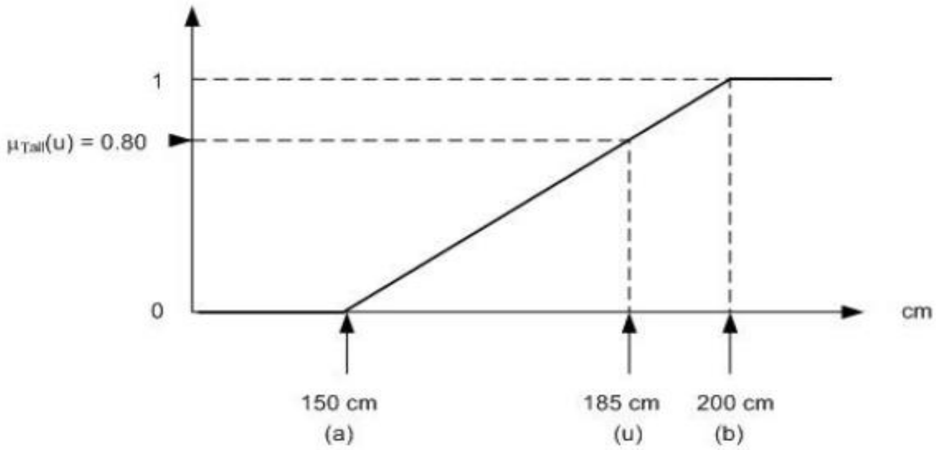

Figure 1. An example of a membership function for the fuzzy set Tall.

A common instance of a fuzzy set is the subset denoting tall individuals, as illustrated in the diagram. This set, referred to as "Tall," exhibits varying degrees of membership. For instance, for individuals with a height less than or equal to 150 cm (at point a), their membership in the fuzzy set Tall is zero, indicating complete exclusion. Conversely, if an individual's height surpasses or equals 200 cm (at point b), they fully satisfy the condition for tallness, resulting in a membership degree of 1. Heights falling between 150 cm and 200 cm represent varying degrees of fulfillment of the tallness criterion. For example, an individual standing at 185 cm (at point u) possesses a membership degree of 0.8 in the set Tall. This membership function, also known as a characteristic function, can be expressed mathematically.

$$\mu_{Tall}(x) = \begin{Bmatrix} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b < x \end{Bmatrix}$$

When a fuzzy set A corresponds to a conventional (crisp) set, the membership function simplifies to

$$\mu_A(x) = \begin{Bmatrix} 1 & x \epsilon A \\ 0 & x \notin A \end{Bmatrix}$$

The mentioned function yields solely two outcomes: 0 or 1. If $\mu_A(x)$ equals 1, then x belongs to A; conversely, if $\mu_A(x)$ equals 0, x is considered a non-member of A. Additional examples of membership functions frequently encountered in literature are illustrated in Figure 2

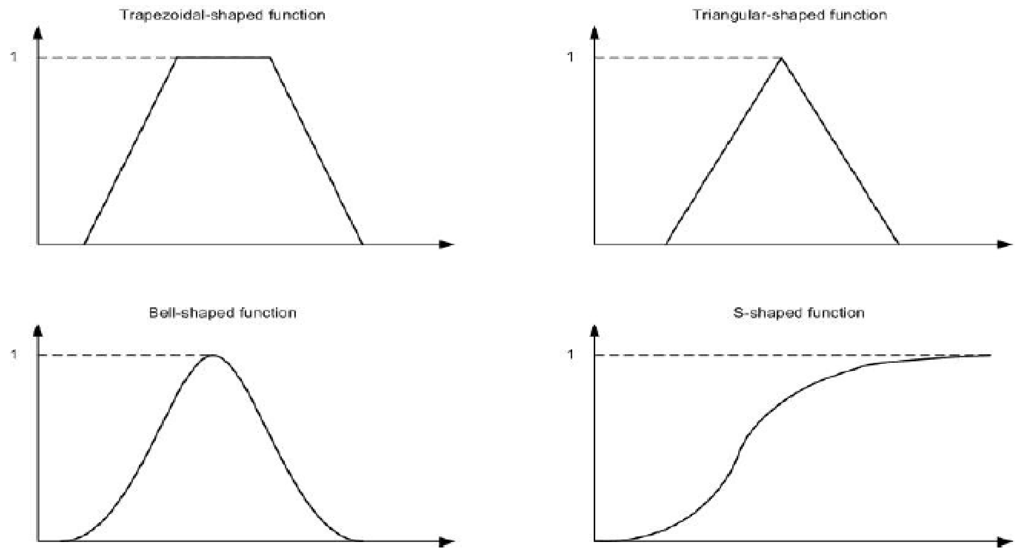

Figure 2. Various shapes of commonly used membership functions.

It's important to acknowledge that fuzzy membership functions may lack symmetry, although this isn't explicitly depicted in the figure. The shape of a membership function can vary depending on the application's requirements. Currently, there's no universal rule or standard for selecting a membership function for a specific type of fuzzy subset. Instead, the decision relies on various factors such as the user's scientific expertise, knowledge, or the practical demands of the application. The choice of membership function is subjective, often reflecting the user's preferences or subjective assessments. Similar to probability theory and statistics, one might assume that a particular function represents a certain property, such as in the case where the membership function in Figure 1 is assumed to describe the attribute of being tall.

F. Alpha Cut

A significant characteristic of fuzzy sets is the concept of alpha cuts (α-cuts). For a fuzzy set A defined on the universal set U and any number within the unit interval, denoted as $\alpha \in [0,1]$, the (weak) α-cut, $A_{\overline{\alpha}}$, and the strong α -cut, A_{α}, represent the crisp sets that adhere to the condition:

$$A_{\overline{\alpha}} = \{x \epsilon A | \mu_A(x) \geq \alpha\}$$
$$A_{\alpha} = \{x \epsilon A | \mu_A(x) > \alpha\}$$

In simpler terms, the α-cut of a fuzzy set A represents the crisp set $A_{\overline{\alpha}}$ that includes all elements from the universal set U whose membership grades in A are greater than or equal to a specified α. This definition holds for both the weak variant A^α and the strong variant A_{α}. Remember that the support of a fuzzy set A within the universal set U is the crisp set containing all elements of U with non-zero membership grades in A. Therefore, for $\alpha = 0$, the support of A is identical to the strong α-cut of A.

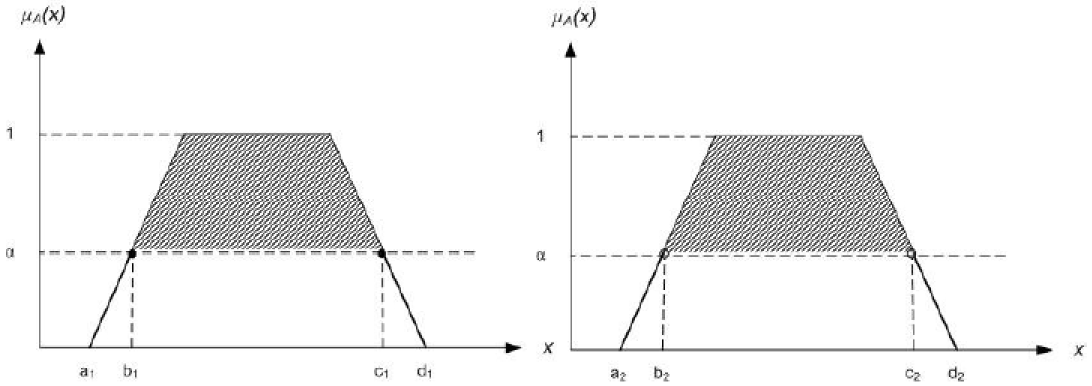

Figure 3. Weak and strong alpha-cut, respectively.

G. Operations on Fuzzy Sets

Classical set theory encompasses three fundamental operations for crisp sets: complement, intersection, and union. These operations also extend to fuzzy set theory, alongside a variety of other operators. The standard fuzzy set operators, complement, intersection, and union, are defined by the equations:

$$\overline{\mu_A}(x) = 1 - \mu_A(x)$$
$$(\mu_A \cap \mu_B)(x) = Min[\mu_A(x), \mu_B(x)]$$
$$(\mu_A \cup \mu_B)(x) = Max[\mu_A(x), \mu_B(x)]$$

Here, A and B represent fuzzy subsets of the universal interval U. The operators min and max denote the minimum and maximum operations, respectively. In these equations, the min and max operations compare corresponding elements between the fuzzy sets μ_A and μ_B. In the case of complement, each membership value of μ_A is subtracted from 1.

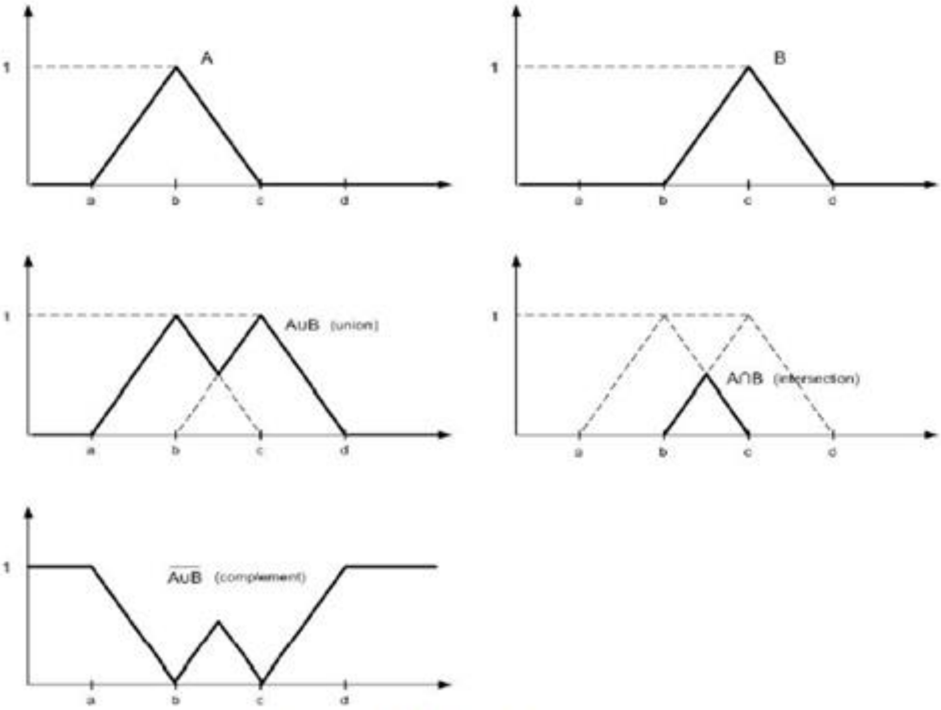

Figure 4 The basic set operations.

H. Fuzzy Numbers

In this section we will briefly review some frequently used classes of fuzzy numbers

Definition 1: Fuzzy Number

A fuzzy number A is defined as any fuzzy subset of the real number line, denoted by IR, with a membership function μ_A possessing the following properties:
(a) μ_A is a continuous mapping from IR to the closed interval [0,ω], 0 ≤ ω ≤ 1;
(b) $\mu_A(x) = 0$, for all x∈[−∞, a]
(c) μ_A is strictly increasing on [a , b]
(d) $\mu_A(x) = \omega$, for all x∈[b , c], where ω is a constant and 0<w≤1
(e) μ_A is strictly decreasing on [c ,d]
(f) $\mu_A(x) = 0$, for all x∈[d ,∞]

In the absence of specific instructions, it is assumed that A, a fuzzy number, is both convex and bounded, meaning that $-{-\infty} < a$, $d < \infty$.. When ω equals 1 in (d), A is termed a normal fuzzy number; when 0<w<1 in (d), A is referred to as a non-normal fuzzy number. For simplicity, the fuzzy number in definition 1 can be represented as A=(a, b, c, d; ω). The opposite of A, denoted as −A, is defined as (−d, −c, −b, −a; w). Property (a) can also be expressed as A: |R → [0, 1]. The membership function of A can be formulated as:

$$\mu_A(x) = \begin{cases} \mu_A^L(x) & a \le x \le b \\ \omega & b \le x \le c \\ \mu_A^R(x) & c \le x \le d \\ 0 & Otherwise \end{cases}$$

$$\mu_A^L(x): [a,b] \to [0,\omega] \;\; And \; \mu_A^R(x): [c,d] \to [0,\omega]$$

Definition 2: Triangular Fuzzy Number

A triangular fuzzy number A is a fuzzy number with a piecewise linear membership function μ_A defined by

$$\mu_A = \begin{bmatrix} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & Otherwise \end{bmatrix}$$

which can be denoted as a triplet (a_1, a_2, a_3)

Definition 3: Trapezoidal Fuzzy Number

A trapezoidal fuzzy number A is a fuzzy number with a membership function μ_A denoted by

$$\mu_A = \begin{bmatrix} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ 0 & Otherwise \end{bmatrix}$$

which can be denoted as a quartet $a_1, a_2, a_3, a_4)$

4th: Evaluate Precision of forecasting Models

1- Mean Absolute Percentage Error (MAPE)

Some statistical tests and measurements used to test the accuracy and the performance of the proposed model, including Mean Absolute Percentage Error (MAPE) In order to predict the accuracy of forecasting model in this paper (Tahir, 2021), Means Absolute Percentage Error (MAPE) index was used to evaluate the performance and reliability of forecasting technique (Pamungkas, 2016). It is defined as follows:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}|\mathrm{PE}_k| * 100\% = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| * 100\% \qquad (12)$$

The forecasting accuracy level can be classified into four grades based on the MAPE of each model as indicated in Table 1 (Lewis, 1982)

Table (1): Categorizing the grade of predicting accuracy.

Grade level	Highly accurate prediction	Good prediction	Reasonable prediction	Inaccurate prediction
MAPE	<10%	10% - 20%	20% - 50%	>50%

The lower the MAPE, the higher precision the forecasting model can achieve.in general the MAPE below <10% is an accurate model and the MAPE in between 10% - 20% is a good model with acceptable accuracy.

2- Precision Rate (p)

Precision Rate, which measures the level of the closeness of the statement of forecast quantity and the actual value, p is defined as follows:

$$p = 1 - MAPE$$

Table (2): Categorizing the grade of predicting accuracy.

Precision rank	Highly accurate	Good	Reasonable	Inaccurate
Precision rate (p)	$p \ge 99.0\%$	$p \ge 95.0\%$	$p \ge 90.0\%$	$p \le 90.0\%$

The higher the precision rate, the higher precision the forecasting model can achieve. In general, the precision rate greater than 99% is an accurate model and the precision rate in between 98.0% and 95.0% is a good model with acceptable accuracy

5th: Application

1- Introduction:

In this study, used the time series data of the fertility rate in Iraq during the year (2005–2023).The data were obtained from the website (macrotrends- population-Iraq-fertility rate), (https://www.macrotrends.net/countries/IRQ/iraq/fertility-rate). In this section we use the two models disculled earlier to analyze the time series to the fertility rate using GM (1,1) and FGM (1,1) for forecasting applied fertility rate in Iraq for (19) years from 2005 to 2023.

2- Variable of this Study

The data used for the GM (1,1) and FGM (1,1) is fertility rate data from 2005 to 2023. Data can be seen in the table below.

Table (3): shows that the Iraq - historical fertility rate data from 2005 - 2023

Year	Fertility rate	Year	Fertility rate	Year	Fertility rate
2005	4.652	2012	4.31	2019	3.635
2006	4.589	2013	4.28	2020	3.588
2007	4.525	2014	4.25	2021	3.54
2008	4.462	2015	4.136	2022	3.493
2009	4.399	2016	4.023	2023	3.446
2010	4.369	2017	3.909		
2011	4.339	2018	3.796		

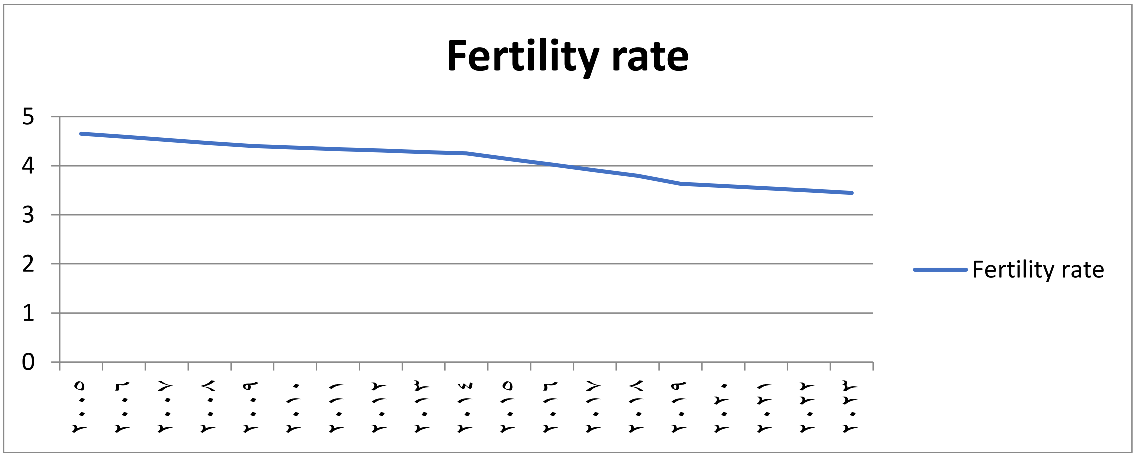

Fig. (5): Plot the data fertility rate from (19) years

Based on Figure 1 of the fertility rate Chart, it can be seen that the data shows obedience in the 2005 period to the 2023 period.

3- Applied Grey Model GM (1,1)

The Grey forecasting model is done to get the forecast value in the next period. GM(1,1) model have been estimated which are intercept and slop of the model with values (398.3463613 and 0.013432874) respectively

Table (4): Results of the GM(1.1) prediction model

Year	Original data	Predicted values	Percentage Error (PE)	Year	Original data	Predicted values	Percentage Error (PE)
2005	4.652	4.652	0	2015	4.136	4.009537195	3.057611333
2006	4.589	4.686650015	2.12791491	2016	4.023	3.94061892	2.047752424
2007	4.525	4.606093128	1.792113324	2017	3.909	3.872885252	0.923887123
2008	4.462	4.5269209	1.454973097	2018	3.796	3.806315831	0.271755288
2009	4.399	4.44910953	1.139111841	2019	3.635	3.740890643	2.913085099
2010	4.369	4.372635628	0.083214183	2020	3.588	3.676590022	2.46906417
2011	4.339	4.297476204	0.956990005	2021	3.54	3.613394638	2.073294866
2012	4.31	4.223608664	2.004439352	2022	3.493	3.551285493	1.668637085
2013	4.28	4.151010803	3.013766291	2023	3.446	3.490243917	1.283920978
2014	4.25	4.079660796	4.007981267				

Table above represents the model value and MPAE by GM (1,1) model. The MAPE value can be found by dividing the total PE by the total data. It is clear seen that the MAPE value for the GM (1,1) is (1.752079612%) and it can be said that the efficiency and accuracy values obtained are (100−MAPE (%) =100− 1.752079612% = 98.24792039%). which means that the forecasting value is considered good prediction because the value of the precision rate in between 98.0% and 95.0%

4- Applied Fuzzy Grey Model FGM (1,1)

Step#1:

In this context, fuzzification is the process of identifying associations between the historical values in the dataset and the fuzzy sets defined in the previous step. Each historical value is fuzzified according to its highest degree of membership. If the highest degree of belongingness of a certain historical time variable, say $F(t-1)$, occurs at a fuzzy set A_k, then $F(t-1)$ it is fuzzified as A_k. A complete overview of fuzzified enrollments is shown in the table.

Table (5): Complete set of relationships identified

Class (From → To)	Fuzzified enrollment	Mid –point	Fuzzified enrollment	FLRG's	Mid -point FLR
3.446 - 3.676	A1	3.561	A1	A1	3.561
3.677 - 3.907	A2	3.792	A2	A2,A1	3.677
3.908 – 4.138	A3	4.023	A3	A3,A2	3.908
4.139 – 4.369	A4	4.254	A4	A4,A3	4.139
4.370 – 4.652	A5	4.511	A5	A5,A4	4.383

Step#2: We use the table above to find the data of the table below, as follows:

Table (6): Fuzzified historical enrollments.

Years	Infertility rate	Fuzzified enrolment	Interval midpoints		FLRG's		
2005	4.652	A5	4.383	Na	Na	>	A5
2006	4.589	A5	4.383	4.383	A5	>	A5
2007	4.525	A5	4.383	4.383	A5	>	A5
2008	4.462	A5	4.383	4.383	A5	>	A5
2009	4.399	A5	4.383	4.383	A5	>	A5
2010	4.369	A4	4.139	4.383	A5	>	A4
2011	4.339	A4	4.139	4.139	A4	>	A4

2012	4.31	A4	4.139	4.139	A4	>	A4
2013	4.28	A4	4.139	4.139	A4	>	A4
2014	4.25	A4	4.139	4.139	A4	>	A4
2015	4.136	A3	3.908	4.139	A4	>	A3
2016	4.023	A3	3.908	3.908	A3	>	A3
2017	3.909	A3	3.908	3.908	A3	>	A3
2018	3.796	A2	3.677	3.908	A3	>	A2
2019	3.635	A1	3.561	3.677	A2	>	A1
2020	3.588	A1	3.561	3.561	A1	>	A1
2021	3.54	A1	3.561	3.561	A1	>	A1
2022	3.493	A1	3.561	3.561	A1	>	A1
2023	3.446	A1	3.561	3.561	A1	>	A1

The model explored in so far is referred to as a first order FTS model. Chen later introduced its high order counterpart which incorporates n-order relationships. In the high order modified, relations of order $n \geq 2$ can be expressed as $A_{i,1}, A_{i,2}, .. A_{i,n} \rightarrow A_{i,n+1}$. For example, a second order relationship is denoted by $A_{i,1}, A_{i,2} \rightarrow A_{i,3}$. A third order relationship is denoted by $A_{i,1}, A_{i,2}, A_{i,3} \rightarrow A_{i,4}$.
The Grey forecasting model is done to get the forecast value in the next period. By using OLS method, the parameters of FGM (1,1). model have been estimated which are intercept and slop of the model with values (398.3463613 and 0.013432874) respectively

Table (7): Results of the FGM (1.1) prediction model

Year	Original data	Predicted values	Percentage Error (PE)	Year	Original data	Predicted values	Percentage Error (PE)
2005	4.652	4.38	0	2015	4.136	3.927930204	0.520814179
2006	4.589	4.481509427	2.256665847	2016	4.023	3.870806651	0.941051434
2007	4.525	4.416335218	0.769555892	2017	3.909	3.81451384	2.38165726
2008	4.462	4.352108833	0.69592715	2018	3.796	3.759039691	2.243963442
2009	4.399	4.288816486	2.140097797	2019	3.635	3.704372298	4.025604011
2010	4.369	4.226444594	2.122112163	2020	3.588	3.650499928	2.512768539
2011	4.339	4.164979771	0.636959011	2021	3.54	3.59741102	1.021934105
2012	4.31	4.104408826	0.826595686	2022	3.493	3.545094179	0.447219252
2013	4.28	4.044718759	2.268866032	2023	3.446	3.493538177	1.895006837
2014	4.25	3.98589676	3.690161563				

Table above represents the model value and MPAE by FGM (1,1) model depending on Eq(11). The MAPE value can be found by dividing the total PE by the total data. It is clear seen that the MAPE value for the FGM (1,1) is (1.652471589%) and it can be said that the efficiency and accuracy values obtained are (100−MAPE (%) =100 − 1.652471589%= 98.34752841%). which means that the forecasting value is considered good prediction because the value of the precision rate in between 98.0% and 95.0%

Table (8): Represents the accuracy of models

Criteria	GM (1,1)	FGM (1,1)
MAPE (%)	1.752079612	1.652471589
Precision rate (p)	98.24792039	98.34752841

This table clearly showed that value of MPAE by GM(1,1) model was (1.752079612%). While, the value of MAPE by FGM (1,1) was (1.652471589%). It is clear seen that the MAPE values of FGM (1,1) are less than MAPE values of GM (1,1), it is means FGM (1,1) model values more accurate than GM (1,1) model values. Hence, FGM (1,1) is strongly suggested for forecast infertility rate in 2024- 2030 in Iraq to compare GM (1,1) model and also the value of P we can select the suitable model for forecasting the precipitation rate. The precipitation rate is expected to range between 98.24792039% and 98.34752841% mm. This range is favorable for the applied fertility rate in Iraq throughout the period from 2024 to 2030. Both models exhibit precision rates within the acceptable range for good accuracy, with the FGM (1,1) model slightly surpassing the GM (1,1) model in precision rate." The forecasted values in 2024-2030 are shown in table (7)

Table (9): The forecasted the fertility rate in Iraq during the period (2024-2030)

Forecasting value	Years	2024	2025	2026	2027	2028	2029	2030
	GM (1,1)	3.43	3.37	3.31	3.26	3.20	3.15	3.09
	FGM (1,1)	3.44	3.39	3.34	3.29	3.25	3.20	3.15

6th: Discussion

It can be seen the table (7) shows the comparison between GM (1,1) and FGM (1,1) models for forecasting applied fertility rate in Iraq during the period (2024-2030) used in this study. In addition, this procedure will forecast future values of fertility rate in Iraq and the data cover (19) years. This research is about forecasting the fertility rate. By using mean absolute percentage error (MAPE) to determine the error and precision rate (p) which measures the precision rate serves as a crucial metric for assessing the accuracy of forecasting models. A higher precision rate suggests that the model can achieve greater accuracy in predicting future values. Generally, a precision rate exceeding 99% indicates a highly accurate model, while rates between 98.0% and 95.0% are considered acceptable with good accuracy

Based on the critical assessment of mean absolute percentage error (MAPE) the results in GM(1,1) model with mean absolute percentage error (MAPE=1.752079612%). While, the value MAPE of the FGM(1,1) model is (1.652471589%) which is less than the value of MAPE in the GM(1,1) model, Then, it can be concluded that FGM(1,1) is the most appropriate method for forecasting fertility rate since the value of MAPE in FGM(1,1) was less than the value of MAPE in the GM(1,1)

Based on the critical assessment of precision rates (p), it is observed that the GM(1,1) model possesses a precision rate of 98.24792039%, whereas the FGM(1,1) model surpasses this with a higher precision rate of 98.34752841%, indicating its accuracy falls within the range considered acceptable with good accuracy (between 98.0% and 95.0%).

This disparity indicates that the FGM(1,1) model may offer superior accuracy in forecasting future fertility rates in Iraq for the period spanning 2024 to 2030 compared to the GM(1,1) model.

In conclusion, the FGM(1,1) model outperforms the GM(1,1) model in terms of precision rate, indicating its potential for providing more accurate forecasts of fertility rates in Iraq for the specified period. Given the significance of precision rate in forecasting accuracy, this suggests that the FGM(1,1) model may be the preferred choice for predicting future fertility trends in the region.

7th: Conclusion and Recommendation

It can be concluded that the comparison between GM(1,1) and FGM(1,1) models based on the criteria is Mean Absolute Percentage Error (MAPE) and precision rate (p) for finding the best method for forecasting applied fertility rate in Iraq.

The comparison of Mean Absolute Percentage Error (MAPE)between the GM(1,1) and FGM(1,1) models indicates that the FGM(1,1) model outperforms the GM(1,1) model in terms of Mean Absolute Percentage Error (MAPE) because the value of MAPE in FGM (1, 1) model was less than the value of MAPE in GM(1,1) model . Consequently, the FGM(1,1) model emerges as a favorable choice for generating more accurate predictions within the specified context.

Both models are deemed accurate according to their precision rates, the FGM(1,1) model shows a slightly higher level of accuracy compared to the GM(1,1) model. This suggests that the FGM(1,1) model may provide marginally more reliable forecasts for fertility rates in Iraq

The study result suggest that the future studies are applying of single Exponential Smoothing, Brown's and Holt's Double Exponential Smoothing and grey model to find the best model and comparing fuzzy single Exponential Smoothing, fuzzy Brown's and Holt's Double Exponential Smoothing and fuzzy grey model.

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