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Maryam Adnan Al-Ethary
Ministry of Education, Directorate of Educational Babylon, Hilla, Iraq,

Maryam Sabbeh Al-Rubaiea
Biology Department, College of Science, Al-Qasim Green University, Babylon 51013, Iraq,

Mohammed H. O. Ajam
Pathological Analysis Department, College of Science, Al-Qasim Green University, Babylon 51013, Iraq.

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ORIGINAL STUDY

New Class Function in Dual Soft Topological Space

Maryam A. Al-Ethary [a,*], Maryam S. Al-Rubaie [b], Mohammed H.O. Ajam [c]

[a] Ministry of Education, Directorate of Educational Babylon, Hilla, Iraq
[b] Biology Department, College of Science, Al-Qasim Green University, Babylon 51013, Iraq
[c] Pathological Analysis Department, College of Science, Al-Qasim Green University, Babylon 51013, Iraq

Abstract

In this paper we introduce a new class of maps in the dual Soft topological space and study some of its basic properties and relations among them, then we study $dual\ Soft_{open}$ and $dual\ Soft_{closed}$ mapping.

Keywords: Dual soft set, Dual soft topological space, Dual soft continuity mapping, *Dual* $Soft_{open}$ mapping and *dual* $Soft_{closed}$ mapping

1. Introduction

Engineering, physics, computer science, economics and many other fields world-wide have many problems.

Scientists created a the model to simplify the work with the real features, but unfortunately these models very complex and do not give accurate results. The cause for not obtaining accurate results was the classical methods due to the lack of knowledge about natural phenomena and the methods which used to measure them, for example decision making in an environment have no a database, so the classical theory that is based on a clear and accurate basis is unable to deal With the vague problems.

Molodtsov in Ref. [7] was presented the concept of Soft set theory as a new mathematical tool to deal with modeling vagueness. As a new mathematical tool in order to deal with uncertainties. The soft set relations were introduced as a sub soft set of the Cartesian product of the soft sets and many related concepts such as equivalent soft set relation by Babitha KV, Sunil JJ [4]. Although Shabir and Naz in Ref. [9] introduced the study of Soft topological space. Maji et al. [8] the researcher has conducted studies regarding to the methods related to the theory of Soft topology and offered a comprehensive theoretical study on it. Pei and Miao [5] He steered a study on the Soft group and demonstrated that it is a type of information system. They in 2021 dual Soft theory were defined by Al swidi L. A., Reyadh D. A., Hadi M. H. [3], for briefness, they will represent it (d.s.). The dual Soft local function and dedoual Soft ideal topological space is studied by investigator Al Rubaie M. and Al Ethary M [1]. In 2023 Al Rubaie M. and Al Ethary M [2]. found a new sort of Soft dual separation axioms. In 2024, Mohammed Abu Saleem [6] introduced the soft covering map on a soft topological space and the notion of a soft local homeomorphism.

2. Preliminaries

Definition 2.1. [4]: Let U_1 and U_2 are initial universes sets and E be the set of all potential boundaries under consideration as for U_1 and U_2. Whereas parameters are descriptions, features or properties of members of the initials universes sets.
The triple (A_D, W, G_D) is dual soft set over U_1 and U_2 where, W, G_D are functions from $\tilde{E}$ to power of U_1 and U_2 respectively, so

$$(A_D, W, G_D) = \{(r, W(k), G(t));\ \forall r \in A_D\} \cup \{(r, \varnothing, \varnothing);\ \forall r \in \tilde{E} - A_D\}.$$

The collection of all dual soft sets is called the dual soft space and is denoted by $Ds(U_1, U_2)\tilde{E}$.

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* Corresponding author.
E-mail address: mariem20132420@gmail.com (M.A. Al-Ethary).

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Definition 2.2. [4]: The dual Soft crossroads of dual Soft sets (A_D, W_1, G_1) and (B_D, W_2, G_2) over a common universes U_1 and U_2 and A_D, B_D are subsets of the parameters $\tilde{E}$ of association for U_1 and U_2, is defined as the dual Soft set (C, H_1, H_2) where $C = (A_D \cup B_D)$ and

$$(p, H_1(p), H_2(p)) = \begin{cases} (p, W_1(p) \cap W_2(p), G_1(p) \cap G_2(p)) & if\ p \in A_D \cup B_D \\ (p, \varnothing, \varnothing) & if\ p \in \tilde{E}/C \end{cases}$$

Denoted by $(C, H_1, H_2) = (A_D, W_1, G_1) \cap_D (B_D, W_2, G_2)$.

Definition 2.3. [4]: The dual Soft connection of dual Soft sets (A_D, W_1, G_1) and (B_D, W_2, G_2) over a typical universes U_1 and U_2 and A_D, B_D are subsets of the parameters $\tilde{E}$ of members for U_1 and U_2, is defined dual Soft set (C, H_1, H_2) where $C = (A_D \cup B_D)$ and

$$(p, H_1(p), H_2(p)) = \begin{cases} (p, W_1(p) \cup W_2(p), G_1(p) \cup G_2(p)) & if\ p \in A_D \cup B_D \\ (p, \varnothing, \varnothing) & if\ p \in \tilde{E}/C \end{cases}$$

Denoted by $(C, H_1, H_2) = (A_D, W_1, G_1) \cup_D (B_D, W_2, G_2)$.

Definition 2.4. [4]: Assume that the sets U_1 and U_2 are the primary universes sets and $\tilde{E}$ are a parameters of members of the sets U_1 and U_2, then

1. $\varnothing_{Ds} = (\tilde{E}, W, N) = \{(r, \varnothing, \varnothing); \forall r \in \tilde{E}\}$ that is $W(r) = N(r) = \varnothing; \forall r \in \tilde{E}$ is called the dual empty Soft set.
2. $X_{Ds} = (\tilde{E}, W, N) = \{(r, \varnothing, \varnothing); \forall r \in \tilde{E}\}$ that is $W(r) = N(r) = X; \forall r \in \tilde{E}$ is called the dual absolute Soft set.

Definition 2.5. [4]: The singular dual Soft point for any subset A of the parameter $\tilde{E}$ of members for the universes sets U_1 and U_2 is $(r, W(k), G(t))$ of the dual Soft set (A_D, W, G)A for any point e in E.

Definition 2.6. [4]: The sub collection StT_E of $DS_{(U_1,U_2)E}$ is called dual Soft topology StT_E on X_{Ds} if satisfy. $\varnothing_{Ds}$, $X_{Ds} \in StT_E$.

1. If FG_A, $f_1G_{1B} \in StT_E$ then $FG_A \cap f_1G_{1B} \in StT_E$.
2. For any index Λ if $f_iG_{iA_I} \in StT_E$. Then $\cup_{i \in \Lambda} f_iG_{iA_I} \in StT_E$.

So $StT_E = \{\varnothing_{Ds}, X_{Ds}, FG_A\}$ is dual Soft topology and (X_{Ds}, StT_E) is dual Soft topological space.

Definition 2.7. Let (X_{Ds}, StT_E) be a dual Soft topological space over X_{Ds} . A dual Soft set (A_D, W, G) over X_{Ds} is said to be a closed dual Soft set in X_{Ds} , if its relative complement $(A_D, W, G)^c$ belong to StT_E.

Proposition 2.8. Let (X_{Ds}, StT_E) be a dual Soft topological space over X_{Ds}. Then the collection $StT_{E\alpha} = \{A_{D\alpha}, W, G| (A_D, W, G) \in StT_E$ for each $\alpha \in E$, defines a topology on X_{Ds}.

Definition 2.9. Let $X = \{x_1, x_2, \ldots, x_n\}, Y = \{y_1, y_2, \ldots, y_m\}, E = \{e_1, e_2, \ldots, e_i\}$, $A = \{e_1, e_2, \ldots, e_j\}, \ni j < i, A \subseteq E$. Then the point e_{xy} is write on this way $e_{xy} = \{e_i, x_n, y_m\}$, such that $e_i \in A$, $x_n \in X, y_m \in Y$.

Definition 2.10. Let (X_{Ds}, StT_E) be a dual Soft topological space over X_{Ds}. A dual Soft set (A_D, W, G) over X_{Ds}. Then the dual Soft closure of (A_D, W, G), denoted by $\overline{(A_D, W, G)}$ is the intersection of all *dual Soft*$_{closed}$ super sets of (A_D, W, G).

Definition 2.11. Aual Soft topological space (X_{Ds}, StT_E) be a over X_{Ds}. A dual Soft set FG_B over X_{Ds}. $e_{xy} \in X_{Ds}$. Then e_{xy} is said to be a dual Soft interior point of FG_B, if there exists a *dual Soft*$_{open}$ set FG_A such that $e_{xy} \in FG_A \subset FG_B$.

Definition 2.12. Let (X_{Ds}, StT_E) be a dual Soft topological space over X_{Ds}. A dual Soft set FG_B ended X_{Ds}. $e_{xy} \in X_{Ds}$. Then FG_B is said to be dual Soft neighborhood of X_{Ds}, if there exists a *dual Soft*$_{open}$ set FG_A such that $e_{xy} \in FG_A \subset FG_B$.

Definition 2.13. Adual Soft topological space (X_{Ds}, StT_E) ended X_{Ds} then *dual soft* $_{interior}$ of set FG_A ended

X_{Ds} is denoted by $(FG_A)^o$ and is Know as the association of all dual Soft sets contained in FG_A.
Thus $(FG_A)^o$ is the largest *dual Soft$_{open}$* contained inFG_A.

3. Doual soft topology on function space

Definition 3.1. Let (A, F, G) be a dual Soft set over X_{DS}, the dual Soft set is called a dual Soft point, denoted by e_{xy} if for element $e \in E, FG_e = \{(e, x, y)\}$ and $FG(e^c) = \emptyset$ for all $e^c \in E - \{e\}$.

Definition 3.2. Let (X_{Ds}, StT_E) and (Y_{Ds}, StT'_E) be two dual Soft topological space such that:
f_{DS}: (X_{Ds}, StT_E) → (Y_{Ds}, StT'_E) be a mapping, for each dual Soft neighborhood FG_B of $f_{DS}(e_{xy})$ if there existis a dual Soft neighborhood FG_A of e_{xy} such that $f_{DS}(FG_A) \subset FG_B$ then f_{DS} is dual Soft continuity mapping at (e_{xy}).
If f_{DS} is dual Soft continuity mapping for all (e_{xy}) then f_{DS} is Know as dual Soft continuity mapping.

Example 3.3. Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\}$
$StT_E = \{X_{DS}, \emptyset_{DS}, FG_A, F_1G_{1A}\}$, $StT_E = \{X_{DS}, \emptyset_{DS}, HK_A, H_1K_{1A}\}$ where

$$FG_A = \{(e_1, \{x_2\}, \{y_1\}), (e_2, X, \{y_2\}), (e_3, \emptyset, \emptyset)\}$$

$$F_1G_{1A} = \{(e_1, X, \{y_1\}), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

$$HK_A = \{(e_1, \{x_1\}, \{y_2\}), (e_2, X, \{y_1\}), (e_3, \emptyset, \emptyset)\}$$

$$H_1K_{1A} = \{(e_1, X, \{y_2\}), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

If we get the mapping f_{DS}: (X_{Ds}, $StT_E, II_{(U_1,U_2)}$) → (X_{Ds}, $StT'_E, II_{(U_1,U_2)}$) defined as $f_{DS}(x_1) = x_2$, $f_{DS}(x_2) = x_1, f_{DS}(y_1) = y_2, f_{DS}(y_2) = y_1$ then since $f_{DS}^{-1}(HK_A) = FG_A$ *and* $f_{DS}^{-1}(H_1K_{1A}) = F_1G_{1A}$, then f_{DS} is a dual Soft continuous mapping.

Theorem 3.4. Let (X_{Ds}, StT_E) and (Y_{Ds}, StT'_E) be two dual Soft topological space such that:
f_{DS}: (X_{Ds},StT_E) → (Y_{Ds},StT'_E,) be a mapping. Then the next conditions are same:

1) f_{DS}: (X_{Ds}, StT_E) → (Y_{Ds}, StT'_E) is a dual Soft continuity mapping.
2) For any *dual Soft$_{open}$* set FG_B over Y_{Ds}, $f_{DS}^{-1}(FG_B)$ is *dual Soft$_{open}$* set over X_{Ds}.
3) For any *dual Soft$_{closed}$* set FG_H over Y_{Ds}, $f_{DS}^{-1}(FG_H)$ is *dual Soft$_{closed}$* set over X_{Ds}.
4) For any dual Soft set FG_A over X_{Ds}, $f_{DS}(\overline{FG_A}) \subset \overline{f_{DS}(FG_A)}$.
5) For any dual Soft set FG_K over Y_{Ds}, $\overline{f_{DS}^{-1}(FG_K)} \subset f_{DS}^{-1}(\overline{FG_K})$.
6) For any dual Soft set FG_B over Y_{Ds}, $f_{DS}^{-1}((FG_B)^o) \subset (f_{DS}^{-1}(FG_B))^o$.

Proof. 1 →2 Let FG_B be a *dual Soft$_{open}$* set over Y_{Ds} and $e_{xy} \in f_{DS}^{-1}(FG_B)$ be any dual Soft point. Then $f_{DS}(e, x, y) = (f_{DS}(e), x, y) \in FG_B$, since f_{DS} is a dual Soft continuous mapping. There exists $e_{xy} \in FG_A \in StT_E$ such that $f_{DS}(FG_A) \subset FG_B$. This implies that $e_{xy} \in FG_A \subset f_{DS}^{-1}(FG_B)$, $f_{DS}^{-1}(FG_B)$ is a *dual Soft$_{open}$* set over X_{DS}.
(2) → (1) Let e_{xy} be a dual Soft point and $f_{DS}(e_{xy}) \in FG_B$ be an arbitrary dual Soft neighborhood. Then $e_{xy} \in f_{DS}^{-1}(FG_B)$ is a dual Soft neighborhood and $f_{DS}(f_{DS}^{-1}(FG_B)) \subset FG_B$. Thus f_{DS} is a dual Soft continuous mapping.
(3) → (4) Let FG_A be a *dual Soft$_{open}$* set over X_{Ds}. Since $FG_A \subset f_{DS}^{-1}(f_{DS}(FG_A))$ and $f_{DS}(FG_A) \subset \overline{f_{DS}(FG_A)}$, we have $FG_A \subset f_{DS}^{-1}(f_{DS}(FG_A)) \subset f_{DS}^{-1}(\overline{f_{DS}(FG_A)})$. By part (3) since $f_{DS}^{-1}(\overline{f_{DS}(FG_A)}$ is a *dual Soft$_{closed}$* set over X_{Ds}, $\overline{FG_A} \subset f_{DS}^{-1}(\overline{f_{DS}(FG_A)})$. Thus $f_{DS}(\overline{FG_A}) \subset f_{DS}(\overline{f_{DS}^{-1}(FG_A)}) \subset \overline{f_{DS}(FG_A)}$, is obtained.
(4) → (5) Let FG_B be a dual Soft set over Y_{Ds} and $f_{DS}^{-1}(FG_B) = FG_A$. By part (4) we have $f_{DS}(\overline{FG_A}) = f_{DS}(\overline{f_{DS}^{-1}(FG_B)}) \subset \overline{f_{DS}(f_{DS}^{-1}(FG_B))} \subset \overline{FG_B}$. Then $\overline{f_{DS}^{-1}(FG_B)} = \overline{FG_A} \subset f_{DS}^{-1}(\overline{f_{DS}(FG_A)}) \subset f_{DS}^{-1}(\overline{FG_B})$.
(5) → (6) Let FG_B be a dual Soft set over Y_{Ds}. Substituting $\grave{FG_B}$ for condition in (5). Then $\overline{f_{DS}^{-1}(FG_B{}^c)} \subset f_{DS}^{-1}(\overline{FG_B{}^c})$ since $(FG_B)^o = (\overline{FG_B{}^c})$, then we have $f_{DS}^{-1}((FG_B)^o) = f_{DS}^{-1}((\overline{FG_B{}^c}))^c = (f_{DS}^{-1}(\overline{FG_B{}^c}))^c \subset (\overline{f_{DS}^{-1}(FG_B{}^c)})^c = (\overline{(f_{DS}^{-1}(FG_B))^c})^c = (f_{DS}^{-1}(FG_B))^o$.
(5) → (6) Let FG_B be a *dual Soft$_{open}$* set over Y_{Ds}. Then since $(f_{DS}^{-1}(FG_B))^o \subset f_{DS}^{-1}(FG_B) = f_{DS}^{-1}((FG_B)^o) \subset (f_{DS}^{-1}(FG_B))^o$, $(f_{DS}^{-1}(FG_B))^o = f_{DS}^{-1}(FG_B)$ is obtained. This implies that $f_{DS}^{-1}(FG_B)$ is a *dual Soft$_{open}$* set overX_{Ds}.

Theorem 3.5. Suppose f_{DS}: (X_{Ds}, StT_E) → (Y_{Ds}, StT'_E,) is a dual Soft continuity mapping, then for each $\propto \in E, f_{DS\propto} : (Y, T_\propto, E) \to (Y, T'_\propto, E)$ is a dual Soft continuous mapping.

Proof. Let $(G, E) \in T'_\propto$ then there exists a *dual Soft$_{open}$* set FG_B over Y_{Ds} such that $G(\propto) = FG_B$. Since f_{DS}: (X_{Ds}, StT_E) → (Y_{Ds}, StT'_E,) is a dual Soft continuity mapping, $f_{DS}^{-1}(FG_B)$ is a *dual Soft$_{open}$* set over X_{Ds} and $f_{DS}^{-1}(FG_B(\propto)) = f_{DS}^{-1}G(\propto) = f_{DS}^{-1}(G, E)(\propto)$ is an Soft open set. This implies that $f_{DS\propto}$ is a Soft continuous mapping.
Now we give an example to show that the converse of above theorem dose not hold.

Example 3.6. Let $X = \{x_1,x_2\}, R = \{r_1,r_2\}, E = \{e_1,e_2, e_3\}, A = \{e_1, e_2\}$
$StT_E = \{X_{DS}, \emptyset_{DS}, FG_A, F_1G_{1A}, F_2G_{2A}, F_3G_{3A}\}$. $Y = \{y_1, y_2\}$, $Q = \{q_1, q_2\}$, $StT'_E = \{Y_{DS}, \emptyset_{DS}, HK_A, H_1K_{1A}, H_2K_{2A}, H_3K_{3A}\}$ such that:

$$FG_A = \{(e_1, \{x_2\}, \{r_1\}), (e_2, X, \{r_2\}), (e_3, \emptyset, \emptyset)\}$$

$$F_1G_{1A} = \{(e_1, \{x_1\}, \emptyset), (e_2, \{x_1\}, \{r_1\}), (e_3, \emptyset, \emptyset)\}$$

$$F_2G_{2A} = \{(e_1, X, \{r_1\}), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

$$F_3G_{3A} = \{(e_1, \emptyset, \emptyset), (e_2, \{x_1\}, \emptyset), (e_3, \emptyset, \emptyset)\}$$

$$HK_A = \{(e_1, \{y_1\}, \{q_2\}), (e_2, Y, \{q_1\}), (e_3, \emptyset, \emptyset)\}$$

$$H_1K_{1A} = \{(e_1, \{y_2\}, \emptyset), (e_2, \{y_2\}, \emptyset), (e_3, \emptyset, \emptyset)\}$$

$$H_2K_{2A} = \{(e_1, Y, \{q_2\}), (e_2, Y, \{q_1\}), (e_3, \emptyset, \emptyset)\}$$

$$H_3K_{3A} = \{(e_1, \emptyset, \emptyset), (e_2, \{y_2\}, \emptyset), (e_3, \emptyset, \emptyset)\}$$

If we get the mapping f_{DS}: $(X_{Ds}, StT_E) \rightarrow (Y_{Ds}, StT'_E)$ defined as

$$f_{DS}(x_1) = y_2, f_{DS}(x_2) = y_1, f_{DS}(r_1) = q_2, f_{DS}(r_2) = q_1.$$

Then f_{DS} is not a dual Soft continuous mapping, because $f_{DS}{}^{-1}(H_1K_{1A}) \notin StT_E$. But $f_{DS}(e_1) : (X_{Ds}, StT_{Ee_1}) \rightarrow (Y_{Ds}, StT'_{Ee_1})$ and $f_{DS}(e_2) : (X_{Ds}, StT_{Ee_2}) \rightarrow (Y_{Ds}, StT'_{Ee_2})$ are dual Soft continuous mapping.

Here $StT_{Ee_2} = \{X_{DS}, \emptyset_{DS}, (e_1, \{x_2\}, \{r_1\}), (e_1, \{x_1\}, \emptyset), (e_1, X, \{r_1\})\}$

$$StT_{Ee_2} = \{X_{DS}, \emptyset_{DS}, (e_2, X, \{r_2\}), (e_2, \{x_1\}, \{r_1\}), (e_2, X, Y), (e_2, \{x_1\}, \emptyset)\}$$

$$StT'_{Ee_1} = \{Y_{DS}, \emptyset_{DS}, (e_1, \{y_1\}, \{q_2\}), (e_1, \{y_2\}, \emptyset), (e_1, Y, \{q_2\})\}$$

$$StT'_{Ee_2} = \{Y_{DS}, \emptyset_{DS}, (e_2, Y, \{q_1\}), (e_2, \{y_2\}, \emptyset), (e_2, \{y_2\}, \emptyset)\}.$$

Definition 3.7. Let $(X_{Ds}, StT_E, II_{(U_1,U_2)})$ and $(Y_{Ds}, StT'_E, II_{(U_1,U_2)})$ be two dual Soft topological spaces, f_{DS}: $(X_{Ds}, StT_E) \rightarrow (Y_{Ds}, StT'_E)$ is a mapping.

a) If the image $f_{DS}(FG_A)$ of any *dual Soft*$_{open}$ set FG_A over X_{Ds} is a *dual Soft*$_{open}$ set in Y_{Ds}, then f_{DS} is called to be a *dual Soft*$_{open}$ mapping.
b) If the image $f_{DS}(FG_B)$ of any *dual Soft*$_{closed}$ set FG_B over X_{Ds} is a *dual Soft*$_{closed}$ set in Y_{Ds}, then f_{DS} is said to be a *dual Soft*$_{closed}$ mapping.

Proposition 3.8. If f_{DS}: $(X_{Ds}, StT_E) \rightarrow (Y_{Ds}, StT'_E,)$ is *dual Soft*$_{open(closed)}$ mapping, then for each $\propto \in E$, $f_{DS\propto} : (X, T_\propto, E) \rightarrow (Y, T'_\propto, E)$ is an *dual Soft*$_{open(closed)}$ mapping.

Proof. The proof of the proposition is direct and it is left to the reader.
To make Note that the notions of dual Soft continuous, *dual Soft*$_{open}$, *dual Soft*$_{(closed)}$ mapping are all independent of any other.

Example 3.9. Let $(X_{Ds}, StT_E,)$ be dual Soft Discrete topological space and (X_{Ds}, StT'_E) be dual Soft Indiscrete topological space. Then 1_{DS}: $(X_{Ds}, StT_E) \rightarrow (X_{Ds}, StT'_E,)$ is a *dual Soft*$_{open}$ and *dual Soft*$_{(closed)}$ mapping. But it is not dual continuous mapping.

Example 3.10. Let $X = \{x_1,x_2\}, Y = \{y_1,y_2\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\}$
$StT_E = \{X_{DS}, \emptyset_{DS}, FG_A, F_1G_{1A}, F_2G_{2A}\}, StT'_E = \{X_{DS}, \emptyset_{DS}, HK_A, H_1K_{1A}\}$ where

$$FG_A = \{(e_1, \{x_2\}, \{y_1\}), (e_2, X, \{y_2\}), (e_3, \emptyset, \emptyset)\}$$

$$F_1G_{1A} = \{(e_1, X, \{y_1\}), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

$$F_2G_{2A} = \{(e_1, X, \emptyset), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

$$HK_A = \{(e_1, \{x_1\}, \{y_2\}), (e_2, X, \{y_1\}), (e_3, \emptyset, \emptyset)\}$$

$$H_1K_{1A} = \{(e_1, X, \{y_2\}), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

If we get the mapping f_{DS}: $(X_{Ds}, StT_E, II_{(U_1,U_2)}) \rightarrow (X_{Ds}, StT'_E, II_{(U_1,U_2)})$ defined as $f_{DS}(x_i) = x_1, f_{DS}(y_i) = y_1, i = 1, 2$. It is clear that:

$f_{DS}^{-1}(HK_A) = \{(e_1, X, \emptyset), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$
$f_{DS}^{-1}(H_1K_{1A}) = \{(e_1, X, \emptyset), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$ then f_{DS} is a dual Soft continuous mapping, but $f_{DS}(FG_A) = \{(e_1, \{x_1\}, \{y_1\}), (e_2, X, \{y_1\}), (e_3, \emptyset, \emptyset)\}$

$$f_{DS}(F_1G_{1A}) = \{(e_1, X, \{y_1\}), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

$$f_{DS}(F_2G_{2A}) = \{(e_1, X, \emptyset), (e_2, X, Y), (e_3, \emptyset, \emptyset)\}$$

Then it is not both *dual Soft*$_{open}$ and *dual Soft*$_{closed}$ mapping.

Example 3.11. Let $X = \{x_1, x_2, x_3\}, R = \{r_1, r_2\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\}$

$StT_E = \{X_{DS}, \emptyset_{DS}, FG_A, F_1G_{1A}\}$,
$Y = \{y_1, y_2\}, Q = \{q_1, q_2\}$
$StT'_E = \{X_{DS}, \emptyset_{DS}, HK_A, H_1K_{1A}\}$ where

$$FG_A = \{(e_1, \{x_1, x_2\}, \{r_1\}), (e_2, X, \{r_2\}), (e_3, \emptyset, \emptyset)\}$$

$F_1G_{1A} = \{(e_1, \{x_2\}, \{r_1\}), (e_2, \{x_2\}, \{r_2\}), (e_3, \varnothing, \varnothing)\}$

$HK_A = \{(e_1, Y, \{q_1\}), (e_2, Y, \{q_2\}), (e_3, \varnothing, \varnothing)\}$

$H_1K_{1A} = \{(e_1, \{y_2\}, \{q_1\}), (e_2, \{y_2\}, \{q_2\}), (e_3, \varnothing, \varnothing)\}$

the mapping f_{DS}: $(X_{Ds}, StT_E, II_{(U_1,U_2)}) \to (X_{Ds}, StT'_E, II_{(U_1,U_2)})$ defined as $f_{DS}(x_1) = y_1, f_{DS}(x_2) = f_{DS}(x_3) = y_2$

$f_{DS}(r_1) = q_1, f_{DS}(r_2) = q_2$

then f_{DS} is a *dual Soft*$_{open}$ mapping , but f_{DS} is not dual Soft continuous mapping because $f_{DS}^{-1}(H_1K_{1A})$ is not dual open set, f_{DS} is not *dual Soft*$_{closed}$ mapping because $(FG_A)^c$ is *dual Soft*$_{closed}$ set but $f_{DS}(FG_A)^c$ is not *dual Soft*$_{closed}$ set.

Example 3.12. Let $X = \{x_1, x_2, x_3\}, R = \{r_1, r_2\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\}$

$StT_E = \{X_{DS}, \varnothing_{DS}, FG_A, F_1G_{1A}\}$,

$Y = \{y_1, y_2\}, Q = \{q_1, q_2\}$
$StT'_E = \{X_{DS}, \varnothing_{DS}, HK_A, H_1K_{1A}\}$ where

$FG_A = \{(e_1, \{x_1, x_3\}, \{r_2\}), (e_2, \{x_1, x_3\}, \{r_1\}), (e_3, \varnothing, \varnothing)\}$

$F_1G_{1A} = \{(e_1, \{x_3\}, \{r_2\}), (e_2, X, \{r_1\}), (e_3, \varnothing, \varnothing)\}$

$HK_A = \{(e_1, Y, \{q_2\}), (e_2, Y, \{q_1\}), (e_3, \varnothing, \varnothing)\}$

$H_1K_{1A} = \{(e_1, \{y_1\}, \{q_2\}), (e_2, \{y_1\}, \{q_1\}), (e_3, \varnothing, \varnothing)\}$

the mapping f_{DS}: $(X_{Ds}, StT_E, II_{(U_1,U_2)}) \to (X_{Ds}, StT'_E, II_{(U_1,U_2)})$ defined as $f_{DS}(x_1) = y_1, f_{DS}(x_2) = f_{DS}(x_3) = y_2$

$f_{DS}(r_1) = q_1, f_{DS}(r_2) = q_2$

then f_{DS} is a *dual Soft*$_{closed}$ mapping, but f_{DS} is not dual Soft continuous mapping because $f_{DS}^{-1}(H_1K_{1A})$ is not dual open set, f_{DS} is not *dual Soft*$_{open}$ mapping because F_1G_{1A} is *dual Soft*$_{open}$ but $f_{DS}(F_1G_{1A})$ is not *dual Soft*$_{open}$ set.

Theorem 3.13. Let (X_{Ds}, StT_E) and (Y_{Ds}, StT'_E) be two dual Soft topological spaces, f_{DS}: $(X_{Ds}, StT_E) \to (Y_{Ds}, StT'_E,)$ is mapping.

a) f_{DS} is *dual Soft*$_{open}$ mapping $\leftrightarrow$ for any dual Soft set FG_A over X_{Ds}, $f_{DS}(FG_A)^o \subset (f_{DS}(FG_A))^o$ is satisfied.
b) f_{DS} is *dual Soft*$_{closed}$ mapping $\leftrightarrow$ for any dual Soft set FG_A over X_{Ds}, $\overline{f_{DS}(FG_A)} \subset f_{DS}(\overline{FG_A})$ is satisfied.

Proof. a) Let f_{DS} be a *dual Soft*$_{open}$ mapping and Let FG_A be an arbitrary dual Soft set over X_{Ds}. $FG_A{}^o$ is *dual Soft*$_{open}$ set and $FG_A{}^o \subset FG_A$. Since f_{DS} is a *dual Soft*$_{open}$ mapping, $f_{DS}(FG_A{}^o)$ is a *dual Soft*$_{open}$ set in Y_{Ds} and $f_{DS}(FG_A{}^o) \subset f_{DS}(FG_A)$. Thus $f_{DS}(FG_A{}^o) \subset f_{DS}(FG_A)^o$ is obtained.
Conversely, let FG_A be an arbitrary dual Soft set over X_{Ds}. Then $FG_A = FG_A{}^o$. From the condition of theorem, we have $f_{DS}(FG_A{}^o) \subset f_{DS}(FG_A)^o$. Then $f_{DS}(FG_A) = (f_{DS}(FG_A)^o) \subset (f_{DS}(FG_A))^o \subset f_{DS}(FG_A)$. This implies that $f_{DS}(FG_A) = (f_{DS}(FG_A))^o$.
b) Let f_{DS} be a *dual Soft*$_{closed}$ mapping and FG_A be an arbitrary dual Soft set over X_{Ds}. Since f_{DS} is a *dual Soft*$_{closed}$ mapping, $f_{DS}(\overline{FG_A})$ is a *dual Soft*$_{closed}$ set over Y_{Ds} and $f_{DS}(FG_A) \subset f_{DS}(\overline{FG_A})$, Thus $\overline{f_{DS}(FG_A)} \subset f_{DS}(\overline{FG_A})$ is obtained.
Conversely let FG_A be an arbitrary dual Soft set over X_{Ds}. From the condition of theorem, $\overline{f_{DS}(FG_A)} \subset f_{DS}(\overline{FG_A}) = f_{DS}(FG_A) \subset \overline{f_{DS}(FG_A)}$. This means that $\overline{f_{DS}(FG_A)} = f_{DS}(FG_A)$.

Definition 3.14. Let $(X_{Ds}, StT_E, II_{(U_1,U_2)})$ and $(Y_{Ds}, StT'_E, II_{(U_1,U_2)})$ be two dual Soft topological spaces, f_{DS}: $(X_{Ds}, StT_E) \to (Y_{Ds}, StT'_E)$ be a mapping , if f_{DS} is a bijective, dual Soft continuous and f_{DS}^{-1} is a dual Soft continuous mapping, then f_{DS} is said to be a dual Soft homeomorphism from X_{Ds} to Y_{Ds}, when a dual Soft homeomorphism f_{DS} exists btween X_{Ds} and Y_{Ds} we say that X_{Ds} is dual Soft homeomorphism to Y_{Ds}.

Theorem 3.15. Let (X_{Ds}, StT_E) and (Y_{Ds}, StT'_E) be two dual Soft topological spaces, f_{DS}: $(X_{Ds}, StT_E) \to (Y_{Ds}, StT'_E,)$ be a bijective mapping. Then the following conditions are equivalent:

(1) f_{DS} is a dual Soft homeomorphism.
(2) f_{DS} is a dual Soft continuous and dual Soft closed mapping.
(3) f_{DS} is a dual Soft continuous and *dual Soft*$_{open}$ mapping.

Proof. It is easily obtained.

4. Conclusion

In this work, a new mapping was introduced using the dual Soft set in the dual Soft topological space, called the dual Soft continuous mapping, and its properties and relationships were studied. Also, the *dual Soft*$_{open}$, *dual Soft*$_{closed}$ mapping and dual Soft homeomorphism was presented.

References

[1] Al Rubaie M, Al Ethary M. On dual Soft local function. Int J Nonlinear Anal 2023;14(1):2617–21. https://doi.org/10.22075/IJNAA.2023.29379.4142.

[2] Al Rubaie M, Al Ethary M. Soft m-separation axioms in dual soft topological space. Int J Sci Trends 2023;3(2):12–5. https://scientifictrends.org/index.php/ijst/article/view/71.

[3] Al swidi LA, Reyadh DA, Hadi MH. About the doual Soft sets thory. J Discrete Math Sci Cryptogr 2022. https://doi.org/10.1080/09720529.2022.2060917.

[4] Babitha KV , Sunil JJ, Soft set relations and functios , Comput Math Appl 60:1840-1849. https://doi.org/10.1016/j.camwa.2010.07.014.

[5] Pei D, Miao D. From Soft sets to information systemsvol. 2. IEEE Inter. Conf.; 2005. p. 617–21. https://doi.org/10.1109/GRC.2005.1547365.

[6] Abu Saleem Mohammed. On Soft covering spaces in Soft topological spaces. AIMS Mathematics 2024;9(7):18134–42. https://doi.org/10.3934/math.2024885.

[7] Molodtsov D. Soft set theory-first results. Comput Math Appl 1999;37:19–31. https://doi.org/10.1016/S0898-1221(99)00056-5.

[8] Maji PK, Biswas R, Roy AR. Soft set theory. Comput Math Appl 2003;45:555–62. https://doi.org/10.1016/S0898-1221(03)00016-6.

[9] Shabir M, Naz M. On Soft topological spaces. Comput Math Appl 2011;61:1786–99. https://doi.org/10.1016/j.camwa.2011.02.006.