AL-Bahir
Journal for Engineering and Pure Sciences
BJEPS

Volume 4 | Issue 2 Article 8

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Recommended Citation
Ayoade I., Adewole (2024) "The Performance of Arima and Arfima in Modelling the Exchange Rate of Nigeria Currency to Other Currencies," *Al-Bahir*: Vol. 4: Iss. 2, Article 8.
Available at: https://doi.org/10.55810/2313-0083.1063

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Source of Funding

No Funding

Conflict of Interest

No conflicts of interest related to this work

Data Availability

Publicly available data

Author Contributions

All authors contributed equally to this work. Each author participated in the conceptualization, methodology, data analysis, and manuscript preparation.

ORIGINAL STUDY

The Performance of Arima and Arfima in Modeling the Exchange Rate of Nigeria Currency to Other Currencies

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Abstract

Economic performance of a nation depends majorly on the stability of foreign exchange rate; the economic viability hangs on the exchange rate of local currencies against other currencies across the globe. Box − Jenkins Approach was employed to model the Naira exchange rate to other major currencies using Autoregressive Integrated Moving Average (ARIMA) and the autoregressive fractional integral moving average (ARFIMA) models. This studies aimed on measuring forecast ability of Autoregressive Integrated Moving Average (ARIMA) (p,d,q) and autoregressive fractional integral moving average (ARFIMA) (p, fd, q) models for stationary type series that exhibit features of Long memory properties. Results indicate autoregressive fractional integral moving average (ARFIMA) is the best model in terms of fit, serial correlation analysis and accuracy measures. The out-sample forecasts confirmed the competence of the autoregressive fractional integral moving average (ARFIMA) models as shown by forecast validation tools. Consequently, the out-sample forecasts result nearly reveal the current economic situation in Nigeria indicating that the autoregressive fractional integral moving average (ARFIMA) model is appropriate and realistic in modeling and forecasting the strength of Naira to other currencies.

Keywords: Exchange rate, Naira, ARIMA, ARFIMA and box − Jenkins

1. Introduction

The significancy of forecasting exchange rate has a crucial impact on macroeconomic fundamentals in Nigeria such as interest rate, unemployment rate, wages, oil price and the rate of economic growth [1]. Exchange rate is a variable tool of economic indicator employed in measuring the overall performance of the economy [2]. Accurate precision of exchange rate regulates the relative prices of local and foreign commodities, as well as the strength of external sector contribution in the international trade, it also measures the domestic worth of an economy in relation to the rate of currencies of most industrialized nations such as, America Dollars, Swiss Franc, Australia Dollar and Canadian Dollar among others [3]. The determinants of Naira to exchange rate of other major currencies globally have been identified in long term as money supply, inflation rates, commodity price level, investment per capita, foreign exchange rates among others [4−6]. Monetary policy is registered as a fundamental instrument over the years for the fulfilment of macroeconomic stability that constitute a major prerequisite in attaining sustainable economic growth. Refs [7−10] described accumulated erratic nature of the naira as a result of irregularities in policies that govern the exchange rate. Refs [11−14] emphasizes that policy makers should place more focus majorly on foreign exchange rate as a determinant of moving Nigeria forward on a sustainable economic growth trajectory. Moreso, several global economic transactions, namely speculation, hedging as well as capital budgeting are related to policies governing exchange rate [15]. Times series prediction can be

Received 10 March 2024; revised 10 April 2024; accepted 12 April 2024.
Available online 10 May 2024

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https://doi.org/10.55810/2313-0083.1063

termed as applicability of specific model to envisage future values built on information of historical observed values [16]. Time series components is structured to decompose observations into trend, seasonal, cyclical and error components for forecast purposes. Researchers have established several times series model in literature to advance its effectiveness and accuracy in terms of modelling and forecasting [17–19], among others.

Autoregressive Integrated Moving Average (ARIMA) model introduced by Ref. [20] is among the utmost widely used statistically itemized recognized process of times series model prediction tool. ARIMA efficiently deliberate serial linear correlation amongst observations, it is distinguished with the characteristics of having advantage of simple applications for forecasting exactness and accordingly require only the endogenous variables without necessarily considering other exogenous variables [21,22], The autoregressive fractional integral moving average (ARFIMA) model is a type of time series model used for modelling time series data; it extends the usage of ARIMA models by allowing non-integer differencing parameter values. ARFIMA can be categorize as a fractional order signal processing approach that generalizes the conventional integer order models of both autoregressive integral moving average (ARIMA) and autoregressive moving average (ARMA) model" [23]. The ARFIMA (p,d,q) model was introduced by Ref. [24], it belongs to the long memory model family that aims the goal of explicitly accounting for persistence in long-term correlations in the series. Researchers have shown the possibilities modelling long memory series of any size using both parametric and non-parametric method of estimation [25–28] among others.

Exchange rate forecasting theory has been in existence for many centuries with different model forecasting results from different models [29]. Several studies had been conducted on modelling and forecasting the exchanged rate of developed and developing countries across the globe in general employing different approaches [30] described an illustrative method of modelling and predicting time series data of official exchange rate of Nigeria using Box-Jenkins methodology. Their results showed that the optimum model for Dollar to Naira prediction was ARIMA (0, 1, 0) obtained from the use of Auto Arima function [31]. Modeled the official naira rate to US Dollar exchange rate over the range of 1960–2017. ARIMA (1, 1, 1) was chosen as the best model for the series which is used to forecast, the result obtained shows that naira will continue to depreciate if adequate and necessary actions were not taking regarding the monetary policy by the Nigeria government [32]. Modeled Volatility of the Exchange Rate of the Naira to vis-a-vis four other currencies, the results obtained from their fitted models showed that most of the parameters are significant and there is persistency in their volatility, symmetric GARCH gives a superior forecasting performance.

However, GARCH model fails to account for asymmetric effect that may occur when modeling financial time series. This led to the extension of GARCH models into TGARCH which accounts for degree of asymmetry, EGARCH which measures the effect of bad news on volatility and PGARCH model also captures asymmetric effects. TGARCH, EGARCH and PGARCH have their distinguished methods of capturing asymmetric effect, but their uniform objective is to capture the asymmetric effect [33], developed the Threshold GARCH (TGARCH) [34], developed the Exponential GARCH (EGARCH) in 1991 did something similar that assume conditional variance as a linear piecewise function [35], discussed the Power GARCH model as another class of ARCH extensive model which is capable of forecasting volatility index. Ref. [36] worked on modeling the exchange rate volatility of Naira against Euro, Pound Sterling, Dollar and West African Unit of Account (WAUA) in Nigeria employing symmetric and asymmetric GARCH models in the presence of Gaussian and Non-Gaussian errors. The results from their works show that symmetric GARCH (1,1) model considered all volatility clustering that has evidence of shock persistence in the exchange rate return series. The asymmetric EGARCH (1,1) and TGARCH (1,1) models yielded responses and leverages impacts in the four exchange rates log return series signifying that positive shocks give more volatility in Nigerian foreign exchange market than positive shocks of other countries with likely magnitude.

Ref. [37] studied over ten year's series of Chinese Yuan (CNY), Indian Rupees (INR), Nigerian Naira (NGN) and Malaysia Ringgits (MYR) daily to the U.S Dollar exchange rate using ARIMA and ARFIMA models. Their results showed that the ARIMA model gives best precision for CNY, INR and MYR to the U.S Dollar exchange rate while ARFIMA method yields suitable model for NGN exchange rate. Ref. [38] recently examined the monthly exchange rate of Naira per Dollar ranging from January 1981 to December 2015 using the ARFIMA method. The presence of a long memory structure was shown by the sample autocorrelation function. Geweke and Porter-Hudak (GPH) method of estimation were used to estimate the long memory parameter 'd' of the ARFIMA models, four models were estimated, the best model chosen was the ARFIMA (1, 0.0868, 1) based on minimum information criteria.

Owing to the significant role played by exchange rate dynamics of Naira in international trade and overall economic performance of Nigeria as a whole, the need for an effective forecasting instrument cannot be undermined. In line with [39] work, the recent financial turmoil naira if facing requires the urgency of perfect information of modelling the exchange rate against other major currencies across the globe. This paper investigates an empirical examination of modelling and forecasting time series data of the official Nigerian Naira to other major currencies exchange rate in the world. The study aims to measure the trend, erect a suitable model, and forecast future exchange rates. Also, this work filled the vacuum in the literatures by comparing the predictability power of ARIMA and ARFIMA to model the exchange rate of Naira against four other currencies. The forecast performance of ARFIMA and ARIMA models were measured using Mean Absolute error (MAE), the Root mean square error (RMSE) and Mean Absolute Percentage error (MAPE) as the validation criteria.

2. Materials and methods

2.1. Autoregressive models (AR)

Autoregressive models are built on the notion that the existing values of the times series x_t can be described as a function of past series $x_{t-1}, x_{t-2}, \ldots, x_{t-p}$ where p denotes the number of phases of the past required to forecast the present values. An autoregressive model of order p, (p) can be expressed as:

$$x_t = \gamma_1 x_{t-1}, +\gamma_2 x_{t-2} +, \ldots, \gamma_p x_{t-p} + \varepsilon_t \tag{1}$$

where x_t is the stationary series, $\gamma_1, \gamma_2\ , \ldots, \gamma_p$ are parameters of AR ($\gamma_p \neq 0$) unless stated otherwise, it is assumed that ε_t is Gaussian white noise with mean zero and variance σ^2. ε_t can be extracted as

$$\varepsilon_t = x_t - \ \gamma_1\ Ux_t - \gamma_2 U^2 x_t -, \ldots, \gamma_p U^p x_t \tag{2}$$

$$\varepsilon_t = x_t \left(\gamma_1\ U\ - \gamma_2 U^2 -, \ldots, \gamma_p U^p \right) \tag{3}$$

which can be simplified as equation (4) below

$$\varepsilon_t = \gamma x_t \tag{4}$$

The Autoregressive operator takes the form of;

$$\gamma(U) = 1 - \gamma_1 U - \ \gamma_2 U^2 - \ldots, -\gamma_p U^p \tag{5}$$

p denotes the order of Autoregressive operators and ϑ_i is the non-seasonal AR parameters, $i\ = 1, 2, \ldots, n$.

2.1.1. Moving average (MA)

A series x_t simply defined as a (q) process or said to assume a moving average process of order q if

$$x_t = \varepsilon_t + \partial_1 \varepsilon_{t-1}, +\vartheta_2 \varepsilon_{t-2} +, \ldots, \theta_p x_{t-p} \tag{6}$$

Employing the backward shift operator U, equation (6) above can be re written as;

$$x_t = \partial(U) \varepsilon_t \tag{7}$$

The moving-average operator in ARIMA model is defined as;

$$\vartheta(U) = 1 - \vartheta_1 U - \ \vartheta_2 U^2 - \ldots, -\vartheta_q U^q \tag{8}$$

q is the magnitude of the MA operator $\vartheta_p, p = 1, 2, \ldots, q$ is the Moving Average parameters and U signifies backward shift operator in such a way that

$$UX_t = X_{t-1} \tag{9}$$

2.1.2. Autoregressive Moving Average Processes (ARMA)

Autoregressive Moving Average Processes (ARMA) comprises the mixture of Autoregressive Moving Average model with p, AR terms and q, MA terms, that is ARMA (p,q). This is defined as:

$$\gamma(U) X_t = \partial(U) \varepsilon_t \tag{10}$$

$$X_t \left(1 - \gamma_1 U - \ \gamma_2 U^2 - \ldots, -\gamma_p U^p \right) \tag{11}$$

which can be simplified as;

$$\gamma(U) X_t = \varepsilon_t \left(1 + \vartheta_1 U + \vartheta_2 U^2 + \ldots, + \vartheta_q U^q \right) \tag{12}$$

2.1.3. ARIMA modeling

The Autoregressive Integrated Moving Average (ARIMA) model as proposed by Ref. [19] comprises of autoregressive as well as the moving average parameters, and explicitly includes differencing in the formation of the model. Moreover, the three major types of parameters involved in the model are: autoregressive parameter (p), the number of differencing passed (d), and the moving average parameter (q). In the notation introduced by Box and Jenkins, models are presented as *ARIMA* $(p,)$;

ARIMA model defined for an average data size can be expressed in general form as;

$$\gamma_p(U)(1-U)^d(X_t)=\vartheta_q(U)e_t \tag{13}$$

d represent the level of modifications; t denotes the definite time and e_t denotes the residual.

ARIMA model can be generalized simplified as;

$$X_t=\gamma_1X_{t-1}+\gamma_2X_{t-2}+,\ldots,\gamma_pX_{t-p}+\epsilon_t-\vartheta_1\epsilon_{t-1}-\vartheta_2\epsilon_{t-2}-,\ldots,\vartheta_q\epsilon_{t-q} \tag{14}$$

γ_p is the autoregressive parameter; ϵ_t denotes the white noise or residual and ϑ_q is the moving average parameter.

2.1.4. Box Jenkins methods of ARIMA model

In identifying a perfect non-seasonal ARIMA model for a specific time series analysis [19], proposed a methodology that consists of four major steps, namely,

i) Model Identification
ii) Estimation of the model parameters
iii) Evaluation of the model fit
(iv) Model forecasting.

The flow chart in Fig. 1 is the iterative methods of Box Jerkins approach of modelling, the methods was adopted from [20].

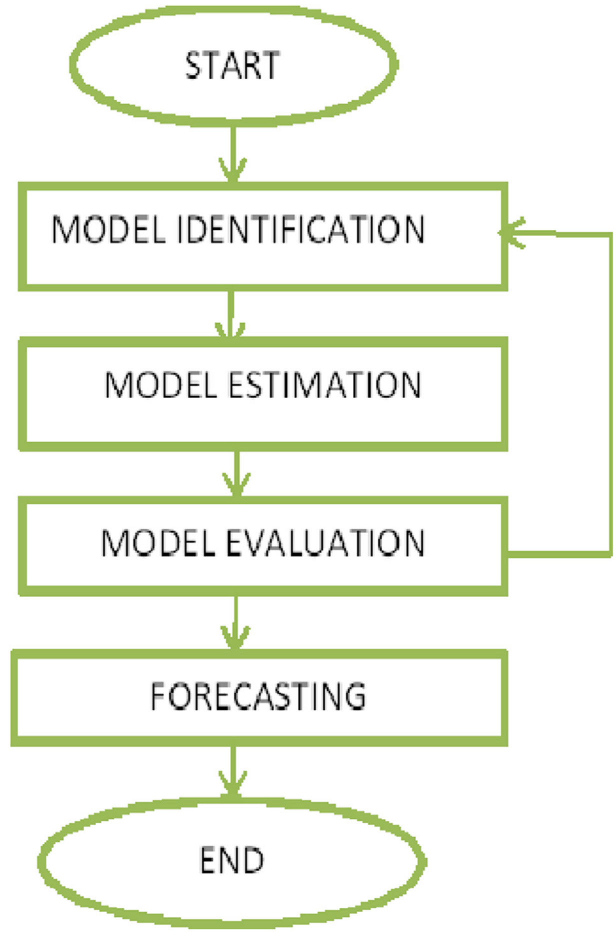

Fig. 1. Flow chart of the ARIMA modelling.

2.2. ARFIMA model process

The generic form of typical Autoregressive Fractionally Integrated Moving Average (ARFIMA).

process is specified as:

$$\gamma(U)\,(1-U)^dX_t=\vartheta(U)\varepsilon_t \tag{15}$$

where, L is the lag operator such that

$$UX_t=UX_{t-1} \tag{16}$$

and the $(1-U)^d$ fractional difference operator replaced the usual standard difference operator $(1-U)$ of a short memory ARIMA process, d is a non-integer parameter that represent the level of the fractional difference. ε_t is independently and identically distributed with mean 0 and variance σ^2, $\gamma(U)$ and $\vartheta(U)$ signify AR and MA components respectively. The method is covariance stationary for the range of $-0.5 < d < 0.5$; involving mean reversion when $d < 1$. This process of ARFIMA can be generalized as the fractional white-noise process illustrated in the works of [23,24]. Where $\gamma(L)$ is established to equal to unity to further analyze the features of the process.

Given that many time series steadily exhibits decaying autocorrelations slowly, the potential virtue of exploiting ARFIMA process with hyperbolic autocorrelation decay patterns in modelling financial time series modeling are numerous compared to modelling the ARMA processes that have exponential or geometric decay. The importance of the hyperbolic decay property can be easily noticeable considering the following expressions;

$$(1-U)^d=\sum_{k=0}^{\infty}(-1)^k\binom{d}{k}(U)^k=1-dU+\frac{d(d-1)}{2!}U^2\frac{d(d-1)(d-2)}{3!}U^3+,\ldots,\sum_{k=0}^{\infty}C_k(d) \tag{17}$$

for any $d > -1$. When $d = 0$, equation (17) above reduces to the classical ARMA (p,q) model.

2.3. Long memory test

Testing whether the observed data series exhibits long memory behavior is a prior process to method of estimating ARFIMA models. This study employed the techniques of Hurst Exponent in checking whether the data conforms to long memory structures.

2.4. Hurst exponent

The Hurst exponent is a representation of a time series' long-memory. The long memory structure happens when the values of H fall in the interval $0.5 < H < 1$. The Hurst exponent can be estimated using the formula;

$$H = \frac{\log\left(\frac{R}{S}\right)}{\log(N)} \tag{18}$$

Where N is the length of the sample data, R is the range of the data series, S is the standard deviation and $\frac{R}{S}$ denotes the matching value of the rescaled evaluation.

2.4.1. ARFIMA process estimation

The estimations of d are usually done in frequency domain. ARFIMA estimators of d are generally categorized into parametric and semi parametric. Parametric method of estimation can estimate all the parameters simultaneously while the semi-parametric method of estimation estimates in two steps.

The parametric methods of estimation attain coherent estimators of d via maximum likelihood estimation of parametric long-memory models. It gives a more precise estimate of d, but generally necessitate needs for the knowledge of the true model. The MLE procedures has the advantages of generating information from large samples under similar conditions and use all information enclosed in a data set to produce good inferences. Sowell's method involves specification of the p and q values, and estimation of the full ARFIMA model restricted to the choices made. This involves the task of choosing a suitable ARMA specification.

This research employed the approaches of the Hurst exponent and semi-parametric approaches of Geweke and Porter–Hudak (GPH) methods to test and estimate long memory parameters, using the following regression,

$$\ln(w_k) = U - d\ln\left[4\sin^2(w_k/2)\right] + n_k \tag{19}$$

Where $w_k = \frac{2nk}{T}, k = 1, 2, ..., n,$ n_k is the residual term and w_k denotes Fourier frequencies. 1 (w_k) represent the periodogram of a time series r_1 and it is defined as

$$I(w_k) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} r_1 e^{-w_k t}\right|2 \tag{20}$$

2.5. Test statistics

Model Identification: Identifying the appropriate model comprises specifying the relevant form of AR, MA or ARMA model in order. Non-stationarity test such as autocorrelation function, partial autocorrelation function, Augmented dickey fuller, Kwiatkowski–Philips–Schmidt–Shin (KPSS) and Phillips–Perron (PP) at 0.05 level of significant level were used to check the stationarity of the data involved.

This research implemented techniques of augmented Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests to investigate linearity assumption on the exchange rate of Naira to other major currencies for unit root test and fractional integration modelling.

Augmented Dickey Fuller Test of Stationarity: ADF test model is expressed as;

$$\Delta X_t = \alpha X_{t-1} + Y_t\varphi + \beta_1\Delta X_{t-1} + \beta_2\Delta X_{t-2} +, \ldots, \beta_p\Delta X_{t-p} \tag{21}$$

where,

ΔX_t denotes the differenced series

ΔX_{t-1} denotes the immediate past observations.

Y_t signifies the optional exogenous regressor which can be constant or be represented as constant trend

α and φ are parameters needed to be estimated.

$\beta_1, \ldots, \beta_p$ signifies the coefficients of the lagged terms.

The ADF test statistic is denoted by

$$t_\alpha = \frac{\widehat{\alpha}}{S_e(\widehat{\alpha})} \tag{22}$$

The test of hypothesis involves;

H_0: $\alpha = 0$, it implies that the series contains unit roots

H_1: $\alpha < 0$, it implies that the series contains no unit roots.

Decision rule: Reject H_0: if t_α is less than asymptotic critical value

2.5.1. Kwiatkowski-Philips-Schmidt-Shin (KPSS) test

Ref. [40] developed the KPSS test of stationarity. The null hypothesis assumes that the Data Generating Process (DGP) is stationary. Considering the following DGP without a linear trend;

$$y_t = x_t + z_t \tag{23}$$

where

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} +, \ldots, \alpha_p x_{t-p} + u_t \tag{24}$$

$u_t \sim \text{iid}(0.\sigma^2)$ and z_t is assume to follow a stationary process.

KPSS test statistic is expressed as;

$$\text{KPSS} = \frac{1}{N^2} \sum_{n=1}^{N} \frac{s_t^2}{\sigma^2 \infty} \tag{25}$$

Where $s_t = \sum_{j=1}^{t} \widehat{m}_j$ with $\widehat{m}_t = x_t - x$ and $\widehat{\sigma}_\infty{}^2$ is an estimator of the long run variance of the stationary process z_t.

2.5.2. Model selection

The model selection was accomplished implementing the optimum selection criteria by choosing the model with minimum Akaike Information Criteria (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criteria (HQIC).

2.5.3. Model estimation

After identifying the tentative best model, the model estimation employed maximum likelihood method to estimate simultaneously all the parameters of the process, the order of integration coefficient and parameters of an ARMA structure.

The likelihood function for the parameters of known observation data is given as;

$$L(\gamma, \vartheta \sigma_\epsilon^2 | X) = (2\pi\sigma_\epsilon^2)^{-\frac{1}{2}} \left(\exp\left(\frac{1}{2} \sigma_\epsilon^2 S(\gamma, \vartheta) \right) \right) \tag{26}$$

2.5.4. Model diagnostics

The white noise, serial correlation and the heteroscedasticity test was examined applying the residual normality test, the Portmanteau test and Autoregressive Conditional Heteroscedasticity Lagrange Multiplier (ARCH-LM) test respectively to validate the appropriateness of the selected ARIMA and ARFIMA models. It is actualized by examining the test of the hypothesis of white noise residuals that assumed to be independently distributed.

Employing the methods of [41] the variance of autocorrelation is defined as

$$\text{Var}(\rho_k(\varepsilon)) = \frac{1}{N(N-2)} (N-K), k = 1, 2, \ldots, K \tag{27}$$

And

$$\left(\sqrt{\frac{N-K}{N(N+2)}} \right)^{-1} \rho_k(\varepsilon) \approx N(0,1)$$

$$Q_{LB} = \left(\left(\sqrt{\frac{N-K}{N(N+2)}} \right)^{-1} \rho_k(\varepsilon) \right)^2 \tag{28}$$

$$= N(N+2) \sum_{k=1}^{K} \frac{[\rho_k(\varepsilon)]^2}{N-K} \approx \chi^2 (K-1) \tag{29}$$

Where K-1 = k-p-q and there is no inclusion of constant term in p + q, N is the sample size and ρ symbolize the autocorrelation coefficient.

Also, model adequacy was investigated by inspecting the spike of sample autocorrelation function of the residual (ACF) and sample partial autocorrelation function of the residual (PACF).

2.5.5. Model forecasting and performance evaluation

This section adopted the last segment of Box Jenkins approach by examining the performance of validation criterion such as; Akaike Information criteria (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criteria (HQIC) for comparing the predicting performances for the appropriate models.

$$\text{AIC} = 2T - m \tag{30}$$

$$\text{SIC} = 2T\log n - \log m \tag{31}$$

$$\text{HQIC} = -2\log m + 2T\log n \tag{32}$$

where T symbolizes the total of estimable parameters, m denotes the maximum likelihood and n is the digits of samples. Moreover, the forecasts accuracy of ARFIMA and ARIMA model are evaluated using Root Mean Square Error (RMSE), the Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE) respectively.

2.5.6. Mean absolute error (MAE)

MAE is the absolute value of the difference between the forecasted value and the actual value. It calculates the average absolute deviation of predicted values from real values. MAE is estimated as follow:

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^{n} |\widehat{y}_f - y_t| \tag{33}$$

2.5.7. Mean Absolute Percentage Error (MAPE)

MAPE is projected as the mean absolute percent error for each time period minus real values divided by real values. It computes the percentage of mean absolute error occurred in the model formation. It is stated as follows;

$$\mathrm{MAPE} = \frac{100}{n} \sum_{t}^{n} \left| \frac{\widehat{y}_{f} - y_t}{y_t} \right| \tag{34}$$

2.5.8. Root Mean Square Error (RMSE)

RMSE illustrate the absolute fit of the model to the observed data, it is computed as follows:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n} \sum_{t=1}^{n} \widehat{y}_{f} - y_t} \tag{35}$$

Where: $\widehat{y}_{f}$ and y_t are the estimated and the real values respectively; n is the sample size. Model with lesser value is likely to have the best precision power of forecast These models are useful for modelling long-memory time series, in which deviations from the long-run mean fades more slowly than is consistent with long memory.

2.5.9. Data

Monthly seasonally adjusted Naira-US dollar, Naira- Canadian Dollar, Naira- Australia Dollar and Naira-New Zeeland Dollar exchange rate data extracted from IMF International financial Statistics, ranging from January 1999 to December 2023 were used to compare the forecast performance of ARIMA and ARFIMA models.

3. Results and discussions

Times series plot of observed values of the major currencies; US Dollar (USD), New Zeeland Dollar r (NZD), Canadian Dollar (CAD) and Australia Dollar (AUD) were presented in Fig. 2. The upward movement (trend) of the time plot reveals that the time series are not stationary.

Table 1 gives the summary statistics of Monthly adjusted Naira-US dollar, Naira- New Zeeland Dollar (NZD), Naira- Canadian Dollar (CAD) and Naira- Australia Dollar (AUD) exchange rate data covering the period of January 1999 to December 2023. The total observation is 300, which is large enough for modeling ARFIMA models.

Table 1. Descriptive statistics

	US Dollar	New Zeeland dollar	Canadian dollar	Australia dollar
Mean	333.54	217.09	270.94	224.60
Median	249.39	155.74	203.50	166.65
Maximum	1142.0	638.70	859.00	851.00
Minimum	146.51	92.530	97.210	60.760
Std. Dev.	209.52	133.64	176.36	180.36
Skewness	2.4523	1.5302	1.6911	1.8505
Kurtosis	1.3376	1.9439	2.1106	3.7363
Jarque-Bera	1.2109	1.8821	1.0784	2.0023
P-value	0.2631	0.3709	0.6609	0.0411
Observations	300	300	300	300

Tables 2 and 3 shows the result of ADF and KPSS test for unit root of Nigeria Naira to US Dollar, Naira-New Zeeland Dollar, Naira-Canadian Dollar, and Naira-Australia Dollar exchange rate. The ADF test statistic tests the null hypothesis that the series has a unit root against the alternative of no unit root (stationary). The decision rule is to reject the null hypothesis when the p-value is less than or equal to 0.05. The KPSS test statistic tests the null hypothesis of stationarity against the alternative that the series has a unit root and to accept the null hypothesis when the test statistic value is less than the critical value. The results of ADF indicate that the time series data integrated at I (1). ADF test accept the

Table 2. Stationarity test results at level

Variables	ADF		KPPS	
	ADF Test Stat	Prob.	KPSS Test Stat.	Prob.
NGN/USD	−2.017	0.114	0.7103	0.029
NGN/NZD	0.2310	0.458	0.2050	0.044
NGN/CAD	−4.1203	0.024	0.3026	0.076
NGN/AUD	0.4523	0.185	0.6252	0.122

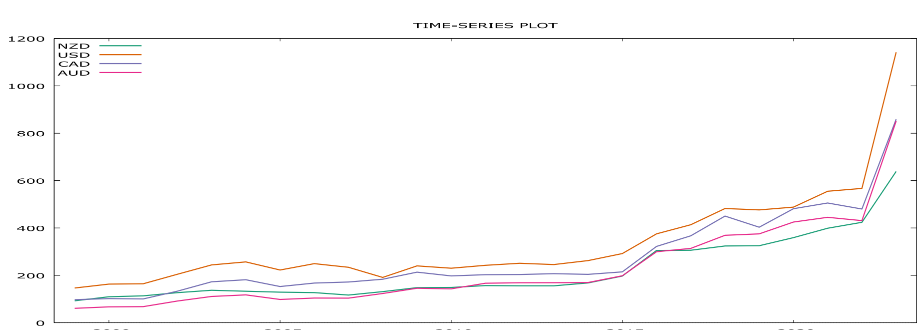

Fig. 2. Times series plots of the major currencies.

Table 3. Stationarity test results at first difference

Variables	ADF		KPSS Test	
	ADF Test Stat	Prob.	KPSS Test Statistics	Prob.
NGN/USD	−1.1172	0.003	0.5947	0.03
NGN/NZD	0.2042	0.000	0.8634	0.025
NGN/CAD	−2.628	0.004	0.6302	0.053
NGN/AUD	−2.0360	0.000	0.6921	0.016

alternative hypothesis that the series is stationary and the KPSS agree with the null hypothesis that the series is non-stationary. Also, there is need to carry out fractional difference on the data.

ARIMA Modeling.

Following the pictures of the correlogram of the series presented in Figs 3–6 above, various speculative ARIMA models for the variables were fitted to the series and the model with the lowest value of AIC, SIC and HQIC was selected as the optimal model amongst the competitors. The matching models and their respective values are tabulated in Table 4 below. The best model selected by the selection criteria is in bold print and asterisk mark for easier recognition. The significant of lowest information criteria in bold print shows that the selected model fits the data better.

Table 5 below presented parameter estimates of the ARIMA fitted models selected from Table 4 above using the selection criteria. The parameter estimation adopted the methodology Box and Jenkins utilizing maximum likelihood method of estimations, which assumes the asymptotic condition of time series observation in line with Brockwell et al.,

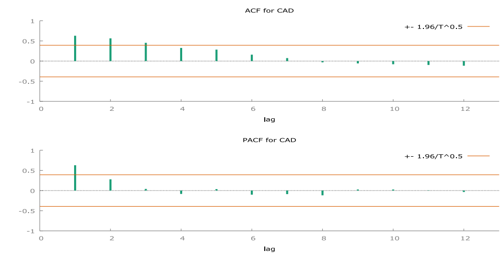

Fig. 5. Correlogram of CAD series.

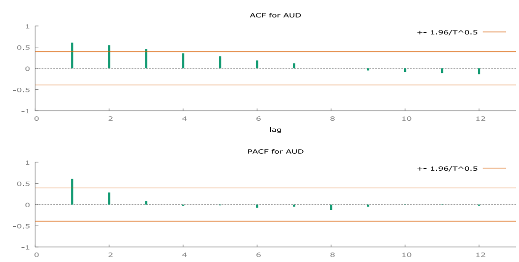

Fig. 6. Correlogram of AUD series.

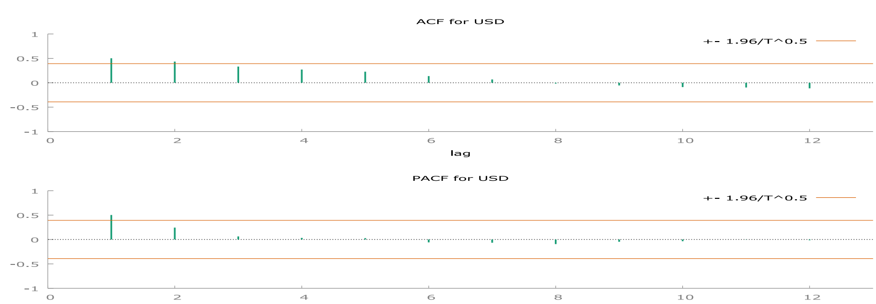

Fig. 3. Correlogram of US Dollar series.

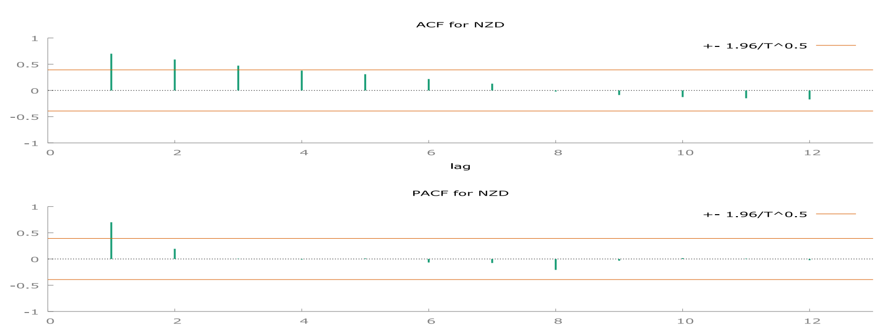

Fig. 4. Correlogram of NZD series.

Table 4. Tentative ARIMA model

Model	Specification (p,d,q)	AIC	SIC	H QIC
NGN/USD				
Model 1	ARIMA (1,1,1)	277.8241	282.7026	279.1794
Model 2	ARIMA (1,1,2)	278.4697	284.5641	280.1600
Model 3	**ARIMA (1,1,3)**	**274.0289**	**281.3411**	**276.0562**
NGN/NZD				
Model 1	ARIMA (1,1,1)	321.3871	326.2626	322.7394
Model 2	**ARIMA (1,1,2)**	**319.9440**	**326.0830**	**321.6341**
NGN/CAD				
Model 1	ARIMA (1,1,1)	304.1879	309.0634	305.5401
Model 2	**ARIMA (1,1,2)**	**301.2220**	**307.3164**	**302.9123**
Model 3	ARIMA (1,1,3)	302.8493	310.1627	304.8770
NGN/AUD				
Model 1	ARIMA (1,1,1)	306.6074	311.4829	307.9597
Model 2	**ARIMA (1,1,2)**	**304.3725**	**310.4669**	**306.0529**

Table 5. Parameter estimates of the ARIMA fitted model

Parameter	Coefficient	Standard Error	Prob.
NGN/USD			
γ_1	0.9306	0.0771	0.0000
ϑ_1	0.3995	0.2843	0.1600
ϑ_2	0.4615	0.2289	0.0438
ϑ_3	0.9603	0.3169	0.0024
NGN/NZD			
γ_1	0.9150	0.1003	0.0000
ϑ_1	0.1278	0.1294	0.3233
ϑ_2	1.0000	0.4458	0.0249
NGN/CAD			
γ_1	0.9140	0.1013	0.001
ϑ_1	−0.2046	0.1175	0.0817
ϑ_2	0.9301	0.4911	0.058
ϑ_3	0.8720	0.4027	0.031
NGN/AUD			
γ_1	0.9470	0.0661	0.0000
ϑ_1	−0.4173	0.1125	0.0002
ϑ_2	1.0000	0.2049	0.0000

2013. Where $\gamma_1, \vartheta_1, \vartheta_2$ and ϑ_3 are the autoregressive parameters of non-seasonal components.

3.1. MODEL diagnostics (ARIMA)

Figs 7–10 shows the residual plot of the exchange rate of Naira to US dollar, New Zeeland Dollar, Canadian Dollar, and Australia Dollar series respectively. From the figures above, the residuals data follows a normal distribution, thus, the selected model for each series is appropriate. Table 6 below revealed that there is no autocorrelation among the residual of the model's forecast errors. Also, the results of heteroscedasticity tests of residuals for the variables presented were all greater than the significance level which indicates the residuals are homoscedastic in nature. The Selected ARIMA

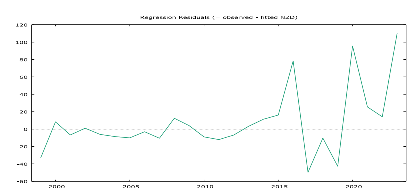

Fig. 8. Residual Plot of NZD series.

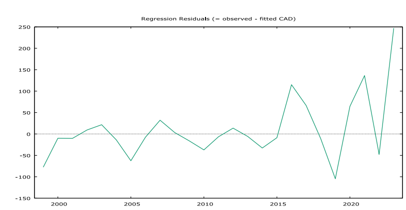

Fig. 9. Residual Plot of CAD series.

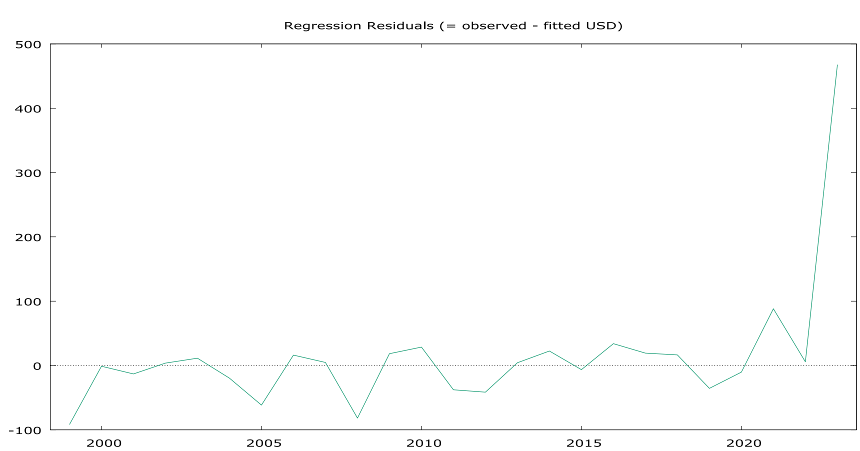

Fig. 7. Residual Plot of USD series.

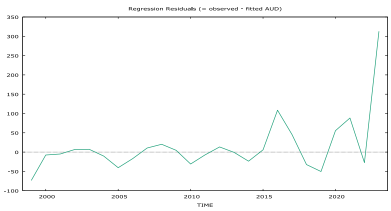

Fig. 10. Residual Plot of AUD series.

model accuracy is further investigated with the forecast accuracy measures.

3.2. ARFIMA model

Table 7 above shows the Hurst exponent values of the exchange rate data using the rescaled Range, Table 7 confirmed the existence of long memory of the series in terms of the exchange rate of naira against the currencies under study. The null hypothesis of no long memory was rejected in lieu of the respective p-values lesser than 0.05 significant level. Also, the Hurst exponent test gives values in the range of $0 < d < 1$.

Table 8 gives the estimates of the fractional difference of the exchange rate series using an automatic initialization of the integration using the Geweke and Porter-Hundlak log-periodogram regression. The competitive estimated models of each series and their respective values for the selection criteria are as tabulated in Table 8. The optimum model for each series is in bold print and asterisk mark for easier identification. Table 9 reported the parameter estimates of the ARFIMA fitted model selected from Table 8 above using the selection information criteria. $\gamma1,\vartheta1,\vartheta2$ and $\vartheta3$ are the autoregressive parameters of non- seasonal components and D is the respective fractional difference of

Table 6. Autocorrelation, Heteroskedacity and Normality test of the selected ARIMA models

Times series	ARIMA (p,d,q)	Autocorrelation Test		Heteroskedacity Test		Normality test	
	Model	Lung Box Q	Portmanteau	Breusch Pagan	White	Jarque Bera Test	Shapiro Wiki
		p-value	p-value	p-value	p-value	p-value	p-value
NGN/USD	ARIMA (1,1,3)	0.1825	0.1904	0.3190	0.5582	0.3328	0.3715
NGN/NZD	ARIMA (1,1,2)	0.1056	0.1129	0.2860	0.2578	0.1837	0.2025
NGN/CAD	ARIMA (1,1,2)	0.2041	0.2203	0.5621	0.6003	0.2692	0.3317
NGN/AUD	ARIMA (1,1,2)	0.1172	0.1250	0.3684	0.3092	0.4412	0.2278
Α		0.05	0.05	0.05	0.05	0.05	0.05

Table 7. Long Memory tests of the ARFIMA models

	CURRENCY			
	NGN/USD	NGN/NZD	NGN/CAD	NGN/AUD
HURST.E/RS	0.8824 (0.007)	0.8912 (0.003)	0.9216 (0.000)	0.6729 (0.001)

Hurst. E/RS is the Hurst Exponent Rescaled Range.

Table 8. Arfima model identification

MODEL	ARFIMA (p,d,q)	D	AIC	SIC	HQIC
NGN/USD					
MODEL 1	ARFIMA 1, d, 1	0.3062	9.215	8.805	7.063
MODEL 2	ARFIMA 1, d, 2	0.1956	7.109	7.428	6.992
MODEL 3	ARFIMA 2, d, 2	0.8945	**5.912***	**5.685***	**5.662***
NGN/NZD					
MODEL 1	ARFIMA 2, d, 1	0.5690	12.002	12.731	11.085
MODEL 2	ARFIMA 1, d, 1	0.8531	12.904	13.116	12.967
MODEL 3	ARFIMA 1, d, 2	0.6623	**11.035***	**11.884***	**12.240***
MODEL 4	ARFIMA 1, d, 0	0.5267	13.342	13.175	12.974
NGN/CAD					
MODEL 1	ARFIMA 1, d, 1	0.8630	**4.804***	**4.279***	**4.006***
MODEL 2	ARFIMA 0, d, 2	0.8421	5.338	6.190	5.929
MODEL 3	ARFIMA 2, d, 2	0.7604	4.919	4.983	5.031
NGN/AUD					
MODEL 1	ARFIMA 1, d, 2	0.2689	21.986	20.964	20.784
MODEL 2	ARFIMA 1, d, 1	0.8523	23.857	25.987	23.986
MODEL 3	ARFIMA 2, d, 0	0.5689	22.764	23.006	22.964
MODEL 4	ARFIMA 2, d, 1	0.7735	**21.236***	**20.883***	**20.643***

Table 9. Parameter estimates of the ARFIMA fitted model

Parameter	Coefficient	Standard Error	Prob.
NGN/USD			
D	0.3062	0.0164	0.0025
γ_1	0.4872	0.0128	0.0000
γ_2	−0.6829	0.1101	0.0007
ϑ_1	0.9387	0.1450	0.0032
ϑ_2	0.3789	0.0484	0.0001
NGN/NZD			
D	0.6623	0.1035	0.0059
γ_1	−0.9567	0.0811	0.0000
ϑ_1	−1.9873	0.1363	0.0018
ϑ_2	0.4890	0.0559	0.0002
NGN/CAD			
D	0.8630	0.1465	0.0011
γ_1	1.0038	0.0480	0.0000
ϑ_1	−0.9822	0.1196	0.0014
NGN/AUD			
D	0.7735	0.1196	0.0014
γ_1	0.2874	0.2819	0.0028
γ_2	−0.3789	0.4782	0.0000
ϑ_1	−0.4492	0.3381	0.0007

the estimates. The parameter estimation adopted the methodology Box and Jenkins utilizing maximum likelihood method of estimations in [20].

3.3. Diagnostics checking of ARFIMA models

Table 10 presents the autocorrelation, Heteroskedacity and the normality check results and the p values for each selected ARFIMA models for the variables. The normality tests revealed that the residuals generated from the selected ARFIMA models are normally distributed, the Ljung-Box and the Portmanteau value for all the variables are greater than the significant level which inferred that there is no autocorrelation among the residual of the model's forecast errors. Also, the results of heteroscedasticity tests of residuals for the variables shows that the residuals are homoscedastic in nature.

Table 11 displayed the ARFIMA and ARIMA selected models for the four currencies exchange rate to Naira forecast accuracy measures. Results comparison of ARIMA and ARFIMA modeling from Table 11 revealed that ARFIMA model is more adequate in modelling the exchange rate of Naira to the currencies under study. The low values of an unbiased statistic MAPE of ARFIMA models in Table 11 revealed the adequacy of the selected ARFIMA models in predicting the future exchange rate of Naira to US dollar accurately. Moreover, the general error measures showed evidence of better forecast with ARFIMA models in forecasting the exchange values of Naira to the aforementioned currency under study.

3.4. Forecasting using the selected ARFIMA model for the variables

However, with the aid of the derived model for the variables, the following forecast were made for

Table 10. Statistical tests of the residuals of selected ARFIMA models

Times series	ARFIMA (p,fd,q)	Autocorrelation Test		Heteroskedacity Test		Normality test	
	Model	Lung Box Q	Portmanteau	Breusch Pagan	White	Jarque Bera Test	Shapiro Wiki
		p-value	p-value	p-value	p-value	p-value	p-value
NGN/USD	ARFIMA (2,0.6745,2)	0.2325	0.1108	0.4227	0.2127	0.3180	0.1427
NGS/NZD	ARFIMA (1,0.6623,2)	0.3174	0.2106	0.1842	0.3052	0.1743	0.3395
NGN/CAD	ARFIMA (1,0.8630,1)	0.2513	0.2618	0.4225	0.5186	0.3425	0.4742
NGN/AUD	ARFIMA (2,0.7735,1)	0.5287	0.5832	0.4412	0.3144	0.5226	0.5084
α		0.05	0.05	0.05	0.05	0.05	0.05

Table 11. Evaluation of selected ARIMA and ARFIMA models forecast accuracy

	ARIMA models				ARFIMA models			
	NGN/USD	NGN/CAD	NGN/AUD	NGN/NZD	NGN/USD	NGN/CAD	NGN/AUD	NGN/NZD
RMSE	0.2579	0.5513	0.3475	0.2089	0.0542	0.0112	0.0362	0.0912
MAE	0.3324	0.4752	0.2972	0.3190	0.0106	0.0557	0.0240	0.0394
MAPE	0.1947	0.6853	0.4417	0.3661	0.0023	0.0341	0.0662	0.0017
R^2	0.8754	0.8558	0.7717	0.8854	0.9421	0.9023	0.8993	0.9135

Table 12. Average annual exchange rate of Naira-USD out of sample forecast

YEAR	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033
FORECAST VALUES	1364	1503.12	1524.48	1592.86	1355.66	1299.35	1234	1181	1132	1089
Lower limits	964.55	1297.41	828.20	676.93	676.93	551.85	445.92	354.86	275.79	206.63
Upper limits	1660.0	1868.82	1981.06	2021.51	2034.40	2032.85	2022.91	2007.95	1990.00	1970.39

Table 13. Average annual exchange rate of Naira-NZD out of sample forecast

Year	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033
FORECAST VALUES	710.27	737.64	818.73	788.72	760.62	734.31	709.69	686.63	665.05	644.84
Lower limits	621.83	615.10	643.16	531.31	448.67	381.43	324.48	275.19	231.91	193.55
Upper limits	768.71	860.19	994.30	1046.12	1072.56	1087.20	1094.89	1098.08	1098.19	1096.14

Table 14. Average annual exchange rate of Naira-CAD out of sample forecast.

Year	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033
FORECAST VALUES	725.79	928.59	884.82	843.82	806.82	772.99	742.09	713.84	688.01	664.41
Lower limits	584.62	755.51	602.14	495.84	412.17	343.23	285.05	235.20	192.06	154.43
Upper limits	866.97	1101.68	1166.46	1191.81	1201.47	1202.77	1192.47	1192.47	1183.96	1174.38

Table 15. Average annual exchange rate of Naira-AUD out of sample forecast

Year	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033
FORECAST VALUES	669.78	968.24	937.24	907.95	880.28	854.13	829.44	806.11	784.06	763.24
Lower limits	524.74	804.02	664.47	566.27	487.18	420.26	362.16	310.92	265.24	224.19
Upper limits	814.82	1132.46	1210.01	1249.62	1273.37	1288.01	1296.71	1301.29	1302.89	1302.30

the year 2024–2033 as shown in Tables 12–15 with their lower and upper limits respectively for average annual exchange rate of Naira to US dollar, New Zeeland Dollar, Canadian Dollar, and Australia Dollar series respectively. The forecasted values are within the lower and the upper limits. The graphical illustrations are presented in Figs 11–14 respectively. The values for all the models are within 95% confidence limits which shows that the models have good predictive abilities.

The result of the out-of- sample forecasts for 2024 to 2028 shows that virtually most of the exchange rate of

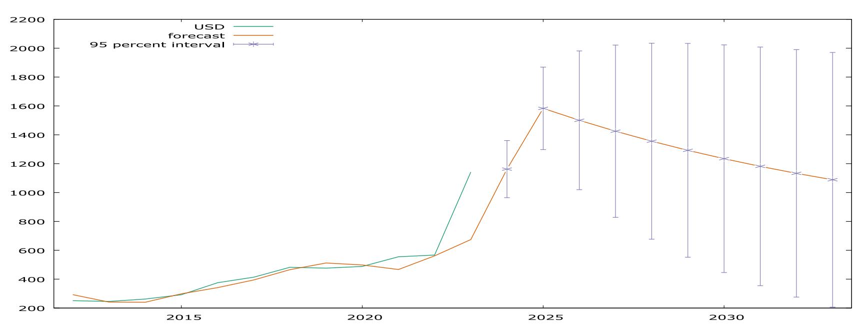

Fig. 11. Forecast rate of Naira-USD.

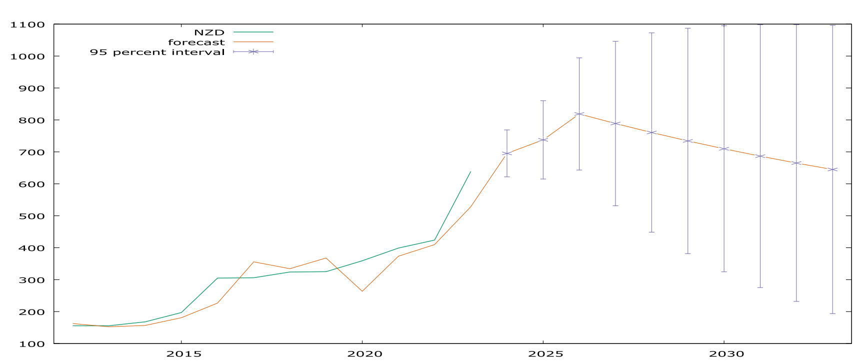

Fig. 12. Forecast rate of Naira-NZD.

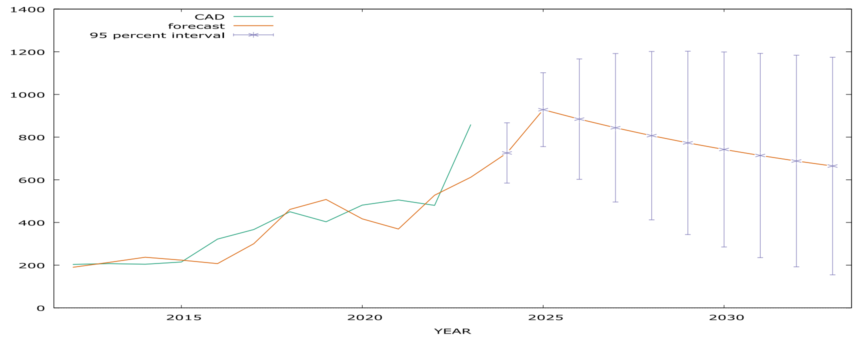

Fig. 13. Forecast rate of Naira-CAD.

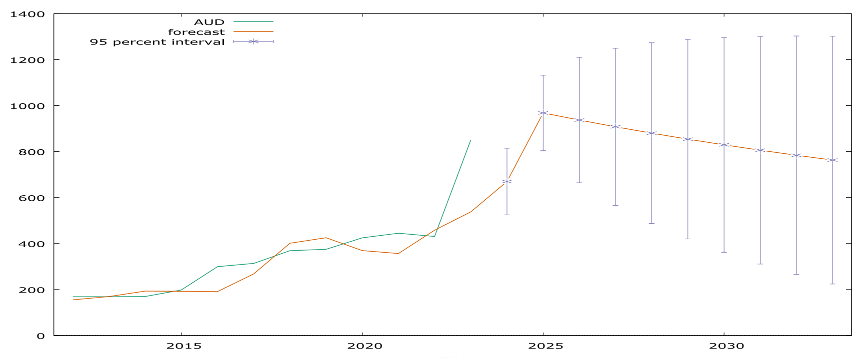

Fig. 14. Forecast rate of Naira-AUD.

Naira to these currencies will continue to depreciate before appreciating its value in later years.

4. Conclusion

This study investigated the exchange strength of Nigerian Naira to US Dollar (USD), New Zeeland Dollar (NZD), Canadian Dollar (CAD) and Australia Dollar (AUD) and fit the best ARIMA and ARFIMA models for the data using Box Jenkins techniques. The stationary tests of the series shows that the time series under consideration is nonstationary in level but become stationary at first difference with an exception of KPSS test, the series exhibit the characteristics of long memory process as indicated in the Hurst exponent test result. The results reveal needs for efficient exploratory exercise in choosing appropriate model to achieve optimum forecast values.

Based on the forecast evaluation measures, ARFIMA models yields the best predictive abilities than ARIMA for all the series, the appropriateness of the model was confirmed that the residuals of the models were normally distributed, random and no presence of errors autocorrelation and forecast for period of ten years terms were made and the forecasted values were all within the confidence limits. The result of the out-of- sample forecasts for 2024 to 2028 revealed that most of the exchange rate of Naira to these currencies will continue to depreciate before appreciating its value in later years. This rise in exchange rates for the short time is relatively steady, the result of the research justifies the result of [37] concerning naira, it is therefore recommended that monetary and policy makers should employ possible policy that will encourage local manufacturing of goods and inflow of foreign capital to reverse the expected trend by stabilizing the Nigeria economy.

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