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ORIGINAL STUDY

Laplace Homotopy Perturbation Method (LHPM) for Solving Systems of N-Dimensional Non-Linear Partial Differential Equation

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Abstract

In this research, we proposed coupling the Laplace transform method and the homotopy Perturbation Method (LHPM). We employed the fusion of the Laplace method to make up for the shortcomings of other semi-analytical approaches like the homotopy perturbation method, variation iteration method, and the Adomian decomposition method. We aim to obtain an approximate and semi-analytic solution of the n-dimensional system of nonlinear partial differential equations. N-dimensional partial differential equations with nonlinear terms are known as nonlinear partial differential equations. They have been used to solve mathematical problems like the Poincaré conjecture and the Calabi-Yau conjecture and describe physical systems, from gravity to fluid dynamics. Therefore, we proffer a semi-analytic solution in the form of a Taylor multivariate series of displacements x, y, and time t using the proposed method. A side-by-side comparison was carried out to compare the exact solution with the new solution using 3-dimensional graphs, and thus the graph analysis followed. Results show excellent agreement, and the emergence of this method as a viable alternative demonstrates its viability by requiring fewer computations and being much easier and more convenient than others, making it suitable for widespread use in engineering as well.

Keywords: Laplace transform, Homotopy perturbation method, System of partial differential equation, Semi analytic approach

1. Introduction

Systems of non-linear partial differential equations arise in many areas of mathematics, physics, and engineering to study real-life phenomena [1–3]. It is necessary to provide solutions to this real-life issue [4,5]. Several methods have been adopted in the past to solve this issue, which include the differential transformation method [6], the deep learning approach [7], the homotopy perturbation method [8–10], the homotopy analysis method [11,12], the fractional homotopy method [13], the elzaki differential transform [14–16], etc.

However, in the quest for a more reliable and efficient method of solution for a system of non-linear partial differential equations, we employed the Laplace homotopy perturbation method. The method involves the merging of two different methods, namely the Laplace transform and the homotopy perturbation method, to provide a more efficient solution for the system [10].

Initially, He [17] combined the traditional homotopy method and the perturbation method to create a new approach to solving linear and nonlinear initial and boundary value problems. Originally developed to make the most of both homotopy and perturbation techniques, the homotopy perturbation method has since been tweaked to achieve faster convergence, less computational overhead, and more precise results. There is a large category of functional equations for

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which the Homotopy Perturbation Method (HPM) has been used [8,9,14,18–29]. HPM and other semi-analytical methods allow for the calculation of partial differential equations over a limited range with many iterations [23,24,30,31].

This paper proposes an LPHM to fix this deficiency. This novel approach meets all conditions. A couple of iterations yield precise outcomes over a broad spectrum. This study solves three n-dimensional systems of nonlinear partial differential equations using the Laplace transform homotopy perturbation method (LHPM). Comparing the current approach to the exact solution proves its efficacy.

The semi-analytic method has been applied to solve several linear and non-linear differential equations [32–34].

2. Laplace homotopy perturbation (method of solution)

Consider the following system of Partial Differentiation Equation

$$P_t U + T_1(u,v) + W_1(u,v) = h_1 \tag{1}$$

$$P_t U + T_2(u,v) + W_2(u,v) = h_2 \tag{2}$$

In operator form; with the initial conditions of the form;

$$U(x,0) = g_1(x) \tag{3}$$

$$V(x,0) = g_2(x) \tag{4}$$

Where ρ, is the first order Partial differential operator T_1 and T_2 are linear differential operators; W_1 and W_2 are the non-linear differential operators and h_1 and h_2 are the non-homogenous terms and U and V are functions of x and t.

Taking the Laplace transform on both sides of equations (1) and (2) and applying the prescribed initial conditions; (3) and (4), we obtain

$$L[P_t U] + L[T_1(u,v)] + L[W_1(u,v)] = L[h_1] \tag{5}$$

$$L[P_t U] + L[T_2(u,v)] + L[W_2(u,v)] = L[h_2] \tag{6}$$

Operating the differentiation property of the Laplace transform gives

$$L[U] = \frac{g_1(x)}{s} + \frac{1}{s}L[h_1] - \frac{1}{s}L[T_1(u,v)] - \frac{1}{s}L[W_1(u,v)] \tag{7}$$

$$L[V] = \frac{g_2(x)}{s} + \frac{1}{s}L[h_2] - \frac{1}{s}L[T_2(u,v)] - \frac{1}{s}L[W_2(u,v)] \tag{8}$$

The operation of the Laplace transform disintegrates the unknown functions $U(x,t)$ and $V(x,t)$ given by an infinite series of components

$$U(x,t) = \sum_{n=0}^{\infty} \rho^n u_n(x,t) \tag{9}$$

$$V(x,t) = \sum_{n=0}^{\infty} \rho^n v_n(x,t) \tag{10}$$

Substituting (9) and (10) into equations (7) and (8) gives;

$$\begin{aligned} L\left[\sum_{n=0}^{\infty} \rho^n u_n(x,t)\right] = \frac{g_1(x)}{s} + \frac{1}{s}L[h_1] - \frac{1}{s}L\left[T_1 \sum_{n=0}^{\infty} \rho^n u_n(x,t) \sum_{n=0}^{\infty} \rho^n v_n(x,t)\right] \\ -\frac{1}{s}L\left[W_1 \sum_{n=0}^{\infty} \rho^n u_n(x,t) \sum_{n=0}^{\infty} \rho^n v_n(x,t)\right] \end{aligned} \tag{11}$$

$$L\left[\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right]=\frac{g_2(x)}{s}+\frac{1}{s}L[h_2]-\frac{1}{s}L\left[T_2\sum_{n=0}^{\infty}\rho^n u_n(x,t)\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right]$$
$$-\frac{1}{s}L\left[W_2\sum_{n=0}^{\infty}\rho^n u_n(x,t)\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right] \tag{12}$$

By applying the Homotopy-Perturbation technique and the linearity of Laplace transform, where ρ an embedding parameter is; in equations (11) and (12) we will have

$$L\left[\sum_{n=0}^{\infty}\rho^n u_n(x,t)\right]=\frac{g_1(x)}{s}+\rho\left\{\begin{array}{l}\frac{1}{s}L[h_1]-\frac{1}{s}L\left[\sum_{n=0}^{\infty}\rho^n u_n(x,t)\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right]\\ -\frac{1}{s}L\left[W_1\sum_{n=0}^{\infty}\rho^n u_n(x,t)\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right]\end{array}\right\} \tag{13}$$

$$L\left[\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right]=\frac{g_2(x)}{s}+\rho\left\{\begin{array}{l}\frac{1}{s}L[h_2]-\frac{1}{s}L\left[T_2\sum_{n=0}^{\infty}\rho^n u_n(x,t)\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right]\\ -\frac{1}{s}L\left[W_2\sum_{n=0}^{\infty}\rho^n u_n(x,t)\sum_{n=0}^{\infty}\rho^n v_n(x,t)\right]\end{array}\right\} \tag{14}$$

Operating the inverse Laplace transform on both sides of equations (13) and (14) and comparing the coefficients of like powers of ρ, we obtain the following approximations:

$$\rho^0: u_0 = L^{-1}\left[\frac{g_1(x)}{s}\right]$$

$$\rho^0: u_0 = L^{-1}\left[\frac{g_2(x)}{s}\right]$$

$$\rho^1: u_1 = L^{-1}\left[\frac{1}{s}L(h_1)\right]-L^{-1}\left[\frac{1}{s}L(u_0,v_0)\right]-L^{-1}\left[\frac{1}{s}L(W_1(u_0,v_0))\right]$$

$$\rho^1: v_1 = L^{-1}\left[\frac{1}{s}L(h_2)\right]-L^{-1}\left[\frac{1}{s}L(u_0,v_0)\right]-L^{-1}\left[\frac{1}{s}L(W_2(u_0,v_0))\right]$$

Proceeding in similar manner, we have a recursive relation for $n \geq 1$ given by;

$$\rho^{k+1}: u_{k+1} = L^{-1}\left[\frac{1}{s}L(h_k)\right]-L^{-1}\left[\frac{1}{s}L(u_k,v_k)\right]-L^{-1}\left[\frac{1}{s}L(W_1(u_k,v_k))\right]$$

$$\rho^{k+1}: v_{k+1} = L^{-1}\left[\frac{1}{s}L(h_k)\right]-L^{-1}\left[\frac{1}{s}L(u_k,v_k)\right]-L^{-1}\left[\frac{1}{s}L(W_2(u_k,v_k))\right]$$

3. Applications

Case 1

Consider the following system of three-dimensional partial differential equation

$$U_t - VU_x - V_tU_y = 1 - x + y + t \tag{15}$$

$$V_t - UV_x - V_yU_t = 1 - x - y - t \tag{16}$$

With initial conditions;

$$U(x,y,0) = x + y - 1$$
$$V(x,y,0) = x - y + 1$$

The exact solutions are given by; $U(x,y,t) = x + y + t - 1$, $V(x,y,0) = x - y - t + 1$

Taking the Laplace transform of the equations (15) and (16)

$$L[U_t] = L[VU_x] + L[V_tU_y] + L(1) - L(x) + L(y) + L(t) \tag{17}$$
$$sL[U(x,y,s)] - U(x,y,0) = L[VU_x] + L[V_tU_y] + L(1) - L(x) + L(y) + L(t)$$
$$L[U(x,y,s)] = \frac{U(x,y,0)}{s} + \frac{1}{s}L[VU_x] + \frac{1}{s}L[V_tU_y] + \frac{1}{s}L(1) - \frac{1}{s}L(x) + \frac{1}{s}L(y) + \frac{1}{s}L(t)$$

$$\begin{aligned} &L[V_t] = L[UV_x] + L\left[U_tV_y\right] + L(1) - L(x) - L(y) - L(t) \\ &sL[V(x,y,s)] - V(x,y,0) = L[UV_x] + L\left[U_tV_y\right] + L(1) - L(x) - L(y) - L(t) \\ &L[V(x,y,s)] = \frac{V(x,y,0)}{s} + \frac{1}{s}L[UV_x] + \frac{1}{s}L\left[U_tV_y\right] + \frac{1}{s}L(1) - \frac{1}{s}L(x) - \frac{1}{s}L(y) - \frac{1}{s}L(t) \end{aligned} \tag{18}$$

Taking inverse Laplace transform for equation for equation (17)

$$\begin{aligned} L^{-1}[L(U(x,y,s))] = L^{-1}\left[\frac{U(x,y,0)}{s}\right] + L^{-1}\left[\frac{1}{s}L(VU_x)\right] + L^{-1}\left[\frac{1}{s}L\left(V_tU_y\right)\right] \\ + L^{-1}\left[\frac{1}{s}L(1)\right] - L^{-1}\left[\frac{1}{s}L(x)\right] + L^{-1}\left[\frac{1}{s}L(y)\right] + L^{-1}\left[\frac{1}{s}L(t)\right] \end{aligned} \tag{19}$$

$$U(x,y,t) = L^{-1}\left[\frac{U_0(x,y,0)}{s}\right] + L^{-1}\left[\frac{1}{s}L(VU_x)\right] + L^{-1}\left[\frac{1}{s}L\left(V_tU_y\right)\right] + t - xt + yt + \frac{1}{2}t^2 \tag{20}$$

$$\begin{aligned} L^{-1}[L(V(x,y,s))] = L^{-1}\left[\frac{V(x,y,0)}{s}\right] + L^{-1}\left[\frac{1}{s}L(UV_x)\right] + L^{-1}\left[\frac{1}{s}L\left(U_tV_y\right)\right] \\ + L^{-1}\left[\frac{1}{s}L(1)\right] - L^{-1}\left[\frac{1}{s}L(x)\right] - L^{-1}\left[\frac{1}{s}L(y)\right] - L^{-1}\left[\frac{1}{s}L(t)\right] \end{aligned} \tag{21}$$

$$V(x,y,t) = L^{-1}\left[\frac{V_0(x,y,0)}{s}\right] + L^{-1}\left[\frac{1}{s}L(UV_x)\right] + L^{-1}\left[\frac{1}{s}L\left(U_tV_y\right)\right] + t - xt - yt - \frac{1}{2}t^2 \tag{22}$$

Suppose the solution of the equations (20) and (22) has the form;

$$U(x,y,t) = \lim_{\rho\to 1}\rho^n U_n(x,y,t) = \sum_{n=0}^{\infty}\rho^n U_n \tag{23}$$

$$V(x,y,t) = \lim_{\rho\to 1}\rho^n V_n(x,y,t) = \sum_{n=0}^{\infty}\rho^n V_n \tag{24}$$

Now applying the Homotopy-Perturbation method to equations (23) and (24) substituting equations (20) and (22) into equations (5) and (6); we obtain

$$\begin{aligned} \sum_{n=0}^{\infty}\rho^n U_n = \; & L^{-1}\left[\frac{U_0(x,y,0)}{s}\right] + \rho\left\{L^{-1}\left[\frac{1}{s}L\left(\sum_{n=0}^{\infty}\rho^n V_n\right)\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial U_n}{\partial x}\right)\right]\right. \\ & \left. + L^{-1}\left[\frac{1}{s}L\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial V_n}{\partial t}\right)\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial U_n}{\partial y}\right)\right] + t - xt + yt + \frac{1}{2}t^2\right\} \end{aligned} \tag{25}$$

$$\sum_{n=0}^{\infty}\rho^n V_n = L^{-1}\left[\frac{V_0(x,y,0)}{s}\right]+\rho\left\{L^{-1}\left[\frac{1}{s}L\left(\sum_{n=0}^{\infty}\rho^n U_n\right)\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial V_n}{\partial x}\right)\right]\right.$$
$$\left.+L^{-1}\left[\frac{1}{s}L\left(\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial U_n}{\partial t}\right)\left(\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial V_n}{\partial y}\right)\right]+t-xt-yt-\frac{1}{2}t^2\right\} \quad (26)$$

Expanding equation (25) and comparing the coefficient of the like powers

$$\rho^0 : U_0 = L^{-1}\left[\frac{U_0(x,y,0)}{s}\right]$$

$$\rho^1 : U_1 = L^{-1}\left[\frac{1}{s}L\left(V_0\frac{\partial U_0}{\partial x}\right)\right]+L^{-}\left[\frac{1}{s}L\left(\frac{\partial V_0}{\partial t}\frac{\partial U_0}{\partial y}\right)\right]+t-xt+yt+\frac{1}{2}t^2$$

$$\rho^2 : U_2 = L^{-1}\left[\frac{1}{s}L\left(V_0\frac{\partial U_1}{\partial x}\right)\right]+L^{-}\left[\frac{1}{s}L\left(V_1\frac{\partial U_0}{\partial x}\right)\right]+L^{-}\left[\frac{1}{s}L\left(\frac{\partial V_0}{\partial t}\frac{\partial U_1}{\partial y}\right)\right]+L^{-}\left[\frac{1}{s}L\left(\frac{\partial V_1}{\partial t}\frac{\partial U_0}{\partial y}\right)\right]+t-xt+yt+\frac{1}{2}t^2$$

$$\rho^3 : U_3 = L^{-1}\left[\frac{1}{s}L\left(V_0\frac{\partial U_2}{\partial x}\right)\right]+L^{-}\left[\frac{1}{s}L\left(V_1\frac{\partial U_1}{\partial x}\right)\right]+L^{-}\left[\frac{1}{s}L\left(V_2\frac{\partial U_0}{\partial x}\right)\right]+L^{-}\left[\frac{1}{s}L\left(\frac{\partial V_0}{\partial t}\frac{\partial U_2}{\partial y}\right)\right]+L^{-}\left[\frac{1}{s}L\left(\frac{\partial V_1}{\partial t}\frac{\partial U_1}{\partial y}\right)\right]$$
$$+L^{-}\left[\frac{1}{s}L\left(\frac{\partial V_2}{\partial t}\frac{\partial U_0}{\partial y}\right)\right]+t-xt+yt+\frac{1}{2}t^2$$

$$\rho^{k+1} : U_{k+1} = L^{-}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}V_n\frac{\partial U_{k-n}}{\partial x}\right]\right]+L^{-}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}\frac{\partial V_n}{\partial t}\frac{\partial U_{k-n}}{\partial y}\right]\right]+t-xt+yt+\frac{1}{2}t^2$$

Expanding equation (26) and comparing the coefficient of the like powers

$$\rho^0 : V_0 = L^{-1}\left[\frac{V_0(x,y,0)}{s}\right]$$

$$\rho^1 : V_1 = L^{-1}\left[\frac{1}{s}L\left(U_0\frac{\partial V_0}{\partial x}\right)\right]+L^{-1}\left[\frac{1}{s}L\left(\frac{\partial U_0}{\partial y}\frac{\partial V_0}{\partial t}\right)\right]+t-xt-yt-\frac{1}{2}t^2$$

$$\rho^2 : V_2 = L^{-1}\left[\frac{1}{s}L\left(U_0\frac{\partial V_1}{\partial x}\right)\right]+L^{-1}\left[\frac{1}{s}L\left(U_1\frac{\partial V_0}{\partial x}\right)\right]+L^{-1}\left[\frac{1}{s}L\left(\frac{\partial U_0}{\partial t}\frac{\partial V_1}{\partial y}\right)\right]$$
$$L^{-1}\left[\frac{1}{s}L\left(\frac{\partial U_1}{\partial t}\frac{\partial V_0}{\partial y}\right)\right]+t-xt-yt-\frac{1}{2}t^2$$

$$\rho^3 : V_3 = L^{-1}\left[\frac{1}{s}L\left(U_0\frac{\partial V_2}{\partial x}\right)\right]+L^{-1}\left[\frac{1}{s}L\left(U_1\frac{\partial V_1}{\partial x}\right)\right]+L^{-1}\left[\frac{1}{s}L\left(U_2\frac{\partial V_0}{\partial x}\right)\right]+L^{-1}\left[\frac{1}{s}L\left(\frac{\partial U_0}{\partial t}\frac{\partial U_2}{\partial y}\right)\right]$$
$$L^{-1}\left[\frac{1}{s}L\left(\frac{\partial U_1}{\partial t}\frac{\partial V_1}{\partial y}\right)\right]+L^{-1}\left[\frac{1}{s}L\left(\frac{\partial U_2}{\partial t}\frac{\partial V_0}{\partial y}\right)\right]+t-xt-yt-\frac{1}{2}t^2$$

$$\rho^{k+1} : V_{k+1} = L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}U_n\frac{\partial V_{k-n}}{\partial x}\right]\right]+L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}\frac{\partial U_n}{\partial t}\frac{\partial V_{k-n}}{\partial y}\right]\right]+t-xt-yt-\frac{1}{2}t^2$$

Then.

$$\begin{cases} U_0 = x + y - 1, \\ V_0 = x - y + 1, \end{cases}$$

$$\begin{cases} U_1 = 2t + \frac{1}{2}t^2, \\ V_1 = -\frac{1}{2}t^2, \end{cases}$$

$$\begin{cases} U_2 = -\frac{1}{6}t^3 - \frac{1}{2}t^2, \\ V_2 = \frac{1}{2}t^2 + \frac{1}{6}t^3 - 2t, \end{cases}$$

$$\begin{cases} U_3 = \frac{1}{3}t^3 + \frac{1}{24}t^4 - \frac{1}{2}t^2 - 2t, \\ V_3 = -\frac{1}{24}t^4 + \frac{1}{2}t^2, \end{cases}$$

$\vdots$

Therefore, the solution is given by;

$$\begin{cases} u = -1 + x + y + 2t - \frac{1}{5040}t^7 + \frac{1}{40}t^5 - \frac{1}{2}t^3 + \cdots, \\ v = 1 + x - y + 2t + \frac{1}{5040}t^7 - \frac{1}{40}t^5 + \frac{1}{2}t^3 + \cdots. \end{cases}$$

Case 2

Consider the following system of equations

$$U_x - VU_t + UV_t = -1 + e^x \sin(t) \tag{27}$$

$$V_x + U_tV_x + V_tU_x = -1 - e^x \cos(t) \tag{28}$$

With the boundary conditions:

$$U(0,t) = \sin(t) \tag{29}$$

$$V(0,t) = \cos(t) \tag{30}$$

The Exact solution is given as

$$u(x,t) = e^x \sin t,$$
$$v(x,t) = e^{-x} \cos t.$$

$$L[U_x] - L[VU_t] + L[UV_t] = -L[1] + L[e^x \sin(t)]$$
$$sL[U(s,t) - U(0,t)] = L[VU_t] - L[UV_t] - L[1] + L[e^x \sin(t)]$$

$$L[U(s,t)] = \frac{U(0,t)}{s} + \frac{1}{s}L[VU_t] - \frac{1}{s}L[UV_t] - \frac{1}{s}L[1] + \frac{1}{s}L[e^x \sin(t)] \tag{31}$$

$$L[V_x] + L[U_tV_x] + L[V_tU_x] = -L[1] - L[e^x \cos(t)]$$
$$sL[V(s,t) - V(0,t)] = -L[U_tV_x] + L[V_tU_x] - L[1] - L[e^x \cos(t)]$$

$$L[V(s,t)] = \frac{V(0,t)}{s} - \frac{1}{s}L[U_t V_x] - \frac{1}{s}L[V_t U_x] - \frac{1}{s}L[1] - \frac{1}{s}L[e^x \cos(t)] \tag{32}$$

Taking the inverse Laplace Transform of equations (3) and (4);

$$\begin{aligned}[L^{-1}[U(x,t)]] = L^{-1}\left[\frac{U(0,t)}{s}\right] + L^{-1}\left[\frac{1}{s}L[VU_t]\right] - L^{-1}\left[\frac{1}{s}L[UV_t]\right] \\ -L^{-1}\left[\frac{1}{s}L[1]\right] + L^{-1}\left[\frac{1}{s}L[e^x \sin(t)]\right]\end{aligned} \tag{33}$$

$$U(x,t) = L^{-1}\left[\frac{U(0,t)}{s}\right] + L^{-1}\left[\frac{1}{s}L[VU_t]\right] - L^{-1}\left[\frac{1}{s}L[UV_t]\right] - x + e^x \sin(t) - \sin(t) \tag{34}$$

$$L^{-1}[L[V(s,t)]] = L^{-1}\left[\frac{V(0,t)}{s}\right] - L^{-1}\left[\frac{1}{s}L[U_t V_x]\right] - L^{-1}\left[\frac{1}{s}L[V_t U_x]\right] - L^{-1}\left[\frac{1}{s}L[1]\right] - L^{-1}\left[\frac{1}{s}L[e^{-x} \cos(t)]\right] \tag{35}$$

$$V(s,t) = L^{-1}\left[\frac{V(0,t)}{s}\right] - L^{-1}\left[\frac{1}{s}L[U_t V_x]\right] - L^{-1}\left[\frac{1}{s}L[V_t U_x]\right] - x - \cos(t) + e^{-x}\cos(t) \tag{36}$$

Suppose the solution of equations (34) and (36) has the form;

$$U(x,t) = \lim_{p\to 1} \rho^n U_n(x,t) = \sum_{n=0}^{\infty} \rho^n U_n \tag{37}$$

$$V(x,t) = \lim_{p\to 1} \rho^n V_n(x,t) = \sum_{n=0}^{\infty} \rho^n V_n \tag{38}$$

Now applying the Homotopy-Perturbation Method to equations by substituting (34) and (36) into (37) and (38), we obtain;

$$\sum_{n=0}^{\infty} \rho^n U_n = L^{-1}\left[\frac{U(0,t)}{s}\right] + \rho \left\{ \begin{array}{l} L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}\rho^n V_n\right)\left(\sum_{n=0}^{\infty}\rho^n \frac{\partial U_n}{\partial t}\right)\right]\right] \\ -L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}\rho^n U_n\right)\left(\sum_{n=0}^{\infty}\rho^n \frac{\partial V_n}{\partial t}\right)\right]\right] - x + e^x\sin(t) - \sin(t) \end{array} \right\} \tag{39}$$

$$\sum_{n=0}^{\infty} \rho^n V_n = L^{-1}\left[\frac{V(0,t)}{s}\right] + \rho \left\{ \begin{array}{l} -L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}\rho^n \frac{\partial U_n}{\partial t}\right)\left(\sum_{n=0}^{\infty}\rho^n \frac{\partial V_n}{\partial x}\right)\right]\right] \\ -L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}\rho^n \frac{\partial V_n}{\partial t}\right)\left(\sum_{n=0}^{\infty}\rho^n \frac{\partial U_n}{\partial x}\right)\right]\right] - x - \cos(t) + e^{-x}\cos(t) \end{array} \right\} \tag{40}$$

Expanding equation (39) and comparing the coefficient of the like powers;

$$\rho^0 : U_0 = L^{-1}\left[\frac{U_0(0,t)}{s}\right]$$

$$\rho^1 : U_1 = L^{-1}\left[\frac{1}{s}L\left[V_0\frac{\partial U_0}{\partial t}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[U_0\frac{\partial V_0}{\partial t}\right]\right] - x + e^x\sin(t) - \sin(t)$$

$$\rho^2 : U_2 = L^{-1}\left[\frac{1}{s}L\left[V_0\frac{\partial U_1}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[V_1\frac{\partial U_0}{\partial t}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[U_0\frac{\partial V_1}{\partial t}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[U_1\frac{\partial V_0}{\partial t}\right]\right]$$

$$\rho^3 \quad U_3 = L^{-1}\left[\frac{1}{s}L\left[V_0\frac{\partial U_2}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[V_1\frac{\partial U_1}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[V_2\frac{\partial U_0}{\partial t}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[U_0\frac{\partial V_2}{\partial t}\right]\right]$$
$$- L^{-1}\left[\frac{1}{s}L\left[U_1\frac{\partial V_1}{\partial t}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[U_2\frac{\partial V_0}{\partial t}\right]\right]$$

$$\vdots$$

$$\vdots$$

$$\rho^{k+1} : U_{k+1} = L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty} V_n\frac{\partial U_{k-1}}{\partial t}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty} U_n\frac{\partial V_{k-1}}{\partial t}\right]\right]$$

Expanding equation (40) and comparing the coefficient of the like powers;

$$\rho^0 : V_0 = L^{-1}\left[\frac{V_0(0,t)}{s}\right]$$

$$\rho^1 : V_1 = -L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_0}{\partial t}\frac{\partial V_0}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\frac{\partial V_0}{\partial t}\frac{\partial U_0}{\partial x}\right]\right] - x + e^{-x}\cos(t) - \cos(t)$$

$$\rho^2 : V_2 = -L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_0}{\partial t}\frac{\partial V_1}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_1}{\partial t}\frac{\partial V_0}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\frac{\partial V_0}{\partial t}\frac{\partial U_1}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\frac{\partial V_1}{\partial t}\frac{\partial U_0}{\partial x}\right]\right]$$

$$\rho^3 \quad V_3 = -L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_0}{\partial t}\frac{\partial V_2}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_1}{\partial t}\frac{\partial V_1}{\partial x}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_2}{\partial t}\frac{\partial V_0}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\frac{\partial V_0}{\partial t}\frac{\partial U_2}{\partial x}\right]\right]$$
$$- L^{-1}\left[\frac{1}{s}L\left[\frac{\partial V_1}{\partial t}\frac{\partial U_1}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\frac{\partial V_2}{\partial t}\frac{\partial U_0}{\partial x}\right]\right]$$

$$\vdots$$

$$\vdots$$

$$\rho^{k+1} : V_{k+1} = -L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}\frac{\partial U_n}{\partial t}\frac{\partial V_{k-n}}{\partial x}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}\frac{\partial V_n}{\partial t}\frac{\partial V_{k-n}}{\partial t}\right]\right]$$

$$\begin{cases} U_0 = \sin t, \\ V_0 = \cos t, \end{cases}$$

$$\begin{cases} U_1 = e^x\sin t - \sin t, \\ V_1 = -x + e^{-x}\cos t - \cos t, \end{cases}$$

$$\begin{cases} U_2 = e^x - e^{-x} - 2x - \frac{1}{2}x^2\cos(t), \\ V_2 = -e^{-x}\cos^2 t + e^x\sin^2 t + 2\cos^2 t + x\cos t - 1, \end{cases}$$

Therefore we have

$$\begin{cases} u = e^x \sin t + e^x - e^{-x} - 2x - \frac{1}{2}x^2 \cos(t) + \cdots, \\ v = -e^{-x}\cos^2 t + e^x \sin^2 t + e^{-x}\cos t + 2\cos^2 t + x\cos t - x - 1 + \ldots. \end{cases}$$

Case 3

$$\frac{\partial u}{\partial t} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial t} - \frac{1}{2}\frac{\partial w}{\partial t}\frac{\partial^2 u}{\partial x^2} = -4xt \tag{41}$$

$$\frac{\partial v}{\partial t} - \frac{\partial w}{\partial t}\frac{\partial^2 u}{\partial x^2} = 6t \tag{42}$$

$$\frac{\partial w}{\partial t} - \frac{\partial^2 u}{\partial x^2} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial x} = 4xt - 2t - 2 \tag{43}$$

Subject to the initial conditions:

$$\begin{aligned} u(x,0) &= x^2 + 1 \\ v(x,0) &= x^2 - 1 \\ w(x,0) &= x^2 - 1 \end{aligned} \tag{44}$$

The exact solution is given by:

$$\begin{aligned} u(x,t) &= x^2 - t^2 + 1, \\ v(x,t) &= x^2 + t^2 - 1, \\ w(x,t) &= x^2 - t^2 - 1, \end{aligned}$$

Taking the Laplace transform, inverse Laplace transform and simplifying equations (41)–(43).
For (41):

$$L[U_t] = L[W_x V_t] + \frac{1}{2}L[W_t U_{xx}] - 4L[xt]$$

$$sL[U(x,s)] - U(x,0) = L[W_x V_t] + \frac{1}{2}L[W_t U_{xx}] - 4L[xt]$$

$$L[U(x,s)] = \frac{1}{s}U(x,0) + \frac{1}{s}L[W_x V_t] + \frac{1}{2s}L[W_t U_{xx}] - \frac{4}{s}L[xt] \tag{45}$$

Taking the inverse Laplace transform of (4)

$$L^{-1}[L[U(x,s)]] = L^{-1}\left[\frac{1}{s}U(x,0)\right] + L^{-1}\left[\frac{1}{s}L[W_x V_t]\right] + L^{-1}\left[\frac{1}{2s}L[W_t U_{xx}]\right] - L^{-1}\left[\frac{4}{s}L[xt]\right]$$

$$U(x,t) = L^{-1}\left[\frac{1}{s}U(x,0)\right] + L^{-1}\left[\frac{1}{s}L[W_x V_t]\right] + L^{-1}\left[\frac{1}{2s}L[W_t U_{xx}]\right] - L^{-1}\left[\frac{4}{s}\cdot\frac{x}{s^2}\right]$$

$$U(x,t) = L^{-1}\left[\frac{1}{s}U(x,0)\right] + L^{-1}\left[\frac{1}{s}L[W_x V_t]\right] + L^{-1}\left[\frac{1}{2s}L[W_t U_{xx}]\right] - 2xt^2 \tag{46}$$

For (2):

$$L[V_t] = L[W_t U_{xx}] + 6L[t]$$

$$L[V(x,s)]-\frac{1}{s}V(x,0)=\frac{1}{s}L[W_tU_{xx}]+\frac{6}{s}L[t] \tag{47}$$

Taking the inverse Laplace transform of (47)

$$L^{-1}[L[V(x,s)]]=L^{-1}\left[\frac{1}{s}V(x,0)\right]+L^{-1}\left[\frac{1}{s}L[W_tU_{xx}]\right]+L^{-1}\left[\frac{6}{s}L[t]\right]$$

$$V(x,t)=L^{-1}\left[\frac{1}{s}V(x,0)\right]+L^{-1}\left[\frac{1}{s}L[W_tU_{xx}]\right]+6\cdot\frac{t^2}{2!}$$

$$V(x,t)=L^{-1}\left[\frac{1}{s}V(x,0)\right]+L^{-1}\left[\frac{1}{s}L[W_tU_{xx}]\right]+3t^2 \tag{48}$$

For (3):

$$L[W_t]=L[U_{xx}]+L[V_xW_t]+L[4xt]-2L[t]-2L[1]$$
$$sL[W(x,0)]-W(x,0)=2L[U_{xx}]+L[V_xW_t]+L[4xt]-2L[t]-2L[1]$$

$$L[W(x,s)]=\frac{1}{s}W(x,0)+\frac{1}{s}L[U_{xx}]+\frac{1}{s}L[V_xW_t]+\frac{1}{s}L[4xt]-\frac{2}{s}L[t]-\frac{2}{s}L[1] \tag{49}$$

Taking the inverse Laplace transform of (8), we have:

$$\begin{aligned}L^{-1}[L[W(x,s)]]=&L^{-1}\left[\frac{1}{s}W(x,0)\right]+L^{-1}\left[\frac{1}{s}L[U_{xx}]\right]+L^{-1}\left[\frac{1}{s}L[V_xW_t]\right]\\&+L^{-1}\left[\frac{1}{s}L[4xt]\right]-L^{-1}\left[\frac{2}{s}L[t]\right]-L^{-1}\left[\frac{2}{s}L[1]\right]\end{aligned} \tag{50}$$

$$L[W(x,s)]=L^{-1}\left[\frac{1}{s}W(x,0)\right]+L^{-1}\left[\frac{1}{s}L[U_{xx}]\right]+L^{-1}\left[\frac{1}{s}L[V_xW_t]\right]+4x\cdot\frac{t^2}{2}-2\cdot\frac{t^2}{2}-2t$$

$$L[W(x,s)]=L^{-1}\left[\frac{1}{s}W(x,0)\right]+L^{-1}\left[\frac{1}{s}L[U_{xx}]\right]+L^{-1}\left[\frac{1}{s}L[V_xW_t]\right]+2xt^2-t^2-2t \tag{51}$$

Suppose the solution of 46, 48, and 51 have the form

$$\begin{aligned}U(x,t)&=\lim_{p\to 1}\rho^nU_n(x,t)=\sum_{n=0}^{\infty}\rho^nU_n\\V(x,t)&=\lim_{p\to 1}\rho^nV_n(x,t)=\sum_{n=0}^{\infty}\rho^nV_n\\W(x,t)&=\lim_{p\to 1}\rho^nW_n(x,t)=\sum_{n=0}^{\infty}\rho^nW_n\end{aligned} \tag{52}$$

Now applying the Homotopy-Perturbation method to equations (46), (48) and (51) by substituting equation (52) we have;

$$\sum_{n=0}^{\infty}\rho^nU_n=L^{-1}\left[\frac{1}{s}U_0(x,0)\right]+\rho\left(\left\{\begin{array}{l}L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial W_n}{\partial x}\right)\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial V_n}{\partial t}\right)\right]\right]\\+L^{-1}\left[\frac{1}{2s}L\left[\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial W_n}{\partial t}\right)\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial^2 U_n}{\partial x^2}\right)\right]\right]\end{array}\right\}-2xt^2\right) \tag{53}$$

$$\sum_{n=0}^{\infty}\rho^n V_n = L^{-1}\left[\frac{1}{s}V_0(x,0)\right] + \rho\left(\left\{L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial W_n}{\partial t}\right)\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial^2 U_n}{\partial x^2}\right)\right]\right]\right\} + 3t^2\right) \quad (54)$$

$$\sum_{n=0}^{\infty}\rho^n W_n = L^{-1}\left[\frac{1}{s}W_0(x,0)\right] + \rho\left(\left\{\begin{matrix} L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}\rho^n\frac{\partial^2 U}{\partial x^2}\right]\right] \\ +L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial U}{\partial x}\right)\left(\sum_{n=0}^{\infty}\rho^n\frac{\partial W}{\partial t}\right)\right]\right]\end{matrix}\right\} + 2xt^2 - t^2 - 2t\right) \quad (55)$$

Expanding (53) and compounding the coefficient of like powers of p

$$\rho^0 : U_0 = L^{-1}\left[\frac{1}{s}U_0(x,0)\right]$$

$$\rho^1 : U_1 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_0}{\partial x}\frac{\partial V_0}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{2s}L\left[\frac{\partial W_0}{\partial t}\frac{\partial^2 U_0}{\partial x^2}\right]\right] - 2xt^2$$

$$\rho^2 : U_2 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_0}{\partial x}\frac{\partial V_1}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_1}{\partial x}\frac{\partial V_0}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{2s}L\left[\frac{\partial W_0}{\partial t}\frac{\partial^2 U_1}{\partial t^2}\right]\right] + L^{-1}\left[\frac{1}{2s}L\left[\frac{\partial W_1}{\partial t}\frac{\partial^2 U_0}{\partial x^2}\right]\right]$$

$$\rho^3 : U_3 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_0}{\partial x}\frac{\partial V_2}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_1}{\partial x}\frac{\partial V_1}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_2}{\partial x}\frac{\partial V_0}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}V_0(x,0)\right] + L^{-1}\left[\frac{1}{2s}L\left[\frac{\partial W_1}{\partial t}\frac{\partial^2 U_1}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{2s}L\left[\frac{\partial W_2}{\partial t}\frac{\partial^2 U_0}{\partial x^2}\right]\right]$$

$$\vdots$$

$$\vdots$$

$$\rho^{k+1} : U_{k+1} = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_{k-n}}{\partial x}\frac{\partial V_n}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{2s}L\left[\frac{\partial W_n}{\partial t}\frac{\partial^2 U_{k-n}}{\partial^2 x}\right]\right]$$

For (54), we have;

$$\rho^0 : V_0 = L^{-1}\left[\frac{1}{s}V_0(x,0)\right]$$

$$\rho^1 : V_1 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_0}{\partial t}\frac{\partial^2 U_0}{\partial x^2}\right]\right] + 3t^2$$

$$\rho^2 : V_2 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_0}{\partial t}\frac{\partial^2 U_1}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_1}{\partial t}\frac{\partial^2 U_0}{\partial x^2}\right]\right]$$

$$\rho^3 : V_3 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_0}{\partial t}\frac{\partial^2 U_2}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_1}{\partial t}\frac{\partial^2 U_1}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_2}{\partial t}\frac{\partial^2 U_0}{\partial x^2}\right]\right]$$

$\vdots$

$\vdots$

$$\rho^{k+1}: V_{k+} = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial W_n}{\partial t}\frac{\partial^2 U_{k-n}}{\partial x^2}\right]\right]$$

For (55), we have:

$$\rho^0: W_0 = L^{-1}\left[\frac{1}{s}W_0(x,0)\right]$$

$$\rho^1: W_1 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 U_0}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_0}{\partial x}\frac{\partial W_0}{\partial t}\right]\right] + 2xt^2 - t^2 - 2t$$

$$\rho^2: W_2 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 U_1}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_0}{\partial x}\frac{\partial W_1}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_1}{\partial x}\frac{\partial W_0}{\partial t}\right]\right]$$

$$\rho^3: W_3 = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 U_3}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_0}{\partial x}\frac{\partial W_2}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_1}{\partial x}\frac{\partial W_1}{\partial t}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_2}{\partial x}\frac{\partial W_0}{\partial t}\right]\right]$$

$\vdots$

$\vdots$

$$\rho^{k+1}: W_{k+1} = L^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 U_k}{\partial x^2}\right]\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial U_{k-n}}{\partial x}\frac{\partial W_n}{\partial t}\right]\right]$$

Therefore.

$$\begin{cases} U_0 = x^2 + 1, \\ V_0 = x^2 - 1, \\ W_0 = x^2 - 1, \end{cases}$$
$$\begin{cases} U_1 = -2xt^2, \\ V_1 = 3t^2, \\ W_1 = 2xt^2 - t^2, \end{cases}$$
$$\begin{cases} U_2 = \frac{1}{2}(16x - 2)t^2, \\ V_2 = \frac{1}{2}(8x - 4)t^2, \\ W_2 = x(4x - 2)t^2, \end{cases}$$
$$\begin{cases} U_3 = 3t^4 + (x(8x - 4) + x(4x - 2))t^2, \\ V_3 = 2x(4x - 2)t^2, \\ W_3 = 2x^2(4x - 2)t^2, \end{cases}$$

The approximate solution is given by;

$$\begin{cases} u \approx 1 + x^2 - t^2, \\ v \approx x^2 - 1 + t^2, \\ w \approx x^2 - 1 - t^2. \end{cases}$$

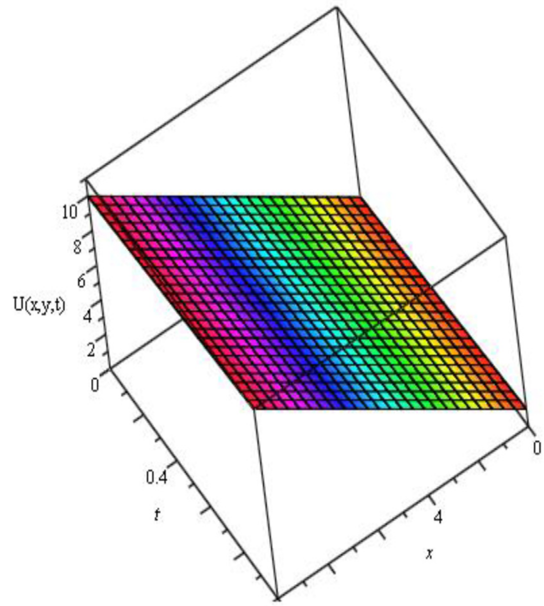

Fig. 1. The approximate solution of U(x,t) using the Laplace Homotopy Pertubation Method for case 1 at t = 0 to 0.1 and x = 0 to 0.1 and y = 1.

4. Numerical simulation

In this section, we checked for the eff, convergence, and authenticity of the proposed Laplace Homotopy Perturbation Method (LHPM) in providing an approximate and reliable solution to the system of n-dimensional partial differential equation by comparing results with the exact solution.

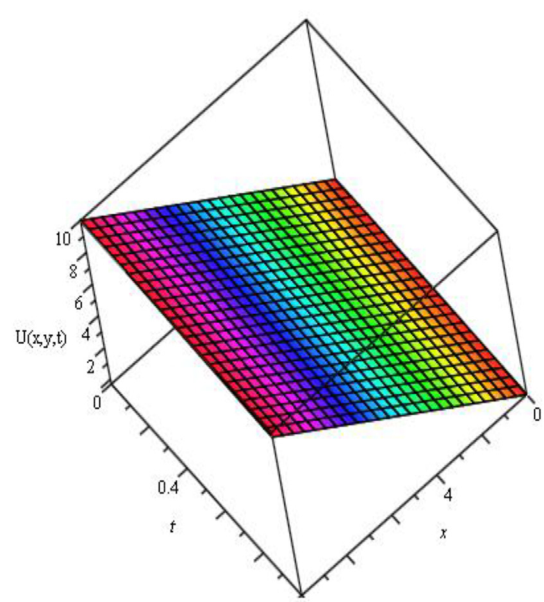

Fig. 2. The Exact solution of U(x,t) for case 1 at t = 0 to 0.1 and x = 0 to 0.1 and y = 1.

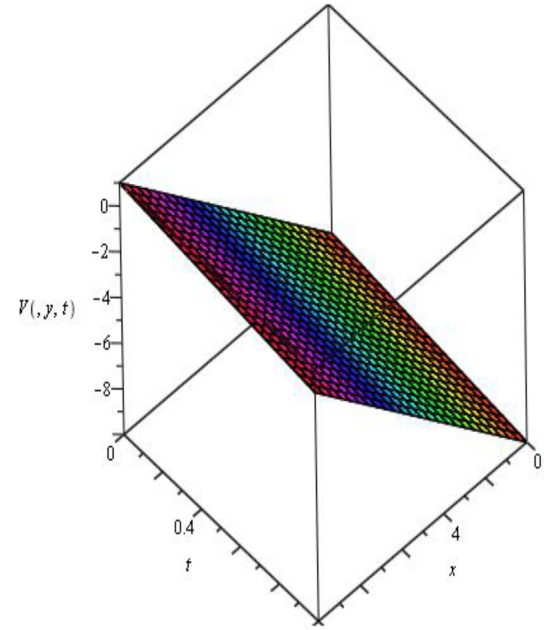

Fig. 3. The approximate solution of V(x,t) using the Laplace Homotopy Perturbation Method for case 1.

5. Discussion of results

In the research work, an efficient hybrid method has been utilized which involves the coupling of the Laplace transformation method and the Homotopy perturbation Method in finding the approximate solution to the system of n = dimension partial

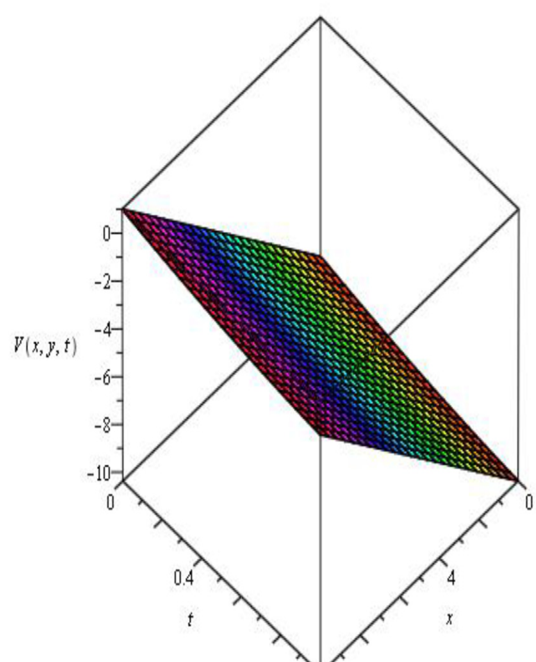

Fig. 4. The Exact solution of V(x,t) for case 1.

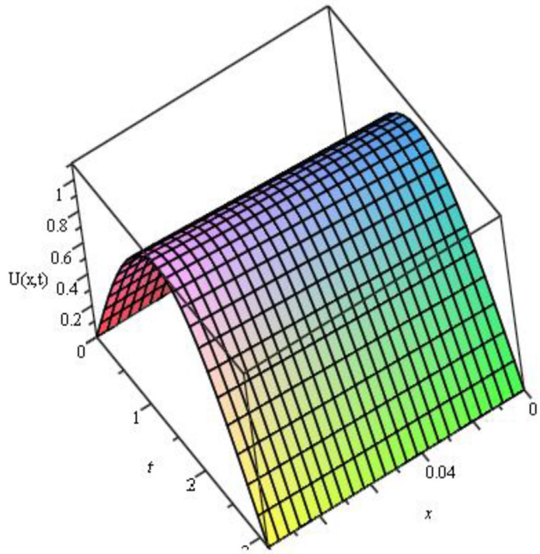

Fig. 5. The approximate solution of U(x,t) using the Laplace Homotopy Perturbation Method for case 2 at $t = 0$ to π and $x = 0$ to 0.1.

differential equation. The Laplace-Homotopy Perturbation method has been implemented excellently on the partial differential equation; thereby obtaining a solution that is highly convergent and accurate. Three different cases with initial conditions were considered. The comparison consists of the exact results extracted from prominent literature that have implanted the normal analytical means and Laplace Homotopy Perturbation results. The results (Figs. 1–14) validated the efficacy and

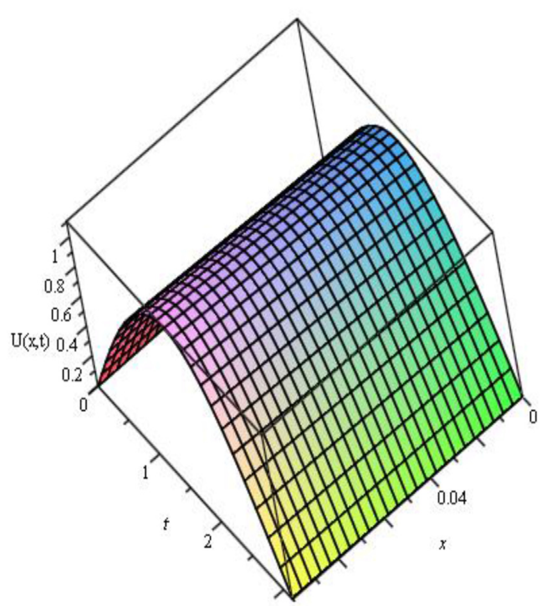

Fig. 6. The Exact solution of U(x,t) for case 2 at $t = 0$ to π and $x = 0$ to 0.1.

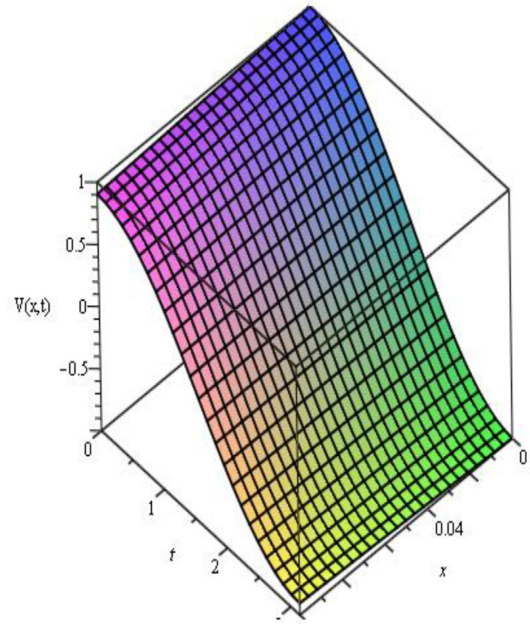

Fig. 7. The approximate solution of V(x,t) using the Laplace Homotopy Perturbation Method for case 2 at $t = 0$ to π and $x = 0$ to 0.1.

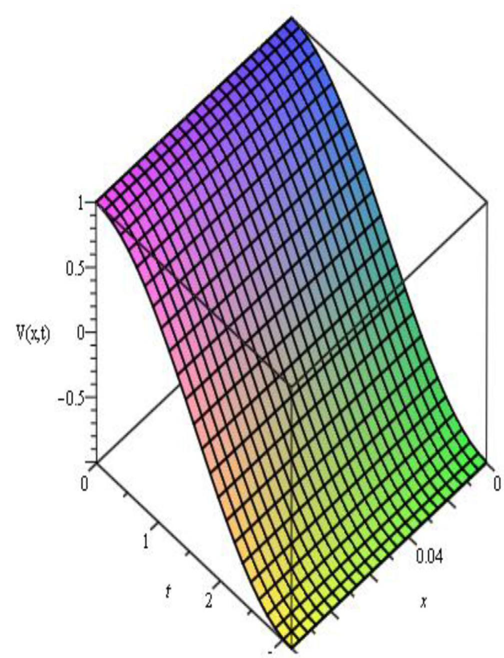

Fig. 8. The Exact solution of V(x,t) for case 2 at $t = 0$ to π and $x = 0$ to 0.1.

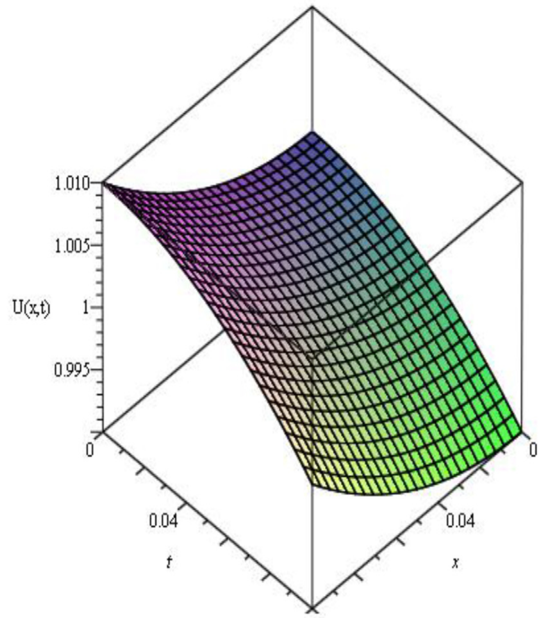

Fig. 9. The approximate solution of U(x,t) using the Laplace Homotopy Perturbation Method for case 3 at t = 0 to 0.1 and x = 0 to 0.1.

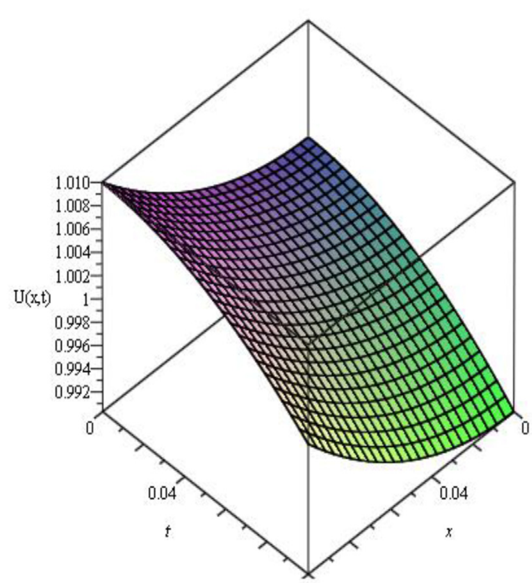

Fig. 10. The Exact solution of U(x,t) for case 2 t = 0 to 0.1 and x = 0 to 0.1.

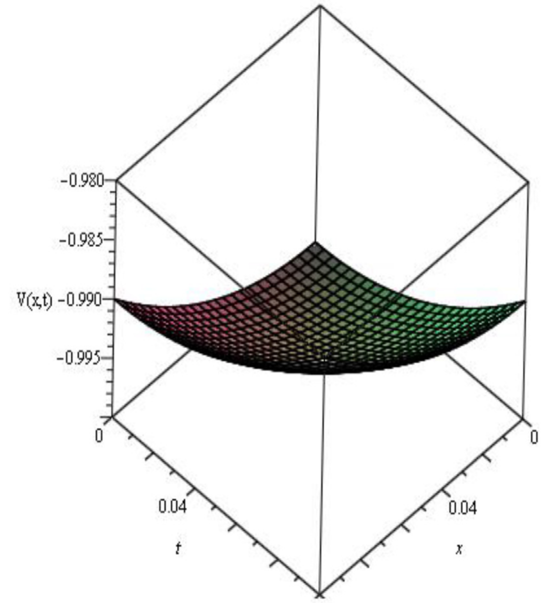

Fig. 11. The approximate solution of V(x,t) using the Laplace Homotopy Perturbation Method for case 3 at t = 0 to 0.1 and x = 0 to 0.1.

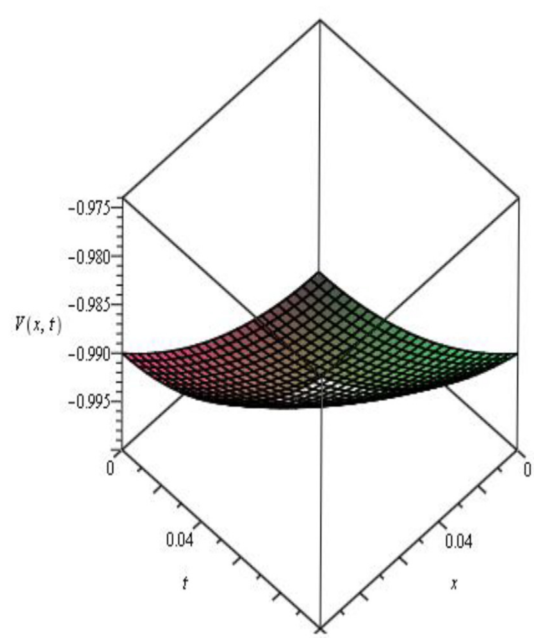

Fig. 12. The Exact solution of V(x,t) for case 2 t = 0 to 0.1 and x = 0 to 0.1.

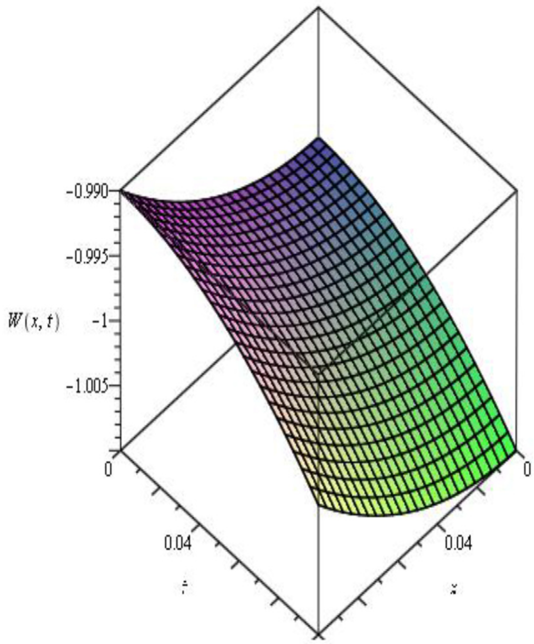

Fig. 13. The approximate solution of W(x,t) using the Laplace Homotopy Perturbation Method for case 3 at t = 0 to 0.1 and x = 0 to 0.1.

reliability of the LHPM by showing a high level of convergence results. Also, the result showed that few iterations yield precise results across a broad spectrum.

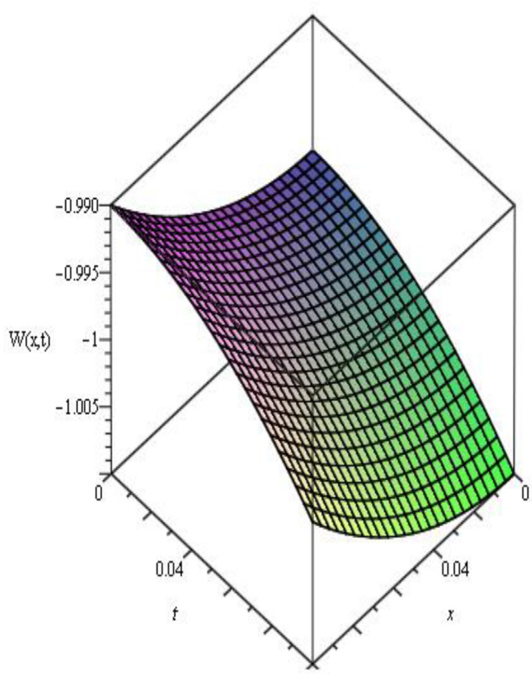

Fig. 14. The Exact solution of W(x,t) for case 2 t = 0 to 0.1 and x = 0 to 0.1.

6. Conclusion

We have applied the Laplace homotopy perturbation method to solve three systems of the n-dimensional nonlinear partial differential equations. Although several methods have been used previously for this purpose, however, to arrive at a more accurate and efficient result, we introduced the new hybrid method. The results obtained using the LHPM showed that the method is valid, reliable, and highly efficient in solving the system of n-dimensional partial differential equations. The result also showed that the method converges within a few iterations compared to all other semi-analytic methods like VIM, HPM, Laplace, etc. As a result of the fast convergence and efficiency of the Laplace Homotopy Perturbation Method, we hereby recommend this method (LHPM) for obtaining an approximate solution, although the exact solution can also be determined from the multivariate series.

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